Some novel features and implications of a general risk-analysis model for new mining ventures

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SYNOPSIS

The scope and salient features of a general risk-analysis model for the main types of new mining ventures are covered in brief outline. The following special features and their effects on the investment criteria are discussed:

— various criteria for viability,
— the maximum shortfall of funds,
— correlations between the grades of up to five ore constituents and between the corresponding product prices,
— differential inflation and escalation rates for costs and prices,
— the estimation, if relevant, of a pay limit based either on the major product only, or on the two major products jointly,
— the likely recovery grades of all the products to be obtained when the ore-body is mined to any such pay limit,
— an option to take account of the effect on the pay limit of any negative taxation on marginal ore,
— investment analyses in current and real terms for individual groups of investors subscribing for equity and any loan capital on differential terms.

SAMEVATTING

Die omvang en uitstaande aspekte van 'n algemene risiko-ontledingsmodel vir die vernaamste soorte nuwe mynbou-ondernemings word in hooftrekke gedek. Die volgende spesiale aspekte en hul uitwerking op die beleggingsmaatstawwe word bespreek:

— verskillende maatstawwe vir lewensvatbaarheid,
— die maksimum tekort aan fondse,
— korrelasies tussen die grade van tot vyf ertsbestanddele en tussen die ooreenstemmende produkpryse,
— differensiële inflasie- en stygingskoerse vir koste en pryse,
— die skatting, waar dit van toepassing is, van 'n rendeergrens wat op slegs die belangrikste produk, of op die twee belangrikste produkte gesamentlik, gebaseer is,
— die waarskynlike herwinningsgrade wat verkry kan word wanneer die ertsliggaam ooreenkomstig sodanige rendeergrens ontgin word,
— 'n opsie om met die uitwerking van enige negatiewe belasting op marginale erts op die rendeergrens rekening te hou,
— beleggingsontledings in algemene en reële terme vir individuele groepe wat op differensiële voorwaardes vir aandele en enige leningskapitaal inskryf.

1. Introduction

In recent years there has been a general and significant increase in the risk element associated with investments in the exploration and exploitation of new mining ventures. This is due *inter alia* to the escalation of uncertainties associated with Government policy, and to intervention, direct and indirect, in a wide range of activities such as taxation, lease consideration, railway and power tariffs, ore beneficiation, export incentives and restrictions, foreign capital investment, and repatriation and undistributed profits. In addition, investment decisions now have to be made in a climate of unstable international currency-exchange rates and differential inflation rates.

Against the background of these uncertainties, some of which cannot be quantified even subjectively, it is essential that critical decisions on capital investments in new mining ventures should be taken only after at least a full assessment has been made of the overall risk associated with those uncertainties that can be quantified on some reasonable basis. Apart from subjective judgement, it is unlikely that there will ever be any alternative to risk analysis for this purpose.

The main objectives of any feasibility study, particularly of a risk analysis, is to provide management with a simulation, as realistic as is practical, of what could happen if the project is proceeded with. This will not provide definite answers for all the problems, but should assist in the decision-making process on

(i) the viability of the project and the chances of success as opposed to the risk of failure.
(ii) the 'best' mining plan, i.e., scale of production, the form in which the products are to be produced, marketing plan, etc., and
(iii) the 'best' scheme for the capitalization of the venture, i.e.
   (a) the general level of capitalization in view of the risk of falling short and having to raise additional funds.
   (b) the level of gearing as associated with the terms for long-term loans, with or without conversion rights, and
   (c) the 'best' mixtures of equity subscription rights and long-term loan commitments for the primary vendor(s) as opposed to any other outside partners respectively.

Note that 'best' is used here not in the sense of statistical optimization, but as the plan or scheme, among all the plans or schemes considered, that is judged to best satisfy the investor's interests.

These basic objectives, and the techniques of risk analysis and its application to certain South African ventures, have been discussed in some detail by Krige[1]

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and Munro[2]. Only some of the practical and novel features of the more general model now developed for the Anglovaal Group will therefore be covered in this paper.

2. Scope of Input Variations

The model caters for *three types of mineral* propositions, either new or already in production:

1. ore is sold directly, or after limited beneficiation and/or subdivision into different ore categories (e.g. coal, iron ore, chromium, manganese), with provision for ore stocks at the mine;
2. metals are recovered direct from the ore in an integrated operation, e.g. gold and uranium;
3. base-metal concentrates and/or mattes and/or refined metals are produced and sold; provision has been made for three types of concentrates, each with up to five saleable metal constituents, and for stocks of concentrates and mattes at the mine and/or en route; individual metals can be sold direct and/or via concentrates and/or mattes.

