A prediction of the long-term future of tin*

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SYNOPSIS

Logistic curves are used in the prediction of the primary production of tin metal from estimated world-wide resources. Such curves are asymptotic to the total cumulative reserves, which are a function of prevailing metal prices and total resources. The figures for tin resources are based on the findings of Robertson and of Brobst and Pratt, the latter authors including estimates of undiscovered deposits.

It is suggested that tin production will slump from the present level to 100 000 long tons per annum by the end of the century, but will reach a new peak of 200 000 long tons some thirty years later. It appears that production will never exceed 300 000 long tons per annum unless substantial new resources, excluding those predicted, are discovered.

SAMEVATTING

Logistiese krommes word gebruik on die primêre produksie van tinmetaal uit die geraamde wêreldwye bronne te voorspel. Sulke krommes is asimptoties ten opsigte van die totale kumulatiewe reserwes wat 'n funksie van die heersende metaalpryse en totale bronne is. Die syfers vir tinbronne is gebaseer op die bevindings van Robertson en Brobst en Pratt. Laasgenoemde skrywers sluit ramings van onontdekte afsettings in.

Daar word beweer dat tinproduksie van die huidige peil sal daal tot 100 000 Britse ton per jaar teen die einde van die eeu, maar ongeveer dertig jaar later 'n nuwe spits van 200 000 Britse ton sal bereik. Dit wil voorkom of die produksie nooit 300 000 Britse ton per jaar sal oorskry nie, tensy daar aansienlike nuwe bronne benewens die wat voorspel is, ontdek word.

Introduction

At the present time world resources of tin appear to be very limited, and the effect of this on future production is of the utmost importance to the tin-mining industry. In this paper, estimates of tin production for the next hundred years are based on six logistic curves that were drawn for various estimates of the total available resources.

However, a consideration of reserves alone yields only a limited amount of information, and the influence of the metal market must also be included in any analysis. In this paper, these two factors are combined to form a basis for the estimation of future trends in the production of tin. Tonnages throughout are in terms of long tons to conform with the data from the literature.

Theoretical Basis for Estimations

For any exhaustible mineral commodity, the rate of production during the initial period of exploitation starts from zero and increases exponentially for a limited length of time. This is followed by a decline in growth rate during which the annual production reaches a peak before slowly returning to zero as the exhaustion of the mineral becomes complete. Thus, the graph of production rate against time takes the form of a bell-shaped curve.

Inherent in such a curve is the fact that the area under the curve from time t_0 to t_1 is a measure of the cumulative production up to t_1, and the total area represents the ultimate cumulative production Q_∞. If the mineral is extracted completely, Q_∞ is equal to the total reserves of that mineral.

If Q_∞ is the only constraint imposed, a family of curves can be drawn with similar areas but different shapes. For example, a curve with a higher peak will have a smaller time span than that of a curve with a lower peak.

A unique curve can be drawn by the imposition of a second constraint effective on past production figures. Since these figures are factual, any predictions based on bell-shaped curves satisfying past production figures will be dependent mainly on estimations of Q_∞; that is, estimations of total reserves. Thus, it is important to realize the factors that influence Q_∞.

Increased demand raises the market price, thus turning a percentage of conditional resources into reserves. (The distinction between resources and reserves is defined later.) Conversely, a decrease in demand depresses the price, turning part of the existing reserves into conditional resources. A decrease in supply due to failing reserves also raises market prices, re-establishing sufficient reserves to meet the demand.

Changes in production costs either increase or decrease the reserves. Such changes are primarily due to technical improvements and variations in the costs of basic materials, notably power and labour.

In this paper, future production is predicted by means of derived logistic curves drawn for different values of Q_∞.

The general equation of the logistic curve is

$$y=\frac{k}{1+e^{a+bx}},$$

where k, a, and b are constants, k being an asymptote. The particular form used here is

$$Q=\frac{Q_\infty}{1+e^{a+bt}}, \qquad (1)$$

where t is time given in ten-year periods and Q represents the cumulative production up to t.

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A rearrangement of equation (1) gives

$$e^{a+bt}=\frac{Q_\infty}{Q}-1,$$

from which it can be seen that

$$a+bt=\ln\left(\frac{Q_\infty}{Q}-1\right).$$

The substition of two known values of Q for two past periods of production yields values of a and b. Equation (1) can then be used in the calculation of the cumulative production for any period desired.

Because Q is the cumulative production,

$Q_{1+t}-Q_1$=total production over period t.

Therefore, the average yearly production$=\frac{Q_{1+t}-Q_1}{t}$, which will occur in mid-period.

In this manner a derived curve of annual production can be developed to cover any length of time required.

Alternatively, the differential of equation (1) can be used to yield annual production figures:

$$Q=Q_\infty(1+e^{a+bt})^{-1}$$

$$\frac{dQ}{dt}=-Q_\infty(1+e^{a+bt})^{-2}.e^{a+bt}.b$$

$$=\frac{-Q_\infty be^{a+bt}}{(1+e^{a+bt})^2}.$$

An example of the calculations for the formulation of a derived logistic curve is given in Addendum 1.

