Three-dimensional finite element stress analysis applied to two problems in rock mechanics

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SYNOPSIS

The finite element method of stress analysis is nowadays widely used in the fields of mining, mechanical and civil engineering. This paper presents a short description of the three-dimensional method, in which the elements used are arbitrary hexahedrons. Two examples of the application of the method are presented, namely, calculation of the elastic stress distributions in a coal pillar and in an opencast mine.

SINOPSIS

Die eindigeelementmetode van drukspanning word deesdae algemeen gebruik op die gebied van mynwese, meganiese en siviele ingenieurswese. Hierdie verhandeling gee 'n kort beskrywing van die drie-dimensionale metode waarin die elemente van arbitrêre sesvlakke gebruik gemaak word. Twee voorbeelde vir die aanwending van die metode word voorgelê, naamlik die berekening van die elastiese drukspanning verspreiding in 'n steenkool pilaar en in 'n oopgroef myn.

INTRODUCTION

The finite element method is becoming increasingly well known as an extremely powerful method of stress analysis of two-dimensional structures. This is due to the fact that it allows arbitrary structural properties to be included in the analysis. This capability is inter alia very useful in the study of the stress distributions around mining excavations in which the structural material is rock, inherently an inhomogeneous and anisotropic medium. It is evident, therefore, that the extension of the finite element method to three dimensions would provide an analytic technique which could treat complex problems in which no two-dimensional axes of symmetry exist. The development of the three-dimensional method has unfortunately been hindered by the fact that extremely large digital computers are required and the computing time becomes considerable. However, the analysis has been applied to the calculations of the stresses in arch dams[1] with good success, and recently also to the stability of rock slopes[2].

This paper presents an outline of the three-dimensional finite element method and its application to two problems in rock mechanics, namely the calculation of the three-dimensional stress distribution in a coal pillar and in an entire proposed open-cast mine.

THE FINITE ELEMENT METHOD

A general outline

The finite element method has been described elsewhere[3] and a detailed explanation will therefore not be presented here. In essence, however, the method may be outlined as follows:

The structure to be analysed is divided suitably into 'finite elements', assumed to be connected only at their corners which are termed nodal points. From the element dimensions and material properties, the stiffness of each element may be calculated. The element stiffness relates the nodal point forces to the corresponding nodal point displacements. The stiffnesses of appropriate elements are superimposed to evaluate the nodal point stiffnesses. If the nodal point forces are known, e.g. from boundary conditions and body forces, the nodal point displacements, and hence element stresses, may easily be calculated. This then provides the stress distribution throughout the complete structure.

Development of the three-dimensional analysis

In the two-dimensional finite element analysis, the simplest procedure is attained using the three node triangular element. Irregular boundaries may be easily fitted with these elements and mesh size may be increased and decreased at will. The tetrahedron, which is the three-dimensional version of the triangle, offers the same advantages, but suffers from the disadvantage that it is extremely difficult to form a mental three-dimensional picture of adjoining tetrahedrons. It is far easier to consider hexahedrons which can be more easily pictured in space.

The element chosen for three-dimensional analysis, therefore, is the arbitrary hexahedron isoparametric element described by Zienkiewicz[3]. This element has eight arbitrary nodes, and faces of the element may be warped. It satisfies the requirements of easy matching of irregular boundaries and easy increase and decrease of mesh size.