The mine products are generally referred to as metals, but can of course also be non-metallic products.

Options for *taxation, royalties, and lease consideration* cover flat rates and the South African sliding type formula, as well as South African type capital allowances for tax and lease payments, export allowances, and excess recoupments on breakup. Provision is made for joint-mining and non-mining assessments, but not for loan levies.

Escalation rates for costs and metal prices are provided for at short-term levels for four years plus a long-term rate thereafter all relative to the estimated inflation of the rand. Separate rates are entered for capital expenditure, and for mining, sundry, transport, smelting, and refining costs. Escalation rates relative to the inflation rate of the rand imply a direct long-term correlation between inflation on the one hand and cost and price escalations on the other hand, with or without specified divergences between them.

Any expected short-term adjustment in, for example, a metal price, that is considered to be out of line with the long-term trend, and any expected short-term cyclical trends, can be catered for in specifying the escalation deviations from the inflation rates individually for the first four years. Justification for this approach under international conditions of differential inflation rates and unstable currency exchange rates was recently covered by Krige[3].

Annual input figures cover tonnages milled, ore grades, capital and other costs, sundry income, and sundry expenses. Full provision is made for a sequence of *equity issues* (including issue expenses, free shares, and premiums), *long-term loans* (including underwriting, interest, repayment, and any conversion details), *building society loans* and *revolving credit loans* (for each product and at variable levels in the short-term). Annual levels for creditors, debtors, and stores can be set relative to current levels of costs (capital and operating), minerals in course of realization relative to current revenue levels, and stock levels (concentrates, and/or mattes) related to monthly production levels.

Risk variables include all the ore grades, metal prices, total ore tonnages, costs (mining, transport, smelting and refining, and capital separately), recovery rates in the treatment process, and the rand inflation rate (short- and long-term separately). Penalties for any metal impurity in a concentrate can, if convenient, be catered for by the inputting of a negative metal price.

The *probability distributions* can be symmetrical (normal) or skew (lognormal), and are specified by entering the likely value and the outer 5 per cent values. Provision is also made for *correlations* between individual ore grades and between the various metal prices.

In the case of the ore grades, the overall likely average grades for the ore-body are input together with the corresponding probability limits and the estimated effective logarithmic variances for the distributions of ore block values on which any eventual selective mining operations will be carried out[4].

The block distributions are assumed to be lognormal, and the block variances and other input parameters will determine whether selective mining is to be done and, if so, whether on a pay limit based only on the main product or on a joint pay limit for the two major products; if desired, both pay limits can also take account of any negative tax effect on submarginal ore. In addition, the annual input figures allow for relative variations in grade over the life of the project to cater for sequential exploitation of the richer and poorer sections of the mine.

If a complete investment analysis is not required, the computer run can be confined to the calculation of the total mill tonnage and average ore grades (and their probability distributions).

Provision is also made for the possibly widely differing interests and participation terms for up to four individual groups of investors. A particular group is identified by its specified share in each block of shares (paid for at par, or at a premium, or free) and in each long-term loan raised (with or without conversion rights) and linked, as an obligation, to the share participation.

Factors such as the eventual scale of operations, the capital financing scheme, etc. are regarded as *decision variables*[1] and can be handled by sequential runs of the program.

3. Main Features of the Computer Program

The program can be run:

(i) as a detailed cash-flow and investment analysis with all the variables at fixed levels; or

(ii) as a *sensitivity analysis* with specified values for all the risk and decision variables but one; the remaining variable, if one of the risk variables, could be entered as such and, if a decision variable, would be specified at various levels in consecutive runs; or

(iii) as a *full risk analysis* in which case a value will be selected on a Monte Carlo basis for each of the risk variables, to satisfy the individual probability distributions and the correlation levels as specified; this process is repeated up to a specified number of reruns of the whole financial analysis.

The average *ore-body grades* are selected as risk variables from the specified (correlated) probability distributions. If the ore-body is to be mined selectively, a pay limit, or joint pay limit, is calculated based on the revenue and the mine working costs (after tax if so specified and excluding major development) — averaged between the position when full production is reached and the end of the mine life. The grades of ore selected for mining above the pay limit are calculated from the selected average ore grades, the block variances, and the levels of correlation between the main product and each of the other products. For this purpose the appropriate bivariate lognormal model is used for each product together with the main product, and the volumes and grades of the distributions as truncated by the pay limit are determined (see Addendum 1).

