An extension of lognormal theory and its application to risk analysis models for new mining ventures

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SYNOPSIS

The lognormal distribution, which remains the most popular distribution model for ore values, is investigated. The *T*-distribution, an integral part of this theory, is dependent on the sample size, *n*, and the unknown population parameter, σ^2, which cannot be integrated out. Hence, σ^2 constitutes a nuisance parameter. In this study, the robustness of the *T*-distribution to changes in σ^2 is examined. It is found that the *T*-distribution is robust for $0,1 \leqslant \sigma^2 \leqslant 2,5$, a domain covering most practical applications of the lognormal theory. By the use of $\sigma^2 = 0,7$, multiplier factors, which facilitate the computation of the confidence limits, are derived for a comprehensive set of confidence levels.

SAMEVATTING

Die lognormale verdeling wat steeds die mees populêre verdeling bly in die bepaling van ertswaardes, word ondersoek. Die *T*-verdeling vorm 'n integrale deel van hierdie teorie en is afhanklik van sowel die steekproefgrootte, *n*, as die onbekende populasie-parameter, σ^2. Laasgenoemde parameter kan nie deur integrasie verwyder word nie, en is derhalwe 'n oorlasparameter. In hierdie studie word die gevoeligheid van die *T*-verdeling vir veranderinge in σ^2 ondersoek. Daar word gevind dat die *T*-verdeling nie gevoelig is oor die gebied $0,1 \leqslant \sigma^2 \leqslant 2,5$. Die oorgrootte meerderheid van die praktiese toepassings van die lognormale teorie val in hierdie gebied. 'n Omvattende stel sekerheidsperke word verkry deur gebruik te maak van die vermenigvuldigers $\sigma^2 = 0,7$. Die berekening van sodanige sekerheidsperke word daardeur vergemaklik.

INTRODUCTION

A number of mining houses have in recent years developed sophisticated simulation models that have been used extensively in the analysis of the risks inherent in new mining ventures. A primary variable included in such a model is the estimated ore value in a demarcated area of virgin land. On the basis of the results obtained from a limited number of boreholes, and estimates of the rate of inflation, the gold price, etc., management must assess the viability of starting up new mines.

Sichel[1] showed that the frequencies of gold values at the Rand Leases Gold Mine approximately followed the two-parameter lognormal distribution law, and many subsequent studies conducted in different parts of the world confirmed that the lognormal model adequately represents observed values in orebodies. In 1966, Sichel[2] developed tables to assist in the derivation of the maximum likelihood estimator for the mean of a lognormal population and constructed multiplier tables for the calculation of confidence limits.

This paper is concerned with the method of ore evaluation. The first part involves a robustness study of Sichel's theory and proceeds to extend his multiplier tables to include a number of levels of confidence.

LOGNORMAL THEORY

As stated, Sichel showed that the frequencies of gold values followed the two-parameter lognormal distribution law, which is given by

$$\lambda\ (z)=\frac{1}{\sqrt{2\pi}}\ \frac{1}{\sigma\ z}\ e^{-\frac{1}{2}\left(\frac{\ln\ z-\epsilon}{\sigma}\right)^2}. \quad (1)$$

Later, Krige[3] showed that certain observed departures from the simple two-parameter law could be overcome by the addition of a third 'location' parameter, a, so that the logarithmic transformation was achieved by

$$x=\ln\ (z+a), \quad . \ . \ . \ . \ . \quad (2)$$

where z = observed ore value
and a = a constant to be added to the observed value.

Thus, the basic problem facing the mathematical geologist lies in estimating the unknown true mean value of a lognormal population, θ, from n observed ore values.

Finney[4] derived a maximum likelihood estimator of θ, but it was only in 1949 that Sichel[5], working independently, arrived at a more tractable form of this unbiased maximum likelihood and minimum variance estimator. This t-estimator is defined as

$$t=e^{\bar{x}}\gamma_n(V), \quad (3)$$

where

$$\bar{x}=\frac{1}{n}\sum_{i=1}^{n} x_i\ , \quad . \ . \ . \ . \ . \ . \quad (4)$$

$$V=\frac{1}{n}\sum_{i=1}^{n}(x_i\ -\ \bar{x})^2\ , \quad . \ . \ . \quad (5)$$

$$\gamma_n(V)=1+\sum_{r=1}^{\infty}\frac{(n-1)^r\ V^r}{2^r\ r!\ (n-1)\ (n+1)\ .\ .\ .\ (n+2r-3)} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (6)$$

and

$$x_i=\ln\ (z_i+a). \quad . \ . \ . \ . \ . \ . \quad (7)$$

Sichel[2] found an excellent approximation to the exact sampling distribution of that t-estimator, $\phi(t)$, which was easy to handle.

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$$\phi(t) \doteq \lambda(t) = \frac{1}{\sqrt{2\pi}\,\sigma_t\, t}\, e^{-\frac{1}{2}\left(\frac{\ln t - \epsilon_t}{\sigma_t}\right)^2}, \quad \ldots\ldots\ldots (8)$$

where

$$\sigma_t^2 = \frac{\sigma^2}{n} + \ln \gamma_n \left(\frac{n-1}{n^2}\sigma^4\right). \ldots (9)$$

and

$$\epsilon_t = \epsilon + \frac{1}{2}(\sigma^2 - \sigma_t^2). \ldots\ldots (10)$$

The mean and variance of this lognormal approximation $\lambda(t)$ are identical to the mean and variance of the exact sampling distribution. Parameter σ_t^2 in (9) can be estimated from

$$\hat{\sigma}_t^2(V) = \frac{V}{n-1} + \ln \gamma_n \left(\frac{V^2}{n-1}\right). \quad (11)$$

The lower confidence limit for the population mean θ is thus

$$\theta_L = t e^{\frac{1}{2}\hat{\sigma}_t^2(V) - T_p \hat{\sigma}_t(V)}, \quad \ldots\ldots\ldots (12)$$

where Tp is a deviate cutting off a proportion p in the tail of the t-distribution and is defined as

$$T = \frac{\ln t - \ln \theta}{\hat{\sigma}_t(V)} + \frac{1}{2}\hat{\sigma}_t(V). \quad \ldots (13)$$

From the point of view of the mining engineer, who must make definite statements about the mineral deposits, this limit is of utmost importance. It indicates the viability of mining that area.

Similarly, the upper confidence limit can be obtained by substitution of $1-p$ for p in (12).

The exact distribution of the T-deviate was given by Sichel[2] as

$$A(T) = C \int_0^\infty \hat{\sigma}_t(V)\, V^{\frac{n-3}{2}}\, e^{-\frac{n}{2\sigma^2}\left\{\left[T\hat{\sigma}_t(V) + \frac{\sigma^2}{2} - \ln\gamma_n(V) - \frac{1}{2}\hat{\sigma}_t^2(V)\right]^2 + V\right\}} dV, \quad \ldots\ldots\ldots (14)$$

for $-\infty \leqslant T \leqslant +\infty$

and $$C = \frac{1}{\sqrt{\pi}\,\Gamma\left(\frac{n-1}{2}\right)} \left(\frac{n}{2\sigma^2}\right)^{\frac{n}{2}}. \quad \ldots\ldots\ldots (15)$$

COMPUTER APPLICATION

Equation (12) can be rewritten as

$$\theta_{L_p} = t\psi_p(V, n), \quad \ldots\ldots (16)$$

where the multiplier

$$\psi_p(V, n) = e^{\frac{1}{2}\hat{\sigma}_t^2(V) - T_p \hat{\sigma}_t(V)} \quad \ldots\ldots\ldots (17)$$

For the derivation of the multiplier factors, ψ_p, used in the calculation of the above-mentioned confidence limits, the distribution $A(T)$ in equation (14) must be integrated out. The slowly convergent power series $\gamma_n(V)$ and $\hat{\sigma}_t^2(V)$ in the integrand influence the selection of the mesh size, dV, as well as the point V, at which the numerical integration is terminated. The empirical rules, illustrated in Table I, were derived by Sichel from preliminary hand calculations and later incorporated into Marting's[6] computer program.

These rules proved to be satisfactory only for 0,3 $<\sigma^2<$1,5, and, when the value of σ^2 was greater than 1,5, certain difficulties arose. For these values of σ^2, the $\int_{-\infty}^{\infty} A(T)dT$ was calculated by the old program as <1, which contradicts the definition of $A(T)$ as a probability distribution.

It was found that this problem arose through the use of the *termination* rules listed in Table I. In these cases, the calculation for the probability densities, $A(T)$, was terminated prematurely, resulting in the integral of the T-distribution being less than unity. To overcome this difficulty, a new formula was derived to replace the termination rules in Table I. This formula was again derived from empirical study.

For each value of T at a specific n and σ^2, the corresponding value of $A(T)$ was determined by Simpson's Rule. Sichel and Marting[6] had previously both selected this rule to do the numerical integration.

Since

$$A(T) = \int_0^\infty \omega(V, T, n, \sigma^2)\, dV,$$

the area from V_0 to V_2 can be defined as

$$I_1 = \int_{V_0}^{V_0+2V} \omega(V, T, n, \sigma^2)\, dV,$$

where V is the mesh size.

Then the integral of $A(T)$ from the origin up to and including V_2 can be defined as

$$I_2 = \int_0^{V_0+2V} \omega(V, T, n, \sigma^2)\, dV.$$

Only when the ratio

$$R = \frac{I_1}{I_2} < 10^{-9},$$

is the integration of $A(T)$ for that T-deviate terminated.

