Flotation research work of the N.I.M. Research Group, and the Department of Chemical Engineering, University of Natal

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SYNOPSIS

The ideas leading to the formulation and utilization of a flotation model are developed. The main theoretical and experimental work that has been carried out by the National Institute for Metallurgy Research Group at the University of Natal is presented. The two experimental flotation plants available to this Group are described.

SINOPSIS

Die idees aanvoerende tot die formulering en gebruikmaking van 'n flotasie model is ontwikkel. Die hoof teoretiese en eksperimentele werk wat deur die Navorsing Groep van die Nasionale Instituut vir Metallurgie aan die Universiteit van Natal gedoen word, word uiteengesit. Die twee eksperimentele flotasie aanleggings beskikbaar tot hierdie groep word beskrywe.

INTRODUCTION

The National Institute for Metallurgy (N.I.M.) Research Group at the University of Natal has devoted a large effort to the development of mathematical models for the flotation process in the belief that this process in all its ramifications is amenable to quantitative description by means of a mathematical model. If such a belief is vindicated — and there is much evidence already available that it will be — and if a practically useful form of the model can be developed, two very important advantages will result. Firstly, a quantitative model will aid in improving the design of flotation plants, particularly the scale-up of batch data from the ore-dressing laboratory to full-scale continuous operation. Secondly, the model will lead to improved plant operation as a result of better control.

Mathematical models have their limitations, and these should be clearly appreciated. Mathematical modelling does not provide a means of circumventing the ore-dressing laboratory, which is the only place in which the ore-dresser can find out how to concentrate a particular ore: no mathematical model, no matter how complex, will tell him that. The model provides an aid in the rationalization of scale-up and design procedures so that best use can be made of the capital invested in the plant. It is not the intention that the Research Group should undertake ore-dressing investigations. The Group undertakes the theoretical and experimental studies that are required for the development of a model and for verification that the model gives a good quantitative description of the operating behaviour of the particular process under study. This must always be expected to be an exacting and time-consuming task, but the ultimate objectives are worthwhile because the work offers a very real opportunity for the improved recovery of valuable metals or minerals from ore obtained at great expense by mining.

The Research Group has concentrated its effort mainly on the flotation process, and this paper reports on the results obtained so far. Major results are highlighted and their significance discussed, and the paper includes a discussion of the utility of the modelling work in practical situations.

HISTORICAL DEVELOPMENT

Many attempts have been made to develop viable models for the flotation process, and many of these have enjoyed a certain measure of success, particularly in the description of single-batch or continuous cells or other very simple configurations of cells. The most obvious gap in this early work was the absence of a model versatile enough to describe quantitatively the behaviour of an operating flotation plant having a complex flow configuration, and it is this gap that the Research Group has attempted, with a large measure of success, to close.

A model that is successful in the description of a complete plant must necessarily provide a precise description of the behaviour of the individual units that make up the plant. Imprecision and errors at the individual cells are compounded when the model is applied to a large plant. Thus a start must be made with a model of the individual cell.

The fundamental basis for the modelling of a flotation cell is that the rate at which material leaves the pulp is a strong function of the amount of that particular kind of material present in the pulp, and an analogy with the model for the rate of chemical reactions is immediately obvious. However, unlike the chemical reaction, which is a process involving identical molecules, the fundamental particles that take part in the flotation process are all different one from another. These differences, because they can easily be misinterpreted, must be carefully allowed for.

The rate of flotation is governed very largely by the rate of collision between mineral particles and bubbles

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in the aerated pulp, and the rate of flotation should accordingly be very nearly proportional to the concentration of mineral particles and the availability of bubble surface area per unit of pulp volume. Quantitatively this can be described by the relation

Rate of flotation$=KC$. . . . . (1)

However, the constant of proportionality or rate constant, K, must be allowed to take different values for the different particles. This gives rise to a so-called distributed — constant model for the process. Imaizumi and Inoue[1] presented the first comprehensive analysis of the flotation process on the basis of the distributed-rate-constant model. They demonstrated that this model explained the apparent anomaly that the data from a batch flotation cell, when plotted as $\ln(C)$ against time, invariably produce a convex curve, whereas the ordinary analogy for first-order chemical reaction would predict a straight line. Fig. 1 shows data obtained by W. Davey plotted in this way, and it is typical of the data that appear in many places in the literature. Such convex curvature has been confused by some researchers with a non-first-order model.

At the time of the appearance of Imaizumi and Inoue's work, B. K. Loveday undertook an investigation in the Department of Chemical Engineering of the flotation of a simple pyrite-silica mixture. This turned out to be an investigation of the utmost importance, which has received very wide recognition. He made three important advances and these were significant in persuading us to mount a comprehensive research programme so that the model could be developed into a practically useful tool. Firstly, he showed that the average value of the rate constant for all particles was invariant with initial pulp concentration and so eliminated the multiple-order-rate model. His data are shown in Fig. 2. The plotting of the data in this way demonstrates one of the most significant features of this model: the average rate constant decreases during the duration of the test because the more easily flotable material is removed rapidly, leaving behind the particles of lower rate constant.