Provision is made for mine costs not related to the total tonnage mined, e.g. the cost of the major development grid across the ore-body, to be specified apart from a direct cost per ton mined. The total cost per ton mined will then vary depending on the percentage of the total tons selected for mining above the pay limit, i.e. costs per unit will increase as the payability (or workability) decreases.

The program calculates on an annual basis all the physical production and sales estimates, as well as all the relevant financial figures. Dividends are equated to the amounts available for distribution with a provision for the automatic cancellation of up to two years' of earlier dividends where necessary to avoid a shortfall of funds in a year when, for example, capital expenditure or loan repayments could be exceptionally heavy. This will also eliminate or limit dividend payments in years when additional capital is raised to cover capital expenditure extending into future years and when capital expenditure cannot be financed fully from profits.

Furthermore, anomalies that can arise in the calculation of the discounted cash flow (D.C.F.) return, or internal rate of return (I.R.R.) for a cash flow changing from positive to negative and *vice versa* more than once, will also be avoided. Provisional estimates for a year are recalculated after a provisional year-end deposit balance has been arrived at and the interest on deposits (or overdraft) has been re-estimated more closely for the year concerned.

The estimates of cash flow for each group of investors are set up to cover all the relevant outgoings (the cost of equity and the long-term loan capital linked to it) and receipts (dividends, loan-capital repayments, and net loan interest after tax). These cash flows are also converted to real terms by discounting at the appropriate rand inflation rates. Where an investor group has taken up equity and has had also to provide part of the long-term loan capital, with or without conversion rights, the indicated returns on the group's combined equity and loan investment can be obtained in both current and real terms on the following bases:

(i) D.C.F. return, i.e. I.R.R.,
(ii) pay-back period and average annual level of receipts after that as a percentage of the total investment,
(iii) average period of investment for every rand invested before it is repaid, i.e. the 'average pay-back period',
(iv) present values of receipts and investments at up to four specified discount rates, and the corresponding potential capital appreciations on the total investment (or P.V. ratios of return/investment).

When not used as a full risk analysis, the program will list, on a tabulated annual basis, all the main production and financial estimates together with the corresponding values for D.C.F., pay-back period, etc. for each investor group.

For the purposes of risk analysis, the final printout covers summaries of the risk variables as selected for each run and of the calculated probability distributions for all the investment criteria of D.C.F. return, etc. for each investor group, as well as for the maximum shortfall of funds (and its timing) over and above the capitalization allowed for in the analysis.

The program is in Fortran IV G and runs on the Wits University I.B.M. 370/158; for a set of 50 iterations, the C.P.U. time used is some 35 to 60 seconds at a cost of R12 to R20.

Addendum 2 gives a brief summary of the sequence of the calculations and of the output format for a full risk analysis.

4. Financing Schemes

The facility for analysing the effective position of up to four investor groups simultaneously can be used profitably in the early stages of negotiating the financing of the venture. In considering the introduction of a partner for example, the effects of trading off an equity participation (at par or at a premium) against some gearing of the equity by way of a long-term loan provided by such a partner can readily be examined. Thus, the return indicated to the original vendor on his equity investment could be significantly improved, and at the same time the prospective partner could be offered an attractive equity plus loan investment, with or without conversion rights.

Repeated risk analyses can also be used to demonstrate the chances of improving the returns on equity for some or all of the investors by gearing at different levels; this has to be weighed against the greater risk of equity capital loss should the venture prove to be marginal[1]. Such analyses would raise in turn the concept of the 'risk aversion' of an investor[5].

5. Inflation and Escalation Rates

It should be stressed that, under the present conditions of high inflation, the most meaningful criteria for economic viability are those expressed in *real terms*, i.e. measures showing the position after elimination of the effects of the depreciation of the currency.

