The development and application of a digital computor method for the solution of strata control problems

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Discussion:

A. Hazell (Associate Member) and **T. J. Kotze** (Graduate Member): The authors must be congratulated on the development of a versatile computer programme for determining the stresses and displacements resulting from mining operations. As a design tool it will no doubt take its rightful place together with the analogue computer.

Union Corporation made use of this digital computer programme whilst still in its development stage to determine the stress distributions which were causing the unexpected failure of the Marievale sub-incline shaft. Two features arising from this investigation may be of interest to members, namely:

A. The solution to the problem of stabilizing a rectangular inclined shaft, which turned out to be contrary to accepted mining practice, and

B. A comparison between the results obtained with the pure digital system and the analogue computer.

A. The shaft is nine ft high and 20 ft wide, dipping at 9° to the horizontal and crossing the bedding planes. Fig. 1 is a plan and Fig. 2 is a section of the sub-incline shaft pillar area showing the relative positions of the shaft and reef as well as their respective dips. The deepest point in the shaft is 3 300 ft below surface and the requirement of a stress free ground surface may not be satisfied. It also means that the original gravitational stresses are low.

A 900 ft wide pillar was left to protect the bottom section of the shaft, the top section being understoped years ago. Extraction of the pillar was started at the beginning of last year, mining operations being carried out in the accepted manner, starting beneath the shaft and mining outwards. The approach did, however, turn out to be the wrong one, because failure of the shaft was noticed soon afterwards. Even small movements were serious in this case due to the low clearance between the skips and the shaft supports. During October, 1968, failure between benchmarks 8 and 10 (see Fig. 1) became so excessive that hoisting speeds had to be reduced. Failure continued to occur and, by the end of March, 1969, failure had propagated up the shaft as far as benchmark 7, and down as far as benchmark 12.

As this shaft had a tight hoisting schedule it was imperative that a quick solution be found to arrest the failure.

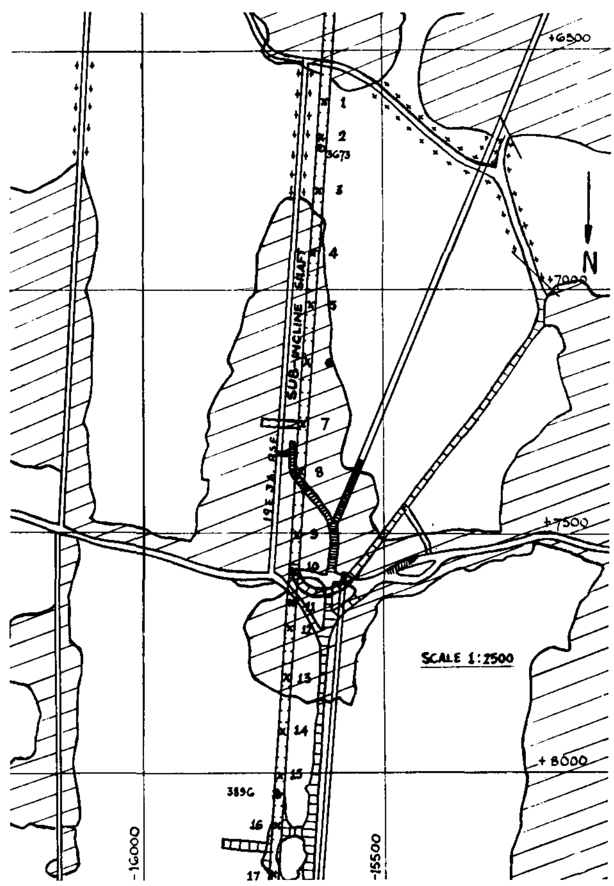

Fig. 1—Showing portion of the sub-incline shaft pillar

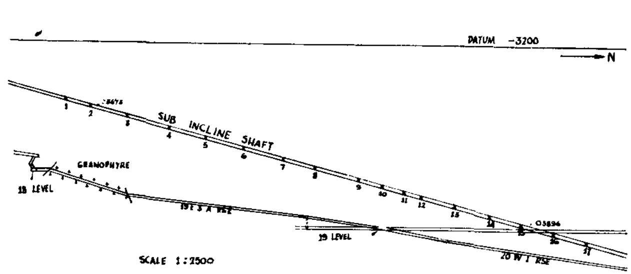

Fig. 2—Section through sub-incline shaft pillar

Two observations concerning the failure should be mentioned:

i. Failure occurred in the hangingwall and footwall and no failure of the sidewalls was observed. Failure of the hangingwall was characterised by the formation of thin slabs parallel to the dip of the shaft intersected by occasional vertical fractures. Failure of the footwall was only noticeable in the upward displacement of the rails.

ii. Failure only occurred in sections of the shaft which were in the process of being understoped.