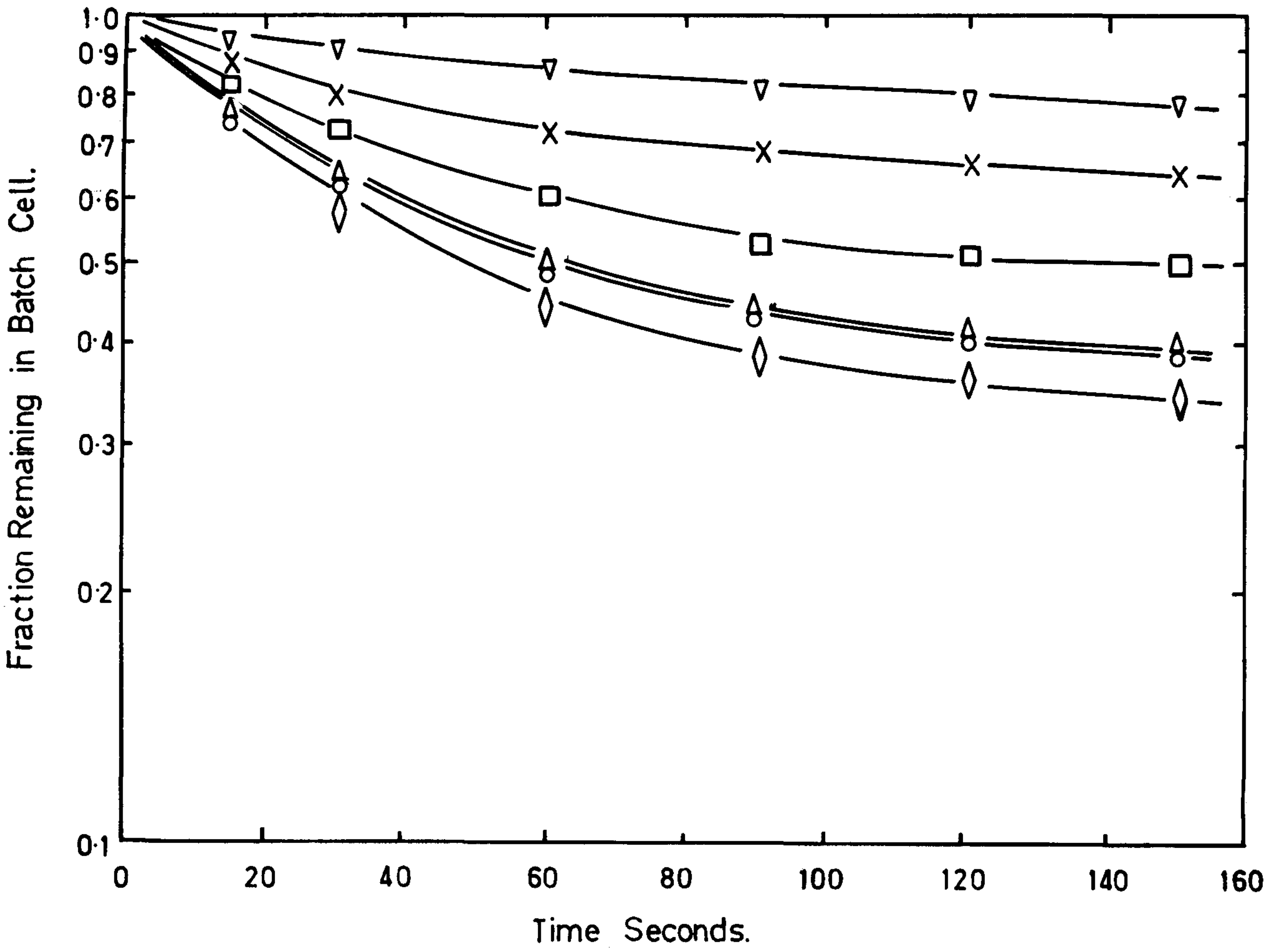

Fig. 1 — Typical data obtained in a batch cell. First-order rate equation would produce straight lines on this plot. Curvature can be due to distribution of rate constants or non-linear rate equation. Different curves result from different conditioning procedures. (Data obtained by W. Davey)

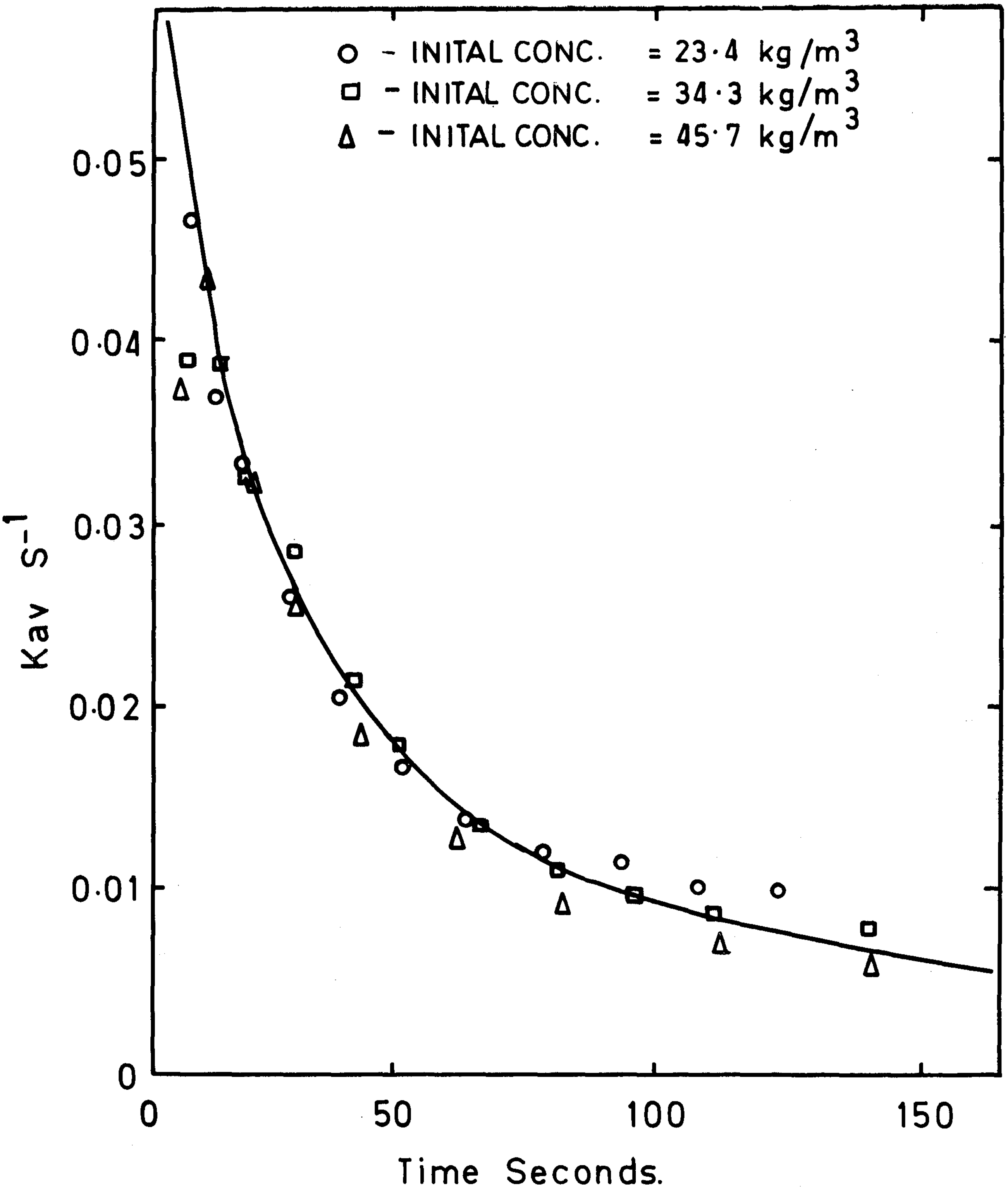

Fig. 2—Variation of average specific rate constant with time in a batch flotation cell. The average rate constant decreases with time as the particles with high specific rate constant are removed preferentially. The invariance of the data with initial concentration eliminates non-linear kinetics as a possible explanation. (Data obtained by B. K. Loveday)

Secondly, he showed that it was possible to make the model computationally tractable in a single batch or continuous cell by the use of a continuous distribution of K values.

