The use of linear programming in the short-term planning of stoping production in gold mines

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SYNOPSIS

This paper describes the application of linear programming to the monthly 'set up' planning of stopers' sections on two gold mines.

On one of these mines, the purpose of the exercise was to maximize the total stoping production (measured in centares per month). On the other mine, the maximum stoping contribution (rands per month) was required. Maximization was to be achieved through improved planning, independently of the various other productivity-improving activities that were taking place at the same time.

The results to date show that, when linear programming is used, an average increase of about 9 per cent in stoping production (or contribution) can be obtained for these two mines.

SAMEVATTING

Hierdie referaat handel oor die toepassing van die lineêr programmeringstegniek op die maandelikse 'opset'-beplanning van die afbouafdelings aan twee goudmyne.

Die doel van die oefening aan een van die myne was om die totale afbouproduksie (in sentaar per maand gemeet) te maksimeer. By die ander myn is maksimum afboubydrae (rand per maand) verlang. Maksimering sou deur verbeterde beplanning verkry word, onafhanklik van ander bedrywighede ter verhoging van die produktiwiteit wat op dieselfde tydstip aan die gang was.

Resultate tot op hede toon dat wanneer lineêr programmering gebruik word, 'n gemiddelde toename van ongeveer 9 per cent in afbouproduksie (of -bydrae) vir hierdie twee myne verkry kan word.

Introduction

It is well known in the gold-mining industry that the productivity of stope production workers is the result of many factors, such as temperature, humidity, supervision, experience and training, gang stability and ethnic composition, stope width, dip, faulting, and so on. Lawrence[1] identifies thirty-eight such factors, which, between them, could explain less than 70 per cent of the variance in the observed performances of stoping gangs.

The maximization of stoping output, whether in terms of centares, profit, or contribution, requires that input–output relationships should be reasonably certain. For the purposes of routine planning, there is a further requirement: that these relationships should be simple to express and manipulate. So it would seem that the maximization of stoping output, through routine planning of optimized stoping labour would not be practicable. However, an approach adopted by Corrie[2] showed that, at the level of the mine overseer and over time-spans of only a few months, strong relationships could be drawn between the concentration of stoping labour per metre of face worked and the consequent rate of advance of that face. Fig. 1 is typical of such a relationship. It was found also that the relationships between various mine-overseers' sections were very often significantly different, leading to the use of the term *characteristic curves* to describe them.

From Corrie's analysis it would seem that, as labour is concentrated in a stoping section, so should the rate of face advance increase, but with diminishing returns for each increment of labour.

The approach illustrated in Fig. 1 was applied on the two mines with which this paper is concerned, not only at the level of mine-overseer's section, but also at the level of stoper's section. A type of relationship similar to that of Fig. 1 emerged in almost every case.

These characteristic curves can be used to test the effects, on the rates of face advance, of various concentrations of stoping labour (per metre of face worked) in various stoping sections. Since the product of face advance rate and face length worked provides the centares of production, it is clear that any variation in the 'input' of labour and face length will produce a fairly predictable variation in centares 'output'. Hence, the means exist with which optimum levels of inputs can be determined for the most probable maximum output, provided that the curves are kept up to date and that planning is confined to periods not exceeding 3 months or so from the most recent up-dating.

Complicating factors, such as grade limits, materials and ore-handling capacity limits, labour-mobility limits, and the different stoping costs applying to the various sections, makes it impracticable to attempt optimization without the assistance of a computer-based optimization procedure, and for this reason the linear programming (LP) technique was used.