The logistic curves given in this paper (Fig. 2) were calculated on a time period of ten years so that short-term effects, which have no real bearing on long-term trends, would be reduced. Fig. 1 shows world production on a yearly and an average ten-yearly basis. The adverse effects of the depression and World War II can be clearly seen.

Values of a and b in equation (1) were calculated from the nine decades from 1890 to 1969, and an average was obtained from the successive values. Since the actual production figures show marked variations from year to year, the curves obtained yielded limited detailed information; however, a more sophisticated method of finding a and b would be no more meaningful and would be much less simple.

A more complete account of the logistic and other curves used in the estimation of mineral production is given in a paper by Onesta *et al.*[1]

TABLE I

THE SUBDIVISION OF MINERAL RESOURCES

Increasing feasibility of economic recovery ↑	IDENTIFIED RESOURCES			UNDISCOVERED RESOURCES	
	Reserves			In known districts	In undiscovered districts
	Measured	Indicated	Inferred	Hypothetical	Speculative
Conditional resources	Paramarginal				
	Submarginal				
	← Increasing certainty of existence				

Estimations of Q_∞

Before an estimate can be made of the amount of tin available for mining, it is necessary to define the terms used in the estimation of a mineral resource (Table I).

Brobst and Pratt[2] define a resource as a concentration of elements from which a mineral commodity can be extracted. Two important parameters are used as a basis of subdivision: economic availability, and the degree of certainty of existence. The latter permits a resource to be divided into *identified* and *undiscovered* parts.

Identified resources are further divided into *reserves*, in which the concentration is high enough to permit profitable extraction at present market prices, and *conditional resources*, in which the concentration is too low for present extraction.

Reserves can be subdivided on the basis of certainty

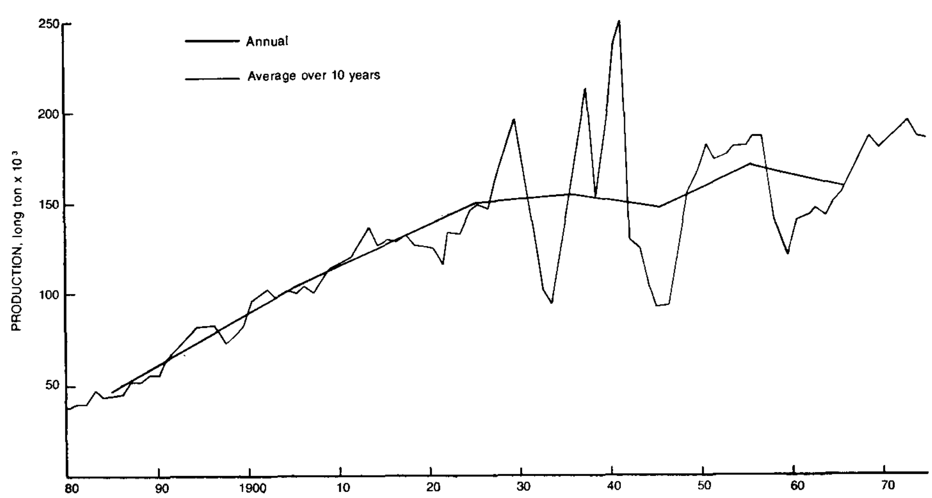

Fig. 1—World primary production of tin in terms of metal

into *measured, indicated,* and *inferred* (proved, probable, and possible), where inferred reserves include unexposed ores that have been definitely identified by geological association.

McKelvey[3] divides conditional resources into *paramarginal* and *submarginal* parts on a basis of economic availability. Paramarginal resources are those that will become reserves at prices $1\frac{1}{2}$ times those prevailing at the time of classification.

Undiscovered resources are divided into *hypothetical* and *speculative* resources, the former being unknown deposits that exist in known areas, and the latter unknown deposits existing elsewhere.

Most estimations of tin resources are based on a report by Robertson[4], who obtained information under the auspices of the International Tin Council by means of a questionnaire that was sent out to all non-Communist producing countries. Modifications were made by Sainsbury[5] and by Brobst and Pratt[2]. Table II is based on the last-named publication, and a number of observations made by the authors are worth mentioning here.

Speculative resources are based on four factors.

(a) Tin has been mined in settled regions for thousands of years, and major new deposits are unlikely to be discovered in these regions. Thus, large new deposits must be sought in remote areas.

(b) History has proved that major deposits are invariably similar to those which have been mined for centuries.

(c) Many of the great mining districts of the world were discovered by Cornish miners, who were unlikely to have overlooked major tin deposits in the areas that they penetrated.

(d) Tin has such a close geological association with acid granitic rocks that tin reserves in new geological environments are highly unlikely.

Hypothetical resources are based on past mining histories and geological evidence.

The large inferred reserves of Indonesia and Thailand are based on a similarity between these deposits and those of Malaysia, which, although still producing large amounts of tin, are of very low grade. It is assumed that Indonesia and Thailand have many years to go before the grade drops to that of Malaysia.

The large speculative resources of Zaire are based on the lack of development of that country and the similarity of their tin field to that of Nigeria. The same reasons exist for the large estimated resources of South America, many of which were discovered only recently.