Furthermore, a real-term criterion will be meaningful only to the extent that realistic assumptions are made of the likely long-term inflation rates, and of (differential) escalation rates of costs and prices. It was recently suggested[6] that, in view of all the uncertainties, costs and metal prices should be escalated only up to the start of production and then left constant. If inflation is unlikely to disappear from the scene at that stage, this approach seems unrealistic and could distort the cash-flow estimates, the indicated financing requirements, and all the estimates of returns, pay back, etc. The following reasons are advanced in support of this view:

(i) when production starts, the capital programme will not have been completed, particularly if the production rate is to be built up over a period of some years;

(ii) tax payment will not have commenced, and redemption (and capital) allowances will therefore on the basis suggested[6] *effectively continue to be calculated in real terms* (relative to the start of production);

(iii) loan repayments are unlikely to have commenced, and these, with loan interest, although in fact to be made in *current terms*, will in effect, as with tax, be deducted from earnings at *real-term levels*;

(iv) distortion of the cash-flow estimates, the financing requirements, and the various criteria for measuring the feasibility of the project would result from the likely incorrect *relative* levels of the estimated capital expenditure, tax payments, loan commitments, and earnings;

(v) any evidence of likely long-term uptrends or downtrends in metal prices and/or in costs *in real terms* will be ignored; historical data indicate the presence of such trends in metal prices — also mining costs in South Africa are likely to continue escalating faster than inflation for some years to come, particularly until the policy of closing the Black–White wage gap is fully implemented.

In the circumstances, the author favours the estimation of the likely cash flow in current terms based on short- and long-term inflation and escalation rates, at differential levels if warranted. The assessment of the short- and medium-term financing requirements should be based directly on this cash flow, and the project's feasibility should be judged on criteria applied to the *real term* equivalent of this flow.

6. Sensitivity Analyses with Risk Cover on Other Variables

In the usual sensitivity analysis, one risk variable is set at different levels while all the other risk variables are fixed — usually at their likely values. Where a full risk analysis has been executed, a more meaningful sensitivity analysis can be effected if the values obtained for the output criterion accepted (e.g. D.C.F. return) are correlated with the corresponding values of the variable whose sensitivity effects are to be analysed. For stable results, the risk analysis might then have to be effected on at least 100 iterations so that 100 pairs of values are available for the correlation; also, the appropriate marginal distribution model(s) (usually normal and/or lognormal) should be used.

On this basis the sensitivity of any one of the investment criteria to variations in any one of the input risk variables can be obtained while all the other uncertain input variables are not fixed but vary according to their probability distributions as defined. Thus, a single risk analysis can also provide all the sensitivity analyses that might be required, and the alternative approach mentioned at the beginning of paragraph 3 above then becomes unnecessary except for decision variables.

This approach was recently used by the author with success in an analysis of the effects of changes in the South African mining-tax structure on the incentive to open up new mining ventures[7].

7. Probability Models for Output Criteria

The program was recently used to do a series of risk analyses for a variety of new gold-mining ventures under South African conditions. Lognormal probability distributions were used both for input grades (gold and uranium) and input metal prices, and normal models were used for all the other risk variables. Although the bulk of the variability in the probability distributions of the returns indicated was attributable to the relatively high variabilities of the four *lognormal* input variables of grade and price — gold grade and price together accounted for more than 93 per cent of this variability — the probability distributions of the more important output variables were all satisfactorily represented by *normal* distributions.

Fig. 1 demonstrates this conclusion on probability paper in respect of one of the analyses done. In cases where this conclusion is valid — and experience indicates that this is likely to be fairly general — the use of the normal model will naturally facilitate further analyses (such as mentioned in Section 6) and improve the estimation of statistical limits corresponding to specified probability levels, particularly when the number of iterations for the risk analysis is limited.

It should be noted that the distributions for D.C.F. returns can depart from normality when a large percentage of the returns is negative.

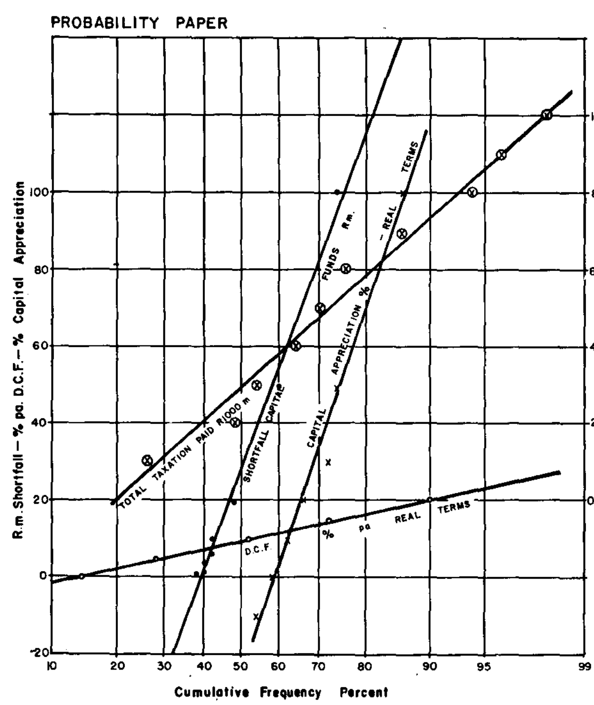

Fig. 1—Normal frequency distributions for some output variables from a risk analysis for a new gold mine based on 50 iterations