Use of the LP Technique

The Problem

For this illustration, it is assumed that a mine has two mine overseer's sections, A and B; 7,5 man-hours of labour are required to produce 1 centare in section A, and 7,0 man-hours of labour are required per centare in section B; 5250 man-hours of labour are available. The ore-handling facilities that serve the two sections can take 600 centares from section A and 900 centares from

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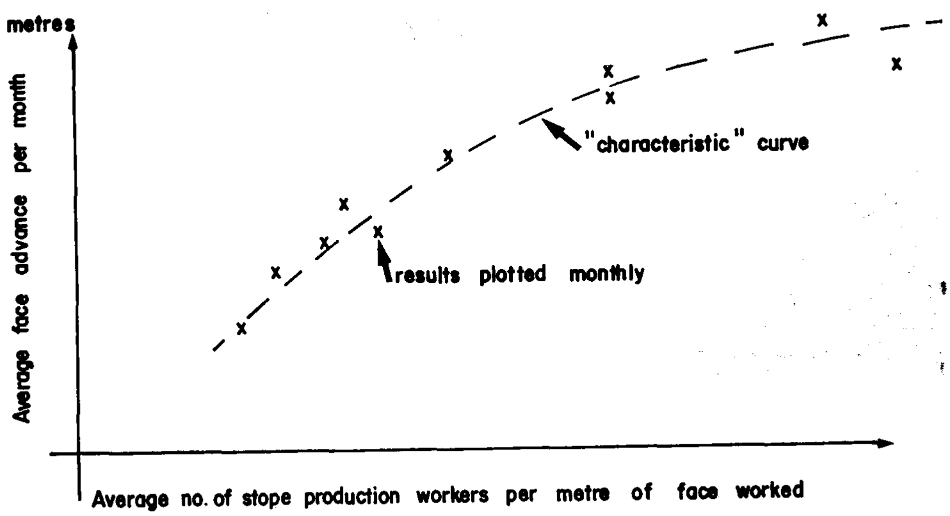

Fig. 1—Relationship between the concentration of stoping labour per metre of face worked and the consequent advance rate of that face

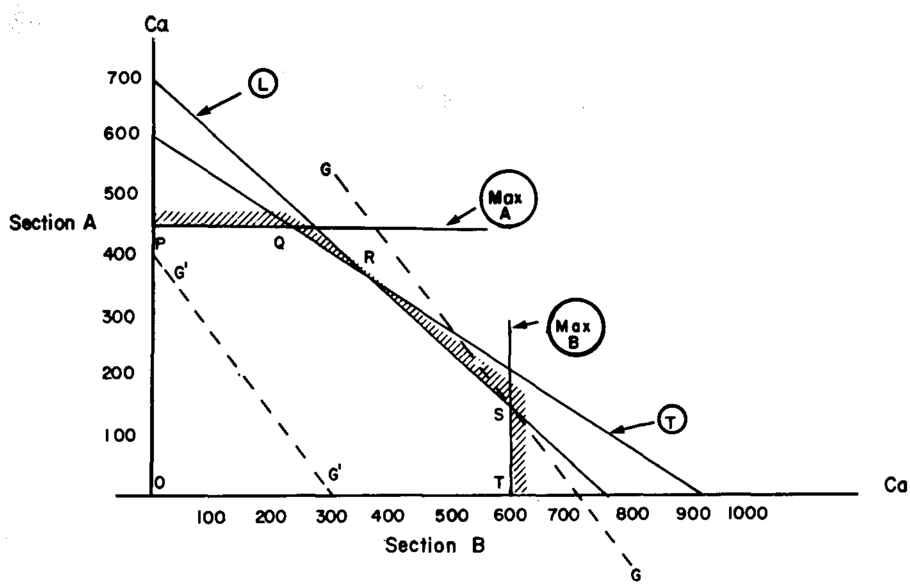

Fig. 2—Graphical solution of the problem used as illustration

section B, or any linearly proportioned combination of these limits. For example, the tramming system could handle 300 centares from section A plus 450 centares from section B. Two further constraints are that the production from section A must not exceed 450 centares and section B's centares must not exceed 600. The reef in section A yields 300 units of gold per centare, and in section B it is 400 units per centare. The objective is to maximize the production of gold.