This limit was established from a study of Normal tables. *Biometrika Tables for Statisticians* (Pearson, E. S. and Hartley, H. O.) lists the ordinate values and the corresponding cumulative frequency for the Normal distribution. By use of the mesh size rules in Table I and the stepwise application of Simpson's rule to the Biometrika tables, the cut-off ratio was found. The ratio of 9^{-8} gave a satisfactory result. In other words, when the ratio of the incremental area as calculated by Simpson's rule to the total cumulative area was $<9^{-8}$, the calculation was terminated. This was because further integration over the curve of the Normal distribution added little to the cumulative area, which already closely approximated unity. However, because of the skewness of the T-distribution and consequently its longer tails, it became necessary to diminish this ratio to the previously mentioned 10^{-9}.

The efficacy of this rule is well illustrated in the following example, which is extreme if not totally unrealistic. For a sample of size $n=30$ and a population variance $\sigma^2=6$, it was found, when the values of $A(T)$ were summed, that

$$\int_{-15{,}00}^{7{,}00} A(T)dT = 0{,}99999808.$$

Although this rule is empirical, the results obtained are entirely satisfactory and allow for experimentation with values of n and σ^2 far in excess of any previously dealt with.

It is interesting to look at this program in its historical context. Marting's original program was the first problem in mathematical statistics to be programmed in FORTRAN in the Republic. It was initially run on an IBM 1620 computer. On account of the slow convergence of the members of the two series and the fine mesh sizes used, the calculation of $A(T)$ for each T required approximately 30 minutes of computer time. This cost prohibited experimentation with different values of n and σ^2. Later Marting developed a FORTRAN IV version of the program, which was run on the IBM 360 computer at the University of the Witwatersrand. The greater speed of this computer reduced the average running time per $A(T)$ value to one minute. However, with the new program, which includes a more refined termination rule, the expense of running the program has again increased. On the IBM 370 Model 145 computer at the University of the Witwatersrand, 61 minutes of CPU time were required for the calculation of the example quoted above. This run alone cost R534,45. When one considers that approximately 25 000 transfers are involved in the derivation of a single value of $A(T)$, one can understand this enormous cost.

TABLE I

RULES DERIVED BY SICHEL

A: Termination Rules

T-interval	Endpoint of V calculations
$-13,00 \leqslant T \leqslant +1,00$	$= 12,823\, e^{-1,239\sqrt{\mid T - 1,5 \mid}}$
$+\ 1,00 < T \leqslant +1,90$	$= 5,339$
$+\ 1,90 < T \leqslant +6,00$	$= 16,444\, e^{-1,802\sqrt{T - 1,5}}$

B: Mesh-size Rules

T-interval	Mesh size for V
$-13,00 \leqslant T \leqslant -\ 0,25$	$= 0,03928\, e^{-1,125\sqrt{\mid T \mid}}$
$-\ 0,25 < T \leqslant +1,15$	$= 0,02280$
$+\ 1,15 < T \leqslant +6,00$	$= 0,26004\, e^{-2,277\sqrt{T}}$

PROBABILITY INTEGRALS

To derive the multiplier factors mentioned earlier, it is necessary to

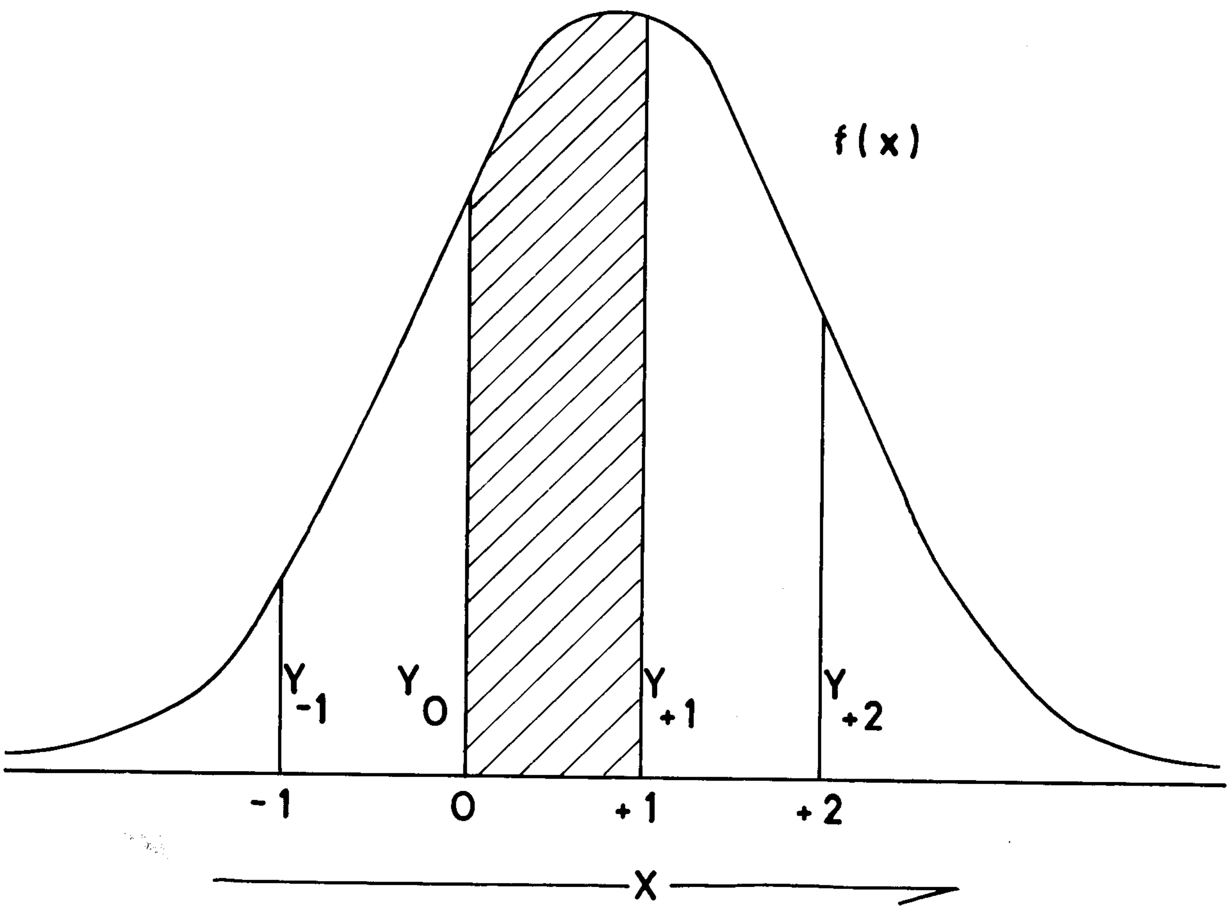

Fig. 1—Graphical representation of quadrature method (A)

TABLE II

$$p = B(T_p,\ n=10, \sigma^2) = \int_{T_p}^{\infty} A(T)dT$$

Lower bound T_p	σ^2							
	0,3	0,4	0,7	1,0	1,5	2,0	2,5	3,0
−10,5	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9999
−10,0	1,0000	1,0000	1,0000	1,0000	1,0000	0,9999	0,9999	0,9998
− 9,5	1,0000	1,0000	1,0000	0,9999	0,9999	0,9998	0,9998	0,9997
− 9,0	1,0000	1,0000	0,9999	0,9999	0,9998	0,9997	0,9997	0,9996
− 8,5	1,0000	0,9999	0,9999	0,9998	0,9997	0,9996	0,9995	0,9994
− 8,0	0,9999	0,9999	0,9998	0,9998	0,9996	0,9995	0,9993	0,9991
− 7,5	0,9999	0,9999	0,9997	0,9996	0,9994	0,9992	0,9990	0,9988
− 7,0	0,9998	0,9998	0,9996	0,9994	0,9991	0,9988	0,9985	0,9982
− 6,5	0,9997	0,9996	0,9994	0,9991	0,9987	0,9983	0,9978	0,9973
− 6,0	0,9995	0,9994	0,9990	0,9986	0,9980	0,9974	0,9968	0,9962
− 5,5	0,9991	0,9989	0,9984	0,9978	0,9969	0,9960	0,9952	0,9944
− 5,0	0,9984	0,9981	0,9973	0,9965	0,9952	0,9939	0,9928	0,9916
− 4,5	0,9972	0,9968	0,9954	0,9942	0,9923	0,9906	0,9891	0,9876
− 4,0	0,9950	0,9942	0,9923	0,9904	0,9877	0,9854	0,9833	0,9814
− 3,5	0,9908	0,9896	0,9866	0,9840	0,9802	0,9771	0,9743	0,9718
− 3,0	0,9829	0,9812	0,9767	0,9730	0,9679	0,9638	0,9602	0,9571
− 2,5	0,9680	0,9655	0,9592	0,9543	0,9476	0,9424	0,9381	0,9344
− 2,0	0,9403	0,9369	0,9288	0,9226	0,9145	0,9083	0,9034	0,8993
− 1,5	0,8903	0,8862	0,8770	0,8700	0,8613	0,8547	0,8497	0,8456
− 1,0	0,8052	0,8014	0,7927	0,7862	0,7782	0,7726	0,7684	0,7654
− 0,5	0,6735	0,6711	0,6654	0,6611	0,6561	0,6531	0,6512	0,6503
0,0	0,4977	0,4968	0,4947	0,4933	0,4924	0,4931	0,4948	0,4973
0,5	0,3073	0,3063	0,3043	0,3037	0,3049	0,3082	0,3128	0.3184
1,0	0,1515	0,1487	0,1428	0,1396	0,1383	0,1405	0,1450	0,1511
1,5	0,0594	0,0559	0,0484	0,0435	0,0394	0,0386	0,0401	0,0433
2,0	0,0198	0,0175	0,0125	0,0095	0,0066	0,0053	0,0050	0,0055
2,5	0,0062	0,0051	0,0030	0,0018	0,0009	0,0005	0,0003	0,0002
3,0	0,0020	0,0015	0,0007	0,0004	0,0001	0,0000	0,0000	0,0000
3,5	0,0007	0,0005	0,0002	0,0001	0,0000	0,0000	0,0000	0,0000
4,0	0,0002	0,0002	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000
4,5	0,0001	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

