Phenomena relating to the failure of hard rock adjacent to an indentor

by M. HOOD*, B.Sc. (Hons.) (Mining Eng.) (Visitor)

SYNOPSIS

It is shown that, when a flat-bottomed indentor is pressed into the surface of a strong rock, the normal force necessary to cause failure of the rock is reduced substantially if a shear force is applied to the indentor. A mathematical analysis is described, which shows that higher stresses are induced in an elastic material close to the free surface of the material ahead of a punch when a shear force together with a normal force is applied to the punch. Therefore, failure of the material would occur with lower forces applied to the punch. Stress trajectories show that fractures develop in the rock along lines of maximum principal stresses.

SAMEVATTING

Daar word getoon dat die loodregte krag wat nodig is om die gesteente te laat breek wanneer 'n platboom-induiker in die oppervlak van 'n sterk gesteente ingedruk word, heelwat verlaag word as daar 'n skuifkrag op die induiker aangewend word. Daar word 'n wiskundige ontleding beskryf wat toon dat hoër spannings in 'n elastiese materiaal naby die vry oppervlak van die materiaal voor 'n pons geïnduseer word wanneer daar 'n skuifkrag saam met 'n loodregte krag op die pons aangewend word. Die materiaal sal dus by die aanwending van laer kragte op die pons breek. Spanningsbane toon dat breuke langs die lyne van die maksimum hoofspannings in die gesteente ontwikkel.

Introduction

The need for rapid excavation in rock has increased in recent years owing to the world's increasing demand for minerals and for underground transport and storage facilities. Consequently, machines are being developed for cutting rock, and at the present time most weak- and medium-strength rocks encountered both in tunnelling and in underground mining situations can be excavated by machine. However, cutting in strong rock presents different problems in terms of higher forces and greater wear on the cutting tools. Advances in the basic technology of cutting rock are necessary for the mechanization of hard-rock mining to be realized economically.

*Chamber of Mines Research Laboratories, Johannesburg.

A programme on the development of methods for the cutting of strong rock was undertaken by the Chamber of Mines, and preliminary experiments conducted as a part of this programme were reported in a paper by Hojem *et al.*[1]. These experiments showed that, from considerations of the strength of the tool, the optimum geometry of drag bits used for hard-rock applications (Fig. 1) differed considerably from the idealized 'wedge-shaped' bit (Fig. 2) generally assumed to be an optimum design for cutting in weak rocks[2, 3].

As a result of this different bit geometry, it was shown that the model proposed by Evans[2] to explain the mechanism of rock fracture adjacent to a 'wedge-shaped' bit is not applicable to a blunt drag bit cutting strong rock[4]. The results obtained showed that a blunt bit breaks the rock by an indentation process and that,

