Some basic considerations in the application of geostatistics to the valuation of ore in South African gold mines

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SYNOPSIS

There is much room for improvement in the statistical procedures that have been used by gold mines over the past decade in the valuation of their ore reserves. Improvement depends on the establishment of the extent to which the patterns of frequency distributions and the directional variograms for the gold values vary from area to area within each section of a mine. The application of the lognormal distribution and of the De Wijsian variogram model for routine ore-reserve valuations requires detailed study before some of the reservations still held by mathematical statisticians can be met. This paper provides some preliminary evidence of the extent to which these models can be used with confidence, as well as suggesting how they can be applied to their full potential.

SAMEVATTING

Die statistiese prosedures wat die goudmyne die afgelope dekade vir die evaluering van hul ertsreserwes gebruik het, kan baie verbeter word. Die verbetering hang af van die vasstelling van die mate waarin die frekwensieverdelingspatrone en die rigtingsvariogramme vir die goudwaardes van een gebied na 'n ander binne elke seksie van 'n myn wissel. Die gebruik van die lognormale verdeling en van die De Wijs variogrammodel vir die routine-evaluering van ertsreserwes vereis 'n uitvoerige studie voordat daar aan sommige van die voorbehoude wat wiskundige statistici nog het, voldoen kan word. Hierdie verhandeling gee voorlopige bewyse van die mate waarin hierdie modelle met vertroue gebruik kan word en doen ook aan die hand hoe hulle tot hul volle potensiaal benut kan word.

INTRODUCTION

The term *geostatistics* was proposed by Matheron[1] in the early 1960's to cover the study of the distribution in space of values such as ore grades and thicknesses, the development of models for such spatial variables, and the application of such models — mainly in ore valuation. In terms of this definition, most of the early work in South Africa by Sichel, Ross, and Krige on the application of mathematical statistics to the valuation of gold ore, and more particularly the weighted moving average technique[2] introduced some ten years ago for the assessment of ore reserves would be classified as geostatistics. The basic concept in geostatistics is that spatially distributed variables are usually not random variables but exhibit a spatial structure of interrelationships, which can be quantified by a variogram function[1] in two or three dimensions, depending on the nature of the variable concerned, e.g., ore values in a relatively thin sedimentary ore-body or in a massive three-dimensional ore-body.

In classical statistical terminology, the analogy to the variogram is found in the two- or three-dimensional autocorrelation or autocovariance patterns used by the author and others[2–4]. The only basic reservation about the application of models based on this approach is that in theory the models used must fit the data adequately, and that the variogram must be stable (or stationary) within the area or volume of the ore-body being valued. In nature, exact stationarity is unlikely but, if variations in the variogram pattern and any departures from any other models used (e.g., lognormal distribution) are such as to have an insignificant effect on the final estimates, the implementation of the approach would be justified provided worthwhile benefits were gained relative to the orthodox approach. Where necessary and practical, the same objective could, of course, be obtained by subdividing the ore-body until reasonable stationarity is achieved within the subdivisions.

It is the object of this paper to provide the first detailed study of the deviations from the average variogram model from area to area within a mine section, as well as of any significant departures from the lognormal distribution model within such smaller areas, in order to gauge the effect on final estimates of ore blocks.

DATA USED FOR ANALYSIS

The data selected for analysis cover a section of the Hartebeesfontein Mine measuring some 3000 m by 900 m and covering mainly mined-out areas on the Vaal Reef. This section was divided into 5 subsections, numbered 301 to 305, as shown on Fig. 1. Subsection 301 was studied in greater detail on the basis of subdivisions into:

(i) 4 areas of about 390 m by 350 m each,
(ii) 8 areas of about 260 m by 230 m each, and
(iii) 33 areas of about 75 m by 75 m each.

Gold ore values (cm.g/t) are available from a computer data bank on the basis of the averages of all underground ore-sample values within basic squares of 25 ft by 25 ft (approximately 7½ m by 7½ m) on plan; the number of individual sample sections varies from zero to 4, and averages about 2 per square.