Different mining configurations were analysed using the digital programme described by the authors. An initial coarse window of 1 unit = 200 ft was used but by scaling up twice, together with the necessary updating a more detailed resolution was obtained.

Fig. 3 shows the variation of the vertical and horizontal field stresses at benchmarks along the shaft for the mining configuration at the end of October, 1968.

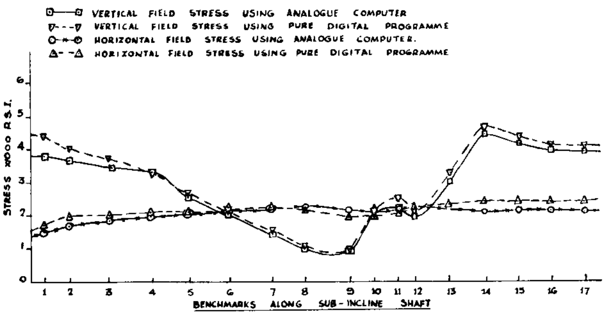

Fig. 3—Vertical and horizontal field stresses at benchmarks along the sub-incline shaft

The curves show that whereas the vertical field stress is sensitive to undermining and decreases rapidly as the mined out span increases, the horizontal field stress is not significantly affected and remains at its original level before mining was started. Failure of the shaft between benchmarks seven and 10 occurs where the vertical field stress is smaller than the horizontal component. Based on these results and those of other mining configurations it was concluded that failure of the hanging and footwall could be minimized by preventing the magnitude of the vertical field stress from dropping below that of the horizontal field stress.

In order to comply with this requirement, the actual mining programme was changed. Instead of starting beneath the shaft and mining outwards, most mining operations are now started at the pillar edges and are directed towards the shaft.

Since the change has been affected, no additional failure has been reported. If failure does eventually occur due to a build up of vertical stress, it should be in the form of sidewall failure, which is much easier to control.

B. In order to check the validity of the Digital programme results, the same mining configuration was analyzed on an Analogue Computer. A scale of one unit equal to 50 feet was used. Using a digital computer programme, the field stresses were again calculated for benchmarks along the shaft. The results from the Analogue Computer are also plotted on Fig. 3. The results as can be seen, are in good agreement with the results of the pure digital programme.

Except over the mined out area, the Analogue results are on the average about 300 lb/in^2 lower than the pure Digital results. This difference is best explained by the fact that in the coarse window (1 unit = 200 ft) of the digital programme, a much larger area was taken into account, providing more accurate boundary conditions during the upscaling process.

H. A. D. Kirsten (Associate): The mining industry is expanding its activities from year to year inasmuch as greater volumes of rock are extracted from the earth's crust at ever increasing rates. A fair proportion of this activity is expected to take place at greater depths than has been experienced to date. An increase in the number and complexity of problems to be met by the industry is therefore expected to occur.

Mining at greater depth will not allow the build up of experience and know-how as has been the case to some extent in the past. Problems will have to be recognised and solved on paper before the project proper can be embarked upon.

The mine designer will find himself in a position where he cannot extrapolate his hard earned experience to these new situations. The successful practice of a technological art, as engineering practice has been referred to, requires implicitly the same governing conditions to be present in the situation towards which the know-how is being extended. Bad engineering guesses may invariably be attributed to the failure to recognise this basic premise to the application of knowledge gained through experience.

Empirical knowledge will be hard to come by in the foreseeable future. It is in this context that the digital computer method as an aid to the control of rock strata will benefit the mining industry mostly. The development of the method by the authors is of tremendous value to the industry. The usefulness of the method[1] together with the scope with which it provides the mining engineer will only be fully realised in the course of time.