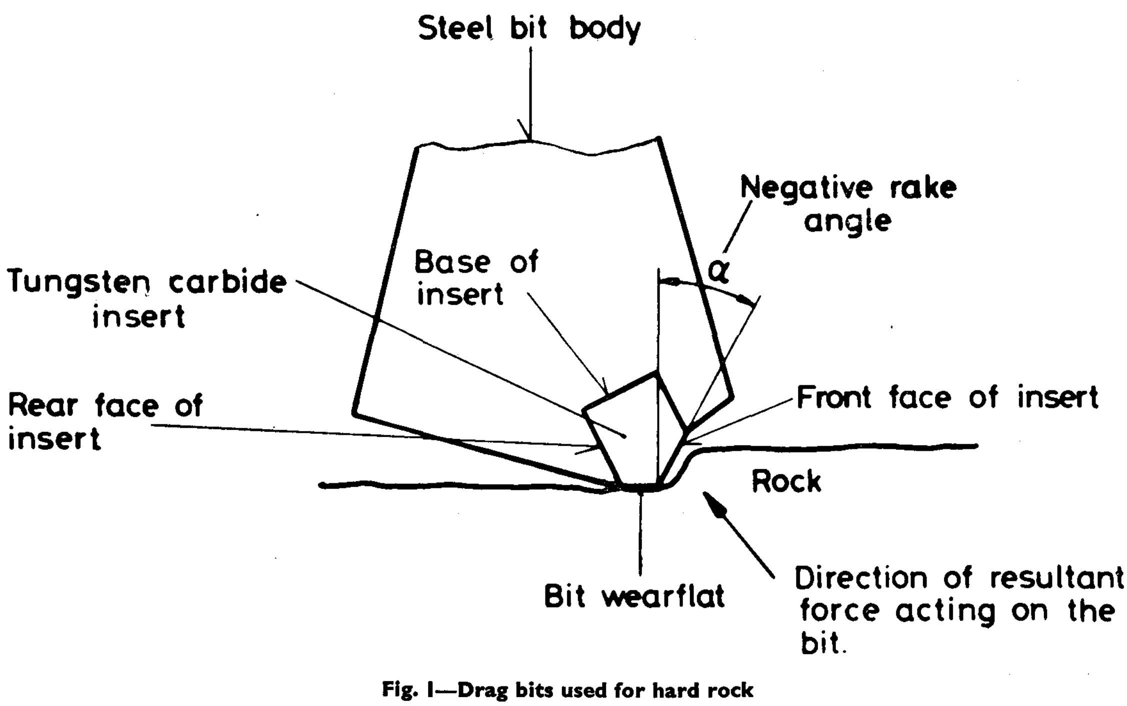

Fig. 1—Drag bits used for hard rock

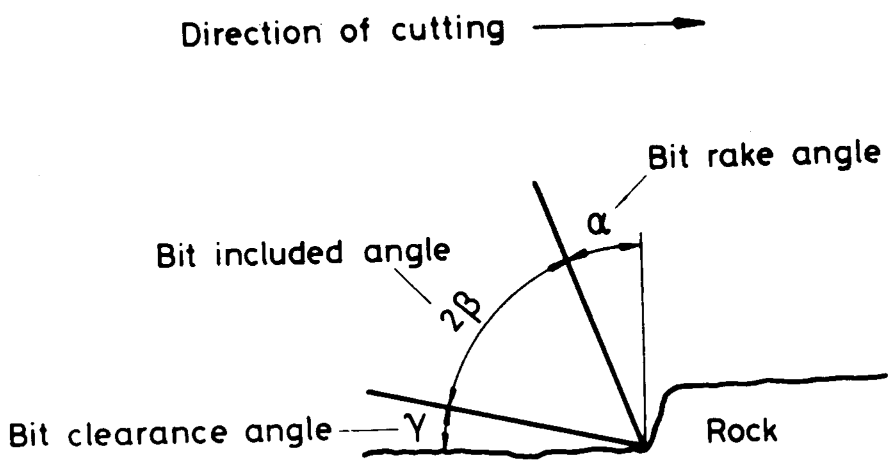

Fig. 2—Geometry of the theoretical model

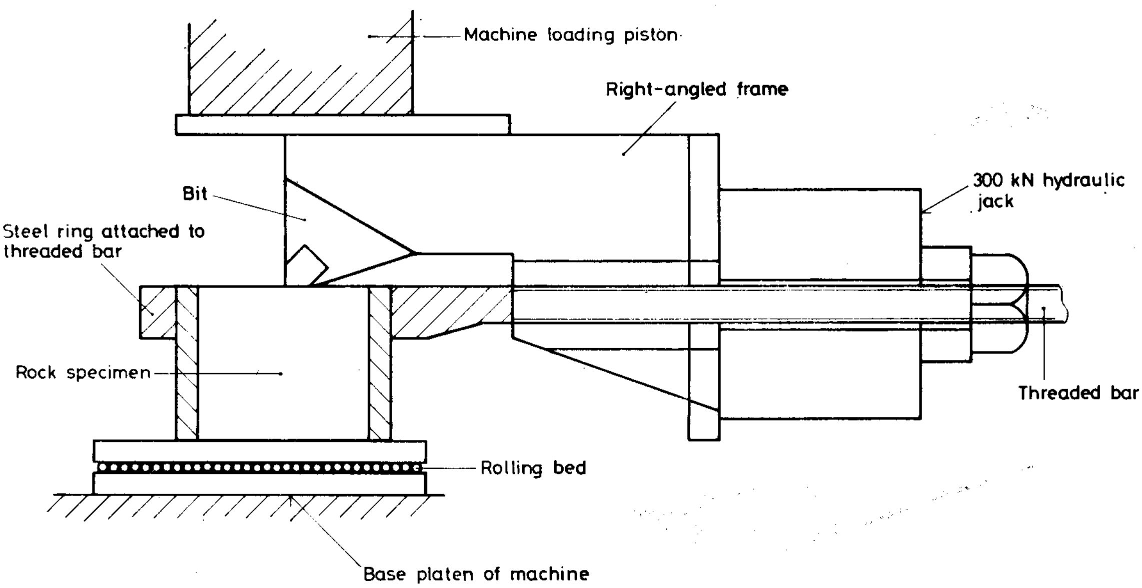

Fig. 3—The apparatus used in the indentation tests in which a shear force was applied to the bit

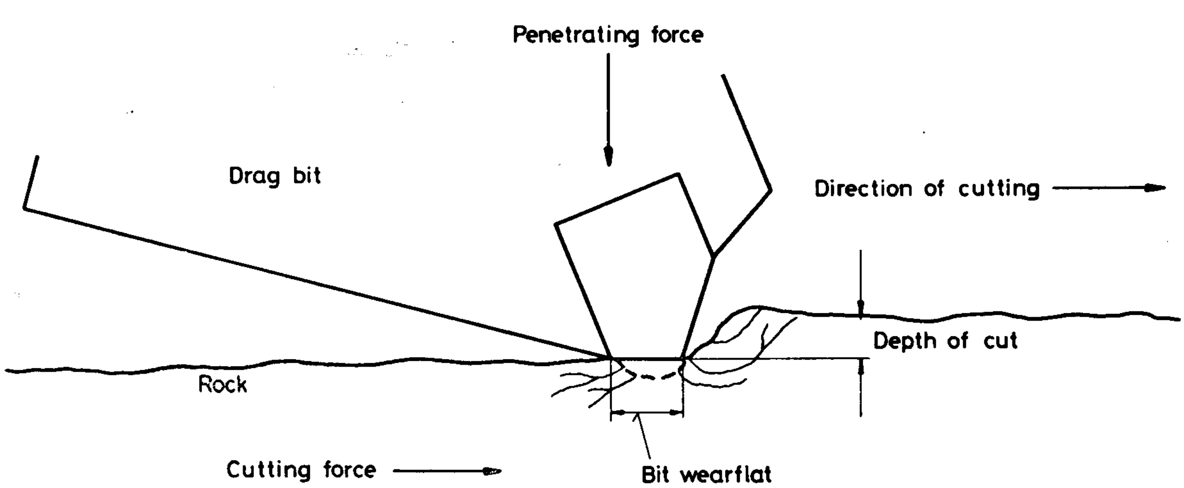

Fig. 4—A cross-section in the plane of the cutting motion, illustrating a drag bit cutting in hard rock

in general, the leading face of the bit is not in contact with the rock during the cutting operation. In addition, it was shown that the mechanism of rock fracture during the cutting process could be studied through a suite of indentation tests in which a bit is pressed into the surface of a rock specimen. The results of these tests demonstrated that rock chips of similar geometries were formed from the quasi-static indentation tests and from the cutting experiments. However, the force normal to the rock surface required to form a rock chip ahead of the bit was approximately twice as great in the indentation tests as in the tests cutting rock. The investigations described in this paper were conducted to examine the differences in these two methods of applying forces and so explain the large difference in the force required to form a rock chip.

Experimental Programme

The indentation tests described by Hood[4] used a stiff compression testing machine to press a drag bit into a rock specimen. In the test series described here, which was conducted subsequently, the apparatus was modified to enable a shear force parallel to the surface of the rock specimen to be applied between the bit and the rock at the same time that a force was applied normal to the rock surface. A schematic diagram of the 300 kN hydraulic jack that was used to apply the shear force is given in Fig. 3.

The experimental procedure involved the application of a force normal to the rock surface by use of the loading piston of the testing machine. As this force was increased, the hydraulic jack was pressurized in a controlled fashion with a hand pump, thereby applying a shear force to the rock. This shear force was measured by monitoring the pressure in the jack. A dial indicator was placed against the rock specimen to accurately monitor the relative movement between the bit and the rock. The shear force was applied using the jack until small movements of the rock, less than $2{,}5 \times 10^{-2}$ mm, were observed to take place. When this occurred, pressurization of the jack was stopped until the normal force, which was applied continuously, had increased. The shear force was applied in this discontinuous manner to ensure that, at any given time, the shear force on the rock was a maximum for a given normal force. The normal and the shear forces were recorded until a large chip of rock formed ahead of the bit.

Eight specimens of norite were tested in this manner, and the rock chips that were formed were found to be geometrically similar to those produced during cutting operations[5]. The indentation force required for the formation of a rock chip ahead of the bit during this experiment varied between 250 kN and 350 kN. The equivalent force measured during the indentation tests when a shear force was not applied was in the range[4] 380 kN to 450 kN. Thus, it was demonstrated that, when an indentor is pressed into a rock specimen, the normal force required to form a rock chip is reduced substantially during the application of a shear force. Mathematical analysis was employed to explain this finding, and calculations were made of the stress distribution in an elastic material underneath an indentor.

Mathematical Analysis

A series of calculations was made of the stresses induced in the rock adjacent to the bit. It was assumed that

