A model for the leaching of non-porous particles

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SYNOPSIS

A model has been developed for the prediction of the rate of leaching for an assemblage of non-porous particles in terms of size-distribution parameters. Expressions are developed for batch and continuous operation, and an example of the application of the model is given.

SAMEVATTING

Daar is 'n model vir die voorspelling van die loogtempo vir 'n versameling van nie-poreuse partikels in terme van grootteverspreidingparameters ontwikkel. Daar word uitdenkings ontwikkel vir lot- en deurlopende bewerking en 'n voorbeeld van die toepassing van die model gegee.

INTRODUCTION

Leaching reactions when the bulk of the material is soluble often result in a rate mechanism that is controlled by the surface area available for reaction. The particle size decreases with time, but, in a continuous operation, fresh material is added to the leaching vessel. A simple model of the leaching mechanism can be used in a prediction of the steady-state size distribution, and hence the overall leaching rate, for a given size distribution of feed material. This approach was used recently at the National Institute for Metallurgy (NIM) in the prediction of the steady-state leaching rate of granulated copper in ammonium carbonate, as part of an investigation of a hydrometallurgical process for copper scrap. The derivation of the model is recorded here for more general use (e.g., corrosion of ferro-silicon, wear of balls in a ball mill).

MODEL FOR BATCH TESTS

The assumption that the leaching rate is proportional to the surface area of the assemblage of particles would be valid if the leaching vessel were well mixed so that mass transfer at the interface was independent of particle size or of position in the leaching reactor. (A similar assumption has been used for gas-solid reactions in a fluidized bed[1].)

For a single particle,

Mass $(M) \propto D^3$

Surface area $\propto D^2 \propto \frac{M}{D}$,

where D is the diameter or a linear dimension describing the particle. Hence

$$\frac{dM}{dt} = -k\frac{M}{D}. \qquad (1)$$

Also,

$$\frac{D}{D_o} = \left(\frac{M}{M_o}\right)^{1/3}. \qquad (2)$$

Substituting in equation (1),

$$\frac{dM}{dt} = k\left(\frac{M_o}{M}\right)^{1/3}\frac{M}{D_o}.$$

Integrating,

$$\left(\frac{M}{M_o}\right)^{1/3} = 1 - \frac{kt}{3D_o}. \qquad (3)$$

or

$$D = D_o - \frac{kt}{3}. \qquad (4)$$

Batch tests can be used in the estimation of the rate constant, k, by use of equation (3) or (4), and the general validity of the model can be tested for different screen fractions.

If the starting material has a distribution of sizes, the characteristic curve for a batch test is given by

$$\frac{M}{M_o} = \int_o^{D_{max}}\left(1 - \frac{kt}{3D_o}\right)^3 f(D_o)dD_o. \qquad (5)$$

There is an analytic solution to this equation only in special cases.

PREDICTION OF STEADY-STATE LEACHING RATES

Consider only one size, D_o, fed continuously to the leaching vessel. All the particles remain in the vessel until they effectively disappear (e.g., in a rotating drum with a steady feed and overflow of leach liquor). The diameter of the particles decreases linearly with time (equation 4). Thus, there is an equal number of particles in each size fraction, and the size distribution at steady-state, based on numbers, is given by

$$f_n(D) = \frac{1}{D_o}.$$

The size distribution based on mass is given by

$$f(D,D_o) = \frac{D^3 f_n(D)}{\int_o^{D_o} D^3 f_n(D)dD} = \frac{4D^3}{D_0^4} \qquad (6)$$

At steady state, the feed rate is equal to the rate of dissolution:

$$\text{Feed rate } (m) = \int_o^{D_o}(\text{rate of dissolution of each size } D)dD.$$

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From equation (1),

$$m=\int_o^{D_o}\frac{k}{D}\,[M_{ss}.f(D,D_o)]dD,$$

where M_{ss}=total mass in the vessel at steady state.

$$m=M_{ss}\int_o^{D_o}\frac{k}{D}\,\frac{4D^3}{D_o^4}\,dD$$

$$=\frac{4}{3}\,\frac{k}{D_o}\,M_{ss}\quad\ldots\ldots\ (7)$$

A comparison of equation (7) with equation (1) shows that the leaching rate at steady state is $\frac{4}{3}$ times the leaching rate based on the feed size, or the inventory of the vessel is only three-quarters of that based on the feed size.

These conclusions can be extended to a distribution of sizes in the feed material. By summing the steady-state mass in the leaching vessel corresponding to each size fraction in the feed,

$$M_{ss}=\int_o^{D_{max}}\frac{3}{4}\,\frac{D_o}{k}\,m\,f(D_o)dD_o$$

$$=\frac{3}{4}\,\frac{D_{av}}{k}\,m,\quad\ldots\ (8)$$

where $f(D_o)$ is the size distribution in the feed material, and D_{av} is the weighted average size.