THE FREQUENCY DISTRIBUTION MODEL

The frequency distribution model generally accepted for gold values in ores is that of the three-parameter lognormal model, where the gold values x, after transformation by the function $\log(x+a)$, are normally

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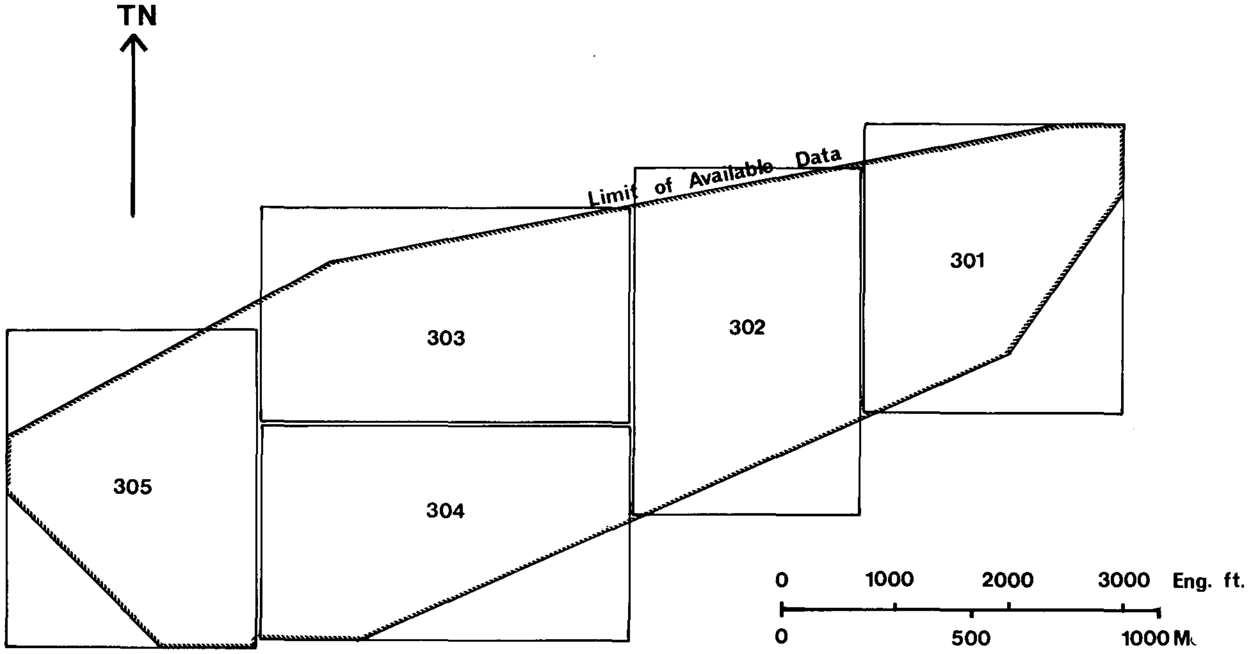

Fig. 1—Subsections 301 to 305 in the Hartbeesfontein Gold Mine

distributed, and where a is an additive constant, i.e., the third parameter of the lognormal distribution. The applicability of this model to the ore values from a wide range of reefs and sizes of areas was studied by the author[5] some fifteen years ago, but, as mentioned above, the study is now extended to smaller sizes of areas.

The more significant patterns are shown graphically on logarithmic-probability paper in Figs. 2 and 3. The evidence is summarized as follows.

(i) With an additive constant a of 150 cm.g/t, the three-parameter lognormal model is adequate for the total mine section and the 5 subsections, as well as for all the sub-areas down to size 75 m by 75 m. In routine valuations of ore reserves, the data for the weighted moving averages, or kriged values[2], is drawn for each grid point from an area roughly of this size, and the lognormal model can therefore be used with confidence. However, the logarithmic variances within such areas appear to vary significantly, and this may affect the application of the complete lognormal kriging model.

(ii) There appears to be a general trend of decreasing values from subsection 301 westwards to 305, as well as a similar trend in the logarithmic variances, thus suggesting a positive correlation between average values and the logarithmic variances.

With only 5 subsections available, the usual statistical tests show this correlation as not necessarily significant. However, in the case of the small 75 m by 75 m areas referred to above, a similar correlation was observed and in this case it was statistically significant.

RELATIONSHIP BETWEEN VARIANCE AND SIZE OF AREA

On the basis of the three-parameter lognormal model with $a=150$, the logarithmic variances of the values of the basic square units within population areas of different sizes up to the main subsections as well as within the mine section as a whole can be adequately represented by the logarithmic de Wijsian model, as shown in Fig. 4.

The mathematics of this model are dealt with in detail by Matheron[6] and Rendu[7]. The model is of the form

$$\text{Variance of population } (\sigma^2) = \text{Nugget effect } (\lambda) + \frac{3a}{2} \log \left(\frac{\text{population area}}{\text{unit area}}\right),$$

where a=absolute dispersion, related to the slope of the line on Fig. 4 and λ=variance at zero lag, i.e., the error variance within unit size areas.