Thirdly, and perhaps most important of all, he showed that it was possible to predict correctly the performance of a cleaner and a scavenger cell processing concentrates and tailings produced from a single continuous cell. This finding confirmed that the model held very real promise as a basis for the description of an entire plant and was the real spur to the Group's attempt to mould the model into a useful and practical tool.

Notwithstanding this successful demonstration by Loveday[2, 3], two major difficulties obviously had to be overcome.

(a) The model should be capable of serving as the basis for the characterization of the flotation behaviour of different ores. To do this, the model would have to be capable of accounting for the mineralogical structure of the separate particles, their size, and their inherent floatability. Because the particles tend to become more homogeneous as liberation increases with increasing fineness of grind, their size and mineralogical properties are related. Floatability is influenced by particle size through several hydrodynamic effects: very small particles will not collide with the bubbles, whereas very large particles, although colliding readily, are easily shaken off the bubble or prevent the bubble from rising to the top of the pulp. Fineness of grind is the single most effective control action that influences the performance of the ore in the plant.

(b) The model must be capable of correctly describing the effect of various control actions that are taken during normal plant operation so that the model can be used for rational control of operating plants. These control variables are conditioner-addition rate, position of conditioner addition, plant configuration, recirculation rates within the plant, and total plant throughput. These variables do not appear explicitly in the fundamental model relation (1), which serves only as a vehicle for the description of their influence on plant operation. This aspect of the work will be amplified in this paper.

MODEL EXTENSIONS AND EXPERIMENTAL CONFIRMATION

During the course of our investigations, the model was developed in accordance with fresh experimental evidence as it came to light.

The first step towards an extension of the model was taken by R. P. Colborn, who investigated its usefulness in industrial-sized equipment. He found that, in this large-scale equipment, the residence-time distribution of the solid played a dominant role, especially because this residence-time distribution is different for particles of different size. Even allowing for the effect of residence-time distribution for individual sizes, he was able to demonstrate that the basic model needed extension. The most effective demonstration that the simple distributed-constant-rate model is unsatisfactory is given by the data in Fig. 3, where the average of the K-distribution is plotted against pulp flowrate through a Denver No. 18 "sub A" flotation machine. The unmistakeable variation with particle size and pulp flowrate is clearly apparent. The experimental procedure developed by Colborn is interesting because it enabled him to measure all the rate parameters from an experiment at a single flowrate — an essential requirement if the effect of varying flowrate is to be unambiguously determined. He made use of a pulse-injection technique using irradiated ore[4], and his work offers a good example of the careful attention to experimental procedure that is required for the successful testing of such models.

Colborn succeeded in showing that the variations present in Fig. 3 could be largely accounted for if proper allowance were made for the particle size in the model. He was able to incorporate specifically in the model a quantitative description of the particle-size effect. He proposed that the rate constant should be split into two factors, one of which accounted for all the effects due to particle size, D. Thus

$$K=k_1\Phi(D). \qquad (2)$$

Colborn was able to correlate his observations with

$$\Phi(D)=(\text{const.}/D)\,(1-({}^{D}/\Delta)^{1,5})\,\exp\,(-\epsilon/D^2), \qquad (3)$$

provided that Δ was allowed to vary with flowrate. The success of this correlation is evident in Fig. 4, which shows the fractional recovery of particles of various sizes in the cell operating continuously at a pulp flowrate of 3,49 m^3/s. Similar agreement was obtained at other flowrates over a three-fold range. The form of equation (3) was established by a hydrodynamic analysis of the motion of a particle close to and on the surface of a bubble.

A further development in the model structure was made by R. I. Edwards, who, in the course of an investigation of the performance of a deep flotation cell, was able to incorporate the effect of variations in aeration rate. It is essential that this effect should be allowed for in the model because the aeration rate is often used by plant operators to control the amount of material that is taken from individual cells in the plant. This practice has a marked effect on plant performance because the circulating loads are largely determined by the flowrates of the concentrates. The calculation of plant performance at various circulating loads is an interesting and important problem that will occupy our attention a little later in this paper. The effect of aeration rate can be modelled in a very simple way by postulating that the rate of flotation is proportional to the available surface area of bubbles per unit volume of pulp. Thus equation (2) becomes

$$K=k_2\mathrm{A}S\Phi(D), \qquad (3)$$

where A is the bubble surface area per unit volume of pulp, and S is the fraction of that area not covered by adhering particles. The relationship between equation (3) and the aeration rate is the simple proportionality.

$$\mathrm{A}=\frac{\sigma\tau G}{V}, \qquad (4)$$

where σ is the bubble surface area per unit volume of bubble, τ the average bubble residence time, and V the