(1) the rock behaves as an ideal, isotropic, elastic

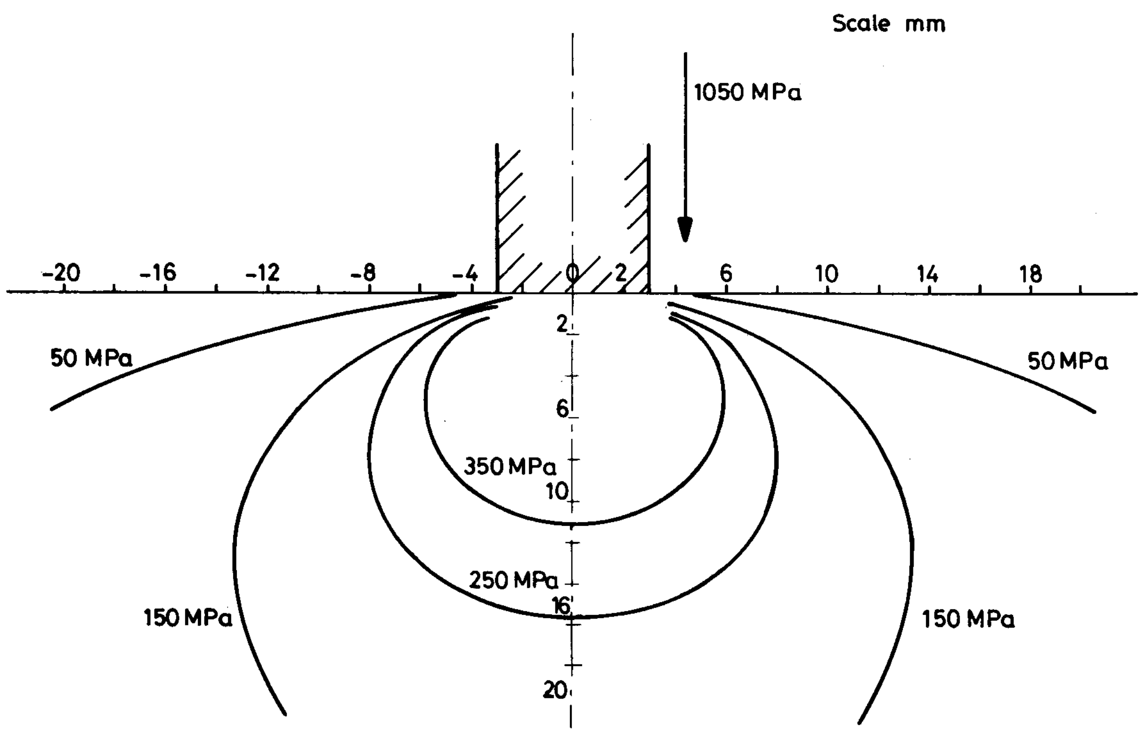

Fig. 5—Plot of maximum principal stress when a pressure of 1050 MPa was applied at the contact area normal to the rock surface —a loading condition that was not sufficient to cause a large rock chip to form

material, and

(2) the contact area between the bit and the rock was loaded uniformly.

It has been shown that most hard rocks exhibit ideal elastic behaviour up to a critical stress level[6], and therefore the first of these assumptions was regarded as valid. The stresses immediately adjacent to the punch depend on whether a uniform load is applied to the punch or whether the punch generates a constant displacement in the rock. Away from the contact area, the stress fields induced by these two loading conditions become virtually identical[6]. For this reason, and since the actual loading condition when a bit is pressed into a rock specimen probably lies somewhere between these two extremes, only one condition, the former, was considered in this analysis.

A two-dimensional analysis was employed, and the solution derived by Jurgenson[7] was used in the calculation of stresses underneath a punch when a load is applied normal to the surface of the elastic body. The stresses were calculated in the plane of the cutting motion (Fig. 4). Additional stress calculations were made for the situation where a tangential or shear force was applied to the punch parallel to the surface of the rock. The solution derived by Scott[8] was used for these calculations.

The results of this work are presented for two conditions of applied loads:

(a) where a normal force only was applied to the punch, representing the condition created during the indentation tests when a normal force only was applied, and

(b) where both normal and shear forces were applied to the punch, representing the cutting situation when both penetrating and cutting forces were applied to the bit (Fig. 4), and also representing the condition created during the indentation tests when both normal and shear forces were applied to the bit.

The analysis was carried out with different forces applied to the punch. These force values were taken from measurements made during previous experimental work of the forces required to cause rock failure. The experiments on cutting in norite[5] had shown that the maximum bit cutting force was about 150 kN and the corresponding bit penetrating force was about 220 kN. Since the solutions of the equations used in the calculation of the stresses in the rock require that the applied loads are expressed in terms of the force per unit area and the area of the bit wearflat was 210 mm^2, these forces represent applied pressures of 714 MPa and 1048 MPa respectively. Similarly, the indentation tests when only a force normal to the rock surface was applied showed that a pressure along the contact area of about 2000 MPa was required to cause a rock chip to form. When both normal and shear forces were applied to the bit in the indentation tests, rock chips were formed with pressures along the contact area of 1480 MPa from the force normal to the rock surface, and 240 MPa from the shear force.

Computer programmes were written for the calculation and plotting of the following:

(i) the stress contours,

(ii) the regions where the induced stresses caused the

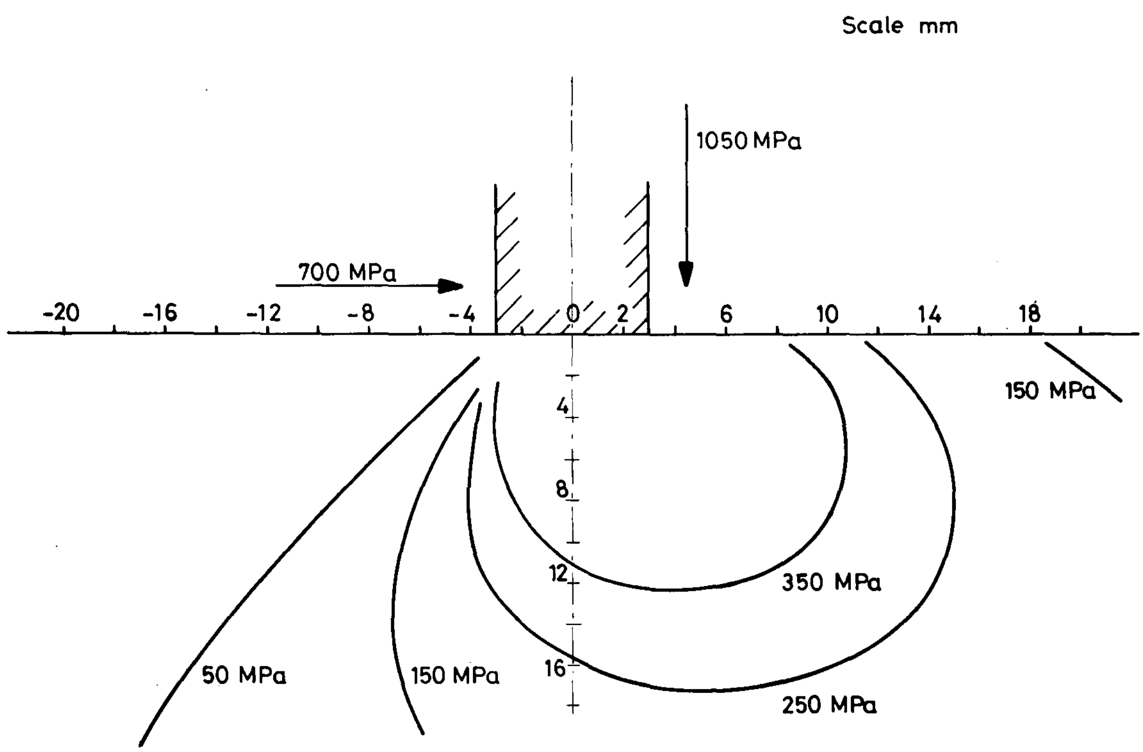

Fig. 6—Plot of maximum principal stress when a pressure of 1050 MPa was applied normal to the rock surface and a pressure of 700 MPa was applied at the contact area parallel to the rock surface — a loading condition sufficient to cause a large rock chip to form