8. Flotation Pay Limits

Based on a full risk analysis followed by a correlation analysis of recovery grade (one main product) as the (lognormally distributed) independent variable and real D.C.F. return to the major investor as the (normally distributed) dependent variable, the following will become available (see Fig. 2):

(i) the average trend of real D.C.F. returns relative to recovery grade, together with the uncertainty of this trend level resulting from all the uncertainties other than grade — i.e. the regression of real D.C.F. return on grade and the corresponding conditional variance,

(ii) the indicated *flotation pay limit* for the venture, i.e. the recovery grade that is estimated to correspond on average (i.e. on the trend line) to a stipulated critical level of D.C.F. return,

(iii) the estimated probability of the venture yielding at least this critical level of return, given *all* the uncertainties including grade — note that this would correspond to the percentage area above the horizontal line through this critical D.C.F. return and *not* to the area to the right of the vertical line through the flotation pay limit as would be assumed if only the grade uncertainty were taken into account.

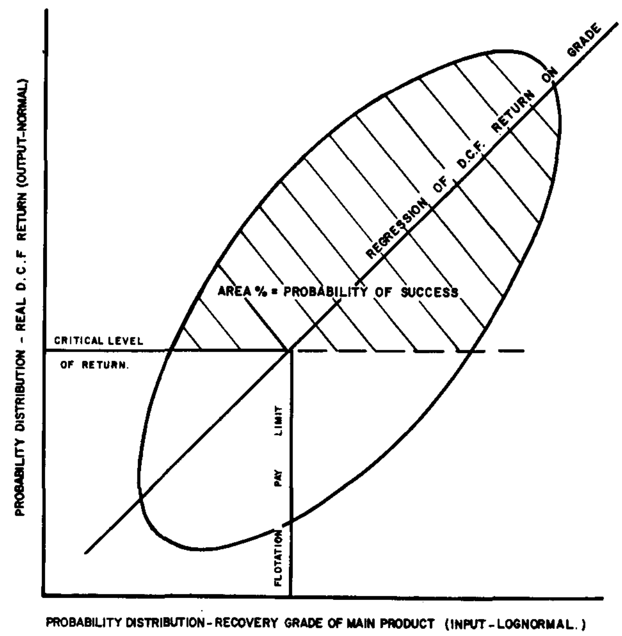

Fig. 2—Flotation pay limit from correlation of DCF return with recovery grade

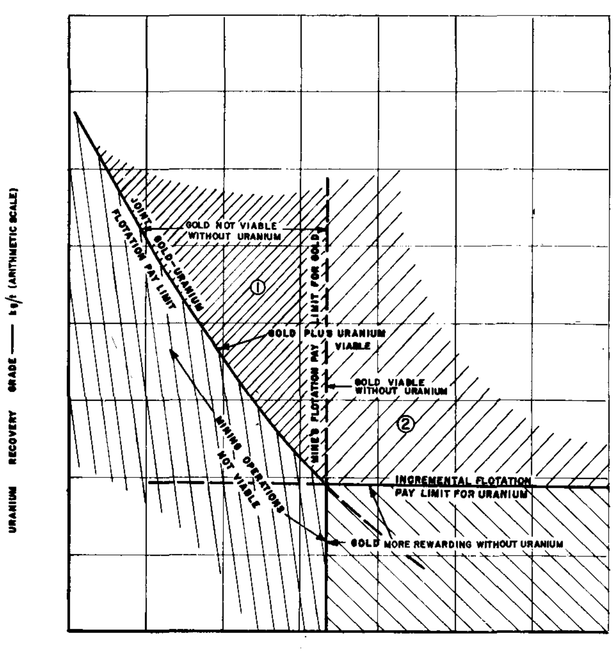

Fig. 3—Illustrating the nature of a joint gold-uranium flotation pay limit for a potential new mine

When two major products are produced and a joint pay limit is involved both at the flotation and operating stages, the position is more complicated. The *joint flotation pay limit* will take the general form shown in Fig. 3, which is reproduced from an earlier paper[8] dealing with joint gold–uranium projects. The *two* main products will then form the independent variables and, following the results of a complete risk analysis, the real D.C.F. return would be the corresponding dependent variable, this requiring a three-dimensional representation.