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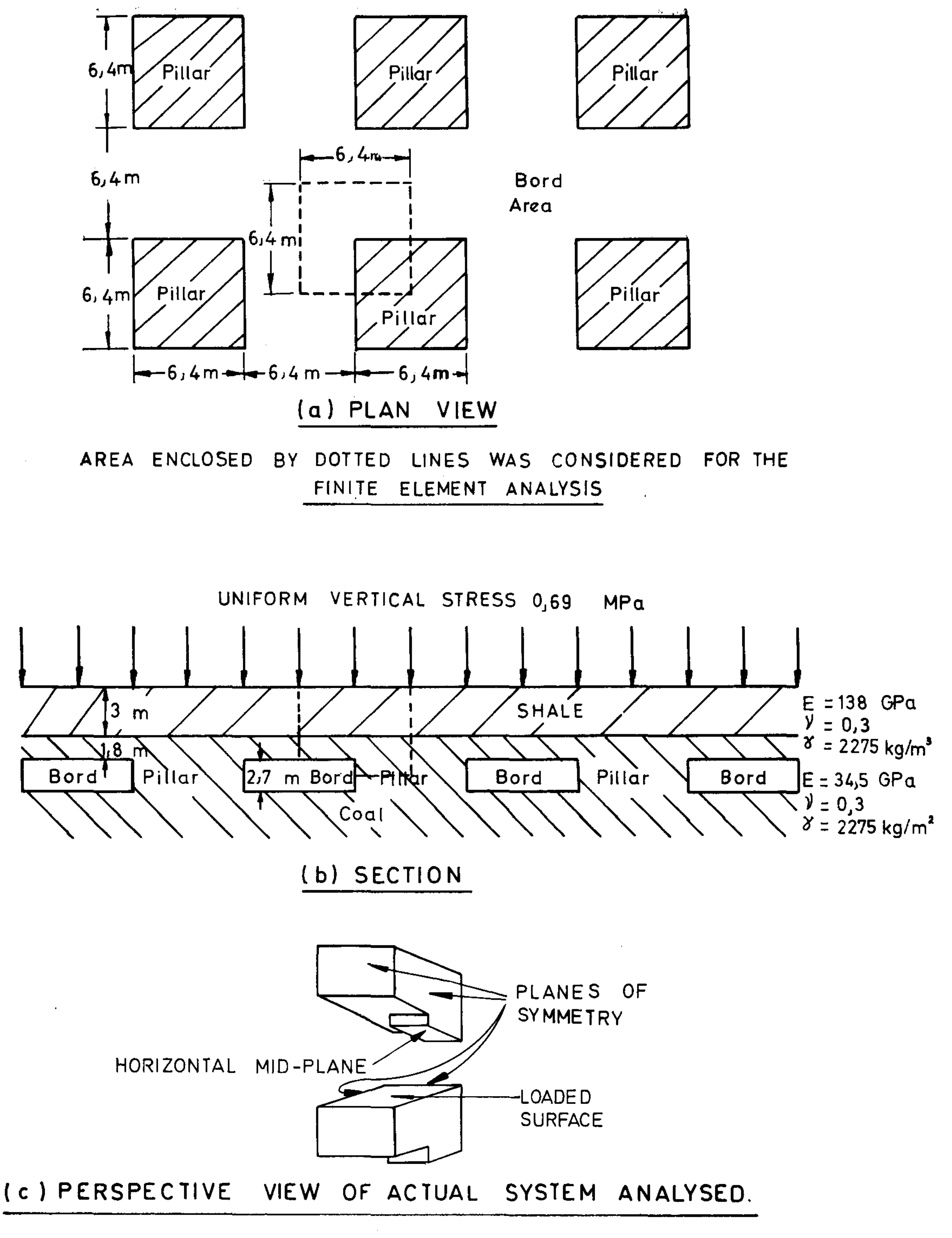

Fig. 1 —GEOMETRY AND DIMENSIONS OF THE REGION CONSIDERED FOR THE FINITE ELEMENT ANALYSIS.
(E=Young's modulus, ν=Poisson's ratio, γ=Density)

The derivation of the element stiffness matrix, perhaps the most critical part of the finite element analysis, has been described by Zienkiewicz[3] and presented elsewhere[4]. The evaluation of the element stiffness is relatively complicated, requiring numerical integration, and a description of this procedure is beyond the scope of the present paper.