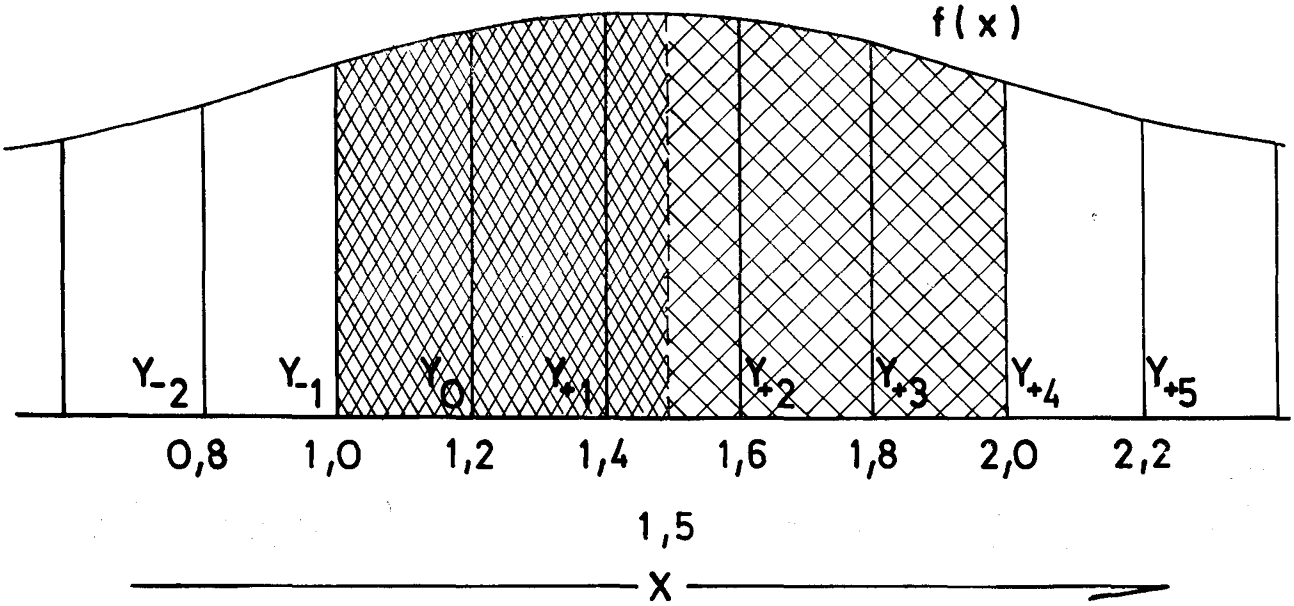

Fig. 2—Graphical representation of quadrature methods (B) and (C)

know the percentage point on the T-scale corresponding to a specific $\int A(T)dT$.

Table II, which is an intermediary table used in the robustness study to be discussed later, contains the cumulative probability distributions for $n=10$ and different values of σ^2. In the calculation of these integrals, quadrature techniques were applied to the ordinate values obtained from the computer program.

For the tails of the T-distribution, $T < -6{,}0$ and $T > 3{,}0$, the formula applied (Figure 1) is the following: (A):

$$\int_0^1 f(x)dx=\frac{1}{24}\,(-y_{-1} + 13y_0 + 13y_{+1} - y_{+2}).$$

However, since the mesh size in this case is not one unit, (A) must be multiplied by h, the mesh size. As the ordinate values had been derived in intervals of $0{,}5, h=0{,}5$, which is neither too coarse nor too refined, and hence is not laborious to use.

In the middle of the distribution, where $-6{,}0 < T < 3{,}0$, the quadrature formulae used are as shown in Figure 2.

For this range, the computer was programmed to calculate ordinate values at intervals of 0,2 units. Not to lose any of this information and still be consistent in recording the probability integrals in steps of 0,5 units, formulae (B) and (C) were used.

(B):

$$\int_{1,0}^{1,5} f(x)dx=\frac{0{,}2}{24}\,(y_{+2} + 23y_{+1} + 25y_0 + 12y_{-1} - y_{-2})$$

(C):

$$\int_{1,5}^{2,0} f(x)dx=\frac{0{,}2}{24}\,(y_{+1} + 23y_{+2} + 25y_{+3} + 12y_{+4} - y_{+5})$$

From Table II one is able to obtain the required T-deviates or percentage points, Tp, by inverse interpolation. The substitution of these values in (12) gives the desired multiplier factors. Confidence limits can then be applied to the maximum likelihood estimate of the population mean of the borehole values that the mining company has obtained.

ROBUSTNESS STUDY

Sichel[2] gave a table of multiplier factors with which the estimates, t, must be multiplied in order to give approximate confidence limits for the lognormal population mean, θ, at nominal confidence levels of $p=0{,}05$, $p=0{,}10$, and $p=0{,}95$.

For these tables, the nuisance parameter $\sigma^2=0{,}7$ was taken. This value had been used in a comparison of the nominal and exact confidence limits achieved when Sichel's tables were used for a sample size of $n=10$. The exercise indicated the robustness of the T-distribution for the domain $0{,}3 \leqslant \sigma^2 \leqslant 1{,}5$, because the exact probability may deviate by $\pm 0{,}01$ from the confidence level aimed at. Such an error is fully acceptable for all practical purposes.

Since the present study was undertaken primarily because the robustness of the T-distribution for $\sigma^2 > 1{,}5$ was called into question, a similar exercise was carried out with a computer program incorporating the new termination rule developed by the writer. Table III presents the results for the extended robustness study for a sample of size $n=10$.

The T-deviates used in this exercise for $n=10$ and $\sigma^2=0{,}7$ are given below:

$T_{0,05} = 1{,}488$
$T_{0,10} = 1{,}182$
$T_{0,95} = -2{,}318$.

It must be noted that the values for $\sigma^2=0{,}0$ were obtained from tables for Student's distribution with 9 degrees of freedom, as Sichel[7] pointed out that, for $\sigma^2 < 0{,}3$, the T-distribution approaches Student's distribution with $n-1$ degrees of freedom.

The results listed in Table III correspond closely to those obtained by Marting and Sichel[6]. It shows that the T-distribution is robust within the domain $0{,}3 \leqslant \sigma^2 \leqslant 2{,}5$. In practice, one seldom encounters populations where $\sigma^2 > 2{,}5$. Thus, one can conclude that the multiplier factors published in the Appendix, and also those published earlier by Sichel[2], are applicable in all practical situations. However, Student's distribution should be used instead of the T-distribution if $\sigma^2 < 0{,}3$.

To complete the study, the robustness of σ^2 for larger sample sizes was tested. Table IV gives the comparable results for a sample of size $n=20$.

The robustness of the T-distribution for the domain $0{,}1 \leqslant \sigma^2 \leqslant 2{,}5$ is even more pronounced in this case. For $\sigma^2 < 0{,}1$, however, one must use Student's distribution with $n-1$ degrees of freedom instead of the T-distribution. In fact, the values for $\sigma^2=0{,}0$ in the table were obtained from Student's tables.

The T-deviates for $n=20$ and $\sigma^2=0{,}7$ are

$T_{0,05} = 1{,}493$
$T_{0,10} = 1{,}185$
$T_{0,95} = -2{,}030$.

TABLE III

EXACT CONFIDENCE LEVELS FOR $n = 10$, $\sigma^2 = 0{,}7$ IN EQUATION (15)

Nominal Confidence level, p	True σ^2 in population							
	$\sigma^2=0{,}0$	$\sigma^2=0{,}3$	$\sigma^2=0{,}4$	$\sigma^2=0{,}7$	$\sigma^2=1{,}0$	$\sigma^2=1{,}5$	$\sigma^2=2{,}0$	$\sigma^2=2{,}5$
0,95	0,9772	0,9598	0,9569	0,9500	0,9446	0,9374	0,9318	0,9272
0,10	0,1336	0,1104	0,1071	0,1000	0,0958	0,0931	0,0942	0,0978
0,05	0,0855	0,0610	0,0576	0,0500	0,0452	0,0410	0,0403	0,0419
0,90 interval	0,8917	0,8988	0,8993	0,9000	0,8994	0,8964	0,8915	0,8853

TABLE IV

EXACT CONFIDENCE LEVELS FOR $n=20$, $\sigma^2=0{,}7$ IN EQUATION (15)

Nominal confidence level, p	True σ^2 in population					
	$\sigma^2=0{,}0$	$\sigma^2=0{,}1$	$\sigma^2=0{,}3$	$\sigma^2=0{,}7$	$\sigma^2=1{,}0$	$\sigma^2=2{,}5$
0,95	0,9717	0,9637	0,9575	0,9500	0,9458	0,9335
0,10	0,1258	0,1156	0,1084	0,1000	0,0960	0,0898
0,05	0,0759	0,0656	0,0581	0,0500	0,0457	0,0383
0,90 interval	0,8958	0,8981	0,8994	0,9000	0,9001	0,8952

MULTIPLIER FACTORS

By use of equation (11) and the standard deviates, T_p, factors $\psi_p(V,n)$ and $\psi_{1-p}(V,n)$ were calculated for

$n = 5(5)20$

$V = 0{,}00\ (0{,}02)\ 0{,}20\ (0{,}1)\ 5{,}0$

$P = 0{,}01\ 0{,}25\ 0{,}05\ 0{,}10(0{,}1)\ 0{,}3$.