Thus, the leaching rate per unit mass in the leaching vessel at steady state (m/M_{ss}) is dependent only on the average of the size distribution of the feed material, and is independent of the spread of sizes. Thus, the surface area per unit of mass in the leaching vessel may be less, or greater, than that of the feed material, as the rate of leaching is defined in terms of the average size only.

Finally, it may be desirable to predict the size distribution of the particles in the leaching vessel at steady state. Equation (6) gives the size distribution that results when a single size, D_o, is being fed. The corresponding expression for a size distribution in the feed, $f(D_o)$, is given by

$$f_{ss}(D)=\frac{\int_o^{D_{max}} D^3/D_0^4\ f(D_o)\ .\ [1-H(D_o)]dD_o}{\int_o^{D_{max}}\int_o^{D_{max}} D^3/D_o^4\ .\ f(D_o)[1-H(D_o)]dD_o\ dD},$$

where H ( ) is the Heavyside function.

This expression does not have a simple solution, but characteristics of the steady-state size distribution can be obtained in terms of moments. The first moment, or mean of the distribution resulting from the feeding of a single size, D_o, is given by

$$Dav_{ss}(D_o)=(\text{1st moment})_{ss}=\int_o^{D_o} D\ .\ 4D^3/D_o^4 dD$$

$$=\frac{4}{5}\,D_o.$$

The resulting mass in the leaching vessel at steady state when a single size, D_o, is being fed is proportional to the product of the feed rate and feed size, i.e., from equation (7),

$M_{ss}(D_o)\propto D_o m f(D_o)$.

Thus, for a given distribution of feed sizes, $f(D_o)$, the average size, or first moment of the steady-state distribution, is given by

$$Dav_{ss}=(\text{1st moment})_{ss}=\frac{\int_o^{D_{max}} Dav_{ss}\ (D_o)\ .\ M_{ss}(D_o)dD_o}{\int_o^{D_{max}} M_{ss}(D_o)dD_o}$$

$$=\frac{\int_o^{D_{max}}\frac{4}{5}\,D_o\ .\ D_o\ .\ f(D_o)dD_o}{\int_o^{D_{max}} D_o f(D_o)dD_o}$$

$$=\frac{\frac{4}{5}\ (\text{2nd moment of feed distribution})}{(\text{1st moment of feed distribution})}\ .\quad\ldots\ldots\ (9)$$

Similarly,

$$(\text{2nd moment})_{ss}=\frac{\frac{2}{3}\ (\text{3rd moment of feed distribution})}{(\text{1st moment of feed distribution})}\ .\quad\ldots\ldots\ (10)$$

The following illustrates the use of the equations developed above. If the size distribution of a feed is well described by a Schuhmann distribution with modulus K and slope n, i.e., cumulative mass

$$\text{fraction}=\left(\frac{D_o}{K}\right)^n,$$

$f(D_o)$ =derivative of cumulative form

$$=\frac{nD_o^{n-1}}{K^n},$$

1st moment

$$(=D_{av})=\int_o^K D_o f(D_o)dD_o\ .$$

$$=\frac{n}{n+1}K$$

2nd moment

$$=\frac{n}{n+2}\,K^2$$

3rd moment

$$=\frac{n}{n+3}\,K^3$$

From equation (8),

$$M_{ss}=\frac{3}{4}\,\frac{n}{n+1}\,\frac{K}{k}\,m.$$

$$\ldots\ldots\ldots\ (a)$$

From equations (9) and (10),

$$Dav_{ss}=\frac{4}{5}\,\frac{n+1}{n+2}\,K.\quad\ldots\ (b)$$

(2nd moment)$_{ss}$

$$=\frac{2}{3}\,\frac{n+1}{n+3}\,K^2.\quad\ldots\ (c)$$

If it is assumed that the steady-state distribution of sizes in the leaching system is also described by a Schuhmann distribution, the appropriate moments can be used for the computation of the parameters of the steady-state distribution. If these parameters are n' and K',

$$\text{1st moment} = \frac{n'}{n'+1} K' = \frac{4}{5}\,\frac{n+1}{n+2} K \quad \text{(d)}$$

$$\text{2nd moment} = \frac{n'\,1}{n'+2} K'^2 = \frac{2}{3}\,\frac{n+1}{n+3} K^2. \quad \text{(e)}$$

Eliminating K's,

$$\frac{(n'+1)^2}{(n'+2)n'} = \frac{25}{24}\,\frac{(n+2)^2}{(n+3)\ (n+1)}.$$

A trial-and-error solution is required for n'. Table I gives the relationship between n' and n. The value K' is then obtained from equation (d).