The computer programme developed for the three-dimensional analysis is based on the two-dimensional version described by Wilson[5]. The analysis of even simple problems using this programme cannot be attempted unless a digital computer with a large core storage is available. For more complicated problems even this is not sufficient, and, for this reason, the programme discussed in the present paper was designed to operate on parts of the structure to be analysed. Thus, the structure is divided into appropriate parts, with a certain overlap which is required for continuity. The implementation of this necessitates the use of magnetic disc storage as well as computer core storage. For each part of the structure, the element stiffnesses and hence nodal point stiffnesses are calculated, and the latter written onto magnetic disc. Therefore, for the calculation of each part, the same core storage can be used. Solution for the nodal point displacements is also carried out in parts. With this method of operation, it is possible to handle a problem involving a large number of elements.

CALCULATION OF THE STRESS DISTRIBUTION IN A COAL PILLAR

A recent investigation by van Heerden[6] into the in-situ stress distribution in a coal pillar prompted the analysis of this problem by the three-dimensional finite element method. Fig. 1 shows the geometry and the dimensions of the pillar and the surrounding region considered for the analysis. The coal seam was 1.8 m thick above the roof. It was overlain by shale right up to the surface. However, only a thickness of 3 m of the shale overburden was taken into consideration and it was assumed that a vertical overburden pressure of 0.69 MPa acted at the system boundary 4.8 m above the roof. The pillars had a 6.4 m square horizontal cross-section, and adjacent pillars were separated by bords 6.4 m wide. The pillar height was 2.7 m. The material properties adopted for the coal and shale are noted on Fig. 1. Both the coal and shale were assumed to be homogeneous, isotropic and to behave perfectly elastically.

The boundary conditions applied to the region considered (see Fig. 1 (c)) were as follows: Horizontal displacement of all vertical in-situ boundary surfaces, i.e. excepting for the free faces of the pillar, was restricted. Vertical displacement of the horizontal midplane through the pillar was restricted. The upper surface of the region was loaded as described above.

The region was divided into a total of 141 elements with 256 nodal points. For computer solution, the problem was divided into two parts.

The distribution of the vertical stresses in the pillar horizontal midplane, along lines from the centre of the pillar normal to the free faces, and along diagonals, are plotted in Figs. 2 and 3 respectively. For comparison purposes the results of in situ stress measurements carried out by van Heerden[6] and also results, calculated by van Heerden, using a theoretical method developed by Salamon and Oravecz[7], are also shown in the two figures. It can be seen that the agreement is close, except near the faces of the pillar. The results of the finite element analysis indicate that the peak vertical stresses acting in the pillar horizontal midplane do not occur at the face, but a little inside. For an elastic analysis at any rate, the form of these results is confirmed by the work of Logie[8], who conducted three-dimensional photoelastic tests on models of bord and pillar excavations. The in situ stress measurements were based on the assumption of elasticity at each measuring point. The elastic properties could, however, vary for different measuring points. For the finite element analysis the elastic properties were assumed to be the same throughout the pillar. This may be the reason for the discrepancy between calculated and in situ stresses.