Bad engineering judgement may be avoided by an appreciation of the limitations attached to the guesswork. Likewise, and appreciation of the limitations of the digital computer method will above all enhance its usefulness. It is with this objective in mind that the following observations and suggestions are tendered to the potential user of the method.

In addition to the method being applicable only to an elastic continuum it should be observed that the rock is also assumed to be homogeneous, isotropic and wholly intact. Geological non-uniformities such as dykes and sills for instance cannot be correctly simulated. However, these structures have more often than not very similar elastic properties to those of the host rock. If then the dyke-to-host rock interfaces are not potential planes of weakness the problem is essentially correctly simulated. Fracturing of the dyke material usually occurs in an entirely different manner to that of the host rock in that the elastic strain energy stored within it tends to be more violently released. One should therefore not be excessively worried in the context of the elastic analysis by the presence of such bodies unless they become excessively highly stressed. Planes of weakness cannot be taken into account, consequently the interpretation of their presence is left to the engineer. The material is furthermore assumed not to be stressed beyond its elastic limit. However unreal this situation may seem, especially in the vicinity of stope faces, the solution is still essentially correct away from the immediate vicinity of the excavation by virtue of St. Venant's principle.

Multiple tabular excavations cannot be simulated. There is, however, no limit to the depth at which excavations may be considered.

The co-ordinate systems employed in the simulation process as well as the appropriate sign convention, notation and the specified input and output formats are described in detail in the reference given.

In extensive practical applications of the simulation system the preparation of the input data could become a task of considerable proportion. The size of the basic window is the first decision to be made. No hard and fast rule can be laid down in this respect. It has been found, however, that a size of basic mesh unit such that the working window envelopes as much as possible of the stoping activity provides a reasonable starting point. This choice of the initial scale will usually not require one to go beyond a third scaling to achieve a desired fineness of mesh. In addition, it also guarantees that full account is taken of the entire stoping configuration inasmuch as very little, if any, stoping will be outside this initial size of window.

The basic deck of computer cards is usually produced from a co-ordinate grid drawn on tracing paper and overlain on the mine plan with the stope profile traced onto it as a series of steps along the mesh units. Following a scaling the profile may always be refined by a series of updated units. This scaling update list may be obtained by reproducing in a similar manner to the basic mesh the stoping profile on a co-ordinate mesh of appropriate scale. The discrepancy in the profiles observed when the scaled mesh is superimposed on the basic mesh constitutes the update list. The number of entries in this list is ultimately dependent upon the length of the profile and the number of times the basic mesh has been scaled. In practice operation in the hands-off mode saves a considerable amount of time, since update lists may be prepared beforehand on cards.

Scaling has been found to have an effect on the accuracy of both the on-reef and off-reef solutions. It may be said from what has been observed that the on-reef solution determined from an n^{th} scale configuration is generally within five per cent of the on-reef solution of an n^{th} scaling of the basic configuration. The corresponding figure as regards off-reef solutions is approximately 10 per cent. The discrepancy from the basic solution of solutions derived from successive scalings amounts in order up to the third scaling to approximately 7½, 10 and 12½ per cent as regards on-reef solutions and to 12½, 25 and 50 per cent as regards off-reef solutions, respectively. Since the energy release is determined from the convergence distribution the effect of the scaling process on its accuracy should be the same as that on the on-reef solution.

When absolute stresses are to be determined due account should be taken of the scaling drift, whereas for a comparative study of alternatives relative stresses would generally be sufficient. For the purpose of determining absolute quantities an appropriate scale adjustment should therefore be applied. The scale adjustment may be defined, since the basic solution of an n^{th} scale configuration is always within 10 per cent of the solution determined from an n^{th} scaling of the same basic configuration, namely, as the difference between the solutions of a basic configuration and an n^{th} scaling thereof, alternatively, as the difference between the basic solutions of the initial and n^{th} scale configurations.

It will be noticed that the second procedure for the determination of the scale adjustment leads to a reduction in both the amount of preparation and the cost of obtaining a final result. Even if only relative stress, displacements or energy releases are of the essence the same procedure may be reverted to.

The energy release option yields the amount of gravitational energy released as a result of a mining change. The area rate of energy release may be obtained by dividing this amount of energy by the number of entries in the update list simulating the mining change times the area of one mesh unit in square feet.