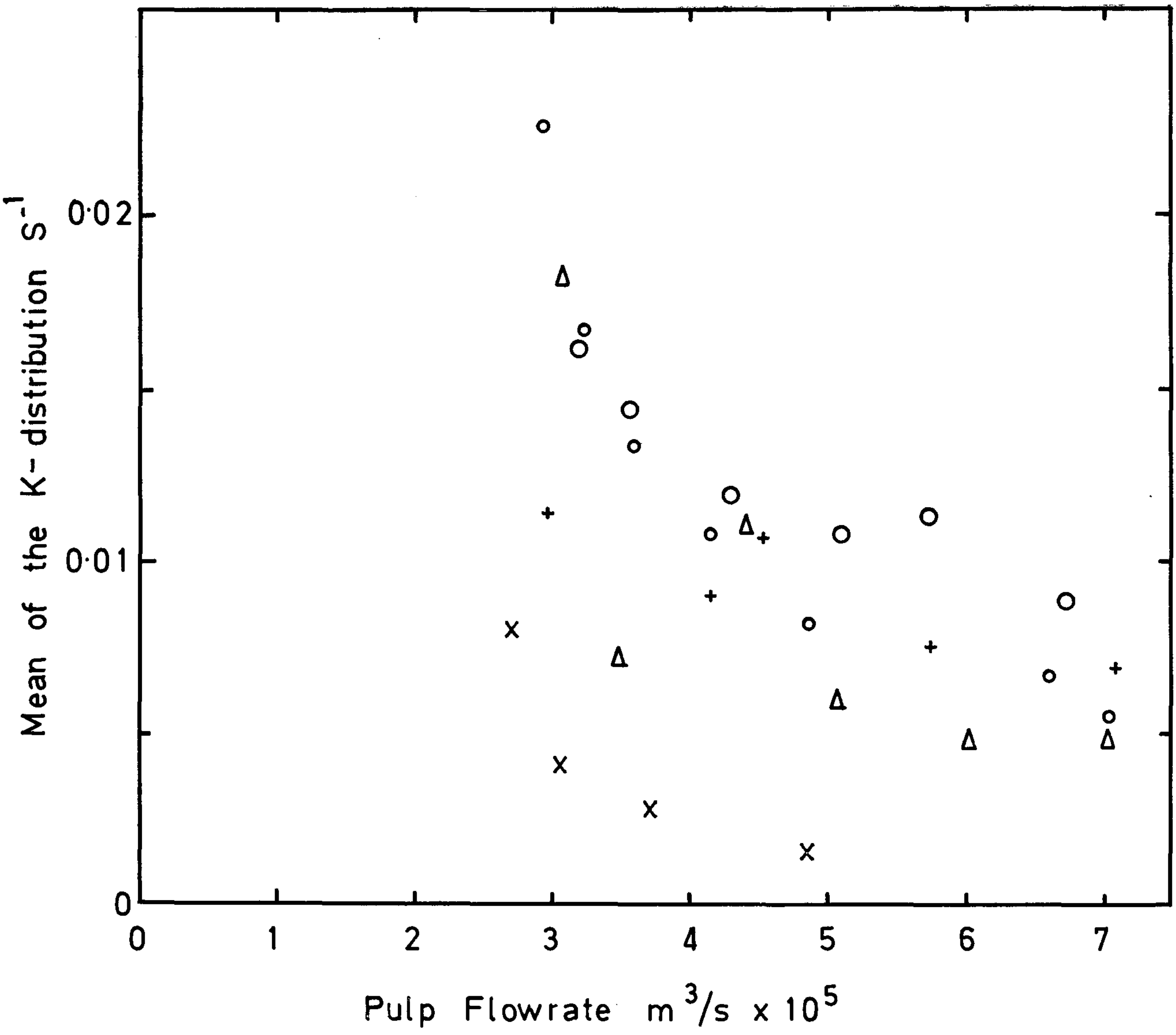

Fig. 3—Mean of the K-distribution, showing variations with particle size and pulp flowrate in an industrial cell. o$<$ 37μm,$+ >$ 37μm to O63μm, 63μm to 125μm, Δ 125μm to 250μm,X $>$ 250μm (Data obtained by R. P. Colborn.)

cell volume. Equation (4) is the main scale-up relation for the flotation cell. τ varies with the size of the cell, and σ, which is determined by the shape and size of the bubble, varies with the size of the cell and the dispersion and agitation mechanism. The usefulness of equation (4) in scale-up predictions is impaired by the difficulty of measuring σ. At present empirical correlations from the literature must be relied on. We have investigated the possibility of relating σ to the volumetric flow of water in the froth, but no quantitative relations have yet emerged. τ is comparatively easy to measure, either from a measurement of air holdup in the cell or by a direct measurement of the residence-time distribution of the air in a conventional tracer experiment.

Equations (3) and (4) indicate that the principal effect of an increase in aeration rate is an increase in the production rate of material from the cell. The rate of water removal in the concentrate is also increased. A similar result can be achieved by variation of the pulp level in the cell, which is also a control variable that is widely used by plant operators. The effectiveness of this control variable depends on the fact that the froth, as it rises in the column above the pulp and before it passes over the froth lip, breaks to a certain extent. Thus material is returned from the froth layer to the pulp. There is also the possibility of selective return of less-floatable material from the froth layer, so leading to an extra measure of enrichment over and above that ob-