The proved and inferred reserves of Europe and Australia result from a revival of tin mining, especially in Australia and England.

The figures quoted for China and Russia are no more than educated guesses based on the remoteness of large unexplored regions in these two countries, such as Siberia and the Himalayas.

Because of the long history of tin mining, resources are not the only parameter that is unknown. Tin smelting has been practised for over 4000 years, which necessitates an estimation of past production up to 1880, when reliable figures were first published. An essay by Benedict[6] was used as a basis for the formulation of a cumulative production of two million long tons up to 1880. Addendum 2 gives a breakdown of the figures.

Logistic Curves

The six logistic curves shown in Fig. 2 were drawn for the following values of Q_∞:

TABLE II
WORLD TIN RESOURCES (IN LONG TONS OF TIN METAL)

Area		Reserves		Conditional resources		Undiscovered resources	
		Measured* indicated	Inferred	Paramarginal	Submarginal	Hypothetical	Speculative
North America	Total	19 400	48 000	14 000	57 000	40 000	317 700
South America							
Bolivia		485 000	500 000		500 000	1 250 000	
Brazil		300 000	300 000	1 074 000		1 674 000	1 000 000
	Total	788 000	803 000	1 074 000	500 000	2 924 000	1 007 000
Europe	Total	143 700	147 700		754 000	1 175 000	
Asia (Non-Communist)							
Indonesia		500 000	1 860 000	540 000	540 000		
Malaysia		600 000	230 000		1 000 000	1 500 000	1 000 000
Thailand		217 000	1 000 000	1 860 000		1 500 000	1 000 000
	Total	1 584 500	3 430 000	2 400 000	1 540 000	3 250 000	2 350 000
Africa							
Nigeria		138 000	138 000		100 000	500 000	
Zaire		65 000	130 000	1 000 000			1 000 000
	Total	320 000	385 000	1 000 000	122 000	500 000	1 330 000
Australia		94 300	94 000	100 000	100 000	100 000	500 000
China		500 000	1 000 000	1 000 000	1 000 000	1 000 000	1 000 000
U.S.S.R.		200 000	420 000	300 000	300 000	300 000	1 000 000
World total		3 649 900	6 327 800	5 888 000	4 373 000	9 289 000	7 504 700

