Mathematical unification of an equation for solute recoveries in countercurrent decantation

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SYNOPSIS

This paper simplifies and provides a strict mathematical proof for an equation developed by R. J. Woody for the calculation of solute recoveries in countercurrent decantation. The equation offers a convenient method for the optimization of flowsheets involving a simple countercurrent-decantation circuit.

SAMEVATTING

Hierdie verhandeling vereenvoudig en verskaf 'n streng wiskundige bewys vir 'n vergelyking wat R. J. Woody ontwikkel het vir die berekening van die herwinning van opgeloste stowwe in teenstroomafgieting. Die vergelyking bied 'n gerieflike metode vir die optimalisering van vloeidiagramme wat 'n eenvoudige teenstroomafgietkring behels.

Introduction

The theoretical recovery of solute by a countercurrent decantation system (CCD) can be calculated by the simultaneous solution of material-balance equations for each of the stages involved if it is assumed that the concentration of solute (on a solid-free basis) in the underflow stream is equal to the concentration in the overflow stream discharged by the same thickener (i.e., an assumption of perfect mixing). However, when the number of stages involved is large, such a procedure of stage-to-stage calculation becomes very tedious and time-consuming. Generally, the problem is to find an optimum flowsheet, and the calculations must be repeated many times for different possible flowsheets.

The problem can easily be solved by the use of a computer and by graphical methods[1]. However, an equation has recently been suggested and discussed[2] for the simplest case of CCD:

$$C_n = \frac{C_o - C_w}{1+R+R^2+\ldots+R^n} + C_w, \quad \ldots\ldots \quad (1)$$

where

C_o = concentration of solute in the feed slurry on a solid-free basis

C_n = concentration of solute in the washed slurry discharged by stage n on a solid-free basis

C_w = concentration of solute in the wash liquor

n = number of CCD stages

$R = \frac{Q_w}{Q_s}$, wash ratio

Q_w = throughput of wash liquor

Q_s = throughput of feed slurry on a solid-free basis (i.e., throughput of liquid in the slurry stream)

This equation represents the quickest and most convenient way of solving the problem for the simple CCD flowsheet shown in Fig. 1, and is based on operation under steady-state conditions and the following assumptions.

(a) The concentration of the underflow is the same for all thickeners.

(b) The feed slurry and the underflow are of a similar concentration.

(c) The influent streams to all the thickeners are perfectly mixed.

(d) Leaching is completed before the CCD circuit.

(e) The wash liquor is fed only to the last thickener (Stage n).

Equation (1) was presented as an assumption without any mathematical proof. It was demonstrated to hold good for 1, 2, 3, and 4 stages by the simultaneous solution of the material-balance equations in a manual stage-to-stage procedure that is assumed to hold good for any number of stages. It was then checked by a comparison between the solution obtained for 11 stages by use of the equation and the computerized solution.

The object of the present paper is to provide a strict mathematical proof for Equation (1), as well as a more simplified form for that equation.

Mathematical Proof

Whereas the flowsheet given in Fig. 1 is the simplest flowsheet for CCD, it does not represent an ideal case but rather a case that is frequently encountered in practice. An equal concentration of underflow in all the thickeners will be aimed at so that no better assumption can be made at the design stage.

Perfect mixing represents an economically favourable design, and is normally closely approached because it is far more economical to improve mixing conditions, if necessary, than to invest in additional CCD stages.

Often leaching in the CCD circuit proceeds to only an insignificant extent, apart from special cases when the recovery of solute and the leaching are integrated in the form of thickener–pachuca stages[3]. However, this entails an entirely different type of flowsheet.

Apart from special cases of complex flowsheets, there is only one wash-liquor stream that is fed to the last stage (Stage n). It should be noted that the wash liquor is not assumed to be entirely free of solute because it is common practice to recycle the effluent of subsequent process stages (solvent extraction, continuous ion exchange, etc.) as the wash liquor.

The only assumption that is not met in most cases is

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that of (b). The feed slurry is often fed at a concentration lower than the underflow concentration as shown in Fig. 2. However, this does not limit the usefulness of Equation (1) because it still holds good for Stages 1 to n, for which the flowsheet is identical with that of Fig. 1. Then Equation (2), the equation for the overall material balance, will form with Equation (1) a set of equations with C_o and C_n as the only unknown, and thus the values of both these variables can be easily calculated.

$$Q_f C_f + Q_w C_w - Q_s C_n = C_o(Q_w + Q_f - Q_s) \quad . \quad . \quad . \quad . \quad (2)$$

Since Equation (1) includes a geometrical series, the proof for this equation will be simplified if the series is replaced by its sum

$$C_n = \frac{R-1}{R^{n+1}-1}(C_o - C_w) + C_w. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (3) is much more convenient for routine calculations. The mathematical proof for Equation (3) can be provided by application of the technique of mathematical induction. The essence of this technique is that the equation holds in general for any n if it can be shown that it holds for $n=1, 2, 3, 4 \ldots$ and, if it holds for $(n-1)$, it can be proved to hold for n as well.

Equation (3) was shown to hold for $n=1, 2, 3$, and 4 by manual stage-to-stage calculations[2]. It thus remains to be shown that, if it is assumed to hold for $(n-1)$ stages, it will hold for n stages as well.

Consider a CCD cascade of $(n-1)$ stages to which another stage, Stage 1, is added to make a cascade of n stages (Fig. 3).