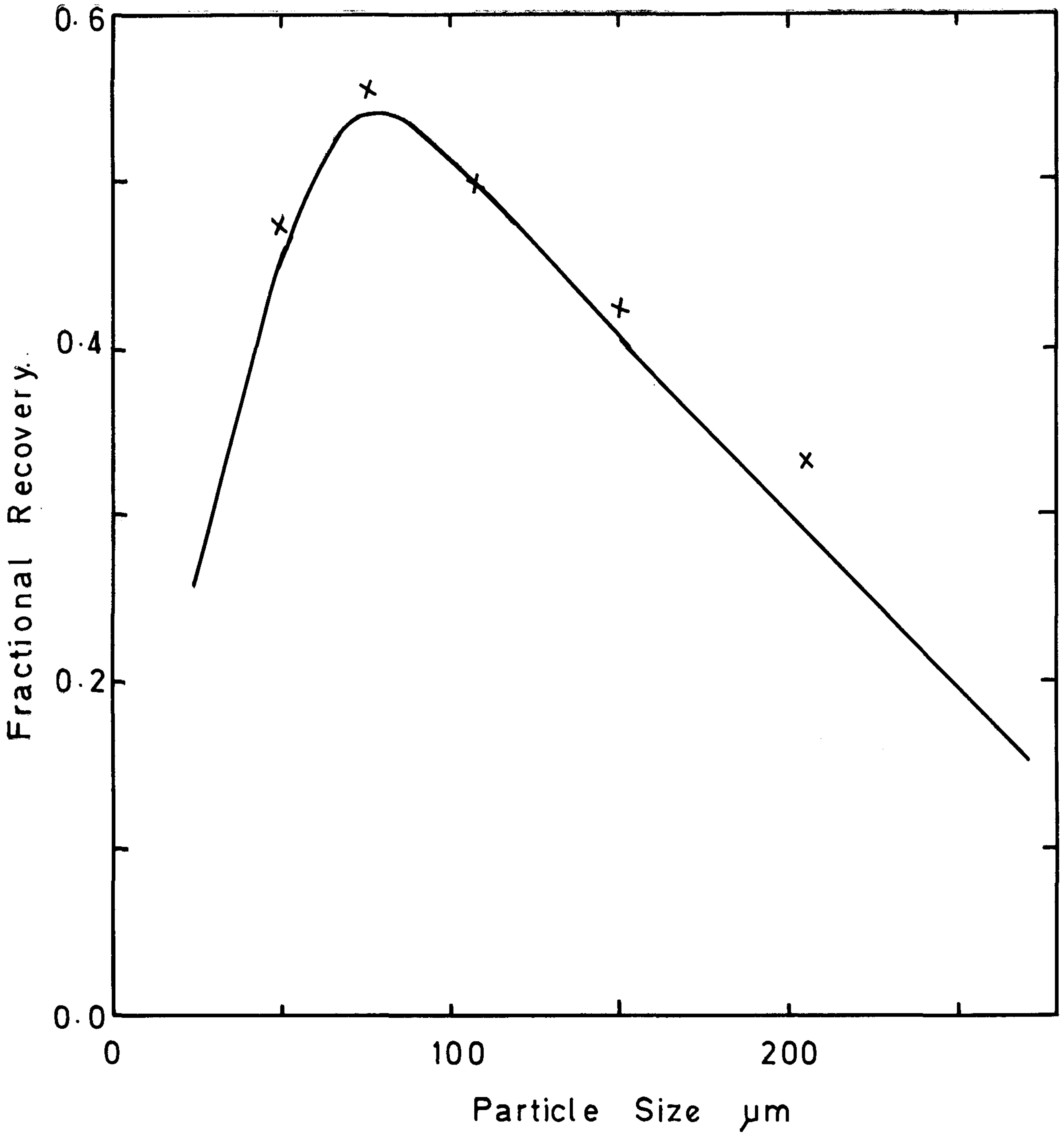

Fig. 4—Fractional recovery as a function of particle size in an industrial cell. The solid line is predicted by the model. (Data obtained by R. P. Colborn)

tained by the flotation process. However, we have not been able to observe this selective return experimentally. I. J. Barker, in a detailed study of the behaviour of the froth column, found that froth columns containing large amounts of very fine solids are very stable and show low rates of breakage.

The data shown in Fig. 5 are typical of the results obtained by Barker[5] for the apatite ore that was used. He concluded that the major effect of increased froth height for stable froths was an increase in the amount of water that drained from the froth back into the pulp. There was very little return of solid material and no detectable increase in grade.

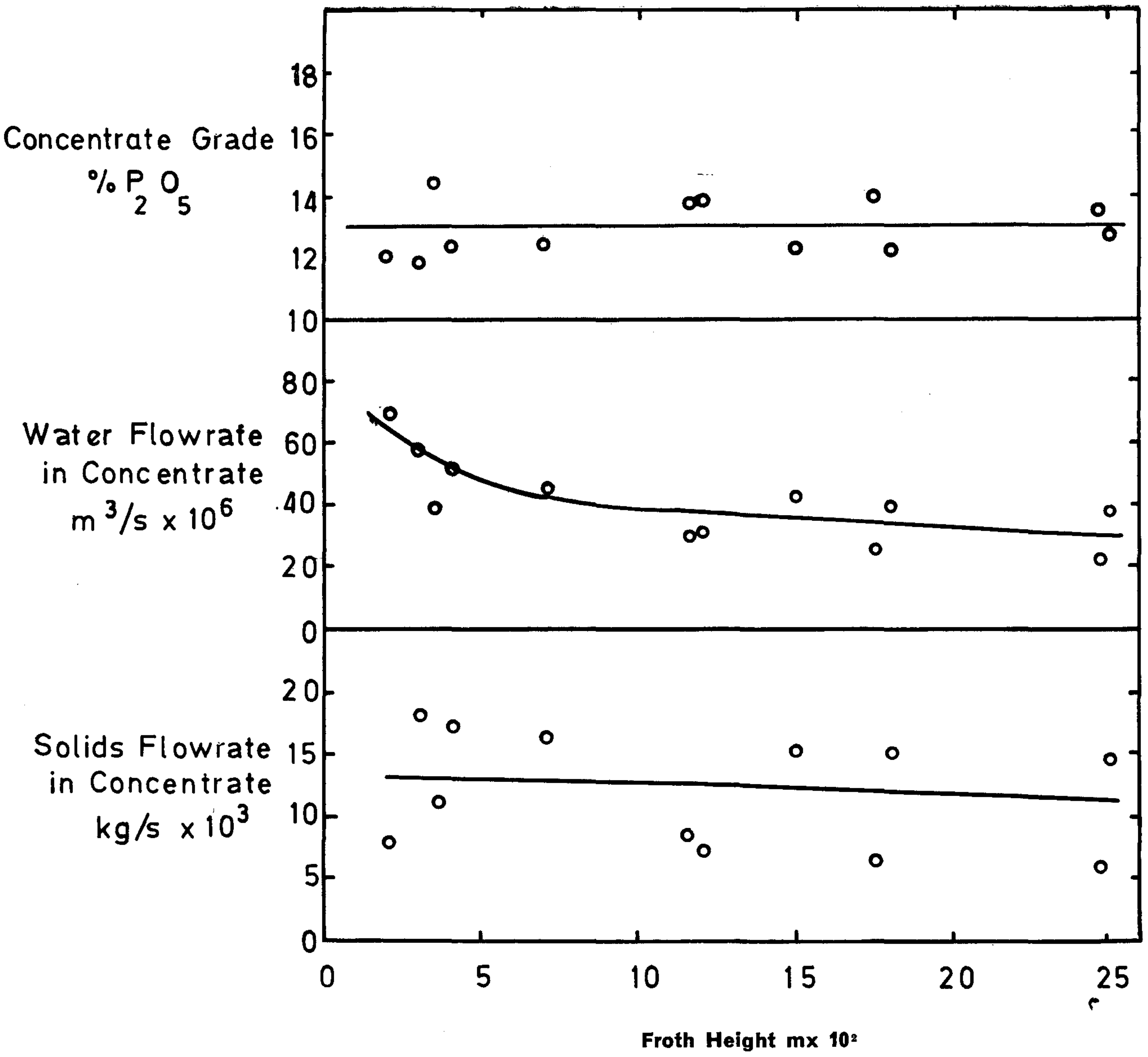

Fig. 5—The effect of froth height on the performance of a single continuous flotation cell. (Data obtained by I. J. Barker)