It should be noted that this solution is only approximate, because the steady-state distribution resulting from the feeding of a Schuhmann-type size distribution is not a Schuhmann distribution. The limiting value of n' is 4,0, which results when the feed is a single size ($n=\infty$) as follows from equation (6). The distribution of sizes at steady state is independent of the rate constant, k, which influences only the hold-up at steady-state.

As mentioned earlier, the model was used in the analysis of the rate On the basis of the Schuhmann parameters for the feed (K=4,9 mm, n=1,83), the predicted size distribution at steady state is given by the Schuhmann parameters K'=4,3 mm, n'=2,06, which is shown as a dotted line in Fig. 1.

The leaching rate per unit mass of hold-up is given by equation (8) in terms of the feed distribution:

$$\frac{m}{M_{ss}} = \frac{4}{3}\,\frac{k}{K}\cdot\frac{n+1}{n}.$$

The leaching rate per unit mass can be computed from the steady-state distribution of sizes:

$$\frac{m}{M_{ss}} = \int_0^{K'} \frac{k}{D}\cdot\frac{n'D^{n'-1}}{K'^{n'}}\,dD = \frac{n'}{n'-1}\,\frac{k}{K'}.$$

In the example given, the appropriate rates are as follows:

Predicted from the feed	$0{,}421k$
Via the predicted Schuhmann distribution	$0{,}453k$
Via the observed Schuhmann distribution	$0{,}576k$

The discrepancy between the first two values indicates that the predicted distribution has less fines or is sharper than the Schuhmann distribution shown by the dotted

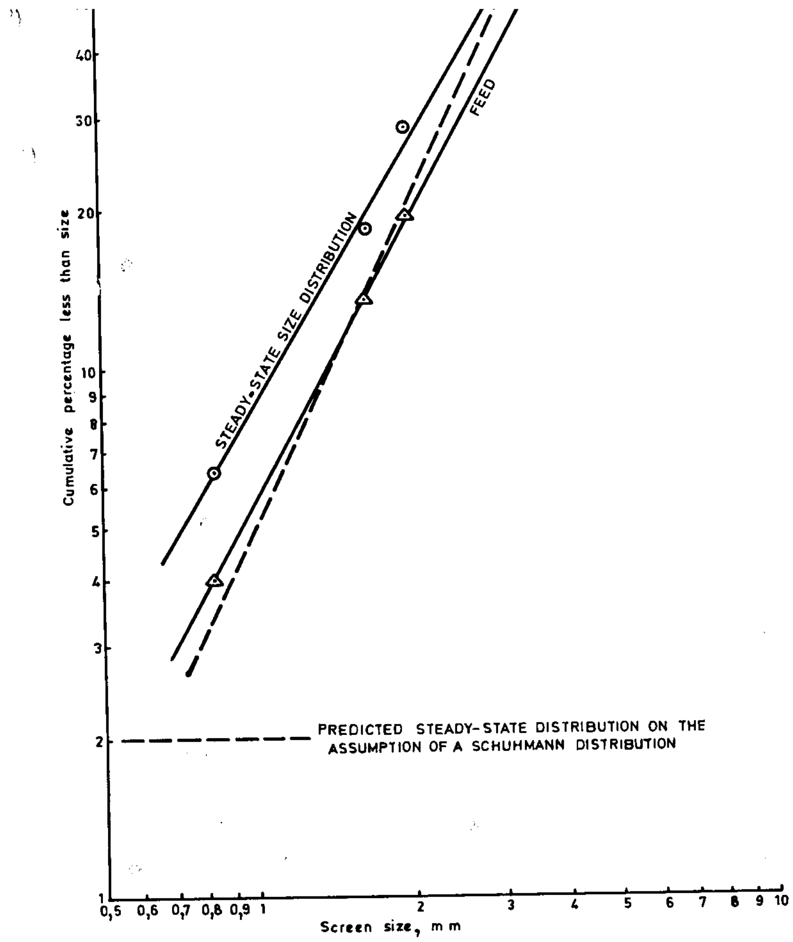

Fig. 1—Size distribution of the feed (copper granules), and steady-state size distribution in the leaching vessel

line in Fig. 1. The upper portion of the line is probably correct, because the method of equating moments is sensitive to the larger sizes. Thus, the accurate prediction is probably a curve dipping more steeply than the dotted line shown in Fig. 1. The discrepancy between the predicted and the experimental distributions may be due to a breakdown of aggregates in the leaching environment, which would result in more fines.

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REFERENCE

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