These are given in the Appendix. The one-sided upper 99 per cent multiplier factors are not included because their immense size makes the limits quite meaningless in practice.

A careful study of the multiplier factors where $a=0{,}2$ and $n=5$, and $a=0{,}3$ and $n=5$, 10, and 15 reveals the rather interesting phenomenon that the lower-limit multiplier factors are greater than 1. Consequently, the lower limit is larger than the estimator, in spite of the estimator being unbiased. A theoretical investigation will show that there is no cause for concern. It can be proved that, when the sampling distribution of the mean is very skew (that is, when the sample size, n, is small), the underlying population is very skew, and a, the level of confidence, is high, the phenomenon holds in general and applies to any parent distribution.

To obtain the values for Tp, Table II and similar tables for $n=5$, 15, and 20 were used and a simple three-point interpolation method was applied. The function decided upon was of the form $x=a_{11}y^2 + a_{12}y + a_{13}$. Though this function is a rough approximation, the values obtained corresponded closely to those published earlier by Sichel. However, it must be emphasized that this method is not optimal. To illustrate this point, one should look at the interpolation results from the function $y=b_{11}x^2 + b_{12}x + b_{13}$. If, for example, the three chosen points lie as shown in Fig. 3, the parabola provides a good fit and it is a simple matter to obtain y_u for a given x_u. In fact, this is the case for the central region of the curve. However, when the three points are distributed as in Fig. 4, certain complications arise. The fitting of a parabola distorts the value of y_u, causing a large deviation. This is the case in the upper end of the cumulative frequency distribution. To guard against such an occurrence, the function $x=a_{11}y^2 + a_{12}y + a_{13}$ was used, with more favourable results (Fig. 5).

Thus, it is evident that an understanding of the behaviour of the distribution is vital in obtaining the best interpolation results.

Although the theory is complicated, the calculations of the confidence limits are extremely simple if the multiplier factors in the Appendix are used.

The following example will serve as a guide to the computation process. It deals with the borehole intersections of the Carbon Leader Reef at East Driefontein Mine. In this case, the location parameter, a, is taken to be zero and it is assumed that there is geological homogeneity.

TABLE V

COMPUTATION OF T-ESTIMATOR AND CONFIDENCE LIMITS FOR THE CARBON LEADER REEF, EAST DRIEFONTEIN MINE

Borehole	Value (z) (cm−g)	$x=\log_{10}z$
EIC	8 435	3,9261
EIE	1 781	3,2506
EIF	1 243	3,0944
EIG	2 520	3,4014
EIH	3 223	3,5083
EIK	811	2,9090
EIL	1 486	3,1721
E2D	1 543	3,1883
E2F	1 713	3,2337
UD5	114	2,0569
UD14	1 925	3,2844
UD15	469	2,6712

It can easily be shown that, for these values,

$\bar{x}=3{,}1414$

and $V=1{,}0233$.

Hence, we have

$e^{\bar{x}}=\text{antilog}_{10}\ (3{,}1414)$

$=1{,}385$.

By referring to Table A in the Appendix to Sichel's paper[2], for a sample of size $n=12$, we get

$\gamma_{12}\ (1,\ 0)=1{,}620$

$\gamma_{12}\ (1,\ 1)=1{,}697$.

Linear interpolation between these tabulated figures yields

$\gamma_{12}\ (1{,}0233)=1{,}6379$.

Therefore, $t=e^{\bar{x}}\gamma_n\ (V)$

$=2269$.

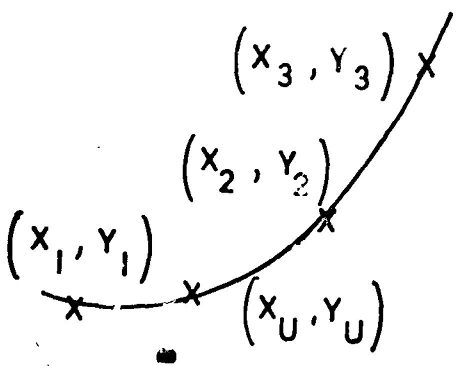

Fig. 3

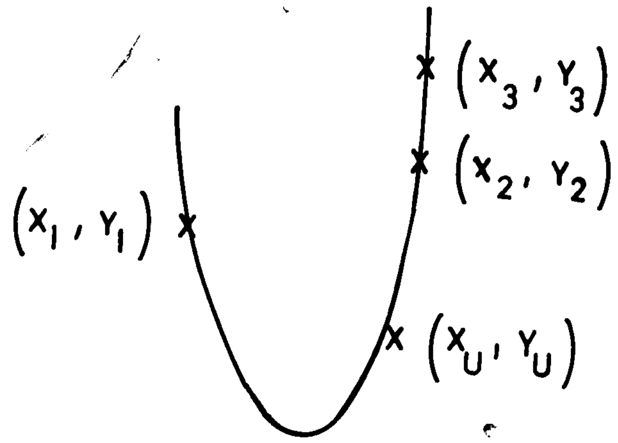

Fig. 4

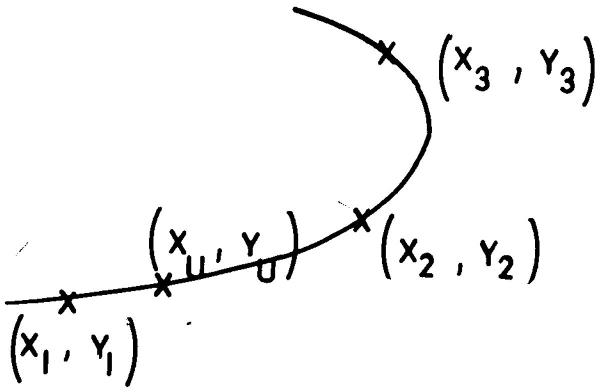

Fig. 5

To obtain the 95 per cent confidence limits, say, we use the Appendix. To derive the multiplier factors for a sample of size $n=12$, one must interpolate linearly, both for n and for V.

We have $\theta_{L,0,95}=1379$
and $\theta_{U,0,95}=5833$.

The probability is thus 90 in 100 that the unknown true mean grade lies between 1379 and 5833 cm – g.

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APPENDIX

FACTOR $\psi_{0,01}$ (V,n) FOR THE ESTIMATION OF THE ONE-SIDED LOWER 99 PER CENT CONFIDENCE LIMIT OF THE MEAN OF A LOGNORMAL POPULATION

V	$n=5$	$n=10$	$n=15$	$n=20$
0,00	1,0000	1,0000	1,0000	1,0000
0,02	0,7157	0,8718	0,9018	0,9171
0,04	0,6219	0,8233	0,8644	0,8826
0,06	0,5575	0,7876	0,8352	0,8574
0,08	0,5084	0,7579	0,8117	0,8361
0,10	0,4685	0,7323	0,7908	0,8180
0,12	0,4345	0,7108	0,7730	0,8020
0,14	0,4054	0,6908	0,7565	0,7872
0,16	0,3796	0,6725	0,7415	0,7737
0,18	0,3569	0,6556	0,7273	0,7608
0,20	0,3362	0,6401	0,7138	0,7484
0,30	0,2579	0,5740	0,6568	0,6968
0,40	0,2040	0,5217	0,6110	0,6550
0,50	0,1646	0,4780	0,5717	0,6179
0,60	0,1347	0,4404	0,5428	0,5855
0,70	0,1114	0,4075	0,5059	0,5559
0,80	0,0928	0,3782	0,4778	0,5290
0,90	0,0779	0,3520	0,4523	0,5043
1,00	0,0657	0,3284	0,4287	0,4811
1,10	0,0557	0,3070	0,4069	0,4597
1,20	0,0475	0,2874	0,3866	0,4396
1,30	0,0407	0,2695	0,3678	0,4208
1,40	0,0349	0,2531	0,3502	0,4031
1,50	0,0301	0,2380	0,3338	0,3865
1,60	0,0260	0,2240	0,3184	0,3708
1,70	0,0226	0,2111	0,3039	0,3559
1,80	0,0196	0,1992	0,2904	0,3418
1,90	0,0171	0,1882	0,2777	0,3285
2,00	0,0150	0,1779	0,2656	0,3158
2,10	0,0132	0,1684	0,2542	0,3037
2,20	0,0116	0,1595	0,2435	0,2923
2,30	0,0102	0,1512	0,2333	0,2814
2,40	0,0090	0,1435	0,2237	0,2710
2,50	0,0080	0,1363	0,2146	0,2611
2,60	0,0071	0,1295	0,2060	0,2507
2,70	0,0063	0,1232	0,1978	0,2427
2,80	0,0056	0,1173	0,1901	0,2341
2,90	0,0050	0,1118	0,1828	0,2259
3,00	0,0045	0,1066	0,1758	0,2181
3,10	0,0041	0,1017	0,1692	0,2106
3,20	0,0037	0,0972	0,1629	0,2035
3,30	0,0033	0,0928	0,1569	0,1966
3,40	0,0030	0,0888	0,1512	0,1901
3,50	0,0027	0,0850	0,1458	0,1839
3,60	0,0024	0,0814	0,1406	0,1779
3,70	0,0022	0,0780	0,1357	0,1721
3,80	0,0020	0,0749	0,1310	0,1666
3,90	0,0018	0,0718	0,1265	0,1614
4,00	0,0017	0,0690	0,1223	0,1563
4,10	0,0015	0,0663	0,1182	0,1515
4,20	0,0014	0,0638	0,1143	0,1468
4,30	0,0013	0,0614	0,1106	0,1424
4,40	0,0012	0,0591	0,1071	0,1381
4,50	0,0011	0,0570	0,1037	0,1340
4,60	0,0010	0,0550	0,1005	0,1301
4,70	0,0009	0,0530	0,0974	0,1263
4,80	0,0008	0,0512	0,0944	0,1227
4,90	0,0008	0,0495	0,0916	0,1192
5,00	0,0007	0,0478	0,0889	0,1158