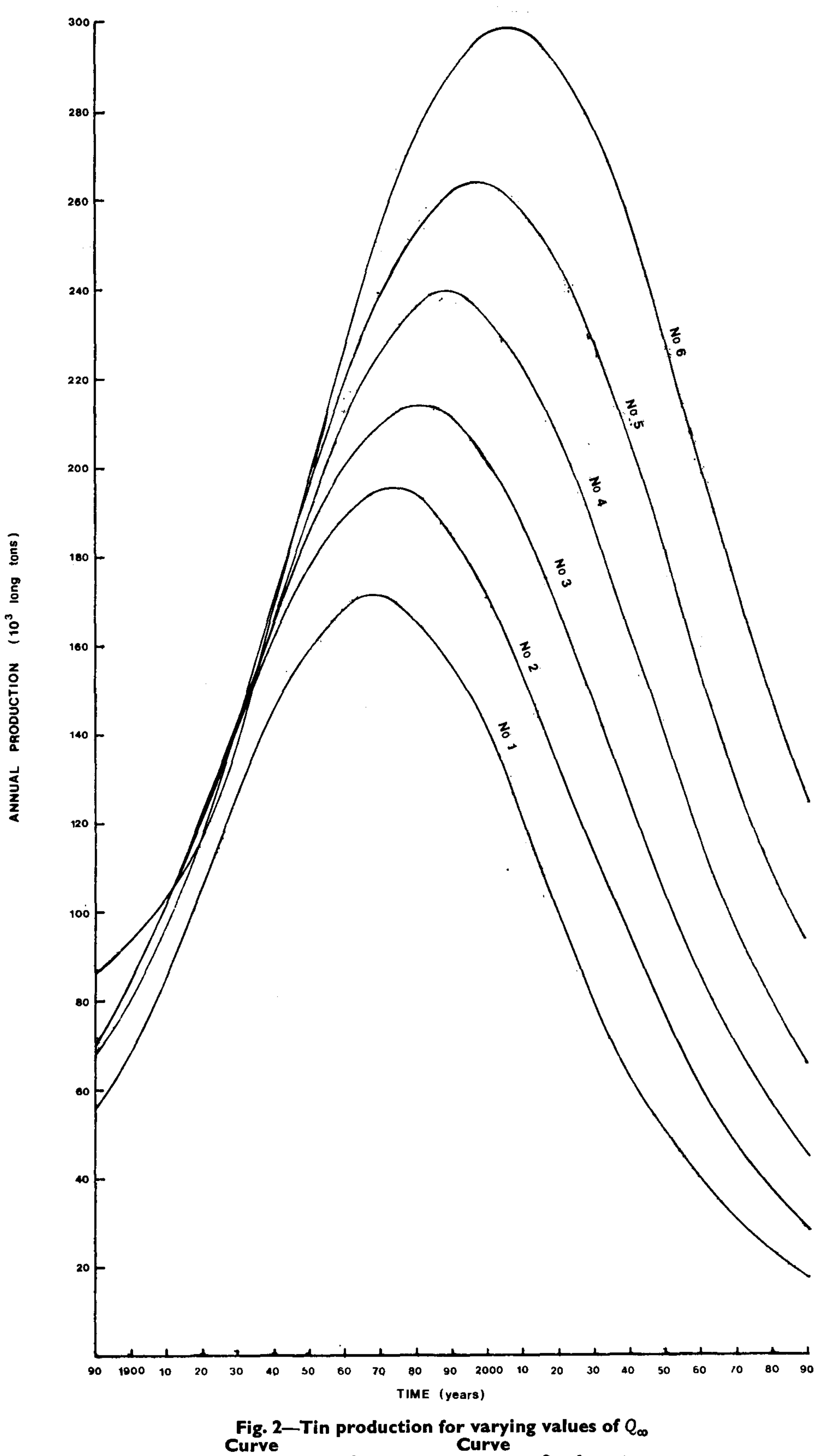

Fig. 2—Tin production for varying values of Q_∞

Curve no.	Q_∞, long ton	Curve no.	Q_∞, long ton
1	22 982 000	4	38 004 000
2	28 870 000	5	43 004 000
3	33 004 000	6	50 036 000

(1) 22 982 000 long tons, comprising 13 004 000 long tons of cumulative production up to 1969 plus 9 978 000 long tons of measured, indicated, and inferred reserves; this reflects the 1970 position as indicated by Robertson's questionnaire;
(2) 28 870 000 long tons, including 5 888 000 long tons of paramarginal conditional resources; Fig. 5 shows that the real prices are approximately $1\frac{1}{2}$ times those prevailing in 1970 (210 cents as against 145 cents per pound), which suggests that the inclusion of the paramarginal resources will reflect a truer picture of the present;
(3) 33 004 000 long tons, including 4 134 000 long tons of submarginal and/or hypothetical resources;
(4) 38 004 000 long tons, including 9 134 000 long tons of submarginal and undiscovered resources;
(5) 43 004 000 long tons, including 14 134 000 long tons of submarginal and undiscovered resources;
(6) 50 036 000 long tons, which comprise the total estimated resources.

Before any conclusions can be drawn from the curves, it is necessary to consider their field of validity.

If it is assumed that a commodity such as tin has an ever-increasing demand, the curves for cumulative production, Q_p, and cumulative discovery Q_d, will be similar in shape but will differ by a time lag (Fig. 3) δt. Both will tend to a final value Q_∞.

The total reserves, Q_r, at any given time can be obtained from the relationship

$$Q_d = Q_p + Q_r.$$

Differentiating with respect to time gives

$$\frac{dQ_d}{dt} = \frac{dQ_p}{dt} + \frac{dQ_r}{dt},$$

that is, the annual discovery of reserves is equal to the annual production plus the growth rate of reserves.

When $\frac{dQ_d}{dt} = \frac{dQ_p}{dt}$, it can be seen that $\frac{dQ_r}{dt} = 0$,

that is, the ore reserves are at a maximum (Fig. 4). Thus, a decline in ore reserves will precede maximum production by a time interval t_a, which is dependent on δt.

A measure of t_a can be obtained if it is realized that all mines must have known ore reserves to keep development and planning far enough ahead of production. The total reserves necessary for the entire tin industry are thus dependent on the number of producing mines. Once Q_r falls below this value, mines will start closing down and production will fall.

However, if the market is working correctly, a decrease in production will cause a shortage, which should be overcome by an increase in market prices to bring conditional resources into the ranks of reserves. Theoretically, prices should continue to rise to keep production equal to demand. Alternatively, prices may remain constant, but improved technical knowhow will increase the reserves.

Although it is impossible to predict future reserves with any degree of accuracy, a great deal of information can be gained from the production curves drawn for various estimates of Q_∞. Because of the ideal nature of the curves, the magnitude of future production figures will become less accurate as more conditional and undiscovered resources are classed as reserves. Therefore, in curves 4 to 6 maximum production is likely to occur later and be somewhat less than predicted.

For the present, the curves are dealt with at face value, the influence of the market being discussed later.

Examination of curve 1 (Fig. 2), in which Q_∞ is based on 1970 reserves, shows a maximum production of 171 500 tons in 1957. Production in 1976 would then be approximately 160 000 tons. This compares favourably with true production up to the mid-1960s, but at present production is remaining steady at around 180 000 tons. It would therefore appear that reserves have increased since the 1965 estimates, owing primarily to a large increase in market prices. This is confirmed by curve 2, in which 5 888 000 tons of paramarginal conditional resources are included in Q_∞. The curve shows a maximum production of 196 000 tons, occurring in 1974. Comparison of this curve and the true production figures suggests a peak of 190 000 tons some time in the next five years, on the assumption that no other reserves are found.

