Computer Applications in Rock Mechanics**

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SYNOPSIS

For some years the Collieries Research Laboratory of the Chamber of Mines of South Africa has been engaged in the development and application of computer programmes to aid in the solution of rock mechanics problems. It is the purpose of this paper to introduce some of the programmes having general applicability.

A system capable of predicting stresses and displacements induced by large-scale tabular excavations is discussed.

A finite element programme especially designed for handling layered continuum problems, and which is being used extensively has been supplemented by an automatic mesh generation facility that reduces problem preparation times considerably.

Some typical applications of these programmes are described.

An entirely new approach, potentially capable of performing linear three-dimensional continuum analysis at practical cost is introduced.

SINOPSIS

Oor die jare was die Steenkoolmyn Navorsings Laboratorium van die Kamer van Mynwese betrokke in die ontwikkeling en aanwending van rekenaar programme as 'n hulp vir die oplossing van probleme betreffende rots meganika. Dit is die doel van hierdie verhandeling om sommige programme voor te lê wat in die algemeen aangewend kan word.

'n Sisteem wat geskik is om drukspannings en verplasings veroorsaak deur grootskaalse tabellariese uitgrawings te voorspel, word bespreek.

'n 'finite element' program spesiaal ontwerp vir die hantering van probleme wat betrekking het op gelaagde solide formasies en wat op groot skaal gebruik word, word aangevul deur 'n outomatiese maas ontwikkelings fasiliteite waardeur die voorbereidings tyd van die probleem aansienlik verminder word.

Sommige tipiese aanwendinge van hierdie programme word beskrywe.

'n Heeltemal nuwe benadering wat potensiëel geskik is om lineare drie-dimensionale analises van solide formasies teen praktiese koste uit te oefen, word voorgestel.

INTRODUCTION

For some years the Collieries Research Laboratory of the Chamber of Mines of South Africa has been engaged in the development and application of computer programmes to aid in the solution of rock mechanics problems throughout the South African mining industry. It is the purpose of this paper to introduce some of these programmes. While some aspects of the underlying theory and system characteristics are described, specific references are quoted for a more detailed and rigorous discussion.

All programmes have been based on linear elastic theory. There is a considerable body of experimental data (e.g. Ortlepp *et al*, 1964) justifying this background in the South African environment. However, it has long been felt that these results could have been anticipated and are valid, certainly in hard rock mining, on the basis of the following argument:

Excluding the fractured zones in the vicinity of excavations, one is generally dealing with a continuous medium. Mining operations normally subject this continuum to progressive displacement changes that are infinitesimal in relation to the size of the configuration, almost in the mathematical sense of the term. Hence, thinking in terms of Taylor expansions, such systems must be describable by means of linear stress-strain relationships and thus linear elastic theory. Here, the emphasis is on *linear*, since the reversibility in behaviour implied by *elasticity* is not often relevant in regard to changes induced by mining.

A system of programmes for the analysis of stresses and displacements around tabular excavations is discussed first. This is particularly suited to the South African gold mining industry. A two-dimensional finite element system designed especially for stratified continuum applications is described next. In view of its wide usage, this has been supplemented by an automatic mesh generation facility. Some typical applications of these programmes are discussed, one in some detail.

Thirdly, an entirely new approach, potentially able to handle general three-dimensional linear continuum analysis, is introduced. An experimental programme employing this method and capable of analyzing hard rock open pit configurations, is discussed. In conclusion, some thoughts on future developments are presented.

A SYSTEM FOR THE ANALYSIS OF THE ELASTIC RESPONSE OF STRATA SURROUNDING TABULAR EXCAVATIONS

The system of programmes to be described in this section has been operational for more than three years. It is known as the *Mining Simulator*, MINSIM for short, to rock mechanics engineers in the South African gold mining industry. It has found considerable application in one-off investigations of the type discussed below and is used routinely in periodically analyzing the effects of advancing faces.

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Background

The development of MINSIM was stimulated by a sequence of results going back to Salamon (1964). Recognizing tabular excavations as an important subset of underground mining configurations, it was realised that the former offer considerable simplifications in analysis when compared with the general three-dimensional problem in linear elasticity. In principle, the solution of the tabular mining problem requires the specification of boundary conditions on two planes, i.e. the surface and the seam being mined. Solutions must vanish at infinity, of course. Further simplifications are possible in the South African gold mining environment.