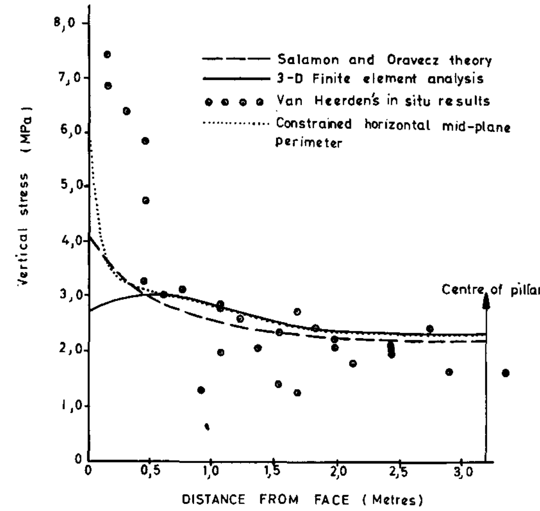

Fig. 2—DISTRIBUTION OF VERTICAL STRESS ALONG NORMAL LINE IN PILLAR HORIZONTAL MID-PLANE

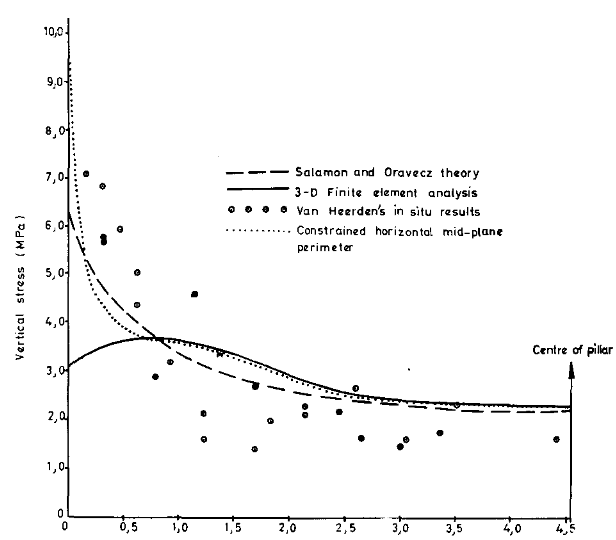

Fig. 3—DISTRIBUTION OF VERTICAL STRESS ALONG DIAGONAL LINE IN THE PILLAR HORIZONTAL MID-PLANE

The in situ results suggest a stress concentration pattern which could be produced by constraint of the boundaries. The finite element analysis was therefore repeated for different boundary conditions, namely restriction of horizontal displacement at the perimeter of the horizontal midplane of the pillar. The resulting vertical stress distributions on the horizontal midplane are given by the dotted lines in Figs. 2 and 3 and show closer agreement with the in situ stresses. Since this boundary constraint is impossible in practice (stresses normal to a free surface must be zero), it is clear that this stress concentration must be produced by latent horizontal stress conditions, material irregularities or a combination of both. Since the analysis was restricted to linearly elastic, isotropic and homogeneous behaviour this problem was not pursued further.

CALCULATION OF THE STRESS DISTRIBUTION IN A PROPOSED OPEN-CAST MINE

A preliminary investigation into the stability of the slopes of a proposed open-cast mine was recently undertaken[9] in which use was made of the finite element method. The investigation took the form of a two-dimensional finite element analysis of a vertical section through the mine and, for comparison purposes, a three-dimensional analysis of the entire mine. Fig. 4 shows a plan of the proposed mine, indicating the section used for the two-dimensional analysis, and Fig. 5 shows this section, outlining the geology. Both figures combined specify the boundaries of the region considered for the three-dimensional analysis. The boundary conditions applied were such that no displacement of the sub-surface boundaries of the region were permitted in a direction normal to these boundaries. The mine was assumed to be stressed by gravity forces only. The material properties adopted for the various types of rocks are noted on Fig. 5.

The three-dimensional finite element idealisation comprised 272 elements and 425 nodal points, and, for computer solution, the problem was divided into two parts. The section shown in Fig. 5 was covered by only 32 hexahedral elements (compared with 701 elements for

