The accuracy of estimation from samples of ore in bulk

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ERRATA:

Equation (1) should read "$\pi = \ldots$"

Equation (3) should read "$\sigma^2 = \sigma_q^2 - 2\pi\rho\sigma_q\sigma_w + \ldots$"

Delete ∧ on $\hat{\pi}$ in third line above "Discussion".

Insert ∧ over π and delete over o in ninth line following "Discussion".

Replace q^j by q_j in tenth line following "Duscussion"

Delete ∧ on first $\hat{\pi}$ in seventh line above "Derivation"

In the line above (8) delete the repetition of "$=(\hat{\pi}-\pi) W/\sigma$".

Insert ∧ over first π in second line above "References"

DISCUSSION

M. DAVID

Professor Venter has proposed a formula to compute the variance of the estimated grade of a lot of broken ore, and interesting comments on the relevance of the formula and suggestions for developments are made by P. R. Janisch.

It might thus be interesting to bring to the attention of readers the work of the French consulting engineer Pierre Gy[1-16], who has specialized in the sampling of broken ore over the past 20 years. Formula (3) (in fact $\sigma^2 = \sigma_q^2 - 2\pi\rho\sigma_q\sigma_w + \pi^2\sigma_w^2$; there must be a printing error in the text) was given with a different notation as early as 1953 and published in the "Revue de l'Industrie Minérale" V. 36, pp. 311-345. The demonstration at that time was very tedious, but a better one is given in the Special Volume of the same journal in January 1967, on page 48. This last demonstration however is probably not as elegant as the one by Professor Venter.

Gy's formula using this notation is:

$$\frac{\sigma^2(a_g)}{\overline{a_g}^2} = \left(\frac{n}{\rho} - 1\right) \sum_{i=1}^{n} \left(\frac{m_i}{M}\,\frac{a_i - a}{a}\right)^2$$

where n is the total number of particles in the lot and

p is the number of selected particles

M is t_w

m_i is W_{Ji} $\qquad Ji=1, \ldots, n$

a_i is $\dfrac{a_{Ji}}{w_{Ji}}$ $\qquad Ji=1, \ldots, n$

a is π

and a_g is the average grade of all the possible lots of size n. In normal circumstances $a_g \neq a$, thus one can rewrite Gy's formula using Professor Venter's notation.

$$\tau^2 = \pi^2 \left(\frac{n}{p} - 1\right) \sum_{i=1}^{n} \left[\frac{w_{Ji}}{t_w}\,\frac{\frac{q_{Ji}}{w_{Ji}} - \pi}{\pi}\right]^2$$

or $$\tau^2 = \left(\frac{n}{p} - 1\right) \sum_{i=1}^{n} \left[\frac{q_{Ji}}{t_w} - \frac{\pi w_{Ji}}{t_w}\right]^2$$

but $$\sum_{Ji=1}^{n} q_{Ji} = \pi \sum_{Ji=1}^{n} w_{Ji}$$

hence $$\tau^2 = \left(\frac{n}{p} - 1\right) \frac{n}{t_w^2} \left(\sigma_q^2 - 2\pi\sigma_{qw} + \pi^2\sigma_w^2\right)$$

$$= \left(\frac{n}{p} - 1\right) \frac{n}{t_w^2}\sigma^2$$

and finally $\left(\frac{n}{p} - 1\right) \frac{n}{t_w^2}$ can be rewritten:

$$\left(\frac{t_w}{W} - 1\right) \left(\frac{n}{t_w.n.\mu_w}\right)$$

or $$\left(\frac{1}{W\mu_w} - \frac{1}{t_w\mu_w}\right) = \frac{1}{\mu_w}\left(\frac{1}{W} - \frac{1}{t_w}\right) \quad . \quad . \quad (A)$$

At this stage note that we have not introduced any condition on W and tw. Also note that P. R. Janisch wonders why the variance is not a function of the original lot mass tw. The answer lies in equation (A). Usually, $tw >> W$ so that $1/tw$ is negligible if compared to $1/W$.

Finally with this assumption, Gy's equation translated into Professor Venter's notation is exactly his equation (3)

$$\tau^2 = \frac{1}{W\mu_w}\left[\sigma_q^2 - 2\pi\rho\sigma_q\sigma_w + \pi^2\sigma_w^2\right]$$

Now the interesting part of the work of Gy is that this formula was only a starting point for him. As P. R. Janisch commented one has to know the granulo-densimetric analysis of the ore to apply the formula. Gy has written a complete book on the subject[15] but he also produced a slide rule which answers any sampling problem[12] making simple approximations.

He finally writes his equation (using rather the relative variance):

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$$\frac{\sigma^2(\pi)}{\pi^2}=\frac{Cd^3}{M}$$

where π is the estimated grade

$\sigma^2(\pi)$ is the variance of π

M is the mass of the sample

d is the diameter of largest ore fragments (Screen opening in centimetres, which retain 5 per cent of the ore)

C is what he calls the sampling constant of the ore. It is a complex function of the granulo densimetric distribution, shape of the particles, and mineralogical composition.