Here, the areas of interest are usually deep enough for surface effects to be negligible. The problem can now be described in terms of two adjoining elastic half-spaces describing the 'roof' and 'floor' strata of the excavations. These spaces are joined rigidly across the unmined areas. The relative normal and tangential displacements of roof with respect to floor strata in the mined regions may be employed as boundary conditions to form a problem having a unique solution. These displacements are referred to, in mining terminology, as closure and ride, respectively. The configuration is symmetric. One need consider only one of the half-spaces bounded by the reef plane. Employing the principle of superposition and the elastic solution of a punch of infinitesimal cross-section pressing into the otherwise rigidly-confined bounding plane of a half-space, as kernel, all stresses and displacements at any point in the medium can be expressed in terms of a set of kernel integrals taken over the reef plane, provided the closure and ride distributions are known.

In particular, this is true for the stresses induced by mining on the reef itself. These stresses must be taken as equal and opposite to the *'virgin'* stress field induced by the weight of the overlying strata before mining took place, in order to ensure a resultant solution that is stress-free on the roof and floor of the excavation. Thus equating the known reversed virgin stress field to the relevant kernel integrals, one can formulate the problem in terms of a set of integral equations involving the unknown relative displacements on the reef. Having solved for the closure and ride, these values can then be used to find stresses and displacements anywhere in the medium.

Reformulation in terms of three potentials further simplifies the problem. In fact, if the Poisson's ratio of the medium is small and the reef is inclined only moderately, the three potentials tend to be proportional to each other. These results led to the development of an electrolytic analogue by Salamon *et al* (1964). This was later superseded by a more reliable resistance analogue developed by Cook *et al* (1965). Output from both analogues are essentially the closure and ride distributions mentioned earlier. To get off-reef results, they are transferred to a digital machine for computation of the relevant kernel integrals.

On the analogues the reef plane has customarily been represented by 60×60 discrete squares that are considered either fully mined or unmined, necessitating the transfer of 3 600 potentials for every solution. This is a cumbersome procedure, particularly if a number of mining changes are to be investigated. It was largely this inconvenience that stimulated the development of MINSIM which produces a completely digital solution of the problem. Such further features as scaling and the facility of storing a solution on disc for immediate retrieval have now rendered the analogue obsolete. Some of the characteristics of MINSIM including the system lay-out and method of solution, are discussed in the following sub-sections. Further details have been given by Plewman *et al* (1969).

General characteristics

The system was viewed initially very much as a digital 'model' of an analogue. In order to simulate as much as possible the graphic capabilities of the analogue, MINSIM is designed to operate in *'conversational' mode*. The user has full control over the machine during his investigations. To facilitate interpretation of results, graphic displays of the mining outline and extent of *total* closure, that is, contact between roof and floor strata within an excavation, have been included.

To facilitate starting up a new configuration or introducing major mining changes, a mine plan may be prepared 'off-line' on cards and then requested from the terminal. This is referred to as *'mixed mode'* operation.

Alternatively, if no on-line communication with the machine is desired, the whole problem including operation and output controls may be entered from cards and run in *'batch mode'*.

A storage feature has been incorporated to enable the user to store any configuration for future retrieval.

The scaling feature allows one to model detailed small-scale mining activities while still taking into account the effects of the complete environment.

At present, the system can cope with moderately-inclined deep tabular excavations. The approach can be extended directly, however, to the treatment of steeply-inclined reefs at shallow depths.

System organisation

The general organisation of the system is shown in Fig. 1.

The *executive phase* forms the primary interface with the user and hands control to the various functional phases on request. After completion of any one of the latter, control is returned to the executive. While the user can choose any functional sequence, the executive will prevent any invalid requests. For example, when starting up the system, all except a *Create* or *Retrieve* request are invalid.

The so-called *tank generator* is called up following a *Create* request. It builds the skeleton data structures that will contain the mine information, in machine memory. These data areas in a way constitute the digital equivalent of the electrolytic tank analogue and are referred to as the *tank*.

The *configuration generator* enters the initial mine configuration, which it reads from cards or accepts from the terminal, into the tank.

The *configuration update phase* enables the user to change the existing mining layout.