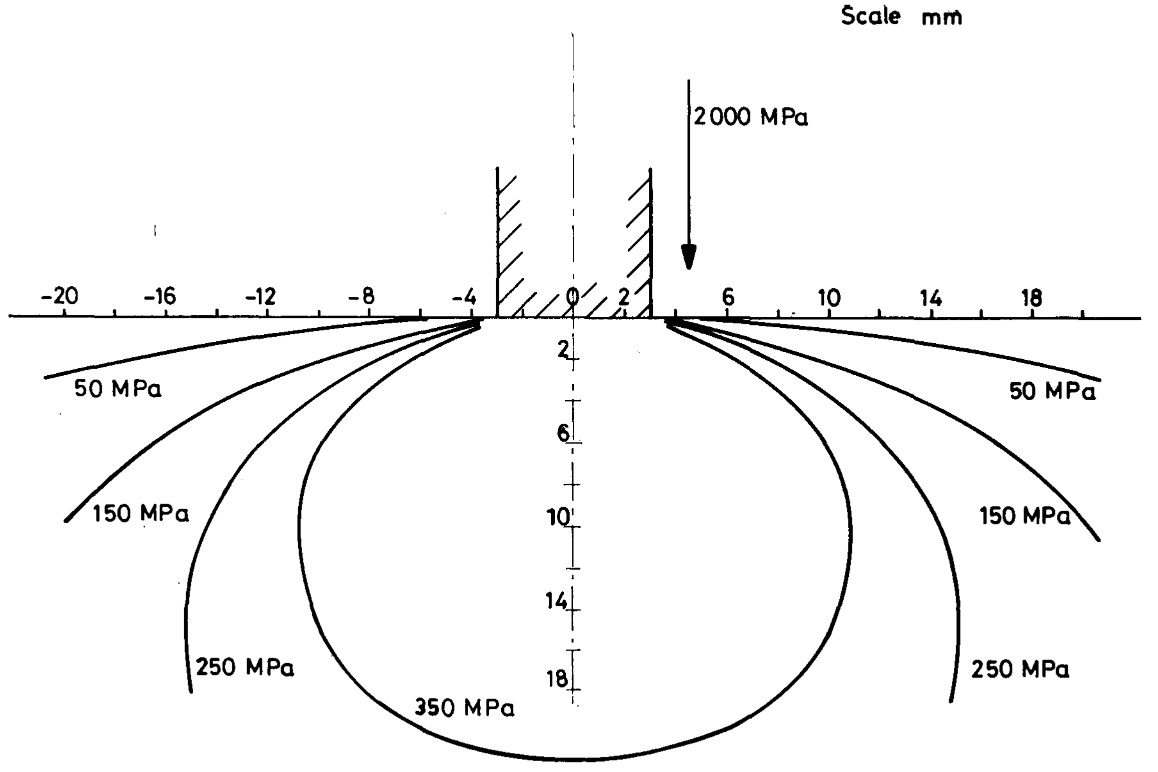

Fig. 7—Plot of maximum principal stress when a pressure of 2000 MPa was applied at the contact area normal to the rock surface — a loading condition sufficient to cause a large rock chip to form

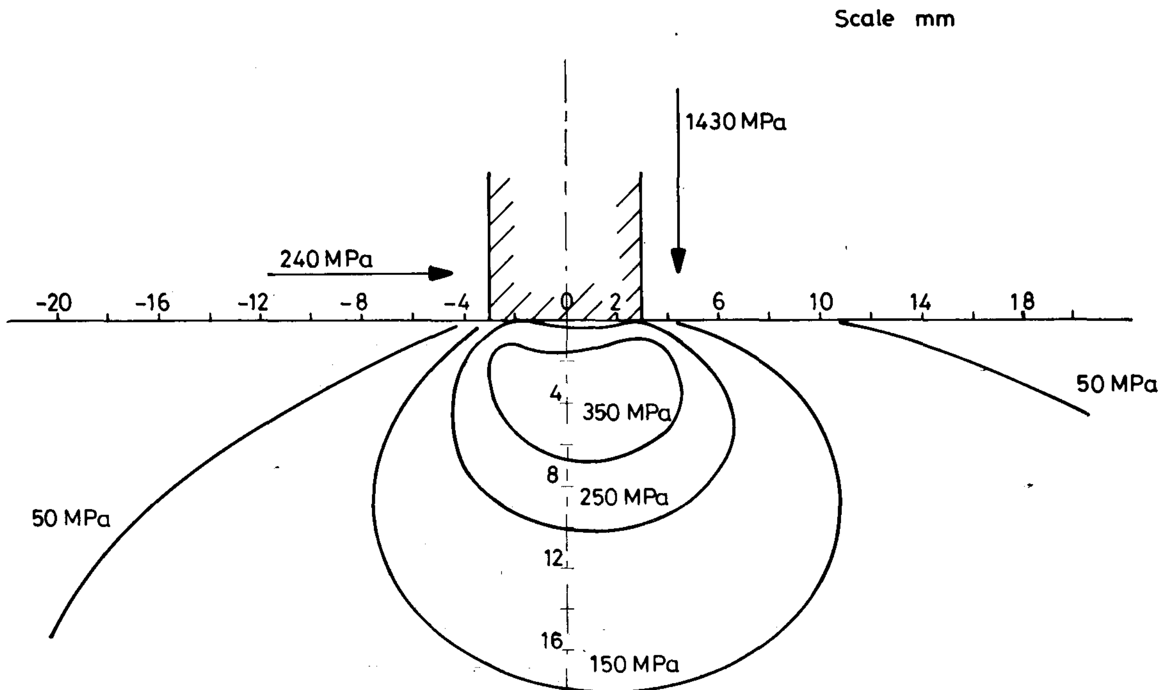

Fig. 8—Plot of maximum principal stress when a pressure of 1430 MPa was applied to the rock surface and a pressure of 240 MPa was applied at the contact area parallel to the rock surface — a loading condition sufficient to cause a large rock chip to form