FACTOR $\psi_{0,025}(V,n)$ FOR THE ESTIMATION OF THE ONE-SIDED LOWER 97,5 PER CENT CONFIDENCE LIMIT OF THE MEAN OF A LOGNORMAL POPULATION

V	$n=5$	$n=10$	$n=15$	$n=20$
0,00	1,0000	1,0000	1,0000	1,0000
0,02	0,8764	0,9204	0,9362	0,9463
0,04	0,8299	0,8894	0,9114	0,9235
0,06	0,7958	0,8661	0,8918	0,9067
0,08	0,7683	0,8465	0,8759	0,8924
0,10	0,7448	0,8295	0,8617	0,8802
0,12	0,7240	0,8150	0,8495	0,8693
0,14	0,7054	0,8014	0,8380	0,8591
0,16	0,6884	0,7888	0,8276	0,8498
0,18	0,6728	0,7771	0,8177	0,8410
0,20	0,6582	0,7663	0,8082	0,8323
0,30	0,5977	0,7192	0,7676	0,7961
0,40	0,5499	0,6806	0,7341	0,7661
0,50	0,5103	0,6474	0,7049	0,7391
0,60	0,4765	0,6181	0,6829	0,7150
0,70	0,4472	0,5917	0,6545	0,6928
0,80	0,4213	0,5677	0,6324	0,6722
0,90	0,3983	0,5457	0,6120	0,6531
1,00	0,3777	0,5253	0,5929	0,6349
1,10	0,3590	0,5065	0,5749	0,6178
1,20	0,3422	0,4889	0,5579	0,6016
1,30	0,3269	0,4724	0,5420	0,5863
1,40	0,3129	0,4571	0,5269	0,5716
1,50	0,3002	0,4426	0,5125	0,5577
1,60	0,2885	0,4291	0,4989	0,5444
1,70	0,2778	0,4163	0,4859	0,5316
1,80	0,2679	0,4042	0,4736	0,5194
1,90	0,2588	0,3928	0,4619	0,5077
2,00	0,2505	0,3821	0,4506	0,4964
2,10	0,2428	0,3719	0,4398	0,4856
2,20	0,2357	0,3622	0,4296	0,4752
2,30	0,2292	0,3531	0,4197	0,4652
2,40	0,2231	0,3445	0,4104	0,4555
2,50	0,2175	0,3363	0,4013	0,4463
2,60	0,2124	0,3285	0,3927	0,4364
2,70	0,2076	0,3211	0,3845	0,4287
2,80	0,2032	0,3141	0,3765	0,4204
2,90	0,1992	0,3075	0,3689	0,4125
3,00	0,1954	0,3012	0,3616	0,4047
3,10	0,1920	0,2952	0,3547	0,3973
3,20	0,1888	0,2895	0,3479	0,3901
3,30	0,1859	0,2841	0,3415	0,3832
3,40	0,1832	0,2789	0,3353	0,3765
3,50	0,1807	0,2741	0,3294	0,3700
3,60	0,1785	0,2694	0,3237	0,3638
3,70	0,1765	0,2650	0,3182	0,3578
3,80	0,1746	0,2609	0,3130	0,3520
3,90	0,1729	0,2569	0,3080	0,3464
4,00	0,1714	0,2531	0,3031	0,3409
4,10	0,1701	0,2496	0,2985	0,3357
4,20	0,1689	0,2462	0,2940	0,3306
4,30	0,1679	0,2430	0,2897	0,3257
4,40	0,1670	0,2400	0,2856	0,3210
4,50	0,1662	0,2371	0,2816	0,3164
4,60	0,1656	0,2344	0,2778	0,3120
4,70	0,1651	0,2318	0,2742	0,3077
4,80	0,1647	0,2294	0,2707	0,3036
4,90	0,1644	0,2271	0,2673	0,2996
5,00	0,1643	0,2250	0,2641	0,2958

FACTOR $\psi_{0,975}$ (V,n) FOR THE ESTIMATION OF THE ONE-SIDED UPPER 97,5 PER CENT CONFIDENCE LIMIT OF THE MEAN OF A LOGNORMAL POPULATION

V	$n=5$	$n=10$	$n=15$	$n=20$
0,00	1,0000	1,0000	1,0000	1,0000
0,02	1,3329	1,1499	1,1063	1,0835
0,04	1,5081	1,2205	1,1541	1,1235
0,06	1,6624	1,2789	1,1947	1,1549
0,08	1,8059	1,3324	1,2300	1,1831
0,10	1,9451	1,3824	1,2632	1,2084
0,12	2,0837	1,4279	1,2932	1,2318
0,14	2,2210	1,4729	1,3226	1,2545
0,16	2,3603	1,5169	1,3506	1,2760
0,18	2,5000	1,5601	1,3781	1,2973
0,20	2,6442	1,6022	1,4057	1,3188
0,30	3,4051	1,8121	1,5367	1,4168
0,40	4,2821	2,0246	1,6634	1,5097
0,50	5,3087	2,2461	1,7914	1,6043
0,60	6,5186	2,4805	1,9005	1,6986
0,70	7,9485	2,7309	2,0618	1,7961
0,80	9,6527	3,0001	2,2056	1,8963
0,90	11,6792	3,2908	2,3554	2,0000
1,00	14,0824	3,6055	2,5144	2,1086
1,10	16,9540	3,9456	2,6823	2,2216
1,20	20,3618	4,3172	2,8613	2,3394
1,30	24,4247	4,7202	3,0498	2,4626
1,40	29,2635	5,1580	3,2499	2,5926
1,50	35,0094	5,6360	3,4627	2,7277
1,60	41,8353	6,1564	3,6890	2,8694
1,70	49,9620	6,7256	3,9301	3,0192
1,80	59,6103	7,3464	4,1856	3,1767
1,90	71,0688	8,0218	4,4580	3,3422
2,00	84,6173	8,7619	4,7502	3,5163
2,10	100,686	9,5708	5,0606	3,7007
2,20	119,721	10,4554	5,3921	3,8937
2,30	142,238	11,4206	5,7464	4,0983
2,40	168,840	12,4771	6,1234	4,3138
2,50	200,309	13,6344	6,5284	4,5399
2,60	237,438	14,8957	6,9582	4,8059
2,70	281,179	16,2783	7,4181	5,0326
2,80	332,769	17,7907	7,9105	5,2996
2,90	393,616	19,4457	8,4358	5,5801
3,00	465,002	21,2577	8,9987	5,8778
3,10	549,148	23,2373	9,5975	6,1924
3,20	647,962	25,4060	10,2461	6,5232
3,30	763,966	27,7830	10,9276	6,8745
3,40	900,114	30,3771	11,6632	7,2444
3,50	1059,47	33,2284	12,4499	7,6355
3,60	1246,37	36,3427	13,2913	8,0493
3,70	1465,26	39,7528	14,1916	8,4853
3,80	1721,26	43,4797	15,1552	8,9486
3,90	2020,17	47,5713	16,1868	9,4372
4,00	2369,87	52,0461	17,2914	9,9545
4,10	2777,47	56,9414	18,4748	10,5024
4,20	3253,41	62,3085	19,7389	11,0806
4,30	3807,75	68,1832	21,0977	11,6932
4,40	4454,55	74,6148	22,5502	12,3400
4,50	5206,51	81,6579	24,1077	13,0257
4,60	6082,26	89,3722	25,7783	13,7526
4,70	7099,61	97,8238	27,5657	14,5234
4,80	8282,13	107,068	29,4785	15,3353
4,90	9656,22	117,180	31,5368	16,1995
5,00	11248,79	128,264	33,7351	17,1135

FACTOR $\psi_{0,05}$ (V,n) FOR THE ESTIMATION OF THE ONE-SIDED LOWER 95 PER CENT CONFIDENCE LIMIT OF THE MEAN OF A LOGNORMAL POPULATION