The *solution phase* forms the core of the system. It

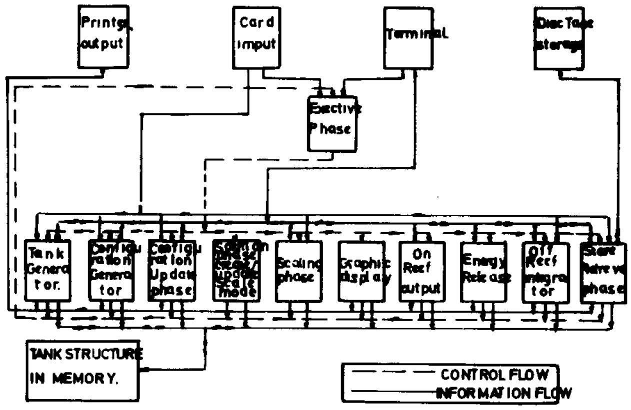

Fig. 1. Organisation of *MINSIM*

produces the closure distributions in the mined areas. This phase may run in *Create*, *Update* or *Scale mode* depending upon which of these functions was performed last. The executive phase ensures the correct mode setting on entry. The method of solution will be discussed in some detail later.

The *scaling phase* allows the user to pick a square window on the reef plane within the present tank and scale it up to full tank size. In doing so he loses direct access to the environment. However, the effect of this environment on the closure distribution inside the scaled-up window is retained. Successive scaling operations are possible.

Normally, a scale-up would first be followed by an update request to refine the mining outline and then by a solution run to adjust the closure distribution. Functional details of the scaling phase are given below.

The *graphic display phase* gives a map of the present mining configuration including an indication of the zone of total closure. A typical example of its output is shown in Fig. 4.

The *on-reef output phase* enables the user to request from the terminal, closure and stress values at selected points on the reef plane. Alternatively, he can have the complete reef plane output on the printer.

The *off-reef integrator* performs the kernel integrations referred to earlier. It computes stresses and displacements at any point in the medium.

Finally, the *store/retrieve phase* handles the permanent storage of a tank on magnetic disc. A configuration may be recalled for further processing by furnishing the name under which it was stored to the system.

Method of solution

When attempting a digital solution to the problem, the natural starting point seemed to be the formulation in terms of the single potential referred to earlier. This potential arises as the solution of Laplace's equation in an infinite half-space, the bounding plane of which is geometrically equivalent to the reef plane. The boundary conditions take the following form:

(a) The potential in the unmined area is zero.

(b) In the mined areas the normal derivative of the potential — the field — on the reef plane is proportional to the normal virgin stresses caused by the weight of the overlying strata.

(c) In mined areas where roof and floor are in contact, i.e. total closure has occurred, the potential is constant and proportional to the initial width of extraction. This condition involves a complication in that the extent of the zones of total closure is not known *a priori*.

Although the problem is basically three-dimensional, the solution is determined largely by conditions on the bounding plane. By utilizing the principle of superposition and the classic Green's function for a half-space, a two-dimensional formulation in terms of an integral equation is obtained. This reduction in dimensionality becomes absolutely essential if one wants to achieve realistic running times when solving practical mining problems. Finite element methods that by definition have to describe the medium as a whole, cannot be used in this application.

Initial experiments employing a 60×60 quantisation of the reef-plane produced discouragingly inaccurate results.

There exists a second basic solution for a half-space, sometimes referred to as the Neumann function, which allows one to specify the normal field on an infinitesimal region of the bounding plane with no effect on the field anywhere else on that plane. This solution also vanishes at infinity. Since the basic formulation is dependent only on the principle of superposition, a mixed integral formulation is feasible. Employing the Neumann and Green functions to represent mined and intact areas, respectively, seemed to constitute the optimum combination in view of the required boundary conditions. The accuracy of this scheme was found to be very satisfactory and matched that of the 'exact' analogues.

Furthermore, it suggested a natural iterative method. Suppose one satisfies the boundary conditions of type (b) above by associating a Neumann function of appropriate amplitude with every elemental area in the mined region. The superposition of these functions results in zero normal fields, but finite potentials, on the reef-plane in the intact and totally closed areas. Hence, in order to satisfy the boundary conditions of types (a) and (c), a compensating set of Green functions must be imposed on the intact and totally-closed areas so as to adjust their potentials back to zero and the width of excavation, respectively. While these Green functions have no effect on the potentials in the mined areas, they do affect the initially imposed fields. Hence it becomes necessary to adjust the amplitudes of the initial Neumann functions to get back to the correct field boundary conditions. This measure in turn necessitates further corrections in the intact and closed areas, and so on. By introducing a static compensation factor into this iterative scheme, very good convergence properties have been achieved. Typically, MINSIM runs on a 64×64-element reef-plane. The corresponding 4 100 equations normally converge to one per cent in approximately ten iterations.