Every mining venture aims inevitably at an ultimate configuration and in so doing an ultimate amount of gravitational energy is to be released. The ratio of this ultimate amount of energy to the area of the ultimate configuration may be defined as the average area rate of energy release. The current rate of energy release may be defined as the total amount of energy released currently, divided by the associated current area of stope advance. The local rate of energy release is that amount of energy released by advancing the stope face locally in relation to the area advanced. A criterion for safe mining, locally, would be to keep the local rate of energy release at a minimum. Since the rockburst hazard is not linearly related to the rate of energy release, the ideal criterion for optimum safe mining would be to excavate such that the current rate of energy release equals the average rate.

The present form of the digital computer simulation method does not permit the ready determination of the average rate of energy release. It would be an additional attribute if the facility for determining the ultimate amount of energy released could be added to the options which are already available. It should also be noticed that this facility would provide an estimate of the storage capacity of stoped out areas.

I may in conclusion report that a fair demand seems to exist for angles of dip exceeding 10° to be reliably handled.

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1. Kirsten, H. A. D. 'MINSIM: Taking the guesswork out of structural stability.' *Coal, Gold and Base Minerals of Southern Africa*, July, 1969.

R. C. More O'Ferrall (Member): The paper, that has been presented this evening, outlines another stage in the advancement of the science of rock mechanics, as applicable to the mining of deep level tabular orebodies.

The rock mechanics engineer has been plagued in the past by having to attempt to solve problems, when they have reached a point where they are practically unsolvable. For this reason we have adopted the approach where possible problems are diagnosed before they have time to materialise. This is done by the staff of the rock mechanics department, who have been trained to recognise and solve problems involving the stoping or development layout. The areas under inspection are subjected to one or other form of analysis, depending on the type of problem. To assist in the solution of the problems a resistance analogue and digital computer are readily available.

This approach was adopted some 15 months ago, when adequate digital computer facilities became available, thereby making this approach feasible. This approach has proved beneficial and is popular with production management.

During the past two and a half years we have been using a damage criteria for square or near square tunnels based on the theoretical vertical stress. These criteria have been established by modelling the stoping geometry associated with the haulage damage in particular geological horizons of the Klerksdorp goldfields. These criteria are used in the solution of stoping layouts involving haulages. The degree of damage expected has been built into these criteria making it possible to predict, not only whether the haulage will suffer damage, but also how much and when in the mining sequence this will occur if it is unavoidable. Based on this information, steps are then taken to install additional support. This is usually done by the support crews operated by the department.

The resistance analogue is used extensively in sequencing the stoping operations and studying the bursting potential of dykes. The techniques employed here have proved successful to date and it is hoped that eventually the occurrence of pressure bursts in dykes will be eliminated and the damage suffered in remnants will be minimised.

The example cited in the paper is of particular interest as we have recently been faced with a similar problem. It is reassuring to find that our results agree fairly closely with those in the example. The main points of difference in the geometry are that the reef plane is some 450 ft above our problem area and the mean dip is 12°.

The stresses computed are considerably lower than those of the example, probably due to numerous residual blocks scattered through the stoped areas. The vertical stress components in our case vary between 6 500 lb/in^2. and 7 300 lb/in.^{2} Single excavations in the geological horizon would normally give severe trouble at 8 000 lb/in^2. However, the area has been honey-combed with excavations, the pillars between which have recently started to fail.

The maximum vertical stress reduction on removing a central reef cut some 450 ft wide was approximately 2 500 lb/in^2, while the maximum increase in the vertical stress in the remainder of the area was approximately 100 lb/in^2. One of the obvious advantages of overstoping the damaged excavations at the distance quoted is, that the reduction in vertical stress extends beyond the limit of the cut as can be seen from Fig. 9 in the paper. A further advantage is that the stress increase ahead of the face in the vital area is minimal.

The computer costs for the basic solution of the convergence distribution is high, when compared with the resistance analogue. The cost of feeding the equivalent amount of information, using the resistance analogue, to the computer by means of punch cards in our case is approximately R7. This technique has the advantage that it does not require the computer for the first phase of the operation, an important point, if computer time is at a premium. It is practice to run the programme during the night shift, which enables the information to be available, if required, within 24 hours. Since it is possible for one member of the rock mechanics department's staff to have as many as six projects in various stages of solution at one time, the rate of output of computer information is in excess of our requirements.