Again, a study of Fig. 4, and of the parameters of the models for each of the 5 subsections 301 to 305 as recorded in Table 1 shows variations from the average relationship for the whole mine section. The Nugget effect averages 0,37 and varies from 0,30 for subsection 304 to 0,41 for subsection 301. The absolute dispersion averages 0,023 and varies from 0,015 for subsection 305 to 0,033 for subsection 302. The higher absolute dispersions appear to be correlated with the higher-grade subsections but, again, owing to the limited number of observations, the correlation is not statistically significant. It is interesting to note that, in the author's study[8] of the Vaal Reef for the whole Klerksdorp

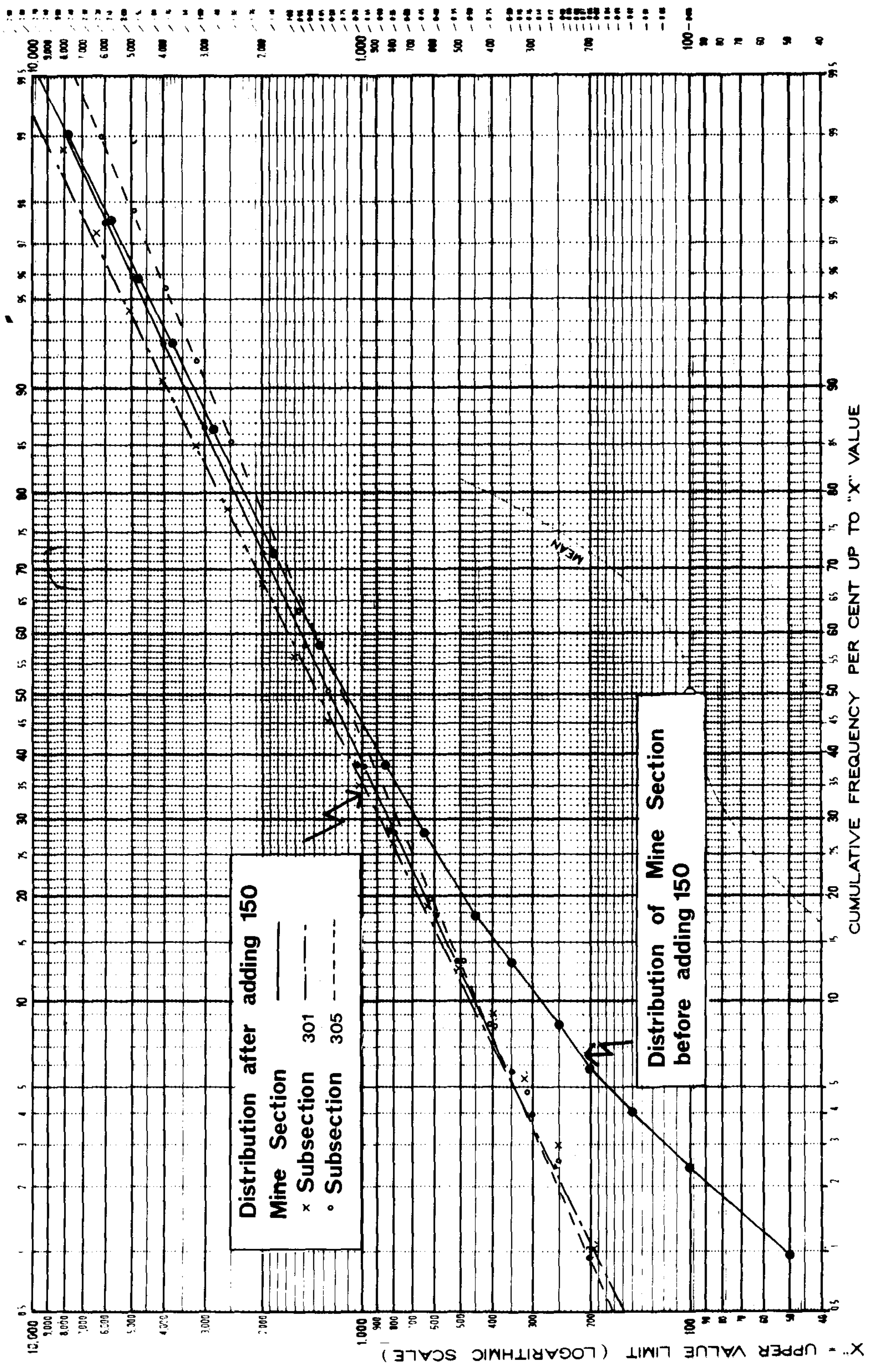

Fig. 2—Frequency distributions for the mine section and two subsections

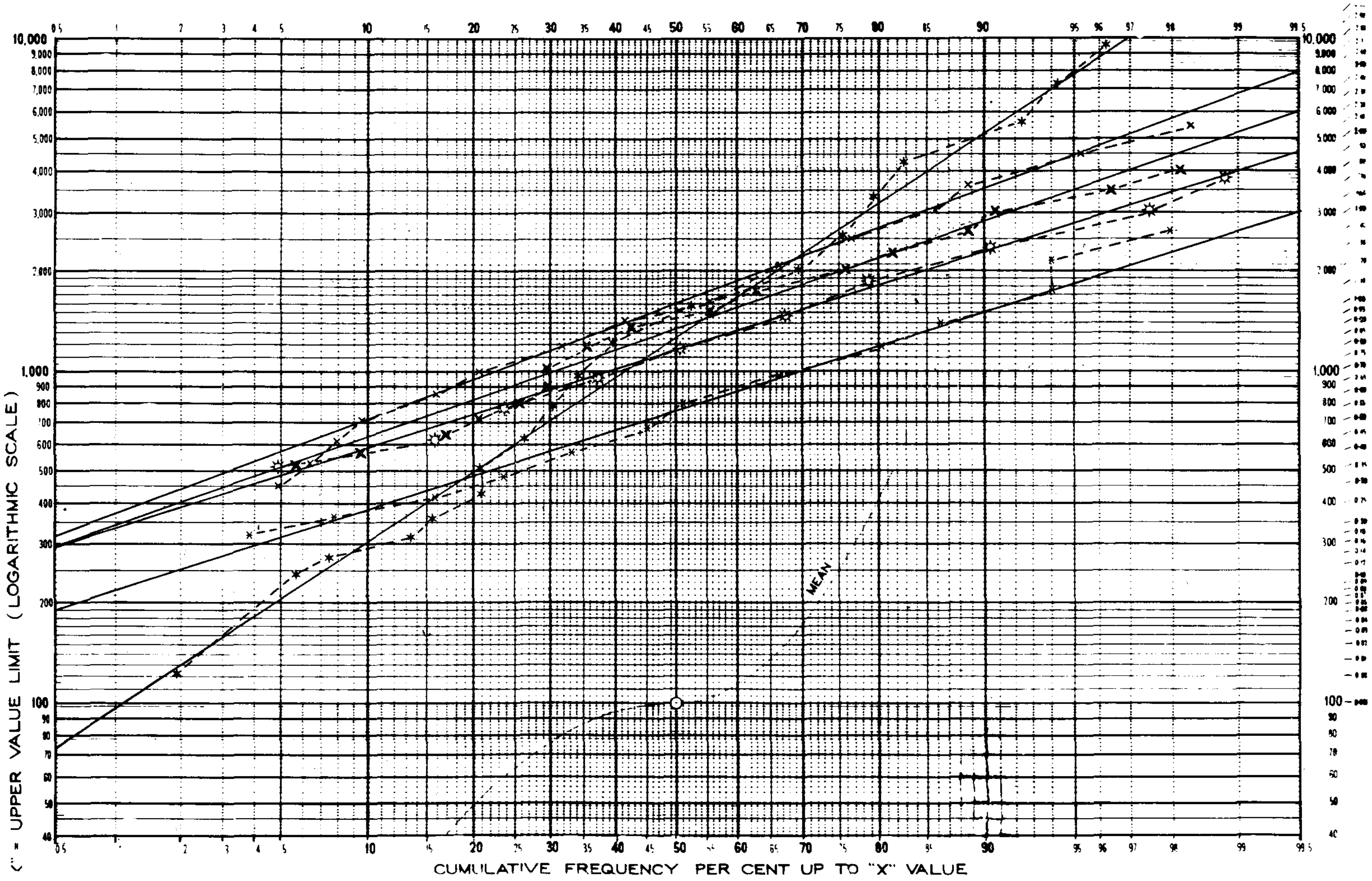

Fig. 3—Four examples of frequency distributions within areas of 250 by 250 ft in Subsection 301