V	$n=5$	$n=10$	$n=15$	$n=20$
0,00	1,0000	1,0000	1,0000	1,0000
0,02	0,8978	0,9333	0,9458	0,9540
0,04	0,8589	0,9071	0,9246	0,9344
0,06	0,8302	0,8874	0,9079	0,9200
0,08	0,8070	0,8708	0,8943	0,9077
0,10	0,7870	0,8563	0,8821	0,8972
0,12	0,7693	0,8439	0,8716	0,8878
0,14	0,7535	0,8323	0,8617	0,8790
0,16	0,7389	0,8216	0,8527	0,8709
0,18	0,7255	0,8116	0,8442	0,8632
0,20	0,7129	0,8023	0,8360	0,8558
0,30	0,6605	0,7618	0,8008	0,8243
0,40	0,6187	0,7284	0,7717	0,7981
0,50	0,5838	0,6995	0,7462	0,7744
0,60	0,5538	0,6739	0,7270	0,7534
0,70	0,5277	0,6508	0,7020	0,7338
0,80	0,5044	0,6297	0,6825	0,7156
0,90	0,4836	0,6103	0,6646	0,6987
1,00	0,4650	0,5923	0,6476	0,6826
1,10	0,4481	0,5756	0,6317	0,6674
1,20	0,4328	0,5599	0,6165	0,6530
1,30	0,4189	0,5452	0,6023	0,6393
1,40	0,4062	0,5315	0,5888	0,6262
1,50	0,3946	0,5186	0,5760	0,6137
1,60	0,3840	0,5065	0,5637	0,6018
1,70	0,3743	0,4950	0,5521	0,5904
1,80	0,3655	0,4842	0,5410	0,5794
1,90	0,3574	0,4740	0,5305	0,5688
2,00	0,3501	0,4644	0,5203	0,5587
2,10	0,3433	0,4552	0,5106	0,5489
2,20	0,3372	0,4466	0,5014	0,5395
2,30	0,3316	0,4385	0,4925	0,5304
2,40	0,3266	0,4308	0,4840	0,5217
2,50	0,3220	0,4234	0,4759	0,5133
2,60	0,3179	0,4166	0,4681	0,5044
2,70	0,3142	0,4100	0,4606	0,4974
2,80	0,3110	0,4039	0,4535	0,4899
2,90	0,3081	0,3981	0,4467	0,4826
3,00	0,3055	0,3926	0,4401	0,4756
3,10	0,3033	0,3874	0,4338	0,4689
3,20	0,3014	0,3825	0,4278	0,4624
3,30	0,2999	0,3779	0,4220	0,4561
3,40	0,2986	0,3736	0,4165	0,4500
3,50	0,2976	0,3695	0,4112	0,4442
3,60	0,2969	0,3656	0,4062	0,4385
3,70	0,2964	0,3621	0,4013	0,4331
3,80	0,2962	0,3587	0,3967	0,4278
3,90	0,2963	0,3556	0,3923	0,4228
4,00	0,2965	0,3527	0,3880	0,4179
4,10	0,2971	0,3500	0,3840	0,4132
4,20	0,2978	0,3475	0,3801	0,4086
4,30	0,2988	0,3452	0,3765	0,4043
4,40	0,3000	0,3430	0,3729	0,4001
4,50	0,3014	0,3411	0,3696	0,3960
4,60	0,3030	0,3393	0,3664	0,3921
4,70	0,3048	0,3378	0,3634	0,3883
4,80	0,3069	0,3363	0,3605	0,3847
4,90	0,3091	0,3351	0,3578	0,3812
5,00	0,3116	0,3340	0,3552	0,3778

FACTOR $\psi_{0,95}$ (V,n) FOR THE ESTIMATION OF THE ONE-SIDED UPPER 95 PER CENT CONFIDENCE LIMIT OF THE MEAN OF A LOGNORMAL POPULATION

V	$n=5$	$n=10$	$n=15$	$n=20$
0,00	1,0000	1,0000	1,0000	1,0000
0,02	1,2411	1,1170	1,0845	1,0671
0,04	1,3623	1,1712	1,1221	1,0990
0,06	1,4664	1,2156	1,1538	1,1239
0,08	1,5613	1,2559	1,1812	1,1462
0,10	1,6518	1,2934	1,2070	1,1661
0,12	1,7405	1,3272	1,2301	1,1845
0,14	1,8270	1,3606	1,2527	1,2023
0,16	1,9136	1,3930	1,2741	1,2191
0,18	1,9993	1,4247	1,2951	1,2356
0,20	2,0867	1,4554	1,3160	1,2523
0,30	2,5320	1,6065	1,4147	1,3279
0,40	3,0191	1,7564	1,5090	1,3988
0,50	3,5628	1,9099	1,6031	1,4705
0,60	4,1756	2,0696	1,6824	1,5412
0,70	4,8696	2,2374	1,7984	1,6137
0,80	5,6634	2,4149	1,9007	1,6877
0,90	6,5703	2,6037	2,0061	1,7635
1,00	7,6047	2,8050	2,1169	1,8424
1,10	8,7946	3,0194	2,2328	1,9239
1,20	10,1548	3,2501	2,3551	2,0083
1,30	11,7177	3,4968	2,4827	2,0958
1,40	13,5127	3,7610	2,6168	2,1874
1,50	15,5693	4,0454	2,7581	2,2819
1,60	17,9276	4,3507	2,9069	2,3804
1,70	20,6390	4,6799	3,0640	2,4838
1,80	23,7488	5,0341	3,2289	2,5916
1,90	27,3182	5,4141	3,4032	2,7042
2,00	31,3985	5,8248	3,5883	2,8217
2,10	36,0792	6,2676	3,7832	2,9454
2,20	41,4437	6,7454	3,9895	3,0739
2,30	47,5859	7,2596	4,2078	3,2091
2,40	54,6113	7,8149	4,4382	3,3506
2,50	62,6606	8,4151	4,6833	3,4979
2,60	71,8613	9,0606	4,9411	3,6700
2,70	82,3661	9,7588	5,2145	3,8156
2,80	94,3775	10,5124	5,5045	3,9860
2,90	108,115	11,3263	5,8112	4,1638
3,00	123,750	12,2057	6,1368	4,3511
3,10	141,632	13,1539	6,4801	4,5477
3,20	162,014	14,1793	6,8461	4,7530
3,30	185,244	15,2885	7,2324	4,9695
3,40	211,722	16,4835	7,6430	5,1958
3,50	241,829	17,7800	8,0781	5,4335
3,60	276,140	19,1781	8,5394	5,6832
3,70	315,198	20,6894	9,0286	5,9445
3,80	359,607	22,3201	9,5476	6,2202
3,90	410,033	24,0878	10,0983	6,5090
4,00	467,417	25,9967	10,6828	6,8126
4,10	532,494	28,0586	11,3034	7,1320
4,20	606,446	30,2911	11,9606	7,4667
4,30	690,289	32,7042	12,6607	7,8188
4,40	785,542	35,3133	13,4026	8,1881
4,50	893,390	38,1351	14,1912	8,5768
4,60	1015,74	41,1878	15,0297	8,9861
4,70	1154,22	44,4911	15,9190	9,4172
4,80	1311,09	48,0599	16,8623	9,8681
4,90	1488,76	51,9165	17,8687	10,3448
5,00	1689,53	56,0924	18,9342	10,8454

FACTOR $\psi_{0,10}$ (V,n) FOR THE ESTIMATION OF THE ONE-SIDED LOWER 90 PER CENT CONFIDENCE LIMIT OF THE MEAN OF A LOGNORMAL POPULATION

V	$n=5$	$n=10$	$n=15$	$n=20$
0,00	1,0000	1,0000	1,0000	1,0000
0,02	0,9192	0,9464	0,9575	0,9635
0,04	0,8882	0,9254	0,9401	0,9479
0,06	0,8652	0,9095	0,9267	0,9364
0,08	0,8466	0,8961	0,9158	0,9265
0,10	0,8307	0,8843	0,9061	0,9181
0,12	0,8164	0,8743	0,8977	0,9106
0,14	0,8036	0,8649	0,8898	0,9036
0,16	0,7919	0,8562	0,8826	0,8971
0,18	0,7811	0,8480	0,8757	0,8910
0,20	0,7709	0,8405	0,8691	0,8849
0,30	0,7283	0,8075	0,8408	0,8596
0,40	0,6944	0,7802	0,8173	0,8385
0,50	0,6659	0,7566	0,7967	0,8194
0,60	0,6415	0,7357	0,7811	0,8024
0,70	0,6202	0,7167	0,7609	0,7865
0,80	0,6014	0,6994	0,7451	0,7718
0,90	0,5847	0,6835	0,7305	0,7581
1,00	0,5698	0,6687	0,7167	0,7450
1,10	0,5564	0,6551	0,7038	0,7326
1,20	0,5444	0,6423	0,6915	0,7209
1,30	0,5337	0,6304	0,6800	0,7098
1,40	0,5241	0,6193	0,6690	0,6991
1,50	0,5155	0,6089	0,6586	0,6890
1,60	0,5079	0,5991	0,6488	0,6793
1,70	0,5011	0,5899	0,6394	0,6700
1,80	0,4952	0,5813	0,6305	0,6611
1,90	0,4900	0,5733	0,6220	0,6525
2,00	0,4855	0,5658	0,6139	0,6443
2,10	0,4817	0,5587	0,6062	0,6364
2,20	0,4785	0,5521	0,5988	0,6289
2,30	0,4760	0,5459	0,5918	0,6216
2,40	0,4740	0,5402	0,5852	0,6146
2,50	0,4725	0,5348	0,5788	0,6079
2,60	0,4716	0,5298	0,5728	0,6007
2,70	0,4712	0,5252	0,5670	0,5952
2,80	0,4713	0,5210	0,5616	0,5892
2,90	0,4718	0,5170	0,5564	0,5835
3,00	0,4728	0,5134	0,5515	0,5780
3,10	0,4743	0,5102	0,5469	0,5728
3,20	0,4762	0,5072	0,5425	0,5677
3,30	0,4786	0,5045	0,5383	0,5629
3,40	0,4813	0,5021	0,5344	0,5582
3,50	0,4845	0,5000	0,5307	0,5538
3,60	0,4881	0,4982	0,5272	0,5496
3,70	0,4921	0,4966	0,5240	0,5455
3,80	0,4966	0,4953	0,5209	0,5416
3,90	0,5014	0,4942	0,5181	0,5379
4,00	0,5067	0,4934	0,5154	0,5344
4,10	0,5123	0,4929	0,5130	0,5310
4,20	0,5184	0,4926	0,5108	0,5278
4,30	0,5249	0,4925	0,5087	0,5248
4,40	0,5318	0,4926	0,5068	0,5219
4,50	0,5391	0,4930	0,5051	0,5192
4,60	0,5469	0,4936	0,5036	0,5166
4,70	0,5551	0,4945	0,5023	0,5142
4,80	0,5637	0,4955	0,5011	0,5119
4,90	0,5728	0,4968	0,5001	0,5098
5,00	0,5824	0,4983	0,4993	0,5078