S. G. Taussig (Fellow): The development of the computer method described by the authors is a considerable aid to the solution of strata control problems. The practical example of the Harmony shaft pillar illustrates the usefulness and versatility of the programme. By using a resistance analogue followed by a digital computer programme such an investigation would have taken more time and, as it was pointed out that comparitively small stress changes may have an appreciable influence on the stability of excavations, the higher degree of accuracy that can be achieved is a great asset of this method. A particularly attractive feature of this programme is the possibility of scaling up mining configurations with ease, thus allowing the computation of displacements and stresses near the plane of the reef.

However, there are occasions where the electrical resistance analogue will hold its own for some time to come. I am thinking of the day-to-day problems encountered on mines where local remnants cannot be avoided. By being able to model mining sequences quickly and by observing the resulting stress changes mine staff are in a position to discuss the selection of the best mining method on the spot. Whilst the cost involved in using the digital programme is fully justified for major problems such as shaft pillars, it is doubtful if this would be the case for less complicated applications. Although for the calculation of displacements and stresses at points not on the plane of the reef the laborious transfer of convergence data from the analogue to the digital computer input is necessary, this can be avoided on more modern types of analogues by attaching an automatic tape punching device.

In the references quoted by the authors it had been shown that the application of the theory of elasticity leads to valid results that can be used to indicate the stability of excavations. This is being done on many mines with good success particularly if the area under investigation is situated in undisturbed ground. However, there is still comparatively little knowledge available about the changes in the field stress caused by various faults and dykes. Experience has shown that there is an increase in the number and also severity of rock bursts in the vicinity of faults. Should future measurements indicate definite trends and patterns in the changes of the stress distribution caused by faults or other geological features then it would be of interest to know if the programme could be extended to incorporate such information.

J. W. Wilson* (Member): The computer programme described by the authors will be extremely valuable to rock mechanics engineers in performing the well established techniques developed on the electrical resistance analogue, and in particular, to those rock mechanics engineers who do not have analogue facilities at their disposal at the areas in which they operate.

For more than two years the Anglo American Corporation Rock Mechanics Department located in Welkom has been the scene of intensive experimentation with the prototype electrical resistance analogue for assessing the applicability of this tool in describing underground conditions in the Group's O.F.S. gold mines. The high degree of success that has attended the prediction of damage and the solution of many rock mechanics problems has shown research workers, as well as production

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personnel, that this instrument is thoroughly reliable, provided sufficient knowledge of local conditions is available.

As the value of the use of the analogue became more apparent to operating personnel the demand for its use increased, and to cope with the volume of work now being handled the digital computer programme described in the paper before us has been modified to suit the Corporation's IBM 360/40 digital computer installation at Welkom. The excellent 'turn around' facilities at the computer section and the repetitive nature of several aspects to the rock mechanics procedures, together with the 'storing' feature of the digital programme, has eased the overall dependence on the "over worked' electrical resistance analogue.

Before describing how the digital computer programme is being applied in the Anglo American mines in the Klerksdorp and O.F.S. goldfields, it will not be out of place to summarise the progress made over the last two years in the 'applied rock mechanics field', with particular reference to the part played by the electrical resistance analogue.[1]

Since its installation some two years ago, the electrical resistance analogue has been used to determine energy release rates caused by scattered mining, which, to some extent, is a measure of the degree of hangingwall fracture. The correlation between predicted energy release rates and variations of actual underground conditions in stopes in the O.F.S. goldfields, has been encouraging enough to define parameters on which future stoping layouts can be based.

With the breast stoping method in vogue in the Anglo American Corporation mines in the O.F.S. and Klerksdorp goldfields, it is impossible to confine all energy release rates to a magnitude less than the defined critical values. However, with the use of the analogue, optimum stoping sequences have been derived for a variety of mining configurations which reduce the number of 'high energy release' areas to a minimum in a given production section of a mine.

An important application of the analogue has been the assessment of the influence of stoping operations on off-reef excavations. Underground conditions at selected damaged and undamaged areas have been compared with elastic stresses and displacements, resulting in the derivation of design parameters for excavations located in the prevailing geological conditions.