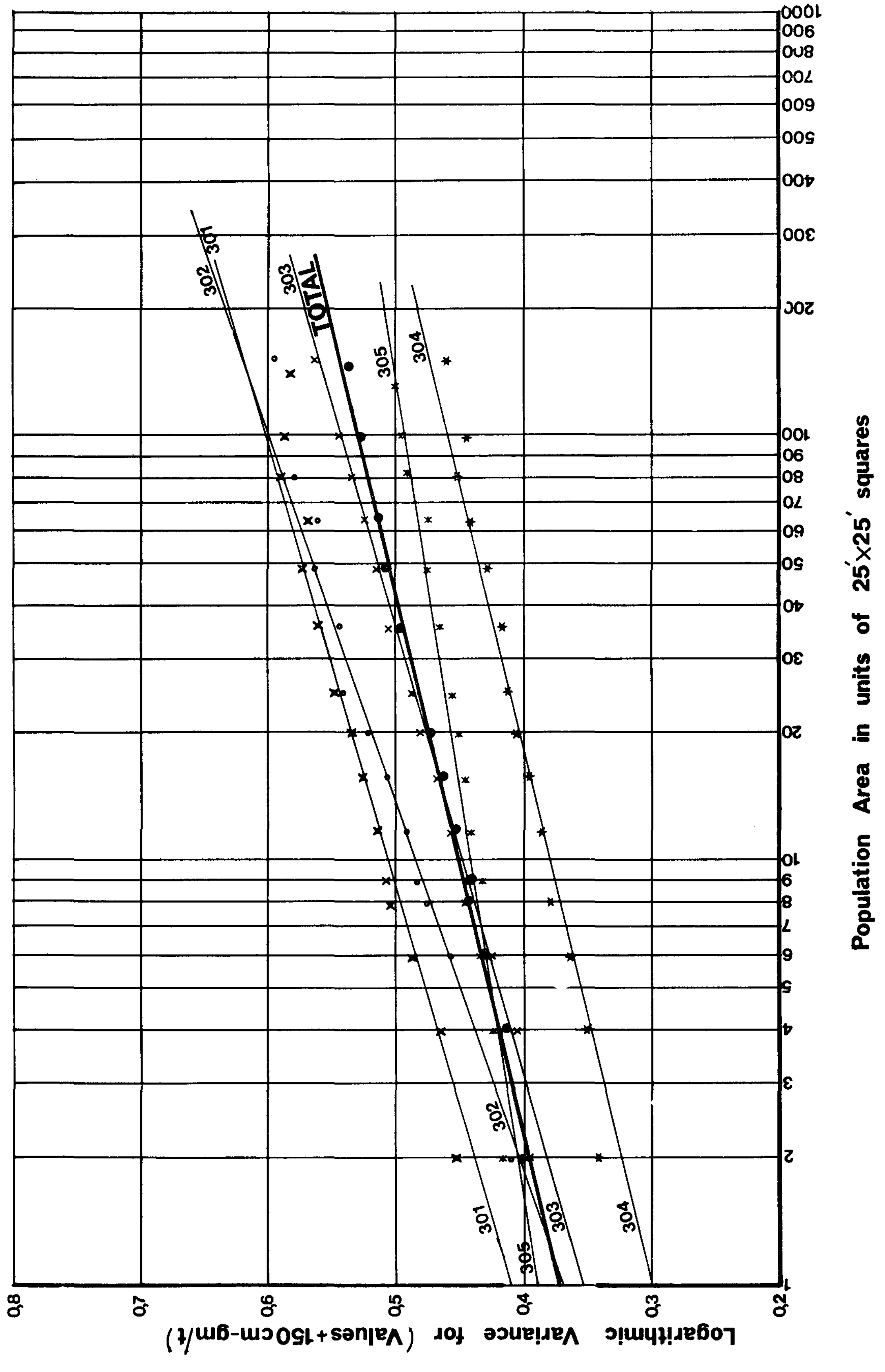

Fig. 4—Relationship between variance and size of area

TABLE I

PARAMETERS FOR THE DE WIJSIAN RELATIONSHIPS BETWEEN VARIANCE AND SIZE OF AREA

Subsection	Nugget effect	Absolute dispersion	Mean cm.g/t
301	0,410	0,028	1 730
302	0,370	0,033	1 587
303	0,355	0,027	1 563
304	0,300	0,023	1 527
305	0,390	0,015	1 394
Mine section	0,372	0,023	1 554

goldfield, the average value of the absolute dispersion in the field was estimated at 0,020, after correction for the factor 3/2, which was omitted in the 1966 paper.