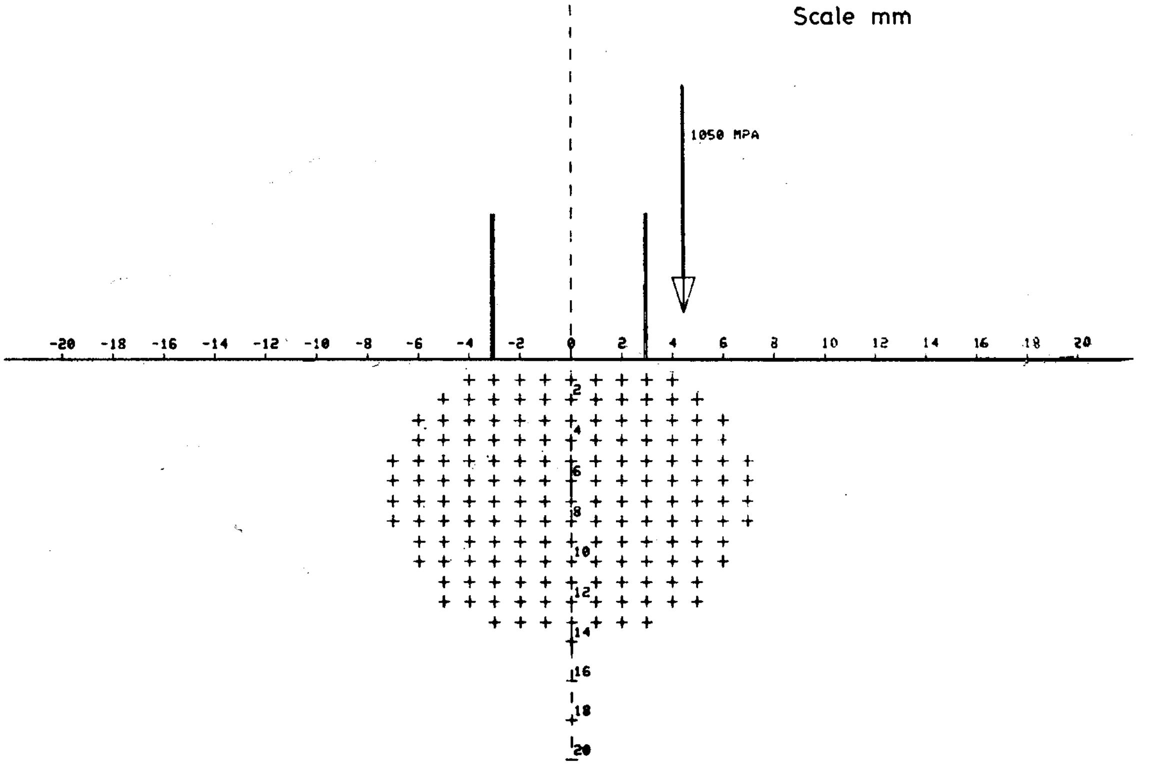

Fig. 9—Region of excess stress when a normal pressure of 1050 MPa was applied to the contact area — a loading condition insufficient to cause a large rock chip to form but producing a zone of crushed rock adjacent to the bit that corresponded roughly with this region of excess stress

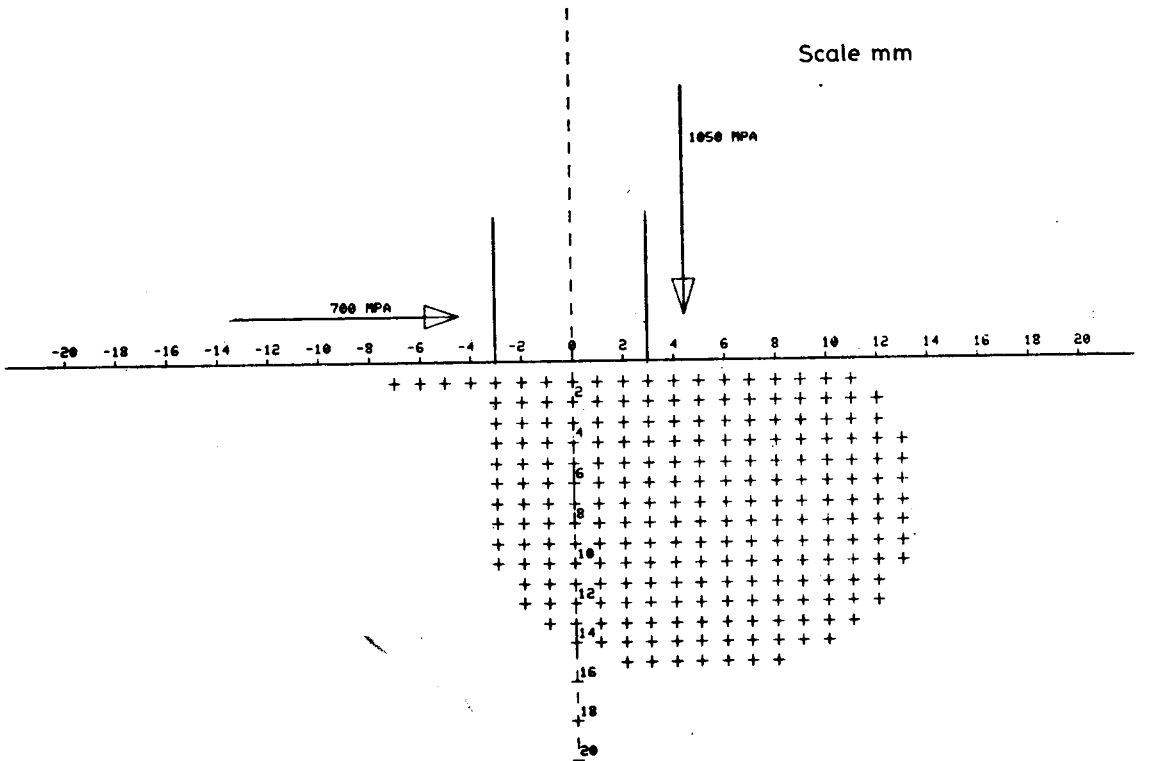

Fig. 10—Region of excess stress when both a normal pressure of 1050 MPa and a shear pressure of 700 MPa were applied to the contact area — a loading condition sufficient to cause a large rock chip to form and producing a region of crushed rock that corresponded roughly with this region of excess stress

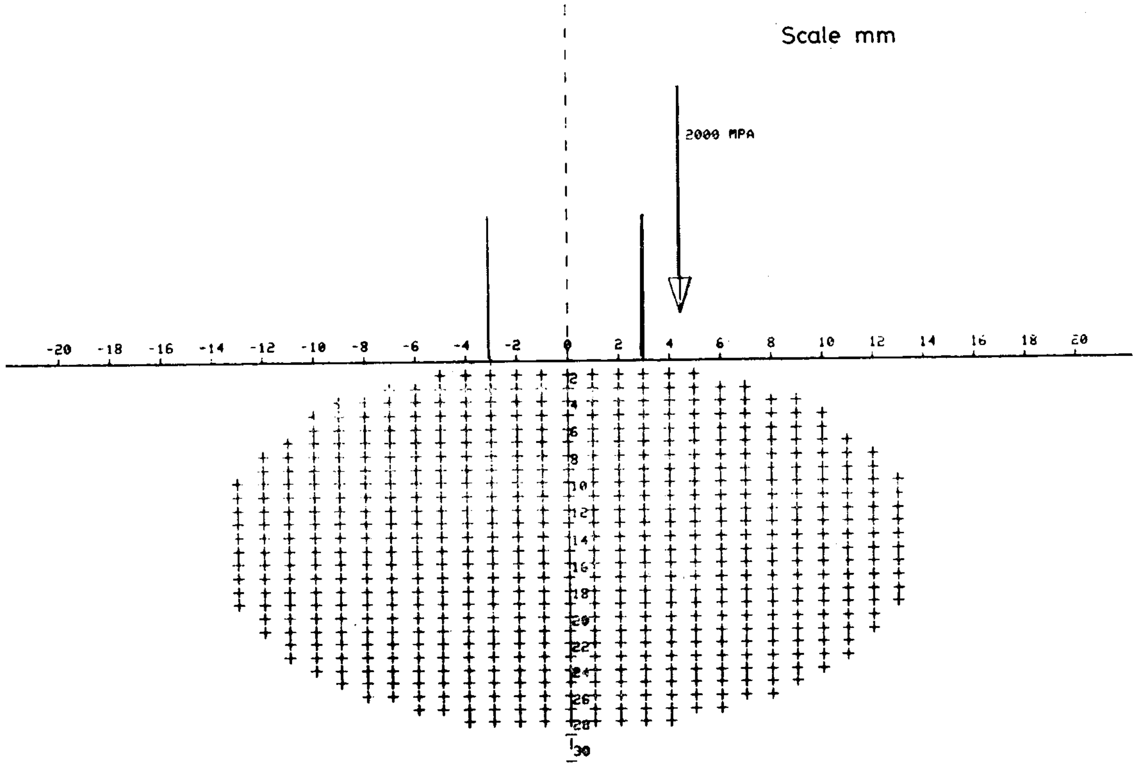

Fig. 11—Region of excess stress when a normal pressure of 2000 MPa was applied to the contact area — a loading condition sufficient to cause a large rock chip to form and producing a region of crushed rock that corresponded roughly with this region of excess stress