Curve 3 (Fig. 2) shows that a growth rate of 3,01 per cent would be necessary to give a maximum production of 215 000 tons in 1982. For this to be acceptable, over 4 000 000 tons of undiscovered and submarginal conditional resources would have to be included in the reserves. This is certainly feasible when the large hypothetical resources of Asia, South America, and China are considered, but, because of the remoteness of many of these tin fields, the high production rate is unlikely to occur before 1990. The substantial submarginal resources of China and Malaysia and the present high price of tin also suggest that a maximum production rate of 215 000 tons per annum is not possible.

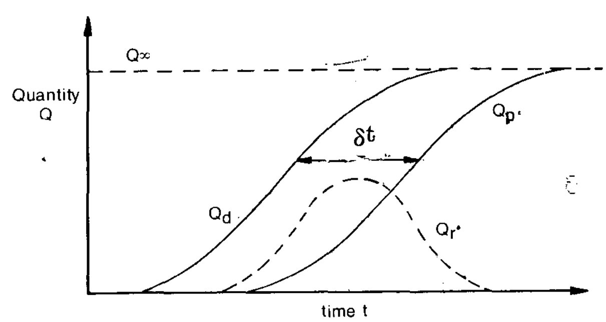

Fig. 3—The relationship between cumulative production, cumulative discovery, and total reserves

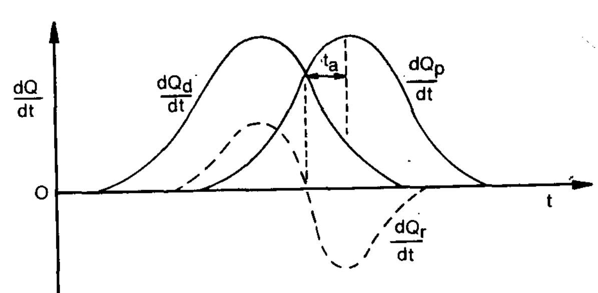

Fig. 4—The relationship between production, discovery rates, and growth rate of reserves

If Q_∞ is increased to contain 9 000 000 tons of submarginal and undiscovered resources, as has been done in curve 4 (Fig. 2), a maximum production rate of 239 000 tons is predicted for 1990. Again, the long lead-in times to bring mines on stream in the remoter areas suggests that a production of this magnitude is not feasible in this century. A growth rate of 1,19 per cent would be necessary to produce 239 000 tons per annum by the year 2000.

Curve 5 (Fig. 2) shows that an increase of 5 000 000 tons of reserves over those estimated for curve 4 will delay maximum production by 6 years. A growth rate of 1,30 per cent would be necessary to produce the expected 265 000 tons by the year 2006.

Curve 6 (Fig. 2), in which Q_∞ is based on total estimated resources, indicates a maximum production of 298 500 tons for 2005. Thus, if predictions of the world's tin reserves are correct, production can at no time exceed 300 000 tons, which is a little more than $1\frac{1}{2}$ times the present production. For this sort of figure to be achieved, vast exploration programmes would have to be launched within the next few years in all the remoter regions. It is considered more likely that production will never exceed 220 000 tons per annum but will continue at this level for at least the first quarter of the next century.

The table below shows the growth rates necessary to achieve a production of 300 000 tons per annum for the following peak years:

Peak year	*Growth rate*
2005	1,77%
2020	1,17%
2030	0,95%
2040	0,80%
2050	0,69%

The average yearly growth rate was 0,87 per cent from 1900 to the present, 0,06 per cent from 1930 to the present, and 0,31 per cent from 1950 to the present.

When the market price necessary to provide reserves of 36 000 000 tons is considered, annual productions in the region of 300 000 tons are in no way feasible.

No final conclusions concerning the future of tin can be reached without a consideration of the role of the metal market.

The Influence of the Market

Although the extent of total reserves is of fundamental importance in the prediction of future production figures, mineral reserves are also a function of economic factors, which must be taken into account when the transfer of resources to reserves is being considered. In the short term, reserves tend to be a function of price, whereas in the long term the opposite applies.

Tin is of special interest in this respect because 90 per cent of the tin consumption in the world is channelled into two products: tinplate and solder. The former is very susceptible to complete substitution by other materials, but solder demands a minimum tin content, substitution in this field being limited and so far having proceeded slowly.

An ideal metal market will allow prices to rise steadily to maintain enough reserves to meet demand. However, as tin prices increase, there will come a time when substitution becomes a viable proposition. When this point is reached, demand will fall and marginal mines will be forced to close down.

When such a situation arises, three things will occur: firstly, demand will fall regardless of what happens to the price subsequently; secondly, the cost of introducing substitutes to an extent large enough to affect the market will prohibit an easy return to the original metal; and thirdly, as substitution nears its maximum potential, the market price must approximate the marginal cost of producing the tin necessary to satisfy world demand.

Unfortunately, markets rarely operate ideally, and a number of factors other than failing reserves will cause marked fluctuations in prices. Although these influences may be short term, their effects may continue well into the future if they allow substitution to be introduced. Because of this, it is necessary to consider the main reasons why markets fail to work competently. Vogely[7] lists five of these reasons.