These conclusions are specific to the particular system that was studied and may not be valid for other systems. It is our intention to make use of the equipment that was developed by Barker to test other ore systems in this way. J. A. Engelbrecht has shown that froths produced by the flotation of pyrite-silica slurries have a tendency to upgrade the more floatable component (pyrite) as the froth height increases. The breakage rate in such froths was shown by Barker to be much greater than in the stable froths produced in the flotation of the apatite ore containing a large proportion of fines. A major achievement of Barker was the development of an experimental technique for the quantitative measurement of the rate of breakage of the froth.

Physical entrainment of particles in the water that enters the froth is another mechanism that we consider to be of importance in determining the performance of the flotation cell. We have not yet developed an experimental technique by which the rates of entrainment and flotation can be measured separately. Barker has shown on theoretical grounds that entrainment rates are likely to be high for particles smaller than 50 μm. The non-selective nature of the entrainment process means that it inevitably degrades performance.

APPLICATION OF THE MODEL TO THE CALCULATION OF PLANT PERFORMANCE

Flotation plants have a complex structure in the sense that they are composed of many individual flotation cells that are linked by various streams. The cells are grouped into stages that have a single feed stream, a single concentrate stream, and a single tailings stream. The feedback of tailings from any stage to the previous one means that the material-balance equations in the plant are highly coupled. Any model for the flotation process will lead to complex calculations of plant performance, and the model that we have advocated and investigated has the added disadvantage that it takes into account many different groups of particles — groups classified by size, mineralogy, and floatability — during their complex passage through the interconnected system of cells. It was by no means certain at the outset that the model could be successfully used for plant calculations, and a large effort was required in the development of suitable numerical techniques for these calculations.

The first effort in this direction was made by Buchalter and Piper[6, 7]. They based their method on an assumed functional form for the K-distribution, but this was found to be inflexible and computationally very inefficient. In addition, it was considered impossible to adapt the method to the computation of a model with the desired three-dimensional spectrum.

It became apparent that a calculation procedure based on individual particle classes was required. In the development of such a procedure, a penetrating study[8] of a linearized version of the flotation model, which was made by Deift, was found to be of great value. Efficiency of calculation was of paramount importance because the assignment of even a modest number of particle classes to each dimension of the three-dimensional distribution leads to a large number of independent particle classes. The chief difficulty in the calculation arises because the individual particle classes do not behave independently of one another and the interclass interactions negate the advantages of the linearity of the original rate models. The essential apparatus for the calculation of plant performance is the material balance. Deift showed that the material-balance equations can be formulated in matrix form.

$$D\,x=F, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

where x is the vector of flowrates in the tailings streams—one element for each stream, F is the vector of feedrates, and D is called the configuration matrix because it specifies the configuration of flow streams among the stages in the plant. Any configuration, no matter how complex, can be described by equation (5). The individual elements of D have the form

$$d_{ik}=-b_{ik}-\nu_k a_{ik}$$

b_{ik} is the fraction of the tailings stream that leaves stage k and enters stage i, and a_{ik} is the fraction of the concentrate stream that leaves stage k and enters stage i.

ν_k is the product of the rate constant K and the holding time θ in stage k if all the froth is removed from the cells. If a deep froth is used, only a fraction, γ, of the floated material is recovered in the concentrate and

$$\nu_k=\gamma_k K_k \theta_k. \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

Here the subscript k deonotes a particular stage. In equation (7), each of the factors γ_k, K_k, and θ_k can assume specific values for each particle class, although it is K_k that accounts for the greatest interclass variation. K_k is a function of the stage number through its dependence on A and S in equation (3).

θ_k is a source of difficulty: it varies fairly significantly with particle size (this was confirmed experimentally by Colborn[4]) but, most important, it depends in each stage on the amount of material that is floated in that stage. There is consequently a strong nonlinear interaction between classes. In a mathematical sense this means that D in equation (5) is a function of x. However, for fixed ν, Deift demonstrated the existence of a positive solution to equation (5) and it proved effective to use the matrix equation as the basis for successive solutions of the linear equations, which, in the limit, approached the solution to the nonlinear equations when this existed. The approach to the limit was found to be rapid, and the computational procedure proved to be efficient, even for large numbers of particle classes. The solution is suitable for the calculation of any quantity of interest in the flotation plant, so that complete design and control calculations can be made. The full details of this work are available in reference 9 and the relevant computer programme is available from the N.I.M.

The plant calculations require data on the characteristics of the ore. These data are best obtained from a series of classifications: the total particle population is classified according to size, the particles within each size group are classified according to mineral content, and the particles of particular mineral content are then classified according to flotation rate constant. The first two classifications can be done by purely physical means (sieving and microscope particle-count methods, for example), but the classification by rate constant requires very special parameter estimation techniques. We are developing standard particle-count methods, and M. Moys has almost completed a comprehensive investigation of the problems associated with the estimation of all the parameters required by the model. In connection with this latter problem, E. M. Buchalter has made the interesting discovery that an ore can be adequately characterized by classification into only two classes of rate constants and that these can be easily estimated from the performance of the successive cells in the rougher bank. This is demonstrated in Fig. 6.

He has also demonstrated that kinetic parameters estimated from batch data can be used in the direct computation of the performance of cells that have the same dimensions and that operate continuously.