THE VARIOGRAM MODEL

The variogram values (or more correctly the semi-variogram values) were calculated for the mine section and each of the 5 subsections from the basic data for a whole range of lags and for 4 directions N–S, E–W, NE–SW, and NW–SE. The results are shown on Fig. 5 for the mine section and 4 of the subsections. The variograms for the whole section and for subsections 301, 302, 304, and 305 appear to be isotropic, i.e., not significantly different in the 4 directions, and Fig. 5 therefore shows only the average variogram in each case; the variogram for subsection 303 overlaps that for the whole mine section and is therefore not shown.

For subsection 302 there appears to be a significantly lower variogram, i.e., a higher level of correlation — in the N–S direction as shown on Fig. 6. Figs. 5 and 6 also show the theoretical linear isotropic variograms[6, 7] corresponding to the linear variance–size of area relationships established above. There is a reasonable agreement in each case between the actual variogram (averaged over the 4 directions) and the theoretical linear de Wijsian model. In practice, the available data will frequently lend themselves more readily to the establishment of the variogram pattern and the weighting system to be used via a study of the variance-size of area relationship. The possibility of standard weighting tables corresponding to specified parameters for this relationship is being examined.

CONCLUSIONS

What are the practical implications of accepting an average variogram model for the whole section and applying the geostatistical lognormal kriging procedures based on such an average model, when variations as shown above do occur between subsections, and when the variance parameter of the lognormal frequency distribution model used also shows variations between and within subsections, i.e., assuming stationarity when not fully justified?

A preliminary study of this problem was done by accepting a fixed pattern of known data to be used in the valuation of a unit area of reef measuring 100 ft by 100 ft, i.e., data available only on one side of the unit area, as shown on Fig. 7.

The optimum weighting systems for this data pattern and corresponding to the distributions and de Wijsian variogram models for the whole mine section and for each of the 5 subsections respectively, were calculated and are shown in Table II. It is evident that the weighting system is reasonably robust, and that the logarithmic error variance of the estimates of

TABLE II

EFFECT OF VARIATIONS IN PARAMETERS ON WEIGHTING SYSTEMS AND ERROR VARIANCES

	Mine section	Subsections on isotropic basis					302 accepting anisotropy	
		301	302	303	304	305	Data E–W	Data N–S
Parameters used								
Population variance	0,620	0,695	0,680	0,620	0,535	0,545	0,695	0,695
Nugget effect	0,372	0,410	0,370	0,355	0,300	0,390	—	—
Absolute dispersion	0,023	0,028	0,033	0,027	0,023	0,015	0,033	0,033
Optimum weights								
Data block 1	0,118	0,117	0,112	0,115	0,119	0,103	0,066	0,103
2	0,169	0,170	0,174	0,171	0,174	0,146	0,257	0,084
3	0,169	0,170	0,174	0,171	0,174	0,146	0,257	0,084
4	0,118	0,117	0,112	0,115	0,119	0,103	0,066	0,103
5	0,015	–0,018	–0,002	–0,018	–0,015	–0,006	0,001	–0,015
6	0,023	0,015	–0,020	0,002	0,014	0,022	–0,044	–0,044
7	0,015	0,005	–0,013	–0,003	0,002	0,026	0,031	–0,023
8	0,015	0,005	–0,013	–0,003	0,002	0,026	0,031	–0,023
9	0,023	0,015	–0,020	0,002	0,014	0,022	–0,044	–0,044
10	0,015	–0,018	–0,002	–0,018	–0,015	–0,006	0,001	–0,015
Pop. mean	0,321	0,423	0,498	0,466	0,411	0,416	0,380	0,792
Error variance of estimates of mean of logs (i.e., geometric mean)								
On optimum weights	0,136	0,162	0,184	0,154	0,132	0,090	0,156	0,215
On mine section weights	0,136	0,162	0,186	0,155	0,133	0,091	0,165	0,237