He devotes a complete chapter of his book to the exact computation of C but derives at the same time the useful simplified formula which appears on the back of his sampling slide rule. He even did a study on the special simplification arising in the case of gold[2, 3] due to the big difference between the specific gravity of the waste and gold itself.

Extended summaries and presentations of his work have already appeared in the Anglo-Saxon literature as well as comments of users about his calculator[17, 18].

Gy's theory is in fact much more developed and the error which we have discussed here, is only what he calls the fundamental error, which assumes a perfect random sampling of a perfectly homogeneous material. He discusses extensively the other causes of errors in his two main books[15, 16].

REFERENCES

1. Gy, P. 'Erreur commise dans le prélèvement d'un échantillon sur un lot de minerai'. Congrès des Laveries des Mines Métalliques Francaises. Paris (Sept. 1953), et Revue de l'Industrie Minérale 36 (1954), pp. 311-345.
2. Gy, P. 'Erforderliche Probemenge — Kurventafeln (Poids à donner à un échantillon — abaques)'. Internationaler Kongress fur Erzaufbereitung — Goslar (8-11 Mai 1955) et Erzmetall 8 (1955), p. B 199-220.
3. Gy, P. 'Poids à donner à un échantillon — abaques'. Revue de l'Industrie Minérale 38 (1956).
4. Gy, P. 'Nomogramme d'échantillonnage; Sampling Nomogram; Probenahme Nomogramm'. Edité par Minerais et Métaux (1956). Paris.
5. Gy, P. 'Eschantillonnage pour analyse granulométrique'. An. Min. (Avril 1956).
6. Gy, P. 'The error committed in size distributions (L'erreur commise dans la distribution granulométrique)'. Congrès de Jamshedpur (Inde) (Fév. 1957). Ind. Min. Jl. (1957).
7. Gy, P. 'A new theory of ore sampling (Une nouvelle théorie de l'échantillonnage)'. Congrès AIME. New Orléans (Louisiana) (Fév. 1957).
8. Gy, P. 'L'échantillonnage'. Tech. de l'Ing. Vol. Mesure et Analyse (1958), pp. 220-1 à 220-14.
9. Gy, P. 'Facteurs ayant une influence dans l'échantillonnage des minerais'. Ann. Min. (Avril 1961), pp. 207-217. Et. et Real. (Sept.-Oct. 1961). pp. 44-47.
10. Gy, P. 'L'échantillonnage des minerais en cours de transport continu'. Revue de l'Industire Minérale 47 (1965), pp. 51-82.
11. Gy, P. 'Sampling of ores and metallurgical products during continuous transport (Echantillonnage des minerais et produits métallurgiques en cours de transport continu)'. Tr. IMM 74 (1965), pp. 165-199.
12. Gy, P. 'Calculateur d'échantillonnage'. Règle à calculs distribuée par la Société de l'Industrie Minérale. Sainte-Etienne (France) (1965), éditée en francais et en anglais.
13. Gy, P. 'Le principe d'équiprobabilité'. An. Min. (Déc. 1964), pp. 779-794.
14. Gy, P. 'La notion d'hétérogénéité'. Bull. B.R.G.M. (1965), 1, pp. 91-99.
15. Gy, P. 'L'échantillonnage des Minerais en Vrac'. Tôme 1 — Théorie Générale. Revue de l'Industrie Minérale. Numéro spécial — (15 janvier 1967).
16. Gy, P. 'L'échantillonnage des Minerais en Vrac'. Tôme 2 — Erreurs opératoires. Revue de l'Industrie Minérale. Numéro spécial — (15 septembre 1971).
17. Cohen-Alloro (1967), 'Note d'utilisation du calculateur d'échantillonnage de Pierre Gy', B.R.G.M., Paris, Document 10.67. Al., 10 p.
18. Ottley, D. J. (1966), 'Pierre Gy's sampling slide rule', Canadian Min. Journal, V. 87, No. 7, pp. 58-62, 65.

AUTHOR'S REPLY

Some minor confusion in the demonstration of David should be pointed out:

"m_i is w_i for $i=1, \ldots, n$" should replace

"m_i is wj_i $\quad J_i=1, \ldots n$".

"a_i is q_i/w_i for $i=1, \ldots n$" should replace

"a_i is qj_i/wj_i for $J_i=1, \ldots n$".

Further a_g is said to be the average grade of all the possible lots of size n; as I see it we are only dealing with one fixed lot of size n (m in my notation). Furthermore, it is stated that $a_g \neq a$ in usual circumstances, but in the next line a_g is equated to π which is a.

Probably a_g stands for the average grade of all possible samples of the prescribed mass W — this would usually not equal π, but for relatively large samples the difference should be negligible and the substitution $a_g=\pi$ therefore valid. A smaller point is that p should probably stand for the average number of particles selected as this number varies from sample to sample; then n/p would be approximately t_w/W.

I would suggest that these points require some attention.

At this time I have no further comments or discussion worth publishing. I am grateful towards David for bringing Gy's work to our attention; I am only sorry that this has not occurred at an earlier stage.