A little reflection on the systems matrix will indicate that it contains two major non-sparse submatrices, embodying the *mined on intact* and *intact on mined* effects. To reduce running times, extensive use has been made of the decay properties of the two kernel functions.

In computing the mutual effects of remote elements, these are first lumped into groups of increasing size as the distance between them increases. By fully exploiting this idea and symmetries in the kernels, running times have been reduced to about five per cent of what would be expected on the basis of a straight matrix multiplication, with practically no loss in accuracy.

A typical lay-out is solved in under two minutes on a 360/65.

Method of scaling

The concept underlying the scaling phase is very simple. Suppose a solution has been found for some configuration and it is intended to implement relatively small mining changes in its central portion. Imagine this central portion enclosed by a window corresponding to half the tank size, say. If the intended changes in the window are small and remote from its boundaries their effect on the area surrounding the window is insignificant. Hence the Neumann and Green functions representing the solution in the environment will remain unchanged and in turn their effect on the interior of the window will remain constant.

In other words, this effect may be accounted for by means of a change in the boundary conditions inside the window. Once the modified boundary conditions have been evaluated for every elemental area, the environment may be dropped and the window scaled up to full tank size. Since each original element is expanded into four of the new tank, error build-up has been minimized by smoothing the modified boundary conditions. The solution must subsequently be 'tightened up' to achieve full accuracy on the finer scale.

A FINITE ELEMENT SYSTEM FOR THE ANALYSIS OF LAYERED MEDIA

In this section another tool, already well known to civil engineers, the method of finite elements, is described. A computer system particularly suited to rock mechanics applications has been implemented.

Whereas MINSIM deals only with flat, narrow excavations, in principle the method of finite elements can solve problems containing any shape of excavation. However, because of the limited speed of presently available computers, the solution of practical mining configurations by this method is restricted to cases that can be represented in terms of two-dimensional idealisations. If certain symmetry conditions such as circular symmetry or periodicity in the third dimension are met, a two-dimensional treatment is still feasible.

The finite element method is based on the following concept. Imagine the medium cut into a large number of elements. The elements are then pin-pointed at their corners, referred to as nodes, and their displacements are constrained so as to preserve continuity along adjacent edges. It can be shown that the response of the structure formed in this manner will be identical to that of the original medium, if the element sizes tend to zero. In practice, a good approximation to the true solution will result if the elements are sufficiently small for stress gradients over them to be negligible.

The computer system referred to as FES below, implemented at the Collieries Research Laboratory, has the following significant features:

(i) The programme is designed to handle two-dimensional configurations.
(ii) Triangular elements are employed allowing easy modelling of complex excavation shapes.
(iii) Configurations containing more than 1 500 nodes can be handled efficiently.
(iv) Programmed machine checks and diagnostics ensure that errors in the input data are detected before a solution is attempted.
(v) The system is particularly suited for handling layered media.
(vi) It is completely open-ended as regards boundary conditions and material properties that can be handled. At present the usual boundary conditions and elastic material properties have been programmed.

Further details are given by Deist *et al* (1968).

SOME TYPICAL APPLICATIONS

A description of a few typical applications that have been handled in the past is given below.

Extraction of a steeply-inclined orebody

The extraction process of a steeply-inclined tabular orebody had to be planned. There was an outcrop on surface. The properties of the ore and surrounding strata were known from core samples. The interaction between a number of proposed mining methods and layouts and the rock mass was unknown. A few computer runs based on various configurations helped to make the decision.

A caving problem in longwall mining

A strong dolerite sill was present in the upper roof strata of a shallow colliery. The feasibility of longwall mining was being investigated. Caving was initially observed to be limited to the strata below the sill. A finite element analysis was run to determine the stresses and displacements resulting from the extraction of a whole panel. It was found that the maximum tensile stress at the base of the sill was exceeding the uniaxial tensile strength of the dolerite. The sill did eventually collapse.

The FES was subsequently used to predict accurately the time of collapse of further panels.

Haulage support at great depths

In a deep gold mine a projected system of twin inclined shafts was being studied. Previous experience in similar situations using a certain type of concrete lining had given unsatisfactory results. A more sophisticated type of support capable of resisting greater tensile stresses was looked for. The important information required in this connection was the change in the stress distribution around the haulage produced by overstoping. This problem was solved by the combined use of MINSIM and FES.