The Solution

Because only two unknown variables are involved, a graphical solution is used. This is shown as Fig. 2, and it is built up as follows. The axes 'Section A' and 'Section B' refer to the values A and B respectively, denoting centares in each section. For the 'labour' constraint, straight line L represents the relationship.

$7{,}5A+7{,}0B \leqslant 5250.$

Straight line T represents the tramming capacity constraint. Lines $MaxA$ and $MaxB$ represent the upper limits of centares for each section.

Examination of the constraints shown in Fig. 2 indicates that no solution that satisfies these constraints can exist outside the region O-P-Q-R-S-T-O, which is known as the *feasible region* of the solution space.

To discover that point of the feasible region that offers a maximum value to the *objective function,* which in this case is 'units of gold', the objective function is plotted as a constraint equal to some value$=k$.

Hence, $300A+400B=k$.

This generates a family of straight lines as k varies, each being a line of constant profit. These straight lines are parallel to one another, as is shown in Fig. 2 by the dotted lines G'-G' and G-G.

In this example, as the objective function value (k) is increased, the constant-profit line moves away from the

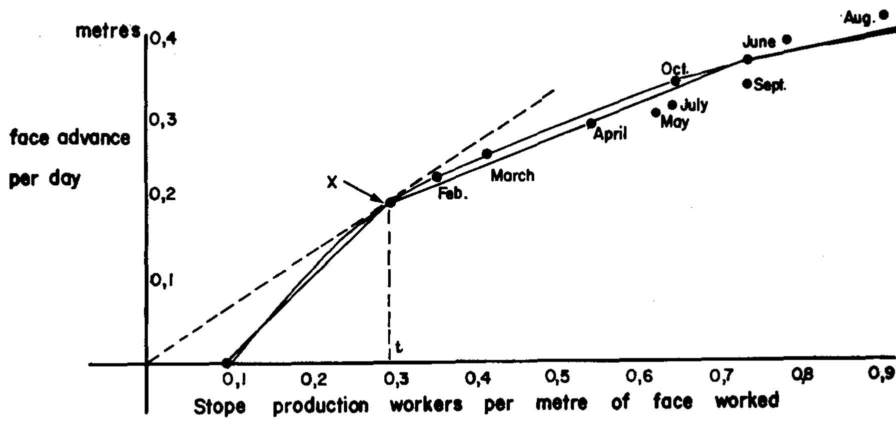

Fig. 3—The characteristic curve for an actual mine overseer's Section

origin. Eventually the constant-profit line leaves the feasible region altogether. The point from which this happens indicates the optimum solution to the problem, since no higher value of the objective function can exist in the feasible region. In the example, this is point S, corresponding to 600 centares from section B and about 140 centares from section A.

Input–Output in Stoping

As stated previously, there is a distinct relationship between the concentration of stoping labour per metre of face worked and the consequent rate of advance of that face. The relationship differs as between stoping sections, but, within any given section, it remains comparatively consistent and characteristic of the section over periods of several months. It might be thought that seasonal variations in temperature could well invalidate this statement. However, analysis on a monthly basis of centares produced per stope production worker shows only a slight reduction in summer, and the most significant effect on productivity is the variation of stope production workers per metre of stope face worked, which is fully taken into account by the characteristic curves, an example of which appears in Fig. 3. This diagram shows a straight-line approximation to the characteristic curve for a particular mine overseer's section on one of the gold mines concerned.

It was found necessary in most cases to use a four-segment straight-line approximation, but the number of segments is not of material importance. The intercept on the 'labour concentration' axis was determined from a simple regression using monthly data from mine overseer's sections over the period of one year. The dotted line through the origin is tangential to the characteristic curve at point X, signifying that, for this mine overseer's section, the corresponding labour concentration, t, would give rise to a maximum value of the index 'centares per unit time per stope production worker (SPW). This is because the average centares per SPW for any given concentration of labour per metre of face is the slope of the line drawn from the origin O to the corresponding point on the characteristic curve. The point X is defined by the average productivity line when it has maximum value, i.e. when it is tangential to the curve.