(1) Influences can be exerted on the market by powerful producers or consumers depressing or raising prices to suit their own needs.
(2) A lack of adequate information can lead to faulty decisions.
(3) Political influence can lead to unforeseen changes in production and consumption.
(4) Differential discount rates for private consumers

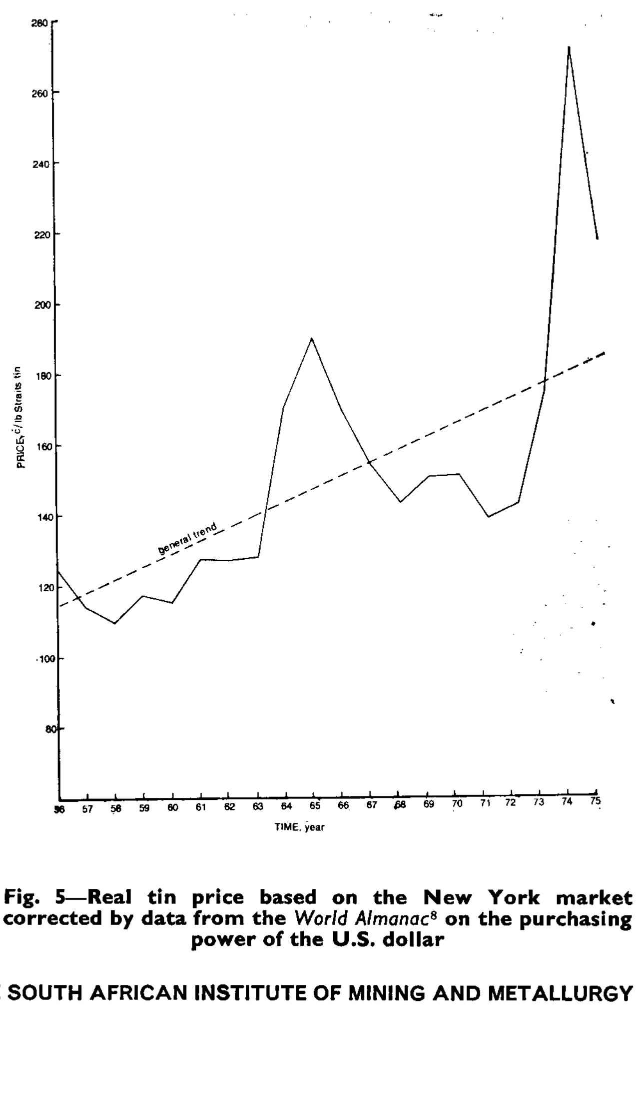

Fig. 5—Real tin price based on the New York market corrected by data from the *World Almanac*[8] on the purchasing power of the U.S. dollar