Use of MINSIM and FES in combination

In those problems where the excavation under study is exposed to the stress field induced by another much larger excavation, such as results from the mining of a seam or reef, the effect of the smaller excavation on the larger can be neglected. A MINSIM run is first made in order to determine the stress and displacement values at the boundaries of an area enclosing the configuration under study. Subsequent FES runs employ the values supplied by MINSIM as boundary specifications for the determination of the stress distribution inside this region which is now modelled in full detail.

It must be kept in mind that the three-dimensional stress and displacement values produced by MINSIM are in general incompatible with a two-dimensional stress distribution as required by the FES. The procedure is strictly applicable only in those situations having little structural variation in one direction. In cases where the departure from a plane strain situation is considerable the results must be interpreted with caution.

Detailed discussion of design considerations for an underground water barrier

A water barrier is essentially a pillar of ore left intact by the mining process and so designed that the highly compressed rock becomes waterproof. The criterion for effectiveness is that the minimum principal stress in the pillar must exceed the hydrostatic head of water behind it.

Fig. 2 represents an idealized section through two adjacent mines. The reason for requiring a barrier was that, should a sudden uncontrollable influx of water occur in the upper levels of the one mine, there would be no danger of flooding the workings in the lower mine. In locating the barrier, advantage could be taken of the presence of a strong 15 m-wide dyke which cut the reef producing a throw of 15 m as shown in Fig. 3. The strata in the vicinity of the reef are composed of five different types of rock. They are indicated in the illustration.

The investigation was intended to establish a pillar width satisfying the following criteria:

(i) The pillar width-height ratio must be acceptably large to ensure pillar stability.
(ii) The zone of failed rock above and below the abutments must not bridge the pillar.
(iii) The minimum principal stress in the central portion of the pillar must be compressive and exceed the hydrostatic head behind the pillar.

It was clear that this type of problem, in which the behaviour of the failed rock mass plays an important part, could not be solved by presently available methods. If anything could be done at all, it was to use MINSIM and/or FES for the approximate determination of the zone of failed rock, i.e. a zone indicated by application of a failure initiation criterion. In other words, the available programmes could be employed to shed some light on criterion (ii) above. They are of little value in the application of criterion (iii) which requires knowledge of the redistributed stress field taking into account the effect of the failed rock mass.

Because of the heterogeneous aspects of the strata and the perturbing effect of the dyke, the reduced problem could not be solved by MINSIM alone. On the other hand, FES, being restricted to two-dimensional situations, could not deal with the complex pattern of mining in the reef plane. The following procedure was adopted to deal with the situation: The general stress levels induced on the barrier area by the effect of mining were supplied by MINSIM. This rough picture was then refined using FES modelling the detailed rock structure on a large scale.

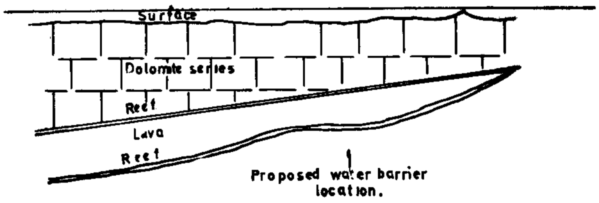

Fig. 2—Idealized vertical section through two mines indicating proposed position of water-barrier

The MINSIM runs were based on an idealized configuration. The rock mass was assumed to be homogeneous and isotropic; no dyke was present. The barrier was simulated on strike and its width varied from run to