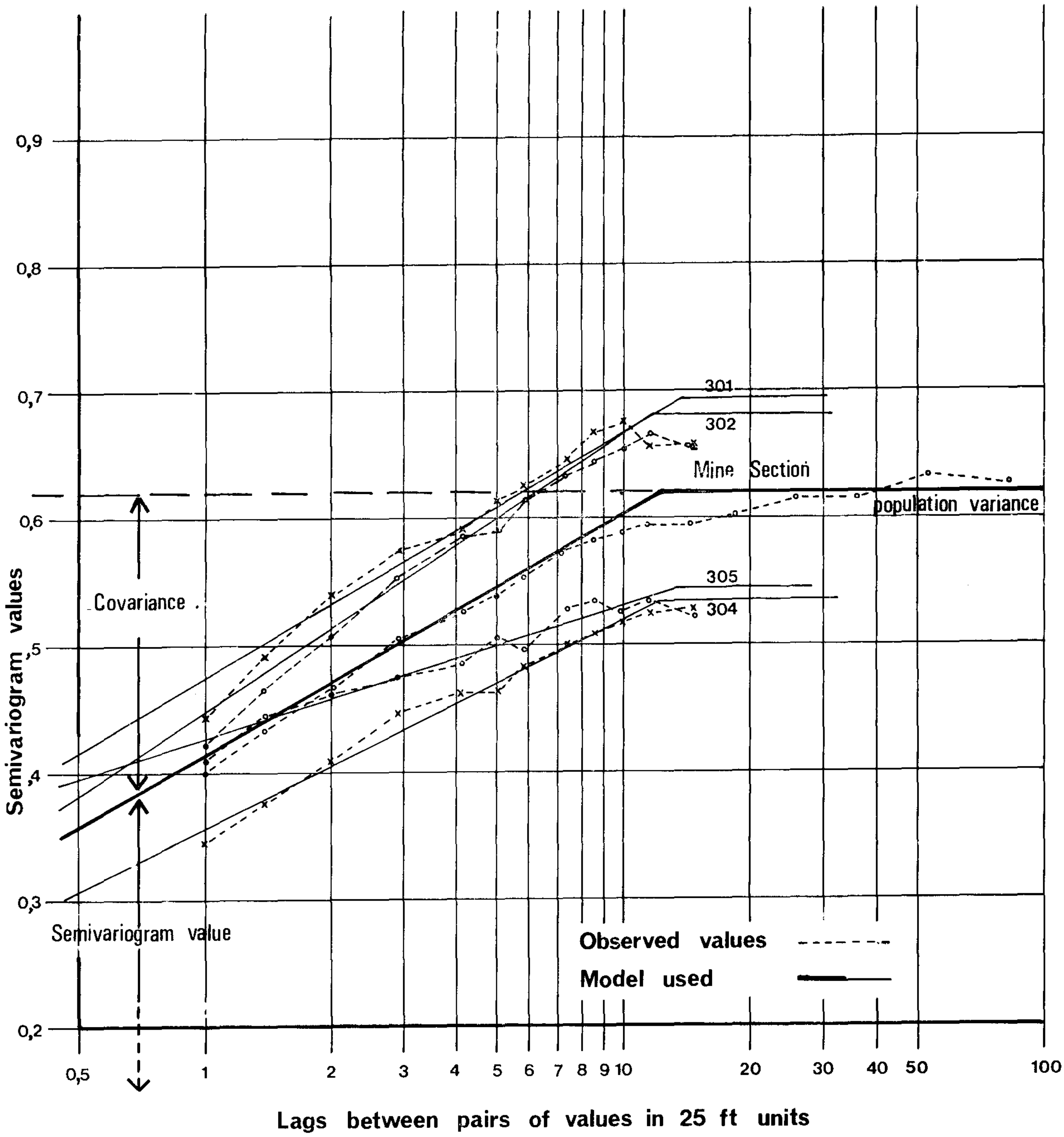

Fig. 5—Variograms for the whole mine section and for Subsections 301, 302, 304, and 305