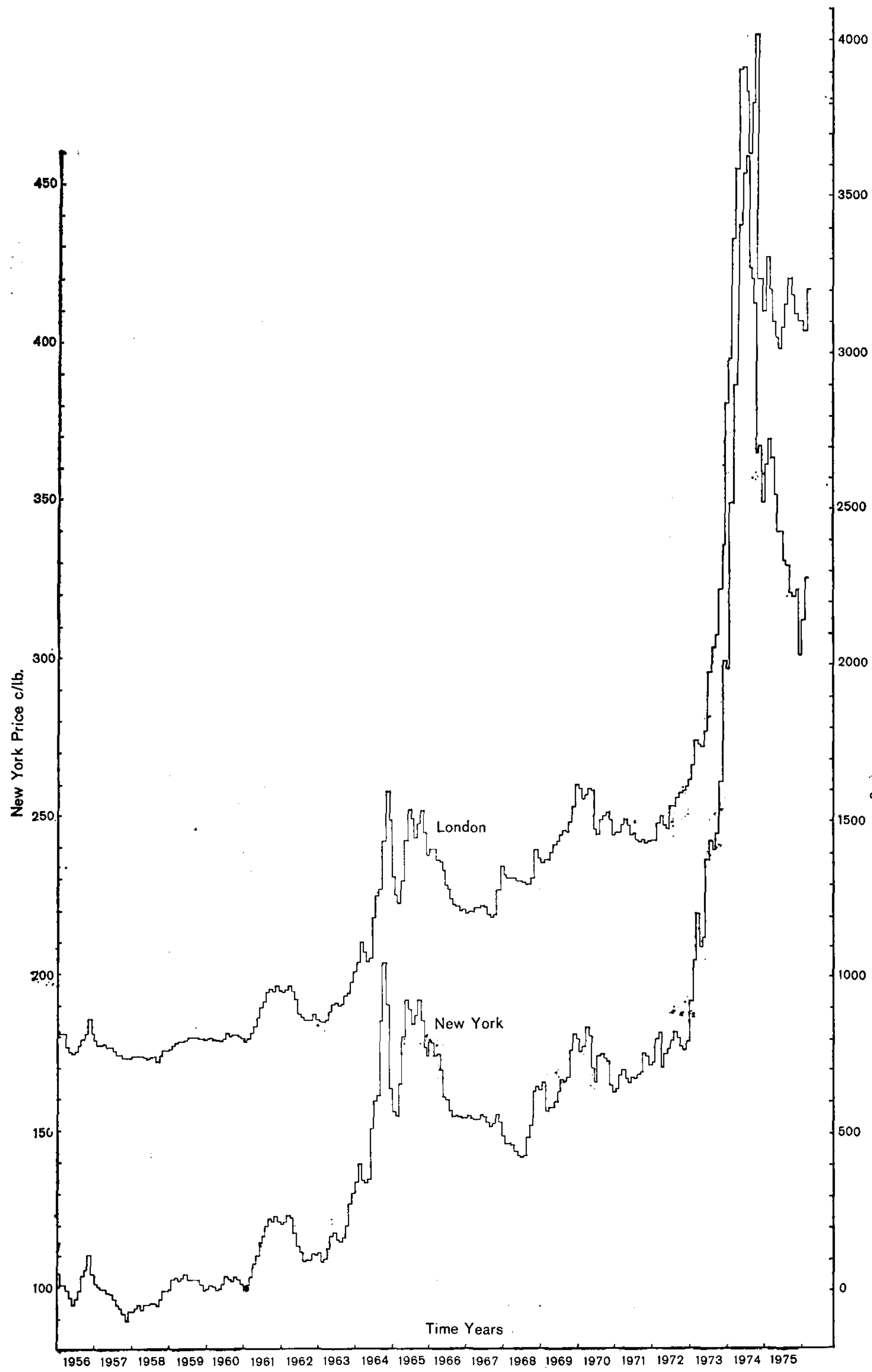

Fig. 6—Average monthly tin prices on the New York and London markets

and society as a whole may increase or decrease the present use of a material.

(5) The cost of external factors that are not reflected in the market price can tend to lead to a faster use of a material because its cost will be somewhat understated.

With regards to tin, the effects of factors (1) and (2) have been reduced to a certain extent by the introduction of the Tin Agreement, but the reluctance of many producers to undertake detailed valuations of reserves has caused a certain amount of speculation.

Political interference has been substantial in the tin industry for many years and looks like increasing in the immediate future. The six leading producing nations — Malaysia, Indonesia, Bolivia, Thailand, Nigeria, and Zambia — all have histories of political instability, which has at one time or another affected their output of tin.

Other spheres of political interference have been the regular sales of tin by the Government Services Administration in the U.S.A., and more recently by large shipments of tin from China. These will, however, be eliminated if and when the U.S.A. and China sign the Tin Agreement.

Factors (4) and (5) can be controlled to a certain extent by world-wide government legislation. Vogely suggests taxes as one remedial concept, and the imposition of standards in construction and manufacture as another.

Examination of Fig. 5 shows how the real price of tin metal has increased over the past twenty years, reflecting the changing picture of reserves. Fig. 6 shows the rise in quoted market prices on both the London and New York Metal Markets, and reflects the sudden changes brought about by political events.

General Conclusions

The present position as indicated by curve 2 of Fig. 2 shows that production cannot continue at around 180 000 tons per annum for any length of time unless reserves are increased substantially. The two possible means of achieving this are the discovery of new deposits and an increase in market prices to turn submarginal resources into reserves. Unfortunately, most of the predicted undiscovered resources are to be found in the remoter parts of the world and will be discovered only when large-scale prospecting of these regions is undertaken. Even if they are discovered within the next year or so, the lack of infrastructure, especially in South America and China, would delay the opening of a mine by at least twenty years, and even then production would grow slowly.

Because of this, it is expected that the demand for tin will outstrip the supply within the next five years, raising the price sufficiently to allow the almost total substitution of tin in tinplate by plastics, aluminium, and light-weight steels. However, at present by far the largest consumers of tinplate are Europe, Japan, and the U.S.A., but Africa, South America, and China, and possibly Russia, may form a very substantial market by the end of the century. If this proves to be the case and their development coincides with the exploitation of their tin deposits, it is likely that tinplate will again be the corner-stone of the container industry in these emerging regions. This is even more apparent when the future crisis that faces the oil and plastics industries is considered.

When tinplate is replaced in the Western World to a large enough extent — a process that has already begun in England and America — production will exceed demand and the price will drop. However, the large hypothetical resources of Bolivia, Malaysia, and Thailand should eventually keep prices reasonably steady, even if at a lower level than at present. Once this has happened, the need to find substitutes for tin in other fields, notably solder, will fall away, and the production of tin will follow the consumption of solder until the development of the South American and Chinese tin fields allows for the re-introduction of tinplate.

If these conclusions are correct, it appears that the production of tin may have two peaks: one in the next five years before substitution is introduced on a wide scale, and one in approximately fifty years' time, when the remoter regions become a major source of tin.

It is no easy matter to estimate future production realistically, but the following is suggested as a broad estimate based on the approach outlined earlier. Production will continue at 180 000 tons per annum for the next five years, and will then decline to 100 000 tons as substitution is introduced over a period of fifteen years. This would suggest a low in 1995, after which production should follow the demand for solder and then pick up again rapidly as more deposits are exploited. A peak of 200 000 tons per annum is estimated for 2030, after which production should decrease as tin becomes scarcer and scarcer.