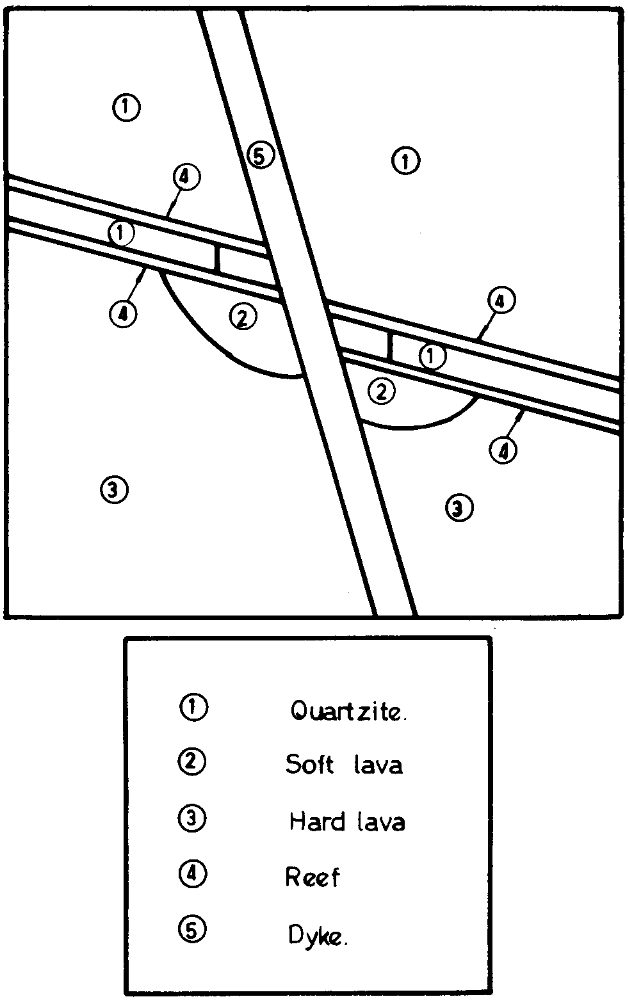

Fig. 3—Detailed vertical cross-section through central portion of proposed water-barrier

run. Fig. 4 shows a MINSIM 'window' of the configuration. A finite element network was drawn up for a vertical cross-section through the barrier. Actually that cross-section with the greatest vertical stress component was selected. The MINSIM output supplied the displacement values at the rim of the finite element network. The components normal to the plane of the network were discarded.

Fig. 5 is a section through the barrier depicting the indicated extent of rock failure for two different pillar widths.

It is seen that the larger of the two pillars satisfies criterion (ii).

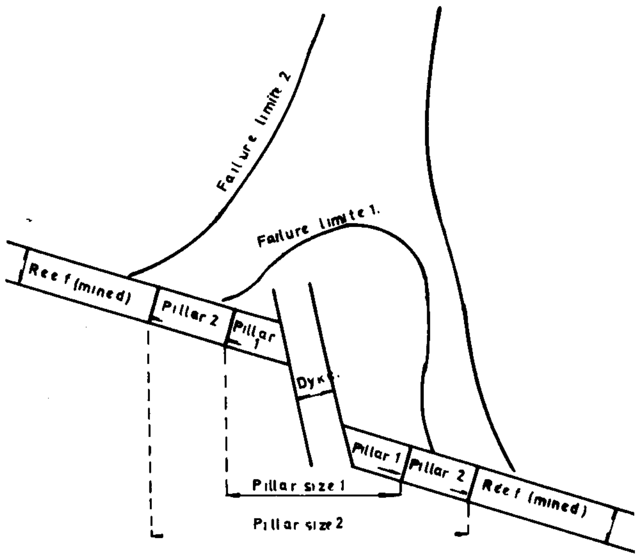

Fig. 5—Indicated extent of failure zone for two different pillar widths

A MESH GENERATION PROGRAMME TO FACILITATE FINITE ELEMENT ANALYSIS

The routine application of powerful solution techniques is frequently hampered by input requirements too subject to human error. Considerable effort and machine time are wasted before results can be accepted with any confidence.

Finite element applications in rock mechanics have been subject to this problem. The preparation and error correction of a typical 1 000 to 1 500-node problem takes approximately two man-weeks on the earlier described system in spite of its relatively convenient form of input and extensive error detection facilities.

In view of its expected wide usage, it was decided to implement a mesh generation facility, relieving the user as much as possible of this task. The mesh, after all, has nothing to do with the basic problem. It is purely an artifice required by this particular method of machine solution.

On examining a considerable number of typical manually-produced meshes, it became clear that the general tendency is to divide the problem area into regions containing fairly regular triangles. Adjacent regions are usually made compatible by introducing additional connections into the boundary layer of the one containing the larger elements.

The same procedure has been implemented by Deist *et al* (1971). The user has to break his problem area into regions which are described in terms of their corner points. The desired triangle size is stated for each such region. Except for possible disturbances in the boundary layers, the programme will produce a regular array of elements in every region.

The *'mesh generator'* operates as a front stage to the finite element system proper. Problem preparation times have been reduced by a factor of at least twenty. Machine costs are normally lower, because most sources of human error have been eliminated.