FACTOR $\psi_{0,90}$ (V,n) FOR THE ESTIMATION OF THE ONE-SIDED UPPER 90 PER CENT CONFIDENCE LIMIT OF THE MEAN OF A LOGNORMAL POPULATION

V	$n=5$	$n=10$	$n=15$	$n=20$
0,00	1,0000	1,0000	1,0000	1,0000
0,02	1,1624	1,0845	1,0621	1,0499
0,04	1,2409	1,1230	1,0894	1,0734
0,06	1,3070	1,1542	1,1124	1,0917
0,08	1,3662	1,1825	1,1321	1,1081
0,10	1,4218	1,2085	1,1506	1,1226
0,12	1,4755	1,2319	1,1671	1,1360
0,14	1,5273	1,2549	1,1832	1,1489
0,16	1,5785	1,2771	1,1984	1,1610
0,18	1,6287	1,2987	1,2132	1,1730
0,20	1,6793	1,3196	1,2280	1,1850
0,30	1,9294	1,4209	1,2971	1,2392
0,40	2,1910	1,5197	1,3623	1,2895
0,50	2,4712	1,6190	1,4266	1,3399
0,60	2,7750	1,7208	1,4803	1,3893
0,70	3,1065	1,8261	1,5580	1,4395
0,80	3,4724	1,9358	1,6257	1,4903
0,90	3,8761	2,0509	1,6949	1,5420
1,00	4,3212	2,1718	1,7669	1,5954
1,10	4,8163	2,2988	1,8416	1,6501
1,20	5,3639	2,4336	1,9196	1,7064
1,30	5,9732	2,5757	2,0002	1,7643
1,40	6,6509	2,7259	2,0842	1,8246
1,50	7,4033	2,8854	2,1718	1,8863
1,60	8,2396	3,0543	2,2634	1,9501
1,70	9,1718	3,2340	2,3590	2,0167
1,80	10,2090	3,4248	2,4586	2,0856
1,90	11,3639	3,6268	2,5629	2,1571
2,00	12,6453	3,8422	2,6726	2,2312
2,10	14,0724	4,0715	2,7872	2,3086
2,20	15,6607	4,3156	2,9073	2,3885
2,30	17,4272	4,5749	3,0333	2,4719
2,40	19,3905	4,8513	3,1651	2,5587
2,50	21,5768	5,1462	3,3040	2,6483
2,60	24,0062	5,4592	3,4488	2,7523
2,70	26,7035	5,7935	3,6010	2,8397
2,80	29,7036	6,1496	3,7611	2,9413
2,90	33,0419	6,5293	3,9288	3,0465
3,00	36,7392	6,9343	4,1053	3,1566
3,10	40,8558	7,3655	4,2897	3,2713
3,20	45,4243	7,8259	4,4846	3,3903
3,30	50,4954	8,3176	4,6884	3,5150
3,40	56,1259	8,8406	4,9031	3,6443
3,50	62,3641	9,4009	5,1287	3,7792
3,60	69,2927	9,9975	5,3657	3,9200
3,70	76,9813	10,6344	5,6148	4,0662
3,80	85,5048	11,3130	5,8767	4,2194
3,90	94,944	12,0395	6,1521	4,3788
4,00	105,442	12,8142	6,4419	4,5452
4,10	117,016	13,6407	6,7469	4,7190
4,20	129,873	14,5246	7,0670	4,8999
4,30	144,101	15,4682	7,4050	5,0888
4,40	159,881	16,4759	7,7600	5,2856
4,50	177,328	17,5525	8,1340	5,4913
4,60	196,659	18,7030	8,5282	5,7064
4,70	218,033	19,9328	8,9426	5,9313
4,80	241,688	21,2455	9,3784	6,1649
4,90	267,871	22,6468	9,8391	6,4101
5,00	296,789	24,1460	10,3227	6,6659

FACTOR $\psi_{0,20}$ (V,n) FOR THE ESTIMATION OF THE ONE-SIDED LOWER 80 PER CENT CONFIDENCE LIMIT OF THE MEAN OF A LOGNORMAL POPULATION

V	$n=5$	$n=10$	$n=15$	$n=20$
0,00	1,0000	1,0000	1,0000	1,0000
0,02	0,9449	0,9641	0,9709	0,9755
0,04	0,9238	0,9500	0,9596	0,9651
0,06	0,9082	0,9394	0,9506	0,9574
0,08	0,8956	0,9304	0,9432	0,9508
0,10	0,8848	0,9226	0,9367	0,9452
0,12	0,8752	0,9160	0,9310	0,9401
0,14	0,8666	0,9097	0,9257	0,9354
0,16	0,8587	0,9040	0,9209	0,9312
0,18	0,8515	0,8986	0,9163	0,9270
0,20	0,8447	0,8936	0,9119	0,9230
0,30	0,8167	0,8719	0,8930	0,9062
0,40	0,7947	0,8542	0,8774	0,8923
0,50	0,7768	0,8390	0,8639	0,8797
0,60	0,7619	0,8256	0,8537	0,8686
0,70	0,7495	0,8138	0,8406	0,8583
0,80	0,7390	0,8031	0,8304	0,8487
0,90	0,7302	0,7935	0,8211	0,8399
1,00	0,7229	0,7848	0,8125	0,8316
1,10	0,7170	0,7769	0,8044	0,8238
1,20	0,7123	0,7697	0,7969	0,8164
1,30	0,7088	0,7632	0,7899	0,8095
1,40	0,7064	0,7574	0,7835	0,8030
1,50	0,7050	0,7521	0,7774	0,7969
1,60	0,7046	0,7474	0,7718	0,7911
1,70	0,7051	0,7433	0,7665	0,7856
1,80	0,7066	0,7397	0,7617	0,7804
1,90	0,7089	0,7365	0,7572	0,7755
2,00	0,7121	0,7339	0,7531	0,7709
2,10	0,7161	0,7317	0,7493	0,7666
2,20	0,7209	0,7300	0,7458	0,7625
2,30	0,7265	0,7287	0,7427	0,7587
2,40	0,7330	0,7278	0,7398	0,7551
2,50	0,7403	0,7273	0,7373	0,7518
2,60	0,7483	0,7273	0,7351	0,7484
2,70	0,7572	0,7277	0,7331	0,7458
2,80	0,7668	0,7284	0,7314	0,7432
2,90	0,7773	0,7296	0,7301	0,7408
3,00	0,7885	0,7312	0,7289	0,7386
3,10	0,8006	0,7331	0,7281	0,7366
3,20	0,8135	0,7355	0,7275	0,7349
3,30	0,8273	0,7382	0,7272	0,7333
3,40	0,8419	0,7413	0,7271	0,7319
3,50	0,8573	0,7448	0,7273	0,7308
3,60	0,8737	0,7486	0,7278	0,7298
3,70	0,8910	0,7529	0,7285	0,7291
3,80	0,9092	0,7575	0,7295	0,7285
3,90	0,9284	0,7625	0,7307	0,7281
4,00	0,9485	0,7680	0,7321	0,7280
4,10	0,9697	0,7737	0,7339	0,7280
4,20	0,9919	0,7799	0,7358	0,7282
4,30	1,0152	0,7865	0,7380	0,7286
4,40	1,0396	0,7935	0,7405	0,7291
4,50	1,0651	0,8009	0,7432	0,7299
4,60	1,0918	0,8087	0,7462	0,7308
4,70	1,1197	0,8169	0,7494	0,7320
4,80	1,1489	0,8256	0,7529	0,7333
4,90	1,1794	0,8346	0,7566	0,7348
5,00	1,2112	0,8441	0,7606	0,7365

FACTOR $\psi_{0,80}$ (V,n) FOR THE ESTIMATION OF THE ONE-SIDED UPPER 80 PER CENT CONFIDENCE LIMIT OF THE MEAN OF A LOGNORMAL POPULATION