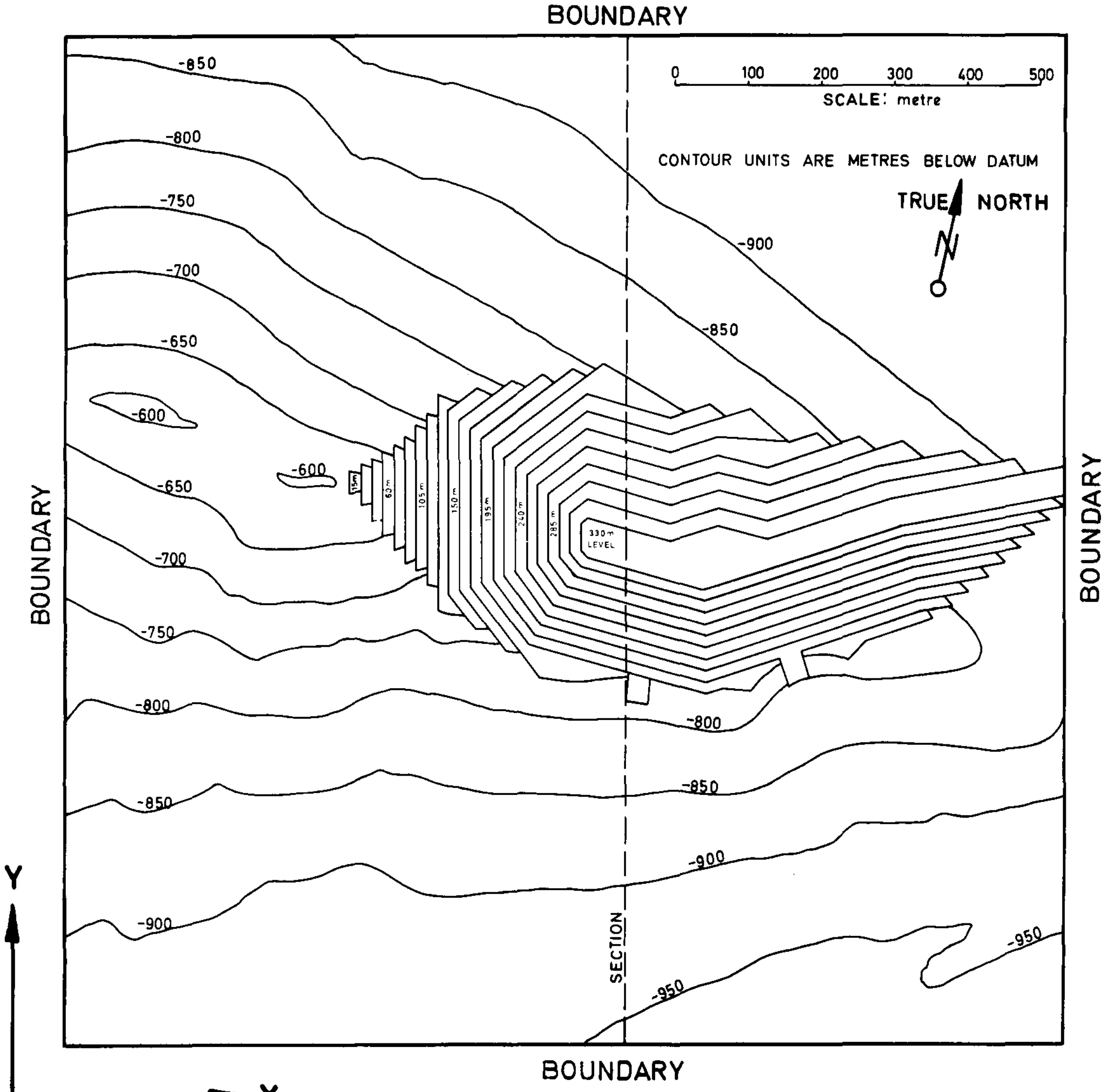

Fig. 4—PLAN OF PROPOSED OPENCAST MINE SHOWING BOUNDARIES OF FINITE ELEMENT ANALYSIS

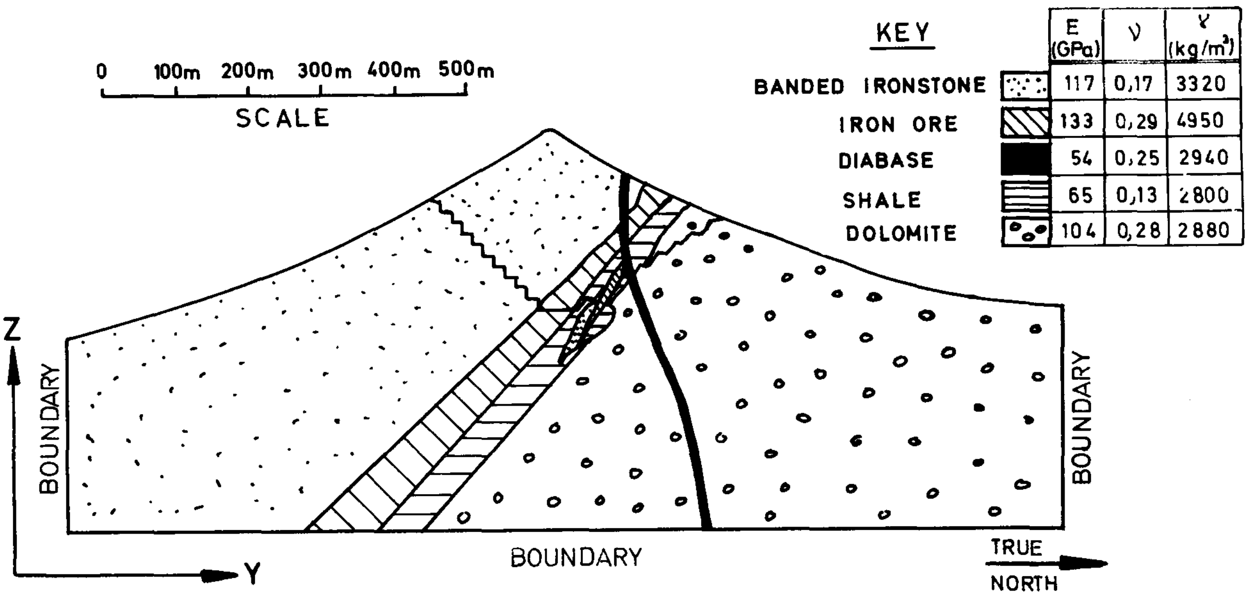

Fig. 5—GEOLOGICAL SECTION OF MINE SHOWING BOUNDARIES OF FINITE ELEMENT ANALYSIS. (E=Young's modulus, ν=Poisson's ratio, γ=Density)

the two-dimensional analysis) and, because of this coarse mesh, the geological features were somewhat simplified, small irregularities being ignored. This meant that there was often more than one type of geological material in each element. In these cases, approximate averages of the material properties were assigned to the element.

Under these conditions of simplification, a detailed description of the stress distribution in the mine could not be expected. The stress distributions across the section are compared in Fig. 6, in which they are presented as contours of the stresses acting in the three orthogonal directions x, y and z, these axes being defined in Figs. 4 and 5. Fig. 6(a) shows the stresses obtained from the three-dimensional analysis, and Fig. 6(b) those obtained from the two-dimensional analysis, assuming conditions of plane strain. The agreement between the vertical stresses (σ_z) is relatively good, but not so good for the horizontal stresses (σ_y) in the plane of the section, though the form of these stress contours is similar. The two-dimensional finite element analysis yielded directly only the stresses in the plane of the two-dimensional section. Using the plane strain conditions, however, the direct stress normal to this section could be calculated from the results of the finite element analysis. As can be seen from Fig. 6, these stresses (σ_z) showed no relation to those calculated three-dimensionally, the latter being much greater in magnitude. This showed that, for the present situation, the two-dimensional plane strain assumption is definitely incorrect.

The solution of this problem emphasises the danger of two-dimensional simplification of a three-dimensional situation. When making a two-dimensional simplification, the validity of the plane strain assumption should first be checked, particularly if the stresses acting normal to the section are to be taken into account since the calculation of these stresses is dependent on the plane strain conditions. This is perhaps of particular importance in the context of slope stability analyses where it is common practice to consider two-dimensional sections through the slope. If the use of a two-dimensional analysis is unavoidable, the results should be verified using a second different type of approach.

Again in this investigation, materials were assumed isotropic and linearly elastic, which may not be a true representation of the actual properties. Therefore stress distributions calculated both two- and three-dimensionally may be in error when compared with the in situ stresses. As a preliminary investigation, however, the analyses do provide some indication of possible stresses to be expected. It is believed that they served a useful purpose in drawing attention to the necessity for care when dealing with two-dimensional simplifications.