For a gold mine there are many such sections, each characterized by its own 'curve'. Because of fluctuations in face length and of labour supply, it is not always possible to plan all sections at their optimum points t. Even if such fluctuations could be controlled, grade constraints, materials-handling capacities, and rock-mechanics limitations would still complicate the issue, and it becomes necessary to formulate a model of the mine suitable for solution by linear programming on a computer.

Model for Allocating Face Length and Labour

The objective is the maximization of output in terms of centares per unit time. However, other objective functions, such as profit maximization, can be specified.

The main constraints in the model are

- labour availability and mobility limits,
- ore-handling and mill capacities,
- face lengths available, and
- upper and lower limits on acceptable average grade for the mine.

These constraints are supplemented by other 'structural' constraints that 'drive' the LP model. The actual detailed mathematical formulation is given in the Addendum.

Use of LP Solutions

The solution obtained from the linear programme contains three items of major importance: the optimum values of the unknown variables, the value of the objective function, and a 'range' report.

Optimum Values of Unknown Variables

For each stoping section within the prescribed limits (or constraints as given earlier), the optimum values of unknown variables show the optimum face length that should be planned and the optimum number of stope labourers.

These results are made available to planning and senior production personnel on each mine, who can use them as a guide when planning the various stoper's sections. However, managers and planners are free to disregard the optimum solution if they see fit, since it is believed that local knowledge and judgement are much more important than the computer's ability to perform calcu-

TABLE I

EXAMPLE OF A 'RANGE' REPORT

PLANNING SHEET FOR OPTIMUM STOPE FACE, LABOUR, AND CONTRIBUTION

MINE........................ MONTH........................ MAXIMIZING Ca/SPW-SHIFT (RESULT)........................

Section or contract	Optimum plan		Expected		Limits observed by the LP						Factors limiting an improved result	Improved value of limiting factor	Increase to value of result	Labour diverted to/from
					Face metres		Centares		Labour					
	Face metres	Stope labourers	Centares per shift	Contribution rand per shift	Max	Min	Max	Min	Max	Min				
1	55	34	20	2 388	55	40	50	20	40	30	Face metres	67	0,93 Ca/shift	
2	120	67	50	6 888	120	100	56	40	80	60	Face metres	135	1,14 Ca/shift	From section 3
3	88	77	40	5 423	100	70	40	20	80	70	Centares	46	0,38 Ca/shift	
Sub-total	263	178	110	14 699										
4	307	231	120	35 500	307	250	140	90	250	200	Face metres	538	7 Ca/shift	
5	292	144	100	30 000	292	200	120	70	150	120	Face metres	352	3,5 Ca/shift	
Sub-total	599	375	220	65 500			220				Centares	221	Negligible	
6	425	136	91	15 363	425	375	100	70	150	120	Face metres	464	2,3 Ca/shift	
7	451	189	115	19 325	451	400	125	80	190	170	Face metres	488	2,3 Ca/shift	From section 12
Sub-total	876	325	206	34 688										
8	30	12	6	1 500	30	20	15	5	15	10	Face metres	73	8 Ca/shift	
9	266	213	62	15 812	266	200	70	50	220	190	Face metres	300	4,5 Ca/shift	
10	416	210	83	22 111	416	350	100	70	220	190	Face metres	555	22,0 Ca/shift	
11	397	229	100	20 150	397	300	100	60	250	200	Face metres	500	9 Ca/shift	
Sub-total	1 109	664	251	59 573			325							
12	169	40	45	2 321	190	150	70	40	50	30				
13	292	105	70	8 080	292	200	100	60	120	90	Face metres	341	4 Ca/shift	From section 12
14	358	104	90	14 160	358	300	100	80	120	90	Face metres	400	7 Ca/shift	
Sub-total	819	249	205	24 561										
15	290	174	134	15 800	380	250	150	100	180	150				
16	319	105	86	11 600	319	250	110	80	110	100	Face metres	200	5 Ca/shift	From section 12
17	175	79	30	6 000	175	150	90	30	80	60	Face metres	200	5 Ca/shift	
Sub-total	784	358	250	33 400			250				Centares	262	4 Ca/shift	From section 12
Total	4 450	2 149	1 242	232 421					2 150	0	Labour	2 262	0,1 Ca/shift per SPB	To/from section 10