However, the complete risk analysis will provide a direct estimate of the probability distribution of the D.C.F. return under such conditions. This distribution in turn leads to the appropriate estimate of the probability of exceeding the relevant critical level of return for the capital to be invested, given two co-products with uncertain recovery grades and all the other uncertainties associated with the other risk variables.

9. Selective Mining

By selective mining to a pay limit based on the main product or to a joint pay limit, the mining of uneconomic ore is avoided as far as is practical, and this should on balance improve the prospects of a new venture and reduce the risks of failure.

The average effects of selective mining will depend on the type of pay limit used, the variability of the ore grades, the level of correlation between the grades for the main and the other products (or metals), and the general level of the likely grades. To demonstrate the type of effect to be anticipated, a typical new gold–uranium project was analysed with average ore-grade variabilities and gold and uranium head grades (no selective mining) of 5 g/t and 100 g/t respectively, and likely prices per kilogram of R6000 and R80 respectively. A full risk analysis with 50 iterations was run at each of three levels of gold–uranium correlation (0, 60, and 95 per cent) and also on the basis of a straight gold pay limit and a joint gold–uranium pay limit.

Selective mining to a straight gold pay limit for the 50 iterations indicated an average of 69/70 per cent of the ore selected for mining at a gold grade some 24 per cent higher than with no selection. The corresponding improvement in uranium grade varied with the degree of gold–uranium correlation, and ranged from 0 (no correlation) to some 20 per cent (95 per cent correlation). The probability of the venture showing a positive real D.C.F. return improved from 46 per cent with no selective mining to 64 per cent on selective mining; furthermore, the degree of gold–uranium correlation had a significant positive effect on the likely D.C.F. return, with an improvement of about ½ per cent p.a. at a high correlation level.

Selective mining on a joint pay limit compared with a straight gold pay limit indicated the ore selected for mining at about the same average percentage and gold grade, but an improvement in uranium grade of some 2 per cent. The probability of a positive real D.C.F. return was similar, but the likely D.C.F. return improved slightly.

In cases where the potential from uranium relative to that from gold is more significant, the advantages from mining to a joint pay limit would be of a much higher order[8].

If the lowering effect of the tax formula on the pay limit is taken into account, the total ore potential is increased by about 12 per cent, and the gold and uranium production by some 7 to 9 per cent, both for a straight gold pay limit and a joint pay limit; the expected D.C.F. return is however slightly reduced.

10. Conclusions

Typical cases executed on the general risk-analysis program show that the following factors can have significant effects on the criteria used for assessing the viability of a new mining project, and should therefore not be ignored unless they are not applicable:

(i) selective mining of the ore-body;
(ii) the choice of a straight or joint pay limit;
(iii) the formula tax effect on the pay limit;
(iv) the correlations between ore grades; and
(v) the use of a long-term inflation rate and the corresponding escalation rates for costs and prices.

It can also be concluded that:

(vi) the output criteria can generally be accepted as having normal probability distributions;
(vii) the risk-analysis program can conveniently be used to provide all the required sensitivity analyses; and
(viii) the risk-analysis program should serve as a useful tool to assist management in assessing and reducing the risks inherent in critical decisions concerning investments in new mining ventures.

Addendum 1: Analysis of Effects of Selective Mining

The model used assumes that the economic ore will be selected for mining on a scale reflected by a practical size of ore block; such ore blocks will be valued by the use of either regression adjustments or geostatistical techniques.

The basic ore-value distributions are assumed to be lognormal so that the effective logarithmic variances for the ore-block distributions for selective mining purposes will be determined by:

the logarithmic variance of the distribution of the *actual* block values (usually estimated geostatistically, e.g. from a variance–size of area or variogram analysis),

less

the logarithmic error variance for the block valuations in terms of the critical parameters of the variance, semivariogram, and likely sampling pattern of the ore values[4].

The effective logarithmic block variance for each of the significant metals or minerals in the ore-body, and the mean ore-body grades as selected from the relevant probability distribution for each case to be calculated, form the input parameters for the relevant subroutine used in the computer program.

In the model used, allowance is made for correlation levels between the block values of the main product and of all the other metals or minerals in the ore-body.

1. No Selective Mining

If no selective mining is to be done, the mean ore-body grades as selected are used direct with provision for relative annual variations (as input) over the life of the mine.