At the present time, the predictive capabilities of the model are being confirmed experimentally in a pilot plant specially constructed to provide various flow configurations. L. A. Cramer has almost completed an extensive study of the effects of the pulp level and aeration rate on the performance of an industrial cell, and this study will provide the necessary data for the precise specification of the effect that these important control actions have on the plant performance. It should be remembered that the model provides the essential link between the variations in the individual cells and the performance of the whole plant. Variations in pulp level and aeration rate manifest themselves on the plant as varying circulating loads around the separate stages, and the chief use of the model is the demonstration of

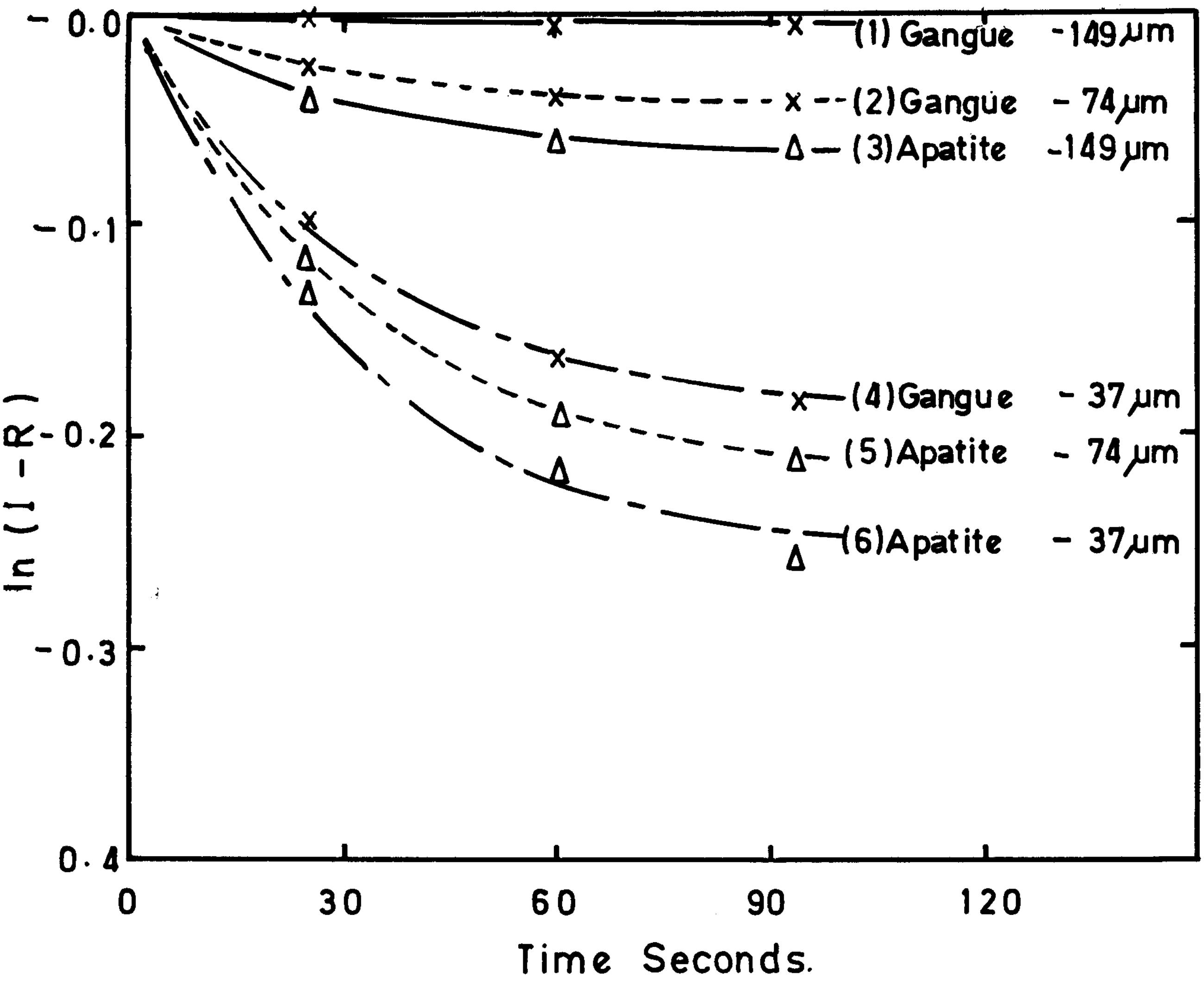

Fig. 6—Data demonstrating the characterization of gangue and apatite of various sizes by two K values. The curves shown were obtained by allocating $K_1=0,04\ s^{-1}$ to a fraction α of the material and a value $K_2=5 \times 10^{-5}\ s^{-1}$ to the remainder. Values of α for the six curves are (1) 0,010, (2) 0,101, (3) 0,145, (4) 0,345, (5) 0,393, (6) 0,443. Data obtained by E. M. Buchalter in first three cells of a continuously operated rougher bank.

how these variations can be used in the control of grade and recovery.

PLANT CONTROL BY CONDITIONER ADDITION

It has long been recognized that the plant will respond sensitively to variations in rates of conditioner dosage, and this has formed the basis of several successful plant control strategies[10, 11] that appear in the literature. We have attempted to establish an empirical but quantitative relation between the concentration of the conditioner in the aqueous phase and the rate of flotation. No such relation has yet emerged, but some progress has been made. B. Addison and H. Coppens[12] have determined the adsorption isotherm for Unitol D.S.R. on phoscorite, and their data are shown in Fig. 7. The unusual shape of the isotherm has not yet been explained.

Careful control is maintained over conditioner-dosage rates in all experimental work. Carefully designed experiments are run in the identification of chemical and physical factors that significantly affect the conditioning of the ore. W. Davey has conducted an extensive series of conventional bench-scale batch experiments in an attempt to quantify the relation between the rate of flotation and the rate of conditioner addition.