The knowledge of critical field stresses for conventional square shaped excavations situated in the common stratigraphic horizons in the vicinity of the reef has been used to define the optimum positions of lateral development for the Anglo American Free State and Klerksdorp gold mines.[2] In addition, nomograms have been constructed to illustrate the amount of over-stoping required to protect inclined shafts and major arterial haulage-ways, and the extent to which waste stoping is required to protect tunnels which traverse fault losses and dykes of varying thicknesses.

The electrical resistance analogue has also been used to assess the vulnerability to bursting of several persistent unmined dykes. The correlation between elastic stresses induced on dykes by adjacent stoping operations, and the behaviour of such dykes underground, has enabled critical field stress levels to be determined for several dykes occurring in the O.F.S. and Klerksdorp mines. The knowledge of such critical parameters enables a realistic assessment to be made of the influence of future stoping on the 'burst' prone dykes. Where high stress conditions are indicated, alternative mining methods can be considered.

The use of digital computer programmes for calculating energy release rates and the elastic stresses and displacements at off-reef positions has played a major role in the establishment of design factors. The close coordination between the Computer Section and the Rock Mechanics Department enables the results of problems modelled during a morning, and punched and processed in the afternoon, to be available early the following morning. According to the statement in paragraph 3.3.1 this appears to be contrary to the experience of the authors. In fact, as a result of job stream scheduling and computer core requirements, it is suspected that in many instances the digital computer programme described in this paper may not speed up the output of results.

Although the establishment of critical field stress and energy release parameters are proving of considerable value in mine planning, limitations to their use occurs when several excavations are situated in close proximity to one another.

In many gold mines problems of this nature occur most frequently within protection zones around vertical shaft systems. Several underground sites have been investigated where varying degrees of damage have occurred in excavations under relatively low field stress conditions. The selective use of a sophisticated two-dimensional finite element method[3,4] to analyse the stress distribution around the tunnel forming these multiple excavation layouts, has shown that a significant correlation exists between the observed fracturing around individual excavations (as observed from the 'discing' of cores obtained from diamond drilling) and the calculated fracture zones derived from the finite element analyses and the Mohr's fracture theory for brittle rock.

The data derived from this work has provided a method of designing improved shaft system layouts for the life of a mine. Furthermore, the stability of existing shaft systems and other multiple excavation areas can be assessed, and remedial action taken if and when necessary.

The definition of critical parameters descriptive of damage in stopes and off-reef excavations as outlined above enables assessments to be made of long and short term stoping forecasts for each mine at regular intervals in time. These assessments highlight probable trouble spots in stopes, as well as the influence of such stoping sequences on pre-developed haulages. Moreover, these investigations provide the opportunity to rephrase stoping sequences to improve hangingwall conditions within stopes or reduce the anticipated damage in off-reef tunnels. It is also possible to demarcate the sections of haulages or crosscuts which will require support, *prior* to the commencement of stoping in an area.

To perform the routine periodic rock mechanics investigations for the Anglo American mines in the Klerksdorp and O.F.S. Goldfields, it was decided to introduce the digital computer programme described in the paper. This was made possible by the increase in storage capacity of the digital computer situated in Welkom, together with some sophisticated-programming techniques by the Corporation's Computer Consultants. Experience to date has shown that the cost of the increased computer time required to solve most mining configurations on the Anglo American computer is more than offset by relatively low computer rates applicable in Welkom.

The underground workings of the seven Anglo American mines operating in the Welkom area have been divided into fifteen mining blocks. The definition of these blocks has been governed largely by major fault zones (not mine boundaries) and great care has had to be exercised in localizing areas where changes of reef dip and strike occur.

Each mining block has been modelled on a 56 × 56 window from a 1:10 000 mine plan using a grid size of 1 square equal to an area of 200 ft × 200 ft. The selection of this base scale permits the use of the convenient 1:2 500 mine plan for up-dating mining sequences under investigation, where one square of the grid now represents an area of 50 ft × 50 ft after two scalings on the digital computer programme.

The solutions to the fifteen basic sections and their related scaled-up areas are stored on five magnetic discs in the Computer Centre in Welkom. By using a suitable catalogue system for these stored solutions it is virtually impossible to accidentally delete important records.

The application of the digital computer is currently being extended to the Anglo American Corporation mines in the Klerksdorp area, where again, major geological anomalies are being used to divide the mine into convenient blocks.