V	$n=5$	$n=10$	$n=15$	$n=20$
0,00	1,0000	1,0000	1,0000	1,0000
0,02	1,0925	1,0512	1,0386	1,0312
0,04	1,1360	1,0743	1,0554	1,0459
0,06	1,1721	1,0930	1,0695	1,0572
0,08	1,2040	1,1097	1,0816	1,0674
0,10	1,2336	1,1251	1,0929	1,0763
0,12	1,2620	1,1389	1,1030	1,0846
0,14	1,2890	1,1524	1,1128	1,0926
0,16	1,3156	1,1654	1,1220	1,1000
0,18	1,3413	1,1780	1,1310	1,1074
0,20	1,3671	1,1901	1,1400	1,1148
0,30	1,4917	1,2485	1,1816	1,1479
0,40	1,6174	1,3046	1,2204	1,1785
0,50	1,7478	1,3602	1,2585	1,2089
0,60	1,8847	1,4165	1,2900	1,2385
0,70	2,0298	1,4741	1,3353	1,2684
0,80	2,1852	1,5334	1,3745	1,2986
0,90	2,3518	1,5947	1,4142	1,3291
1,00	2,5304	1,6585	1,4551	1,3604
1,10	2,7235	1,7246	1,4973	1,3923
1,20	2,9312	1,7939	1,5410	1,4249
1,30	3,1560	1,8662	1,5858	1,4582
1,40	3,3994	1,9416	1,6321	1,4927
1,50	3,6624	2,0208	1,6801	1,5278
1,60	3,9469	2,1036	1,7297	1,5640
1,70	4,2557	2,1906	1,7813	1,6014
1,80	4,5902	2,2819	1,8345	1,6399
1,90	4,9531	2,3775	1,8898	1,6796
2,00	5,3452	2,4781	1,9475	1,7205
2,10	5,7708	2,5839	2,0072	1,7629
2,20	6,2324	2,6952	2,0693	1,8065
2,30	6,7328	2,8120	2,1340	1,8517
2,40	7,2749	2,9350	2,2011	1,8984
2,50	7,8635	3,0646	2,2712	1,9463
2,60	8,5014	3,2005	2,3437	2,0015
2,70	9,1923	3,3439	2,4192	2,0477
2,80	9,9419	3,4948	2,4980	2,1009
2,90	10,7558	3,6537	2,5799	2,1557
3,00	11,6355	3,8211	2,6653	2,2127
3,10	12,5916	3,9972	2,7538	2,2717
3,20	13,6275	4,1829	2,8466	2,3325
3,30	14,7503	4,3788	2,9428	2,3957
3,40	15,9679	4,5847	3,0433	2,4609
3,50	17,2856	4,8024	3,1479	2,5285
3,60	18,7155	5,0315	3,2569	2,5984
3,70	20,2660	5,2731	3,3705	2,6707
3,80	21,9460	5,5274	3,4889	2,7458
3,90	23,7646	5,7963	3,6123	2,8235
4,00	25,7383	6,0796	3,7411	2,9040
4,10	27,8738	6,3781	3,8755	2,9875
4,20	30,1899	6,6935	4,0153	3,0738
4,30	32,6972	7,0261	4,1616	3,1633
4,40	35,4178	7,3770	4,3140	3,2559
4,50	38,3611	7,7474	4,4731	3,3520
4,60	41,5526	8,1384	4,6393	3,4518
4,70	45,0065	8,5513	4,8126	3,5555
4,80	48,7489	8,9868	4,9932	3,6623
4,90	52,8046	9,4461	5,1825	3,7738
5,00	57,1912	9,9316	5,3794	3,8892

FACTOR $\psi_{0,30}(\hat{V},n)$ FOR THE ESTIMATION OF THE ONE-SIDED LOWER 70 PER CENT CONFIDENCE LIMIT OF THE MEAN OF A LOGNORMAL POPULATION

V	$n=5$	$n=10$	$n=15$	$n=20$
0,00	1,0000	1,0000	1,0000	1,0000
0,02	0,9647	0,9772	0,9816	0,9846
0,04	0,9515	0,9684	0,9744	0,9781
0,06	0,9420	0,9618	0,9689	0,9734
0,08	0,9343	0,9563	0,9643	0,9694
0,10	0,9279	0,9516	0,9603	0,9660
0,12	0,9222	0,9476	0,9569	0,9629
0,14	0,9173	0,9438	0,9537	0,9601
0,16	0,9128	0,9404	0,9508	0,9575
0,18	0,9087	0,9373	0,9480	0,9551
0,20	0,9049	0,9343	0,9454	0,9527
0,30	0,8902	0,9220	0,9344	0,9428
0,40	0,8798	0,9124	0,9255	0,9349
0,50	0,8724	0,9045	0,9181	0,9278
0,60	0,8674	0,8980	0,9126	0,9217
0,70	0,8643	0,8926	0,9059	0,9163
0,80	0,8630	0,8882	0,9009	0,9114
0,90	0,8632	0,8845	0,8966	0,9070
1,00	0,8649	0,8817	0,8928	0,9030
1,10	0,8679	0,8795	0,8895	0,8994
1,20	0,8722	0,8780	0,8866	0,8962
1,30	0,8777	0,8771	0,8842	0,8934
1,40	0,8845	0,8769	0,8822	0,8909
1,50	0,8925	0,8772	0,8806	0,8886
1,60	0,9016	0,8780	0,8794	0,8867
1,70	0,9120	0,8795	0,8785	0,8851
1,80	0,9235	0,8814	0,8781	0,8837
1,90	0,9363	0,8839	0,8779	0,8827
2,00	0,9502	0,8869	0,8782	0,8818
2,10	0,9653	0,8904	0,8787	0,8813
2,20	0,9816	0,8945	0,8797	0,8810
2,30	0,9992	0,8991	0,8809	0,8810
2,40	1,0180	0,9042	0,8825	0,8812
2,50	1,0381	0,9098	0,8844	0,8816
2,60	1,0596	0,9159	0,8867	0,8824
2,70	1,0823	0,9226	0,8892	0,8833
2,80	1,1065	0,9298	0,8922	0,8845
2,90	1,1321	0,9375	0,8954	0,8859
3,00	1,1590	0,9458	0,8990	0,8876
3,10	1,1876	0,9546	0,9029	0,8895
3,20	1,2177	0,9639	0,9071	0,8917
3,30	1,2494	0,9739	0,9117	0,8941
3,40	1,2827	0,9843	0,9166	0,8968
3,50	1,3178	0,9954	0,9218	0,8996
3,60	1,3546	1,0070	0,9274	0,9028
3,70	1,3934	1,0193	0,9333	0,9062
3,80	1,4340	1,0321	0,9396	0,9098
3,90	1,4766	1,0456	0,9462	0,9137
4,00	1,5213	1,0597	0,9531	0,9178
4,10	1,5681	1,0744	0,9605	0,9222
4,20	1,6172	1,0898	0,9682	0,9268
4,30	1,6686	1,1059	0,9762	0,9317
4,40	1,7225	1,1227	0,9847	0,9368
4,50	1,7788	1,1402	0,9935	0,9422
4,60	1,8379	1,1584	1,0027	0,9478
4,70	1,8997	1,1774	1,0123	0,9538
4,80	1,9644	1,1971	1,0223	0,9600
4,90	2,0321	1,2176	1,0328	0,9664
5,00	2,1029	1,2389	1,0436	0,9732

FACTOR $\psi_{0,70}$ (V,n) FOR THE ESTIMATION OF THE ONE-SIDED UPPER 70 PER CENT CONFIDENCE LIMIT OF THE MEAN OF A LOGNORMAL POPULATION

V	$n=5$	$n=10$	$n=15$	$n=20$
0,00	1,0000	1,0000	1,0000	1,0000
0,02	1,0538	1,0308	1,0234	1,0190
0,04	1,0792	1,0447	1,0336	1,0280
0,06	1,1002	1,0561	1,0423	1,0350
0,08	1,1187	1,0662	1,0497	1,0413
0,10	1,1359	1,0756	1,0566	1,0468
0,12	1,1524	1,0840	1,0628	1,0519
0,14	1,1681	1,0922	1,0689	1,0569
0,16	1,1834	1,1001	1,0745	1,0615
0,18	1,1983	1,1078	1,0801	1,0661
0,20	1,2131	1,1152	1,0856	1,0706
0,30	1,2845	1,1508	1,1113	1,0913
0,40	1,3559	1,1850	1,1354	1,1104
0,50	1,4292	1,2190	1,1590	1,1294
0,60	1,5053	1,2532	1,1785	1,1479
0,70	1,5851	1,2882	1,2066	1,1667
0,80	1,6697	1,3241	1,2309	1,1856
0,90	1,7593	1,3611	1,2555	1,2047
1,00	1,8542	1,3995	1,2808	1,2244
1,10	1,9556	1,4391	1,3069	1,2443
1,20	2,0634	1,4805	1,3338	1,2648
1,30	2,1786	1,5235	1,3614	1,2856
1,40	2,3018	1,5682	1,3898	1,3072
1,50	2,4332	1,6148	1,4192	1,3291
1,60	2,5736	1,6634	1,4496	1,3517
1,70	2,7241	1,7142	1,4810	1,3750
1,80	2,8850	1,7672	1,5134	1,3989
1,90	3,0574	1,8224	1,5469	1,4235
2,00	3,2412	1,8802	1,5817	1,4489
2,10	3,4381	1,9406	1,6177	1,4751
2,20	3,6490	2,0039	1,6550	1,5019
2,30	3,8746	2,0698	1,6937	1,5298
2,40	4,1160	2,1389	1,7337	1,5584
2,50	4,3746	2,2113	1,7753	1,5878
2,60	4,6513	2,2867	1,8182	1,6215
2,70	4,9471	2,3657	1,8628	1,6496
2,80	5,2639	2,4484	1,9090	1,6820
2,90	5,6035	2,5350	1,9569	1,7152
3,00	5,9659	2,6256	2,0067	1,7496
3,10	6,3548	2,7203	2,0580	1,7851
3,20	6,7708	2,8195	2,1116	1,8216
3,30	7,2160	2,9235	2,1669	1,8595
3,40	7,6928	3,0321	2,2244	1,8984
3,50	8,2023	3,1463	2,2841	1,9385
3,60	8,7483	3,2655	2,3460	1,9800
3,70	9,3331	3,3904	2,4102	2,0227
3,80	9,9589	3,5210	2,4768	2,0670
3,90	10,6281	3,6582	2,5459	2,1125
4,00	11,3454	3,8018	2,6176	2,1596
4,10	12,1122	3,9520	2,6921	2,2083
4,20	12,9338	4,1097	2,7693	2,2584
4,30	13,8126	4,2749	2,8497	2,3102
4,40	14,7548	4,4479	2,9330	2,3636
4,50	15,7620	4,6293	3,0196	2,4188
4,60	16,8413	4,8196	3,1096	2,4759
4,70	17,9956	5,0191	3,2030	2,5350
4,80	19,2318	5,2280	3,2998	2,5957
4,90	20,5560	5,4468	3,4008	2,6588
5,00	21,9718	5,6766	3,5053	2,7238