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Addendum 1—Calculation of a Derived Logistic Curve

Q_∞=38 004 000 long tons

t	*Period*	*a*	*b*	*Q*	*Difference*
0	1880–89			2 890 917	
1	1890–99	2,672 391	−0,281 538	3 640 084	749 167
2	1900–09	2,693 652	−0,302 800	4 558 194	918 110
3	1910–19	2,680 270	−0,296 108	5 669 663	1 111 469
4	1920–29	2,638 568	−0,282 208	6 995 467	1 325 804
5	1930–39	2,495 255	−0,246 380	8 549 327	1 553 860
6	1940–49	2,295 585	−0,206 446	10 333 318	1 783 991
7	1950–59	2,355 317	−0,216 401	12 333 694	2 000 376
8	1960–69	2,148 786	−0,186 897	14 518 137	2 184 443
9	1970–79	2,497 478 (mean)	−0,252 347 (mean)	16 835 730	2 317 593
10	1980–89			19 220 513	2 384 783
11	1990–99			21 598 433	2 377 920
12	2000–09			23 896 274	2 297 841
13	2010–19			26 050 395	2 154 121
14	2020–29			28 013 118	1 962 723
15	2030–39			29 755 612	1 742 494
16	2040–49			31 267 285	1 511 673
17	2050–59			32 552 671	1 285 388
18	2060–69			33 627 132	1 074 461
19	2070–79			34 512 531	885 399
20	2080–89			35 233 578	721 047
21	2090–99			35 815 161	581 583
22	2100–09			36 280 626	465 465

Generally, $a+bt=\ln\left(\frac{Q_\infty}{Q}-1\right)$.

When $t=0$ $a=\ln(38\ 004\ 000/2\ 455\ 906-1)$
$=2{,}672\ 391$ . . . . (1)

When $t=1$ $a+b=\ln(38\ 004\ 000/3\ 187\ 506-1)$
$=2{,}390\ 852$ . . . . (2)

equation (2) – (1) $b=-0{,}281\ 538$
$a=2{,}672\ 391$

When $t=2$ $a+2b=\ln(Q_\infty/4\ 190\ 452-1)$
$=2{,}088\ 053$ . . . . (3)

equation (3) – (2) $b=-0{,}302\ 800$
$a=2{,}693\ 652$

When $t=3$ $a+3b=\ln(Q_\infty/5\ 428\ 282-1)$
$=1{,}791\ 944$ . . . . (4)

equation (4) – (3) $b=-0{,}296\ 108$
$a=2{,}680\ 270$

When $t=4$ $a+4b=\ln(Q_\infty/6\ 877\ 882-1)$
$=1{,}509\ 737$ . . . . (5)

equation (5) – (4) $b=-0{,}282\ 208$
$a=2{,}638\ 563$

When $t=5$ $a+5b=\ln(Q_\infty/8\ 375\ 951-1)$
$=1{,}263\ 357$ . . . . (6)

equation (6) – (5) $b=-0{,}246\ 380$
$a=2{,}495\ 255$

When $t=6$ $a+6b=\ln(Q_\infty/9\ 801\ 238-1)$
$=1{,}056\ 911$ . . . . (7)

equation (7) – (6) $b=-0{,}206\ 446$
$a=2{,}295\ 585$

When $t=7$ $a+7b=\ln(Q_\infty/11\ 455\ 444-1)$
$=0{,}840\ 510$ . . . . (8)

equation (8) – (7) $b=-0{,}216\ 401$
$a=2{,}355\ 317$

When $t=8$ $a+8b=\ln(Q_\infty/13\ 004\ 000-1)$
$=0{,}653\ 614$ . . . . (9)

equation (9) – (8) $b=-0{,}186\ 897$
$a=2{,}148\ 786$

In the table above, the difference column represents the production of each ten-year period. Therefore, the annual production at mid-period is as follows:

Year	*Production*
1895	74 917 long tons
1905	91 811
1915	111 147
1925	132 580
1935	155 386
1945	178 399
1955	200 038
1965	218 444
1975	231 759
1985	238 478
1995	237 792
2005	229 784
2015	215 412
2025	196 273
2035	174 249
2045	151 167
2055	128 539
2065	107 446
2075	88 540
2085	72 105
2095	58 158
2105	46 547

Addendum 2—Tin Production Before 1880

(Based on an essay by Benedict[6])

1.	Cornwall and Devon	
	Pre-history	10 000 long tons
	1200 – 1300 at 150 long tons per annum	15 000
	1300 – 1600 at 500 long tons per annum	150 000
	1600 – 1650 at 1 000 long tons per annum	50 000
	1650 — 1740 at 1 400 long tons per annum	126 000
	1740 – 1880 at 4 300 long tons per annum	600 000
2.	East Indies	
	1850 – 1860	500
	1860 – 1870	3 000
	1870 – 1880	40 000
3.	Malaya (Malaysia)	
	1800 – 1860 at 4 000 long tons per annum	240 000
	1860 – 1880 at 20 000 long tons per annum	400 000
4.	Australia	
	1870 – 1880 at 4 000 long tons per annum	40 000
5.	Other areas	
	From pre-history onwards	100 000
Total		1 774 500

i.e. approximately 2 000 000 long tons.