EXPERIMENTAL FACILITIES

The Research Group has two well-instrumented pilot plants available for laboratory testing work, each of the

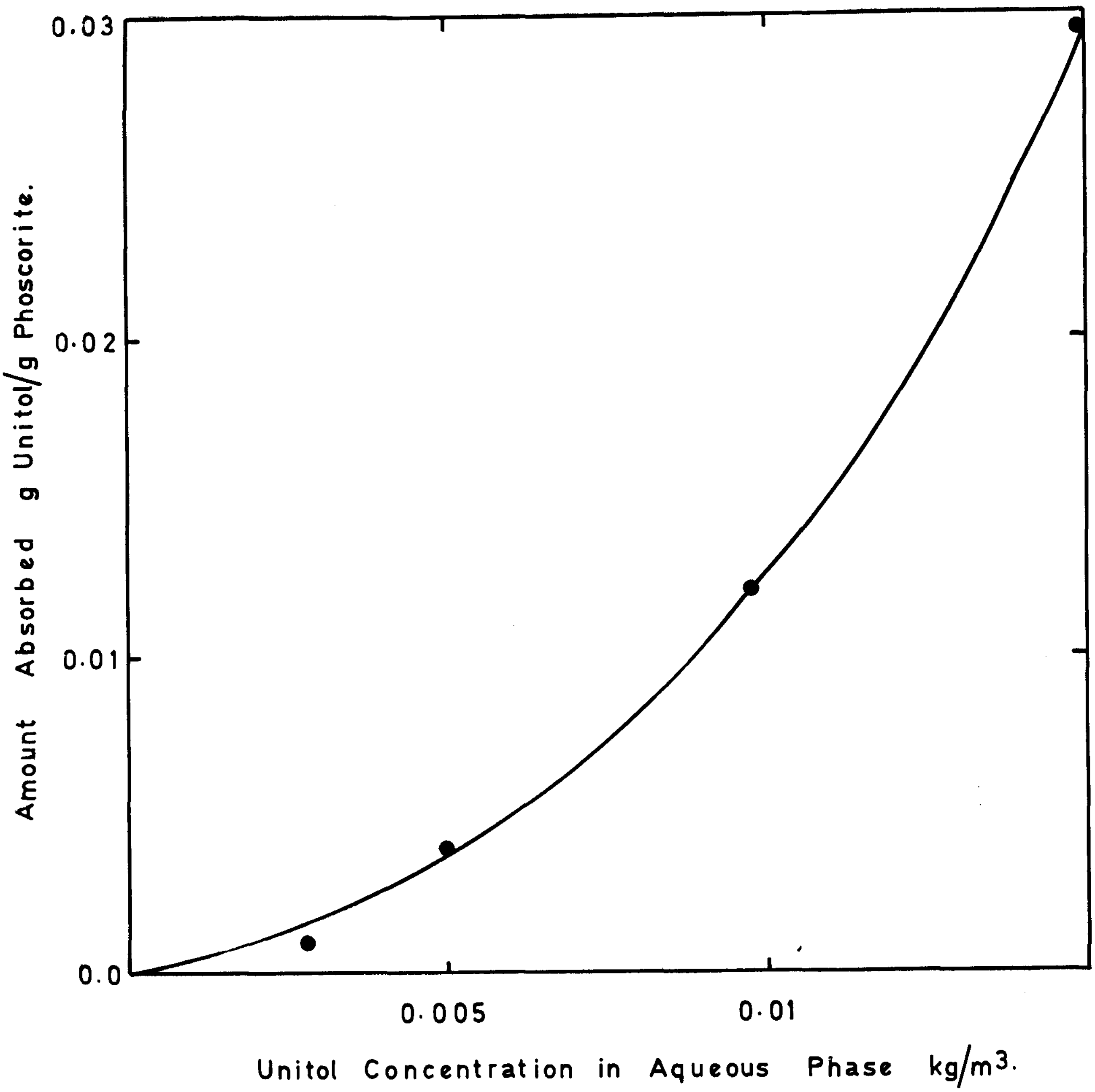

Fig. 7—Adsorption equilibrium between aqueous solutions of Unitol D.S.R. and phoscorite. (Data obtained by B. J. Addison and H. J. Coppens)

plants having cells of widely differing size. One plant is composed of eight separate cells of 0,025 m^3 each, together with four similar cells arranged as a single hog trough, which is used for scavenging. This plant can be operated in a very wide variety of configurations and is used for studies on parameter estimation and model verification, particularly those aspects of the model that influence the performance of the entire plant configuration. Particular attention has been paid to the provision of accurate and reliable sampling devices. Wigwag samplers are provided on all the concentrate streams. The flowrates of feed streams are measured by magnetic-inductance flowmeters, and the density of the plant feed is metered by a gamma-ray attenuation gauge. The feed-rate and density of the plant feed are very closely controlled to ensure stationary operating conditions during test work.

The second plant is constructed around two Denver No. 18 (0,7 m^3) cells. These can be connected as a rougher-scavenger or rougher-cleaner configuration. Facilities are provided on one of the cells for the control and measurement of aeration rate and pulp level. On this plant provision is also made for the collection of accurate samples by means of wigwag samplers on the concentrate streams and sample cutters on the tailing streams. The feedrate and density of the plant feed are also tightly controlled. This plant is used for the testing of scale-up relations and the effect of control variables, such as aeration rate and pulp level, on cell performance.

Other special-purpose flotation cells are available. R. I. Edwards investigated the performance of a deep cell, and J. A. Engelbrecht has investigated the effect of gas precipitation on the flotation of pyrite. These studies, although of very great interest, do not, at this stage, play a major role in the development of the flotation model and are consequently given no prominence in this review. The results of this work will, of course, be published elsewhere.

We attach a great deal of importance to the interplay between experimental and theoretical modelling work. By maintaining an active experimental programme, we believe that the models developed by the Research Group will not only have a sound theoretical basis but will be useful to the practising engineer.

We are exploiting the most modern methods of experimental design to ensure that the conclusions drawn from our experimental work effectively test our theoretical model structures. I. Copelowitz has demonstrated that data obtained in the batch cell discriminate overwhelmingly in favour of the distributed-rate-constant model against possible nonlinear rate models, and he will apply the same theoretical technique to the data obtained on the continuous plant.

CONCLUSIONS

This paper is a very brief summary of the work done by the several investigators who have contributed and are contributing to the programme. I have attempted to trace the main thread of the investigation, and further details must be sought in the papers and theses given in the list of references. Much of the work is still unpublished, and the most important part of the experimental programme is still to be completed.

The major conclusion of the work so far is that an ore can be sufficiently well characterized for its behaviour in a complex plant to be predicted. Only further experimentation on pilot and full-scale plants will indicate just how precise these predictions are and whether they are useful as a basis for practical plant-control techniques.

We have shown that it is necessary, in characterizing the behaviour of an ore, to make an unconventional set of observations. These include froth-breakage rates, bubble size and residence-time distribution, solids residence-time distribution, individual particle identification and counting, grades in different size fractions, and so on. The models provide a framework within which these observations can be put to practical use.

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