2. Selection Based on the Main Product Only

(a) *Main Product*

If all the products other than the main one are regarded as byproducts, the *in situ* block values of which will not influence selective mining decisions, the position for the *main product* is first determined.

The distribution of ore-block values — three-parameter lognormal with mean recovery grade as selected and effective logarithmic variance and additive constant as input — is set up, and that fraction, with its grade, above the 'recovery' pay limit is determined[4,9]. A provisional pay limit (in grams per ton or as a percentage) is calculated for this purpose:

$$(\mathbf{w}-\mathbf{b})/\mathbf{c}$$

where **w**, **b** and **c** are the values for the first year of full production with

w = working costs per ton milled excluding the costs of any major development grid, which has to be done in any case before any block valuation is possible;
b = revenue credits per ton milled from byproducts (taken provisionally at their mean ore-body grades); and
c = the price per gram or per ton for the main product,

and the effective lowering of the pay limit due to taxation (e.g. by the gold tax formula) can also be imposed as an option.

The 'payable' fraction of the main product above this pay limit is determined from the formula applicable for lognormal distributions[4,9]:

$$\text{Fraction} = \frac{1}{2\sqrt{\pi}} \int_{w_i}^{\infty} \exp\,(-w^2/2)dw, \qquad \ldots\ldots \quad (1)$$

$$\text{where } w_i = \frac{1}{\sigma}\Big[\log_e(z_i + a_1) - \epsilon\Big]$$

z = untransformed lognormal variable
z_i = pay limit as calculated
ϵ = mean of $\log_e (z_i + a_1)$
$= \log_e$ of $(m + a_1) - \sigma^2/2$

m = ore-body mean grade = $\bar{z}_i$

a_l = additive constant = 3rd parameter for main product block values

σ^2 = logarithmic block variance as input.

This fraction, as determined, is applied to the total ore-body tonnage as selected from its probability distribution so that the total mill tons, and hence the life for the proposition, can be estimated. The pay limit for the final year of the mine's life is then calculated as above, and the average of the two pay limits is determined. This average pay limit is used in the recalculation of the final payable tonnage and life.

The average payable grade (main product) of this tonnage is calculated from the formula[4]

$$\text{Payable grade} = (m+a_1)\frac{\int_{w_i-\sigma}^{\infty} \exp(-w^2/2)dw}{\int_{w_i}^{\infty} \exp(-w^2/2)dw} - a_1 \quad . \quad . \quad (2)$$

(b) *Byproducts*

If the correlation between a byproduct and the main product is input as zero, selective mining of the main product will have no effect on the byproduct grade, which is therefore accepted at the average ore-body grade for that product as selected from its probability distribution.

If the correlation is not zero, the regression equation for the byproduct on the main product is set up[4]:

$$\log_e (\text{expected byproduct value} = p + a_2)$$

$$= \log_e(\overline{p+a_2}) + (r/2)\sigma_p(\sigma_m - r\sigma_p) + r(\sigma_p/\sigma_m)\left[\log_e(z+a_1) - \log_e(m+a_1)\right] \quad . \quad . \quad . \quad . \quad (3)$$

where p = block value of byproduct

z = block value of main product

m = mean value of main product

a_1 and a_2 = additive constants for main-product and byproduct block values

σ_p^2 = block variance for byproduct after addition of constant a_2

σ_m^2 = block variance for main product after addition of constant a_1

r = logarithmic correlation coefficient for block values of byproduct and main product (after addition of constants).