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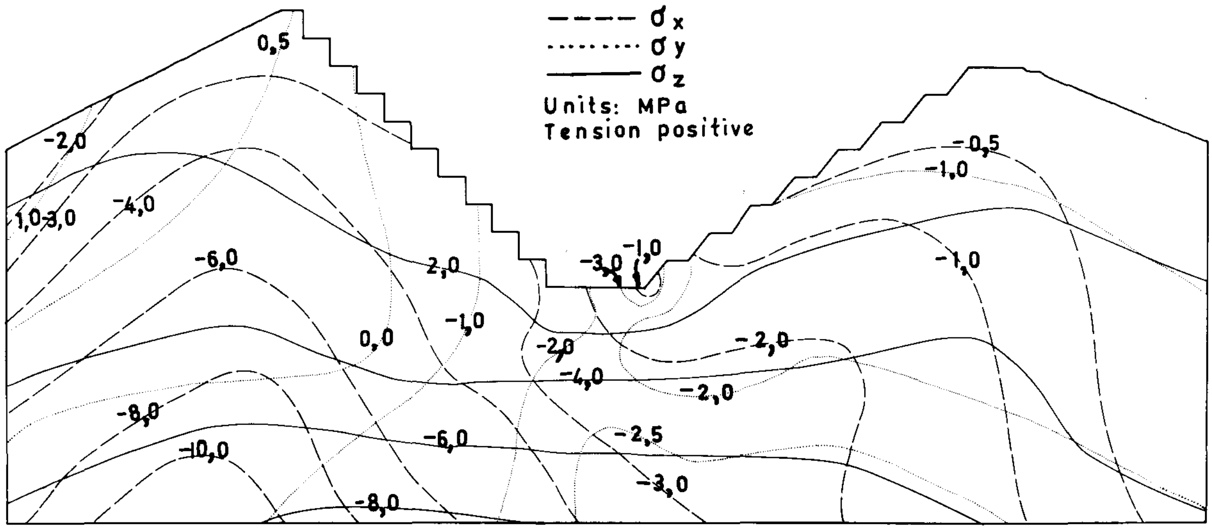
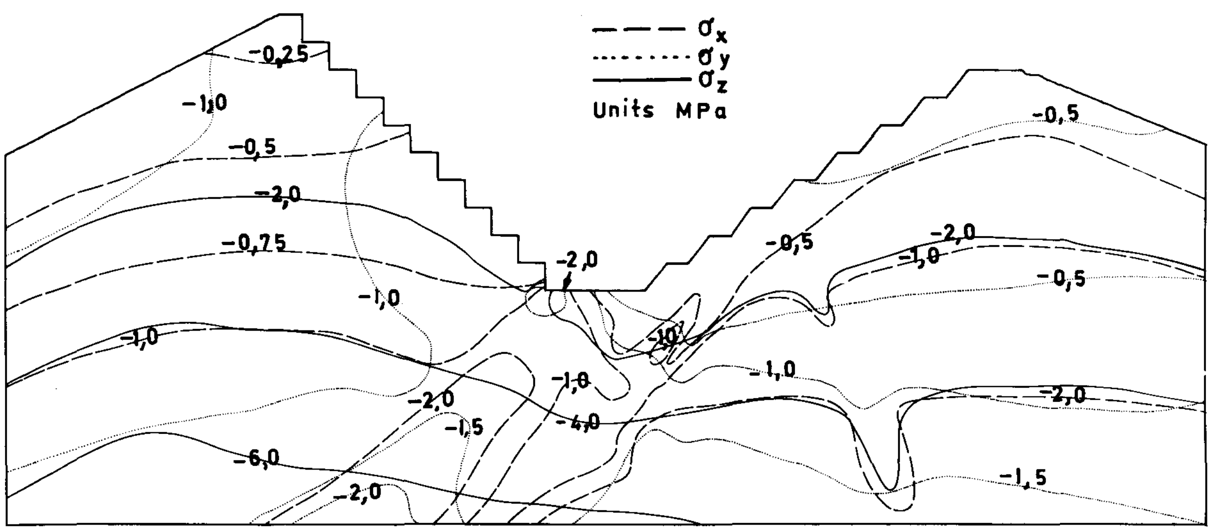


(a) Three-dimensional analysis



(b) Two-dimensional analysis

Fig. 6—STRESS CONTOURS FOR TWO- AND THREE-DIMENSIONAL ANALYSES

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