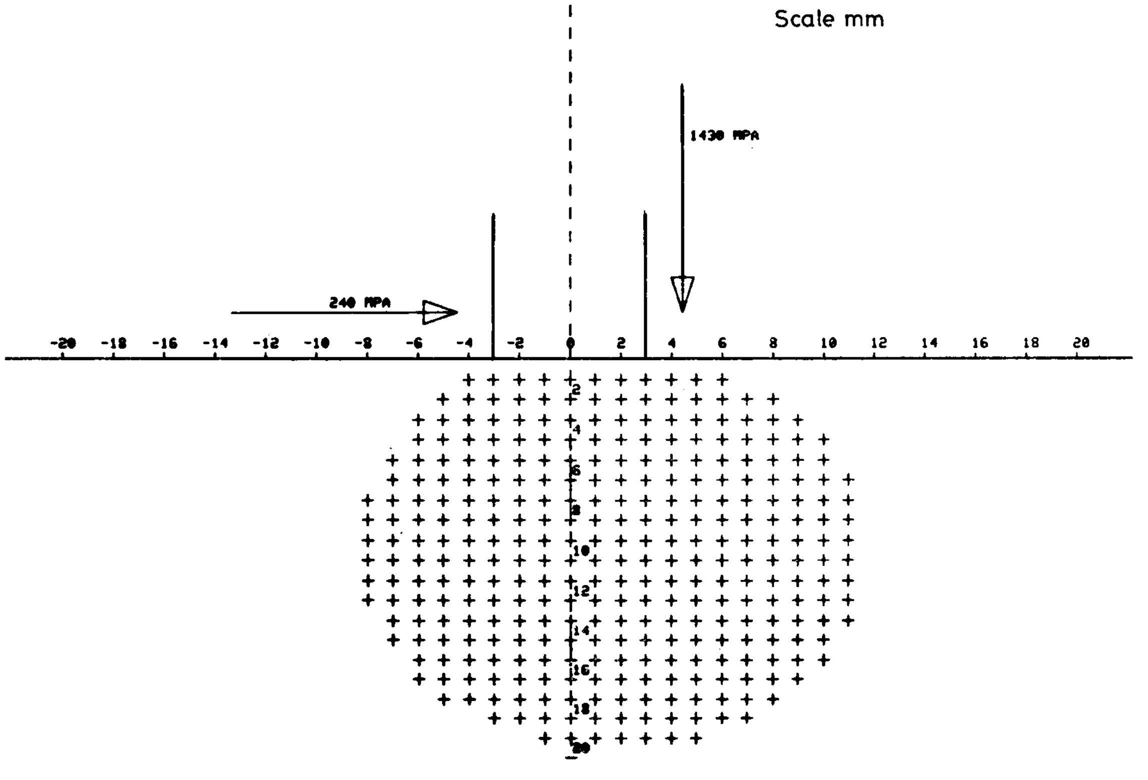

Fig. 12—Region of excess stress when both a normal pressure of 1430 MPa and a shear pressure of 240 MPa, were applied to the contact area — a loading condition causing a large rock chip to form and producing a region of crushed rock that corresponded roughly with this region of excess stress

strength of the rock to be exceeded, termed regions of excess stress, and

(iii) the stress trajectories.

Stress Contours

The equations that were used in the calculation of the stresses are given later in this paper.

The effect of a shear force to the bit on the stress distribution was studied from a series of graphs giving the maximum principal stress and the maximum shear stress. Examples of these graphs are given in Figs. 5 and 6. The applied normal stress was taken as 1050 MPa and the applied horizontal stress was taken as 700 MPa. These graphs show that the stress distributions, which are symmetrical about the axis when the normal force only is applied, are skewed over when the shear force is applied also. This has the effect of inducing high stresses closer to the free surface ahead of the punch.

When different forces are applied to the punch, plots of these stress components (as shown in Figs. 6 and 7) show the following.

(a) A very high stress field is induced in the regions both immediately underneath the punch and on either side of the punch when no shear force component is applied (Fig. 7).

(b) These stresses are reduced considerably in the regions underneath and behind the punch when a relatively small shear force is applied to the punch (Fig. 8). However, because the application of a shear-force component causes the stress field to be skewed over, this results in high stresses being induced ahead of the punch (Fig. 8).

Regions of Excess Stress

It can be shown that, in the regions immediately underneath and in front of the punch, the elements in the rock mass generally are in a state of triaxial compression[9]. Therefore, for an assessment of the significance of the state of stress at a given point, it is necessary to calculate the damage that these induced stresses would have caused to the rock at that point. This assessment was achieved by the computation of the Mohr circle from the principal stresses at that point. When this circle fell outside the Mohr envelope for norite, the rock at that point was considered to have exceeded the strength of the rock. Details of the equations used in the establishment of this criterion are given later.

Figs. 9 to 12 show these regions of 'excess stress' for different applied loads. It is evident from these diagrams that the extent of these regions ahead of the leading face of the punch is similar for the different load conditions, being some 11 mm to 13 mm ahead of the centre line of the punch. On the other hand, underneath the punch these regions extend from 11 mm to 28 mm below the contact area, depending on the loads applied.

These regions of excess stress correspond fairly well to the zones of crushed, powdered rock that were observed underneath the bit after the indentation tests.

Stress Trajectories

Observations made by other research workers[10] have indicated that brittle rock tends to fracture along the line of the maximum principal stress. To investigate whether this phenomenon occurred when rock fracture was caused by an indentation process, stress trajectories were plotted in the region underneath the punch. The equations from which these trajectories were calculated are given later.

A comparison of the graphs showing the stress trajectories for the two loading situations (Figs. 13 and 14) shows that the area behind the punch, along the y axis in the negative direction, is distorted when the shear force is applied. Ahead of the punch, along the y axis in the positive direction, the stress trajectories are altered only slightly by the superimposition of the shear force. Therefore, the expected crack orientation and mode of failure of the rock would be similar whether only a normal force was applied or whether both normal and shear forces were applied to the rock surface. However, since the stresses near to the free surface ahead of the punch are calculated to be higher when the shear force is added, cracks would be expected to develop with lower values of the normal force in this situation.

Background Theory for the Mathematical Analysis

Stress Contours

The limiting values of the stress components in an elastic body underneath an indentor when a normal force is applied to the punch are given by

$$\sigma_x = \frac{P}{\pi}\{\alpha+\sin\alpha\cos(\alpha+2\delta)\} \quad \ldots\ldots \quad (1)$$

$$a_y = \frac{P}{\pi}\{\alpha-\sin\alpha\cos(\alpha+2\delta)\} \quad \ldots\ldots \quad (2)$$

$$\tau_{xy} = \frac{P}{\pi}\{\sin\alpha\sin(\alpha=2\delta)\} \quad \ldots\ldots\ldots \quad (3)$$

where P is the normal force per unit area and α and δ are as defined in Fig. 15 (after Jurgenson[7]).