lations. Further, it is only by analysis of the differences between the preferences of the mine's planning managers and the computer's solutions that improvements can be (and have been) made to the LP models.

Dependent variables, such as 'expected centares' and 'expected contribution' are also given per section. The figure for 'expected centares' enables the planners to check that the LP is producing realistic results. The 'expected contribution' can be compared with overhead costs to help managements decide when particular sections of the mine are approaching 'unpay' conditions. It amounts, in point of fact, to having differential 'pay limits' for various sections of a mine.

Value of the Objective Function

This shows the conclusion reached by the computer that some given value probably cannot be exceeded unless the constraints are revised. This knowledge at the commencement of a period is important. The management may expect that, because of seasonal variation in the labour supply, the level of production in, say, July 1978 will be much the same as in July 1977. However, if the section characteristics have altered in the interim, and if the mix of face lengths (not to mention total face length) has also altered, a different level of production ought to be expected. The LP optimization will place an upper limit on this level, which can be exceeded only if uncharacteristically easy breaking conditions or 'spurts' in labour productivity are experienced.

Alternatively, if the upper limit as given is unsatisfactory, the management has an early warning that action needs to be taken, and the highlighting of appropriate bottlenecks in the plan is assisted greatly by the LP's 'range' report.

'Range' Report

The 'Range' report shows how the solution would be changed if individual constraints were removed.

Management is given, on the same form that contains the optimum solution, those factors that limit an improved result, the suggested improved value of that limiting factor, the expected change in the value of the objective function if the improved value is allowed, and (if labour is affected) the places to which or from which stope labour should be diverted as a result of the improvement. Table I is an example of such a report.

Costs and Benefits

For a model in which about twenty characteristic curves are represented, the direct cost of an optimization run is about R30. The time involved in the maintenance of such a model and in the preparation of data for each run should be about one day per model, for a numerate person. A model of this size could either represent a large mine taken at the level of mine overseer, or a large underground manager's section taken at the level of 'stoper'.

The benefits are very high. The results to date indicate that an increase in total centares or total contribution of at least 2 per cent can be expected under conditions of plentiful labour supply (conditions of least benefit) with an average improvement of about 9 per cent. The contribution to overheads and profit per month on a medium-sized mine could be around R2 000 000 per month. A 2 per cent improvement would be R40 000 per month.

Results

The linear programmes were run in parallel with the manual system of planning for several months.

The method used to check the accuracy of the model as a predictor of stoping-labour productivity was to feed into the model the plan that had been produced by the manual system, and to allow the model to derive an expectation of centares. At the end of the month, when actual centares produced had been measured, the predictability of the manual system was compared with that of the LP model, using the index

$$\frac{\text{Predicted centares per predicted SPW shift}}{\text{Actual centares per actual SPW shift}} \times 100\%.$$

During December 1977 and January 1978, 254 sets of results were obtained from stoper's sections on the two mines, and the distribution of the index was found to be approximately lognormal. Table II presents comparisons based on the assumption of lognormal distribution.

TABLE II
COMPARISON OF RESULTS: LP MODEL VERSUS MANUAL PLANNING

	Geometric mean %	Median %	log variance
Manual planning	113,2	107,6	0,23
LP model	98,3	100	0,28
Ideal result	100	100	

It can be concluded from Table II that, at the 5 per cent level of significance, the variances are not different and that the mean of the LP model has less bias. Therefore, the LP model offers a valid alternative method of planning.