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3. DEIST, F. H., and ORAVECZ, K. I., 'The finite element method and its application to problems in rock mechanics.' Chamber of Mines Research Report No. 60/68.
4. WILSON, J. W. 'The use of the finite element method in assessing the stability of excavations situated in close proximity to one another.' Anglo American Corporation of South Africa, Internal Report, August, 1969.

P. C. Pirow (Fellow): The authors of this paper must be congratulated on a first class effort. They have tackled a problem that we attempted on the Leo 3 computer many years ago. We gave up our attempt owing to the very great complexities of mathematics that were involved. The logic and mathematical developments that have been introduced into the current programme are very noteworthy indeed, and represent a piece of logical reasoning equal to anything the mining industry has produced.

Arising out of the development of the mathematics is one question that I would like to ask the authors. They mention that their simplified linear scheme minimises the detrimental effect caused by an over-estimate of the spread in the eignvalue. They state that this spread has been reasonably well established for a variety of practical configurations. I would be very interested to compare the estimates as calculated by the programme with measurements of the values as obtained by experimentation and I feel that publication of these figures would add considerably to the force of the paper.

One of the most impressive things about this paper is that the mathematics no longer limit us to the basic Laplace equations but enable us to bring in other parameters. To cite an example of this, in some earlier work I found on City Deep, Limited[1] some evidence of plastic deformation especially in the behaviour of the surface relative to underground mining excavations. It is quite possible to envisage this plasticity being allowed for by expanding the mathematics to a fourth dimension, the fourth dimension being time.

We should ask ourselves what are the long term implications of this model. It is not difficult to envisage a situation in which each mine is equipped with a computer terminal coupled to one large central computer and in which each mine regularly updates the information on its current stoping measurements through the medium of the terminal. (Despite the successful work undertaken by Anglo American Mines in the Orange Free State I submit that one large machine is preferable on cost consideration.) This would mean that each mine could keep detailed records on the central computer as stoping progresses. Not only would this enable the stresses and movements around mining excavations to be determined and so eliminate some of the difficulties arising out of pressure conditions, but such a system would also provide the information necessary for stope measurements, tonnage returns, and a great deal of regular mining information.

Actual measurements of closure, stress, Youngs Modules, Poisson's ratio, etc. could be used for regularly reviewing the values used in the model. In a large computer it would not be very difficult to couple this information with that needed for the moving average trend surface developed by Dr D. G. Krige thus enabling a complete up to the minute picture of the mine to be available to management. It may well be felt that such a system is extremely expensive and this is of course true today.

At a conference that I attended last year it was mentioned that a factor of 100 was possible in computer improvements in the course of a decade. This could mean that in ten years' time it will be possible to obtain 100 times the computing power for the same cost. One speaker at this international conference in fact went so far as to say that the figure could be 1 000 fold improvement in the cost performance ratio in the course of ten years.[2] If we bear this in mind the computer requirements necessary to provide information for such an integrated system will certainly not be beyond the means of most of the mines that will still be in existence in 10 years' time.

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Dr M. D. G. Salamon (Fellow): I listened with special interest to the presentation of this interesting paper. I have a certain amount of personal involvement in this issue, since the authors have proved me a liar.

Some years ago—in 1963—I started also with my friend, Dr J. A. Ryder, to develop the solution of problems associated with tabular excavations by a digital method. The principle of our approach was very simple. We knew then that the stresses and displacements at any point in the rock mass can be calculated from known distributions of convergence and ride. For example, if the reef is horizontal, when the effect of ride is negligible, the vertical stress (σ_z) can be expressed as follows:

$$\sigma_z = \iint_A s_z(\xi, \eta) K[(x-\xi), (y-\eta), z] d\xi d\eta \quad \ldots (1)$$

In this integral A is the area of mining; s_z represents the convergence distribution; x, y are the horizontal co-ordinates; z is the perpendicular distance from the reef; ξ, η are auxiliary co-ordinates and KdA ($dA = d\xi d\eta$) defines that vertical stress at point x, y, z which is induced by an unit convergence in a small portion, dA, of the mined out area.