In the calculation of the stress components when both normal and shear forces are applied to the punch, the solution given by Jurgenson[7] is combined with the solution for the shear force only given by Scott[8].

This yields

$$\sigma_x = \frac{P}{\pi}\{\alpha+\sin\alpha\cos(\alpha+2\delta)\} + \frac{T}{\pi}\{\sin\alpha\sin(\alpha+2\delta)\} \quad \ldots\ldots\ldots \quad (4)$$

$$\sigma_y = \frac{P}{\pi}\{\alpha-\sin\alpha\cos(\alpha+2\delta)\} + \frac{T}{\pi}\{\log_e\left({R_1}^2/{R_2}^2\right)-\sin\alpha\sin(\alpha+2\delta)\} \quad . \quad (5)$$

$$\tau_{xy} = \frac{P}{\pi}\{\sin\alpha\sin(\alpha+2\delta)\} + \frac{T}{\pi}\{\alpha-\sin\alpha\cos(\alpha+2\delta)\} \quad \ldots\ldots \quad (6)$$

where T is the shear force per unit area and R_1 and R_2 are as defined in Fig. 15.

The principal stresses are given by

Maximum principal stress $\sigma_1 = \frac{1}{2}(\sigma_x+\sigma_y) + \{{\tau_{xy}}^2+\frac{1}{4}(\sigma_x-\sigma_y)^2\}^{\frac{1}{2}}$ (7)

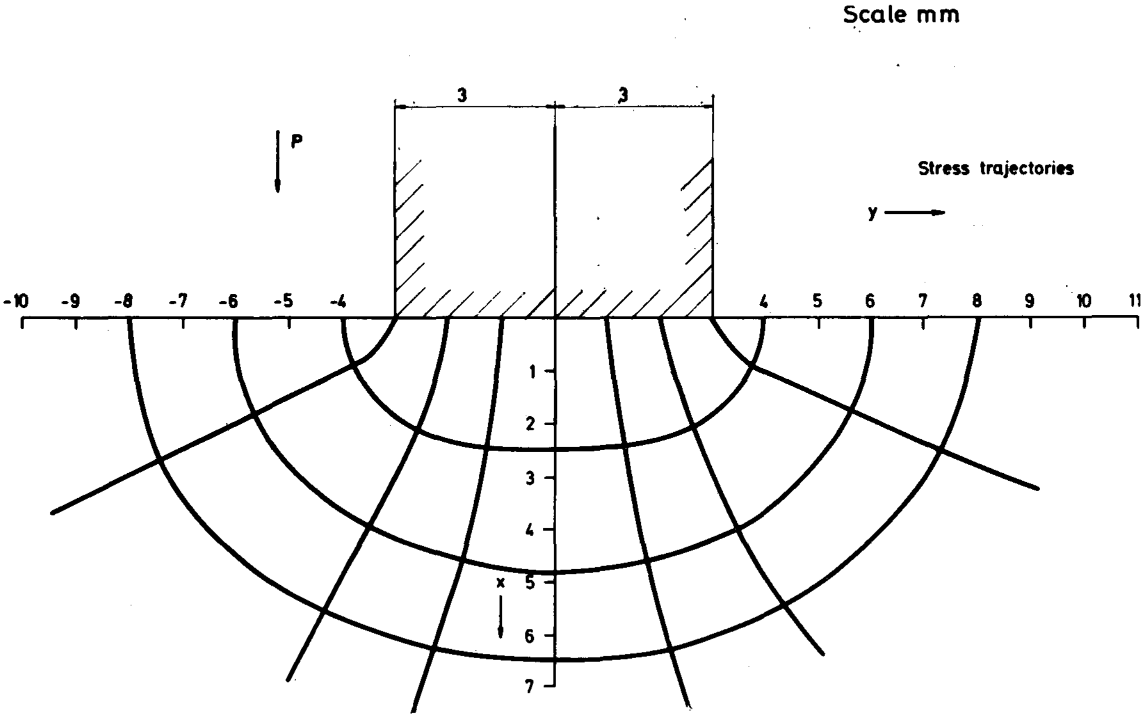

Fig. 13—Stress trajectories when a normal pressure was applied to the contact area

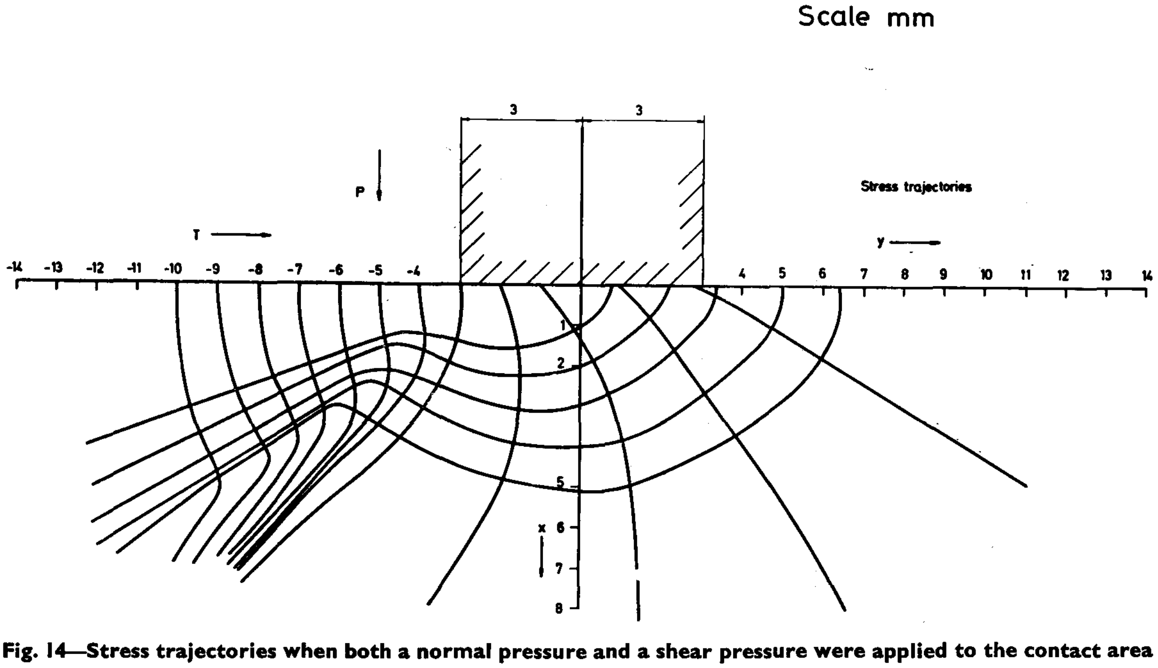

Fig. 14—Stress trajectories when both a normal pressure and a shear pressure were applied to the contact area