Table III compares the centares/shift per SPW that, according to the characteristic curves, were to be expected from the manually planned allocation of stoping labour, with those which (with equal probability) would have been achieved if the same labour had been allocated by the LP method.

The average improvement indicated in Table III is about 9 per cent, which is a measure of the scope for optimization. This increase in production would be obtained without the violation of grade or any other constraints present in the model.

The introduction of the model as a planning tool in the gold mines of Anglo-Transvaal Consolidated Investment Co. Ltd. is at an early stage. However, parallel planning has been done at shaft level by stoper's sections and at mine level by mine overseer's sections with encouraging results in terms of acceptance by mining officials.

Conclusion

Table II shows that the LP models that are based on characteristic curves are fairly good predictors of actual performances. Table III shows that, when the LP models are used to maximize overall performances, such as

TABLE III

OPTIMIZATION POTENTIALS

		Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.
Ca per SPW shift	Expected	0,522	0,566	0,517	0,506	0,554	No	0,51	0,50	0,52
	Optimum	0,552	0,594	0,59	0,55	0,563	results	0,59	0,56	0,58
Scope of LP model		Whole mine at mine overseer level					Model undergoing revision	Individual shafts at 'stoper' level		

stoping production and stoping contribution (to overheads and profit), an average increase of some 9 per cent can be expected in those values.

It is believed that the conditions of labour supply under which the South African mining industry operates and may continue to operate for some time impose on mine managements the need to use the available labour at all times to the best advantage. For this reason, managements should be able to react, and to adjust labour allocations underground, swiftly and effectively in the short and long term. The use of the LP technique can be of assistance in the achievement of this aim.

Acknowledgements

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References

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2. Corrie, W. J. Monthly production planning. Western Deep Levels Limited, *Report* no. MS/WDL/212/1. Aug. 1976. (Unpublished interim report.)

Addendum: Mathematical Formulation

This formulation, which is very straightforward, would be applicable to most gold mines with only minor modifications.

Variables

F(i, j) =Face metres allocated to mine overseer section i at productivity rate denoted by the jth segment of the characteristic curve
SL(i) =Labour allocated to section i
C(i) =Centares broken in section i
PF(i) =Planned face length for section i
T(i) =Tons broken in section i (1 ton=1000 kg)
AU(i) =Gold produced in section i
CONT(i) =Profit contribution of section i
TOTTON=Total mine tonnage
TOTAU =Total gold production
TRD =Total contribution
TSPW =Total stope production workers
FACE =Total face metres expected

Description of Coefficients

ZMDY(i, j)=Labour allocation in section i at productivity rate j (from characteristic curve of section i)
ZMAD(i, j)=Expected advance increments (from characteristic curve of section i)
FLF(i) =Face length factor for section i (for the relation between planned and actual face length)
SW(i) =Stope width in section i
GDE(i) =Grade of section i
ZLCS =Labour cost per worker-shift
ZMCC(i) =Materials cost per centare in section i
RPGRAM =Gold price in R/g
ZMAXAU =Maximum average grade
ZMINAU =Minimum average grade
UPF(i) =Maximum face to be planned in section i
UPL(i) =Maximum labour to be allocated in section i
LOL(i) =Minimum labour to be allocated in section i
MILLA =Capacity of mill located at shaft A
MILLB =Capacity of mill located at shaft B
UPTSPW =Availability of stope production workers
0,367 =Cubic metres per ton of rock