To determine the convergence distribution, it can be argued that in the mined out area ($z=o$) we know the induced vertical stress so the only unknown in (1) is the convergence distribution. (The argument is only slightly more complex if in portions of A complete closure takes place.) Hence, the formula in (1) can be regarded as an integral equation for $s_z(x,y)$, the solution of which can be reduced to the solution of a system of linear equations where the unknowns are convergence values at nodal points of a rectangular grid.

When we tried this approach in practice we found that the solution was too slow and, therefore, we concluded, as I wrote to a German friend in 1966, 'that it would be unpractical to use it for the solution of problems involving irregular workings'. We then turned our attention to the development of the electric analogue, which seemed more suitable for the determination of the convergence and ride distributions.

I think this episode, in the light of the present paper, illustrates well that sometimes an idea can be made to work by skill and persistence and also it shows that we research workers must be careful with our conclusions.

I would like to point out now a basic difference between the analogue and the digital approaches. As the authors suggested in their paper, the digital method could be extended to the solution of problems where the analogue is not applicable. The reason for this is that the validity of the electric analogue as we know it today—I must be careful with my statements—hinges on the applicability of the Laplace equation. The digital method, however, does not have this limitation in principle. In other words, function K in (1) need not satisfy the Laplace equation, although the authors have considered only such problems so far were this is in fact the case.

There does not appear to be any fundamental reason which would prevent the extension of the digital method to the solution of the following practical problems:

(i) mining in a seam or reef which is near to the surface,

(ii) mining in two or more seams or reefs in close vicinity,

(iii) mining in seams which are not parallel with each other.

None of these problems can be solved directly with the aid of the electric analogue, except perhaps through a tedious and unpractical method of successive approximation. I would like to hear the opinion of the authors with regard to the practical feasibility of extending the digital method in these directions.

Authors' Reply

We thank those members who made contributions to our paper, and we have these comments to offer:

In replying first to Dr Salamon's contribution, we should like to start on a philosophical note regarding his remarks about 'making an idea work by skill and persistence'. The basic approach to problems such as this one—namely that of reducing the dimensionality of a linear system (from three to two in our case) by employing a set of 'characteristic' or Green Functions that satisfy the boundary conditions—is so familiar and is used implicitly or explicitly in so many areas of applied science as to suggest itself as the obvious line of attack. In fact, as is almost invariably the case in the computing field, the effort yielding the principles of the solution of the problem—the 'idea' if you like—was found trivial compared to all the 'sub-ideas' that had to be evolved before a practical method materialised.

It has been mentioned in the paper that the approach may be extended to shallow and steeply inclined reefs. We agree with Dr Salamon that multiple and disrupted seams can also be solved on a practical basis. These examples may possibly be viewed in a more general context: it seems likely that within a few years any rock mechanics problem that is linear and 'quasi-two-dimensional' in nature can be solved at reasonable costs on a digital computer. All examples cited fall into this category. in principle, of course, the approach can be extended to describe any fully three-dimensional situation. The required computing effort would largely be related to the total surface area of all significant excavations. As such, the approach offers attractive advantages over, for example, a three-dimensional finite element method which is dependent on the total volume of the system to be analysed. Without being over-optimistic, using this approach the analysis of the detailed *local* behaviour of a system of ancillary excavations that is being loaded by stoping activities in the vicinity, can probably be attempted within the next five years, if computing costs take the plunge that has been predicted. The case study reported in the paper forcefully demonstrates the need for such investigations to become practical.

We wholeheartedly agree with Mr Pirow's vision of an integrated information system describing all aspects of a mine or group's activity. In fact, one need not stop at that level. The machine can be drawn into the direct planning effort. This would enable management to investigate many more alternatives than can ever be handled on a manual basis. The design and implementation of such a system should obviously be tackled on an industry level. Its applicability need not be restricted to one sector of the industry only.

The eigenvalue estimates referred to in the appendix are significant only to the internal workings of the programme. They cannot be compared with any physical measurements. The model as a whole assumes a homogeneous isotropic elastic rock-mass and hence relies on the same experimental unification as the various analogues.

It seems that Mr Kirsten has built up a considerable body of experience in the use of the programme. He has, not unexpectedly, felt the need for a number of features that should be incorporated, among them rate of energy release rather than release itself. This can easily be added. We are convinced that there must be many others. In fact the present system should be considered a prototype completely open to improvements and extensions.

We are grateful to the other contributors for providing additional examples or discussing other potential applications of the computer method.