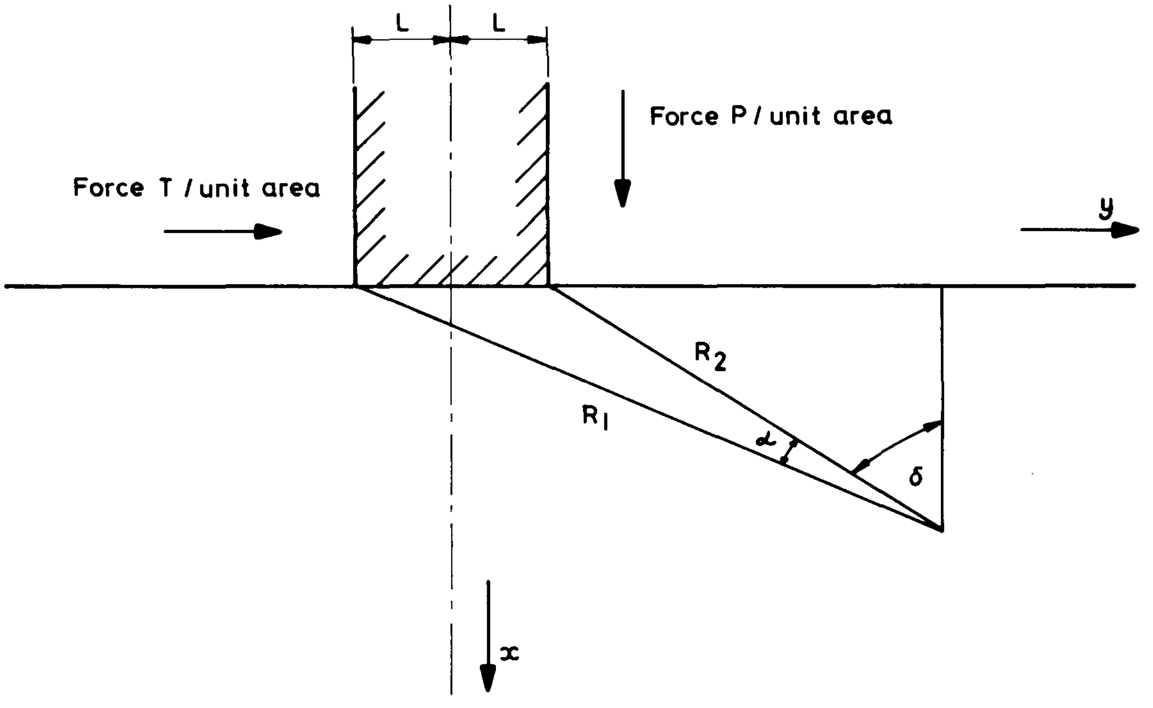

Fig. 15—Nomenclature used in the mathematical analysis

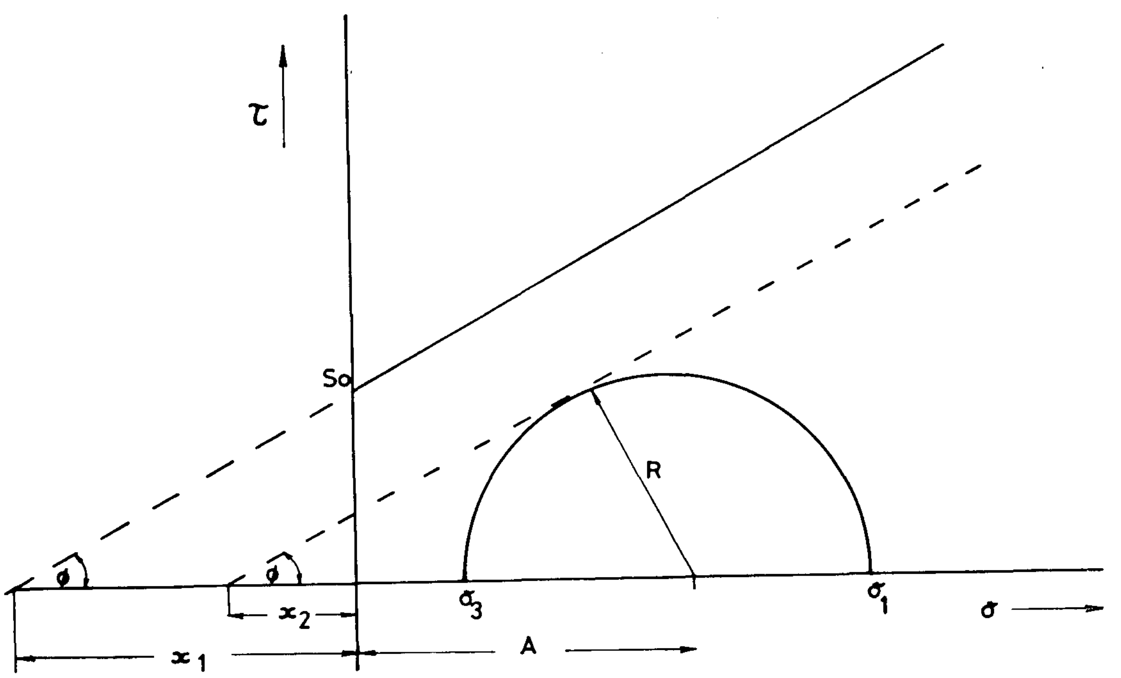

Fig. 16—Criterion used in the determination of regions of excess stress

Minimum principal stress $\sigma_3 = \frac{1}{2}(\sigma_x + \sigma_y) - \{\tau_{xy}^2 + \frac{1}{4}(\sigma_x - \sigma_y)^2\}^{\frac{1}{2}}$ (8)

Maximum shear stress $\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_3)$ . . . . . (9)

(after Jaeger and Cook[11]).

Regions of Excess Stress

The Mohr envelope for norite was plotted from a large number of triaxial tests performed with specimens of this rock (Fig. 16).

If the intercept of the Mohr envelope with the y axis is given by So and the angle that the linear portion of this curve makes with the x axis is given by ϕ (Fig. 16), linear extrapolation of this curve gives the intercept on the x axis as

$$x_1 = -\frac{So}{\tan\phi}.$$

If the principal stresses are calculated at a point in the rock underneath the punch from equations (1) to (3), the Mohr circle describing the state of stress at that point can be plotted. The rock is defined as failed if the circle either crosses or touches the envelope.

From Fig. 16 the intercept on the x axis from the Mohr circle is given by

$$x_2 = \frac{\frac{1}{2}(\sigma_1 - \sigma_3)}{\sin\phi} - \frac{1}{2}(\sigma_1 + \sigma_3)$$

$\therefore$ If $x_2 < x_1$, the rock has not failed.

But, if $x_2 \geqq x_1$, the rock has failed.

This criterion was used to define regions of excess stress underneath the punch.

Stress Trajectories

It can be readily shown that, if the principal axes make an angle θ with the axes that were used in the calculation of given stress values,

$$\tan 2\theta = \frac{2\,\tau_{xy}}{\sigma_x - \sigma_y}$$

(Jaeger and Cook[11]).

To determine the stress trajectories underneath a punch, values of σ_x, σ_y, and τ_{xy} were calculated from equations (1) to (3) substituted into this equation.

Conclusion

It was shown that the application of a relatively small transverse force, in addition to the penetrating force, to an indentor causes a severe distortion in the stress field in the rock under the indentor. As a consequence, the penetrating force necessary to cause the rock to break is greatly reduced with the addition of a small transverse force. This explains why the penetrating force that is necessary for cutting rock with a drag bit is lower than the penetrating force that is necessary to punch into rock. This also has important implications for roller cutters, and suggests that lower penetrating forces would be required if transverse forces were applied to the indentors whether they be buttons or discs.

Acknowledgement

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