Other attempts at producing mesh generators seem so far to have considerably more restricted objectives, e.g. Frederick *et al* (1970).

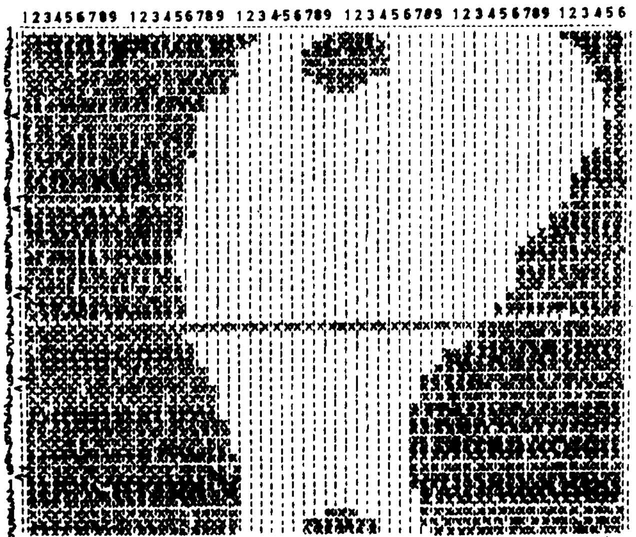

Fig. 4—MINSIM graphic printout of idealized mine lay-out

A NEW APPROACH TO THE SOLUTION OF THREE-DIMENSIONAL ELASTIC PROBLEMS

A practicable applicable method for the elastic analysis of general three-dimensional configurations is not available at the present time. As a consequence, particularly in the field of rock mechanics, a great number of pressing problems remain unsolved.

In principle the finite element method is extendable to three dimensions and it has been applied to solve certain problems. However, in continuum applications of the rock mechanics type, running times would take on astronomic proportions — if a solution could be obtained at all. Also the problem preparation effort, even if aided by a three-dimensional mesh generator, would become enormous.

The inherent shortcoming of finite difference and element methods is that they require a representation of the whole medium. Since the problem is, strictly speaking, defined completely in terms of conditions at the boundaries, these approaches must be unnecessarily cumbersome when applied to linear analysis.

The development of MINSIM points to a new and promising approach. Its extension to general three-dimensional situations would have the significant advantage of a quasi-two-dimensional treatment. Clearly, its fundamental basis, the principle of superposition, carries through without change. The practical difficulty lies in making an appropriate choice of basic elastic solution distributions to be associated with the various surface elements, in order to obtain an integral formulation suitable for efficient machine treatment.

In the experimental work carried out so far, an attempt has been made to minimize the mutual influence between adjacent surface elements. Since these, except in the vicinity of corners, will tend to lie in the same plane, it was natural to evolve a set of half-space solutions allowing independent specification of all types of boundary conditions occurring in practice. Typically, one such basic solution enables one to impose unit normal stress on an elemental surface. It induces no shear stresses on the same element and leaves all other elements in its plane free of normal and shear stresses.

In the general problem curved boundaries and mixed boundary conditions will result in a partially dense system matrix, tending to offset the potential advantage of the quasi-two-dimensional formulation. However, all kernel functions decay rapidly with distance, enabling one to lump elements together for the computation of remote influences. This mechanism amounts to an application of St. Venant's principle. In practice, the scheme results in drastic running time reductions as experienced with MINSIM. Also matrix preparation times and the time per iteration will tend to be proportional to the number of surface-elements rather than to their square.

Another point worth noting is that all basic solutions employed exhibit singularities on the infinitesimal surface elements to which they are applied i.e. one is dealing with singular integral equations. Fortunately, where required, suitable limiting values can be derived by approaching the bounding plane from inside the medium. The experimental programme referred to below works directly with these singular kernel functions.

The singularities can be avoided by choosing suitable kernel amplitude distributions for each element and then solving for the parameters defining these amplitude distributions. This approach would, of course, necessitate prior evaluation of 'averaged' kernel functions and increased machine time in the matrix formation stage. On the other hand, it is possible that bigger surface elements are acceptable for the same accuracy. Here, the two possible extremes, point kernels on the one hand and averaged kernels completely free of singularities on the other, have been mentioned. In practice, a large number of intermediate alternatives can be thought of. One may, for instance, employ the simpler point kernels in the solution and their more sophisticated averaged versions when evaluating stresses and displacements at selected points in the medium. The surface amplitudes found in the solution will first have to be smoothed for this purpose.