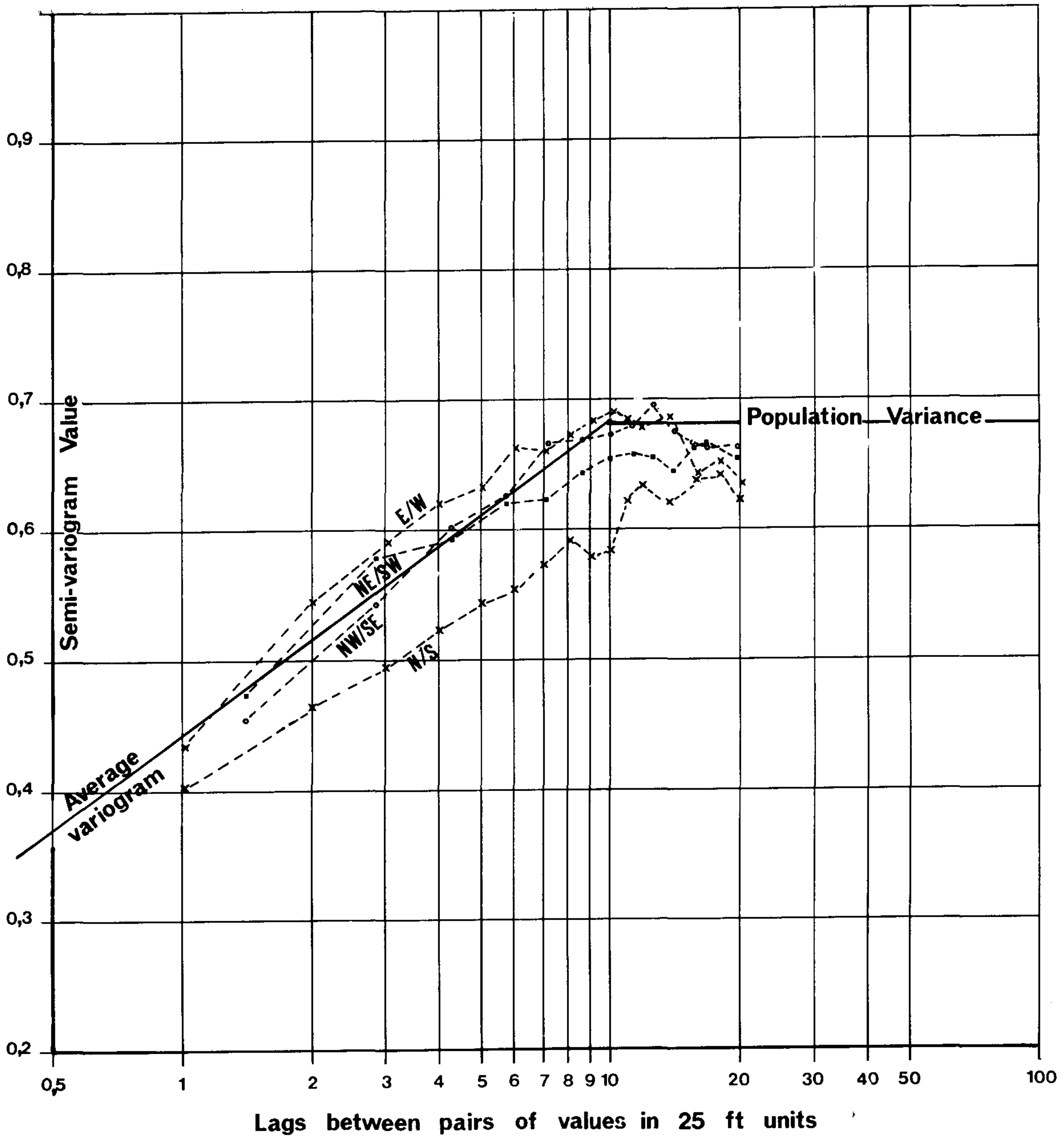

Fig. 6—Directional variograms for Subsection 302, showing significant anisotropy

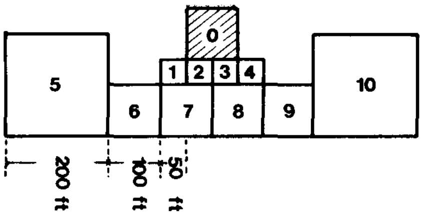

Fig. 7 — Pattern of ten available data squares (1/10) for valuation of 100X100ft (0) square

the mean of the logs is only slightly increased by using for the subsections the non-optimum weights as determined for the section as a whole. For practical purposes, subdivision into the 5 subsections will therefore not improve the position significantly. This is reassuring as far as the present system used on the Anglovaal mines is concerned, i.e., the use of such a weighting system for a whole mine section, and applying it directly to the untransformed values. A significant improvement in efficiency can, however, be obtained in subsection 302 if the N–S value trend is recognized, and a mine section should therefore be subdivided wherever significant changes in value trends occur within the section.

Further improvements in efficiency should result from the application of weighted averages (or kriging) to the transformed values, i.e., to the logs of the values plus a constant, and then to transform the estimates back to the straight cm.g/t values.

The theoretical formula for this retransformation involves the assumption of true lognormality, the weighted average value on the log basis (i.e., the logarithmic kriged value), its error variance, as well as the variances of the populations of the various sizes of data units and of the ore units to be valued[7]. In the case of gold values such as those analysed in this paper, for which the variance within the relatively small areas from which the known data are drawn varies significantly from area to area, problems and the danger of biassed estimates arise. These can be properly studied and overcome only by detailed practical simulation tests. These are being conducted at present, and the results will be disclosed in due course.

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