If Equation (3) holds good for the cascade of $(n-1)$ stages, it can be written as

$$C_n = \frac{R-1}{R^n-1}(C_1 - C_w) + C_w. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

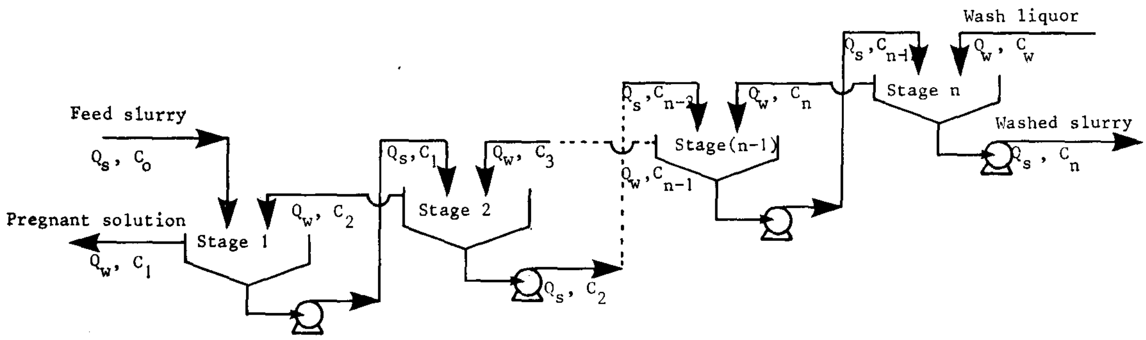

Fig. 1—A simple CCD flowsheet

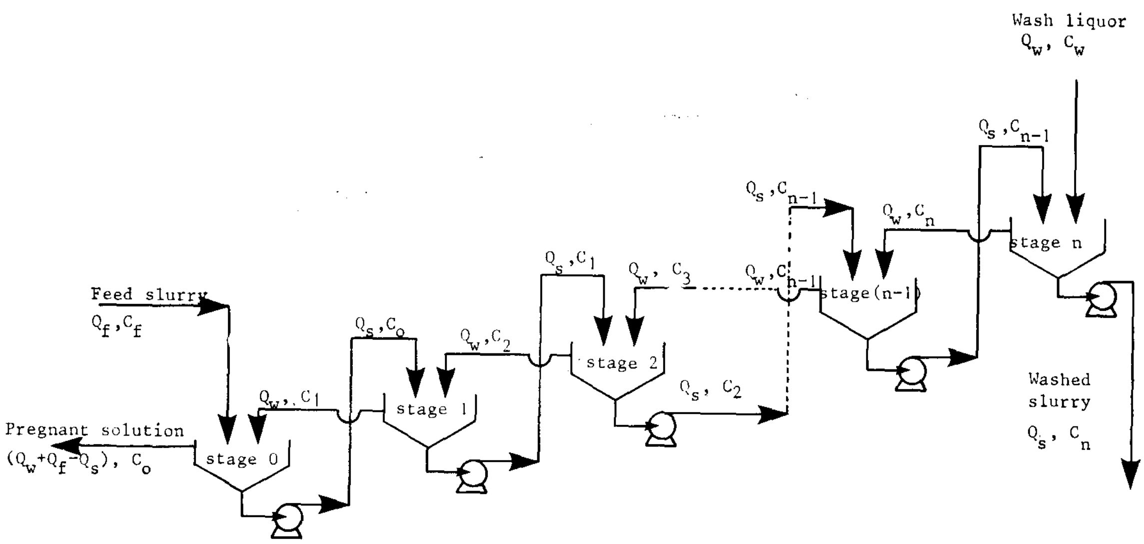

Fig. 2—A common CCD flowsheet

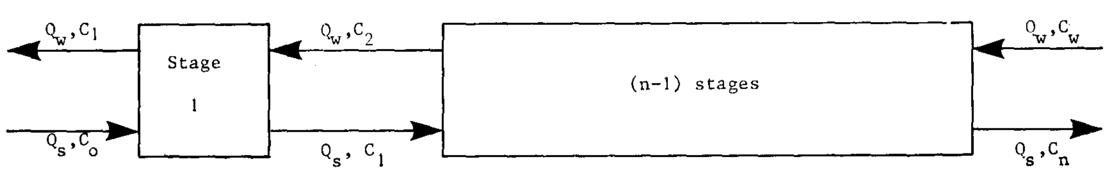

Fig. 3—An illustration of the mathematical proof

An overall material balance over the entire cascade, including Stage 1, yields

$$C_1 = C_w + \frac{C_o - C_n}{R}. \quad \ldots \ldots \ldots \ldots \quad (5)$$

By substitution of Equation (5) in Equation (4),

$$C_n = \frac{(R-1)\ (C_o - C_n)}{(R^n - 1)\ R} + C_w, \quad \ldots \ldots \ldots \quad (6)$$

and, by rearrangement of Equation (6),

$$C_n \frac{R^{n+1}-1}{R^{n+1}-R} = \frac{R-1}{R^{n+1}-R} C_o + C_w = \frac{R-1}{R^{n+1}-R}(C_o - C_w) + \frac{R^{n+1}-1}{R^{n+1}-R} C_w. \quad \ldots \ldots \ldots \quad (7)$$

Division of Equation (7) by the term $(R^{n+1}-1)/(R^{n+1}-R)$ yields Equation (3), which was what had to be proved.

Conclusion

A mathematical proof of the equation for the calculation of solute recovery in a simple multi-stage CCD circuit has been provided by application of the technique of mathematical induction. In addition, the equation has been expressed in a more convenient form. This equation offers a convenient method by which common flowsheets involving a simple CCD circuit can be optimized.

References

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