Objective Function

Maximize centares produced

$$\text{MAX} \sum_{i} C(i)$$

Constraints

1. Total labour allocated to each section
$$\sum_{j} \text{ZMDY}(i,j) \times F(i,j) = \text{SL}(i) \qquad \text{for all } i$$
2. Total production in each section
$$\sum_{j} \text{ZMAD}(i,j) \times F(i,j) = C(i) \qquad \text{for all } i$$
3. Logic constraint, which allows the same face length to be allocated at different levels of labour productivity in each section
$$F(i,j+1) - F(i,j) \leqslant 0.0 \qquad \text{for all } i, \text{ for } j=1,2,3$$
4. Relation between planned face length and actual face length
$$\text{FLF}(i) \times \text{PF}(i) = F(i,1) \qquad \text{for all } i$$
5. Tons broken in section i
$$\text{SW}(i) \times C(i) = 0.367 \times T(i) \qquad \text{for all } i$$

6. Gold production in section i
$GDE(i) \times T(i) = AU(i)$ for all i
7. Profit contribution of area i
$RPGRAM \times AU(i) - ZLCS \times SL(i) - ZMCC(i) \times C(i) = CONT(i)$ for all i
8. Lower bound on average grade
$ZMINAU \times TOTTON \leqslant TOTAU$
9. Upper bound on average grade
$ZMAXAU \times TOTTON \geqslant TOTAU$
10. Total contribution
$$\sum_{i} CONT(i) = TRD$$
11. Total tonnage
$$\sum_{i} T(i) = TOTTON$$
12. Total gold production
$$\sum_{i} AU(i) = TOTAU$$
13. Total stope production workers
$$\sum_{i} SL(i) = TSPW$$
14. Total face length allocated
$$\sum_{i} F(i,1) = FACE$$
15. Capacity of mill located at A shaft
$$\sum_{i \; MillA} T(i) \leqslant MILLA$$
16. Capacity of mill located at B shaft
$$\sum_{i \; MillB} T(i) \leqslant MILLB$$
17. Maximum amount of labour to be allocated to each section
$SL(i) \leqslant UPL(i)$ for all i
18. Minimum amount of labour to be allocated to each section
$SL(i) \geqslant LOL(i)$ for all i
19. Maximum face length to be planned in each section
$PF(i) \leqslant UPF(i)$ for all i
20. Maximum total stope production workers
$TSPW \leqslant UPTSPW$

Student prizegiving

The second annual student prizegiving function of the South African Institute of Mining and Metallurgy was held in the Van der Byl Hall, University of Pretoria, on Thursday, 10th March, 1978. The main address (see opposite page) was given by Dr T. F. Muller, Chairman of Iscor.

The Institute introduced this annual function into its calendar to promote the development of student interaction in the departments of mining and metallurgy in the universities, and to encourage the recruitment of students into these departments.

The training of students is not limited to academic functions. A mining or metallurgical engineer requires more than a B.Sc. degree if he is to make a positive contribution in his chosen field of engineering. Leadership, resourcefulness and perseverance, and the ability to welcome challenge and meet adversity with fortitude are qualities that should be nurtured during the formative years at university.

The wish to recognize these qualities has led to the institution of prestige prizes to students, in the third or fourth year of study, who display the potential sought by the industry. In addition to academic performance, an important consideration is the contribution made to student affairs and interaction with the department of mining or metallurgy. Book prizes are awarded to fourth-year students with the best academic records in a specialized field of mining or metallurgy.

The prizewinners for 1977 are as follows:

Prestige Prizes

R. Gould	Department of Mining Engineering, University of the Witwatersrand
Miss D. Wiggill	Department of Metallurgy, University of the Witwatersrand
A. W. de Villiers	Department of Mining Engineering, University of Pretoria
H. J. Marais	Department of Metallurgical Engineering, University of Pretoria

Book Prizes

M. J. de Beer	Department of Mining Engineering, University of the Witwatersrand
K. Maske	Department of Metallurgical Engineering, University of the Witwatersrand
J. R. de Villiers	Department of Mining Engineering, University of Pretoria
J. C. Strydom	Department of Metallurgical Engineering, University of Pretoria.