A number of alternatives will have to be investigated before settling on an approach best suited for a production system.

Certain of the basic solutions contain line singularities on the outward normal to the surface elements on which they are centred. Again a choice has to be made when handling multiply-connected regions. On the one hand one can employ only kernels with point singularities and tolerate strong influences between adjacent elements. A denser system matrix and a larger number of iterations will be the consequence. Alternatively, it is possible to cut the medium suitably and rejoin by a double layer of elements along the cut. A larger number of equations will result. However, the matrix will tend to be more sparse. As before, the correct choice can be made only on the basis of experimentation.

Incidentally, stratified media can be handled in essentially the same way. The boundaries between adjacent homogeneous blocks will be represented by double layers of surface elements each associated with an appropriate basic solution.

Many rock mechanics applications include boundaries at infinity. Solutions must vanish far away from excavations. Since the basic kernel functions all have this property, unlike finite element methods, the approach does not require explicit modelling of boundaries at infinity.

An experimental programme intended for the treatment of open pit configurations has been implemented. Fig. 6 shows one of the test cases that have been run. The pit has been elongated purposely in order to approximate to plane strain conditions in the central vertical cross-section. The configuration was represented by 360 surface elements. A comparative two-dimensional finite element analysis modelling this cross-section and employing approximately the same number of elements shows excellent agreement in the stresses. The displacements which in the two-dimensional problem have to be locked artificially at an arbitrary point, should not be compared. As with MINSIM, test problems have so far required a very small number of iterations, usually under ten. Again, this is very encouraging compared with the convergence rates obtainable on large stiffness matrices of the finite element type.

Finding non-trivial known solutions for test purposes is one of the difficulties in this work. While the developments described above were in progress, it was found, not unexpectedly, that other workers in the field have appreciated the usefulness of an integral formulation as regards machine solution of these problems (Cruse, 1968). However, their work is based on well known integral results and as such linked to particular kernel functions. The very general approach taken in this paper really contains these formulations as special cases and as such, it is thought, for the first time offers sufficient manoeuvrability for the development of a practical system to perform general three-dimensional linear analysis.

Finally, it is clear that a successful production system will have to contain sophisticated surface element generation facilities, allowing the user to specify his problems in the simplest possible terms.

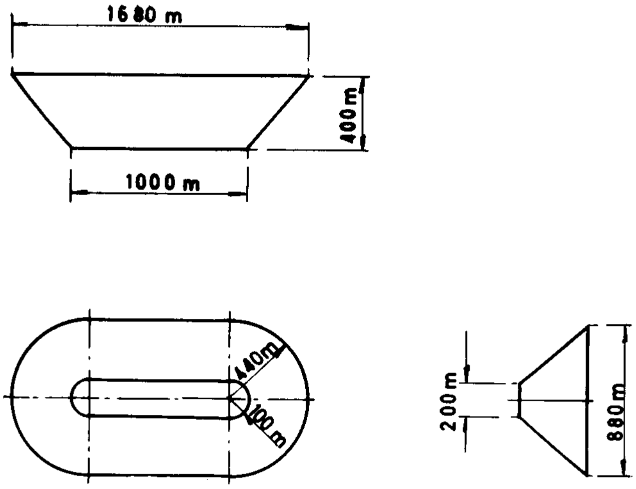

Fig. 6—Three-dimensional test configuration of the open pit type

Most ideas presented in this section are being pursued actively at present.

CONCLUSIONS AND SOME THOUGHTS FOR THE FUTURE

In summary, it has been the objective of this paper to give an account of the computer programmes of general applicability that have been developed by the authors to aid in the solution of rock mechanics problems.

The programmes are all based on linear elastic theory as motivated in the introduction. On conclusion of the development of a practical viable system for the analysis of three-dimensional configurations along the lines described in the previous section, the time has come to try to incorporate the inevitably nonlinear effects of failing material in the vicinity of excavations into these models. Some earlier attempts (e.g. Deist, 1965, 1966) t expressing the behaviour of failing material in a form suitable for the machine analysis of practical mining configurations, are presently being followed up. Such developments will have to take place in conjunction with extensive field experimentation, if they are to be of value.

A considerable amount of work lies ahead before any really useful results can be expected. However, it is felt that the 'mere crushing of a piece of rock in the laboratory' stage has certainly been passed.

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