The effect on the byproduct grade of selective mining

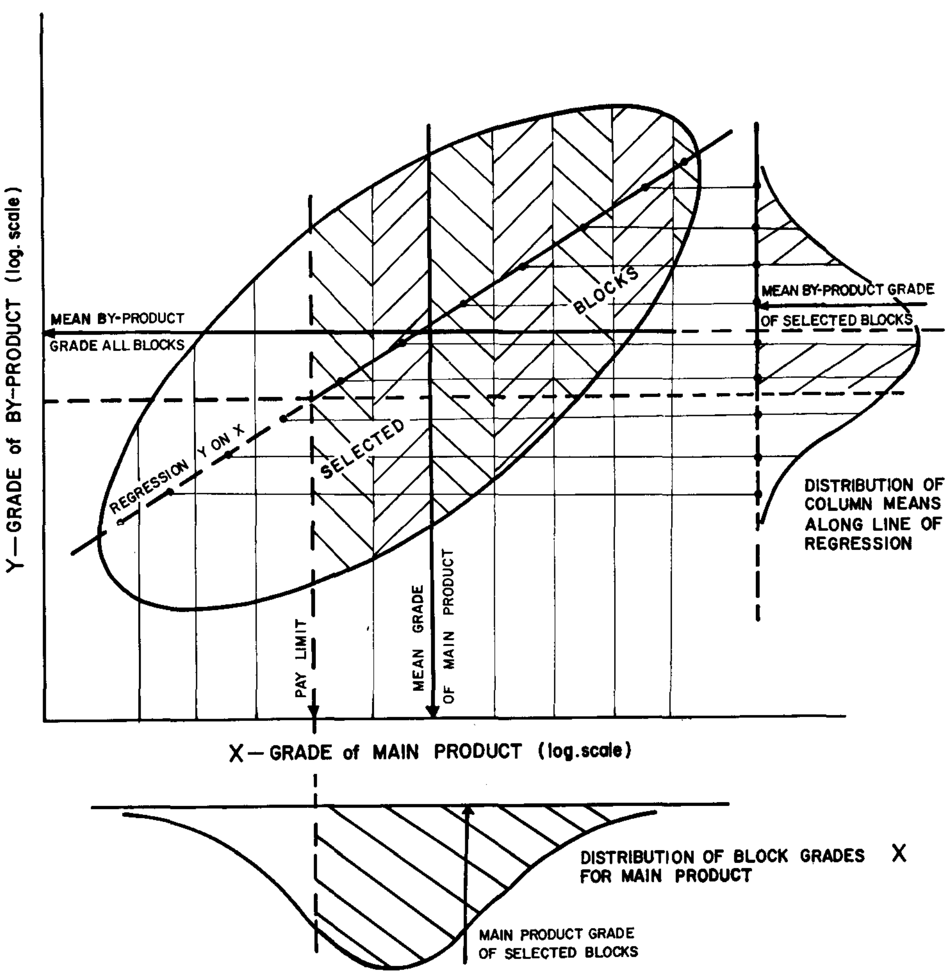

Fig. 4—Lognormal model for estimating tonnage and grades of ore selected for mining on main product pay limit

of the main product is demonstrated in Fig. 4. The relevant average mined grade for the byproduct is the average of all the ore blocks with main-product grades above the pay limit. This will therefore be the average of all the means of the byproduct conditional distributions in the columns to the right of the pay-limit line, i.e. the average of the means as distributed along the line of regression as defined above (with variance$=r^2\sigma_p{}^2$) and lying to the right of that byproduct value on the regression line corresponding to the main product pay limit. This *effective byproduct pay limit*, say p_q, is given by p in the above regression formula on substitution of the main product pay limit value for z. The average of the pay fraction for the byproduct distribution with mean as selected, logarithmic variance $r^2\sigma_p{}^2$, and effective pay limit p_q can now be calculated from formula (2) above with a_2 substituted for a_1 and $(\overline{p+a_2})$ for $(m+a_1)$.

(*c*) *Adjustments*

This process is completed for all the byproducts, and the 'pay' grades as calculated are substituted in the provisional pay-limit calculation for the main product (see (a) above). If the latter changes by more than $2\frac{1}{2}$ per cent from the original (which was based on the average ore-body grades for the byproducts), the calculations from (a) above are repeated, now using the estimated 'pay' grades for the byproducts.

3. Selection on Joint Basis

(*a*) *Two Major Products*

The relevant joint pay limit will be of the form

Revenue per ton for two major products

$=$costs per ton $-$ credit per ton from other products with revenue, costs, and credits all either before or after tax,

or

$$aG_1+bG_2=c, \quad (4)$$

where a, b, and c are constants, and G_1 and G_2 are the recovery grades of the two main products.

As the bivariate distribution of block values for the two main products is accepted as bivariate lognormal, the above straight-line joint pay limit (4) will plot as a curve on double logarithmic paper, and the determination of the pay fraction and the pay grades becomes more laborious.

As shown in Fig. 5, the marginal distribution for main product 2 is subdivided into 8 grade categories representing frequency intervals of 5, 10, 15, 20, 20, 15, 10, and 5 per cent respectively, and the average grade for each category is determined by use of formula (2). For each of these categories, the corresponding average conditional value of main product 1 is now calculated using the relevant regression formula similar to (3) above, thus defining the means of 8 conditional distribu-

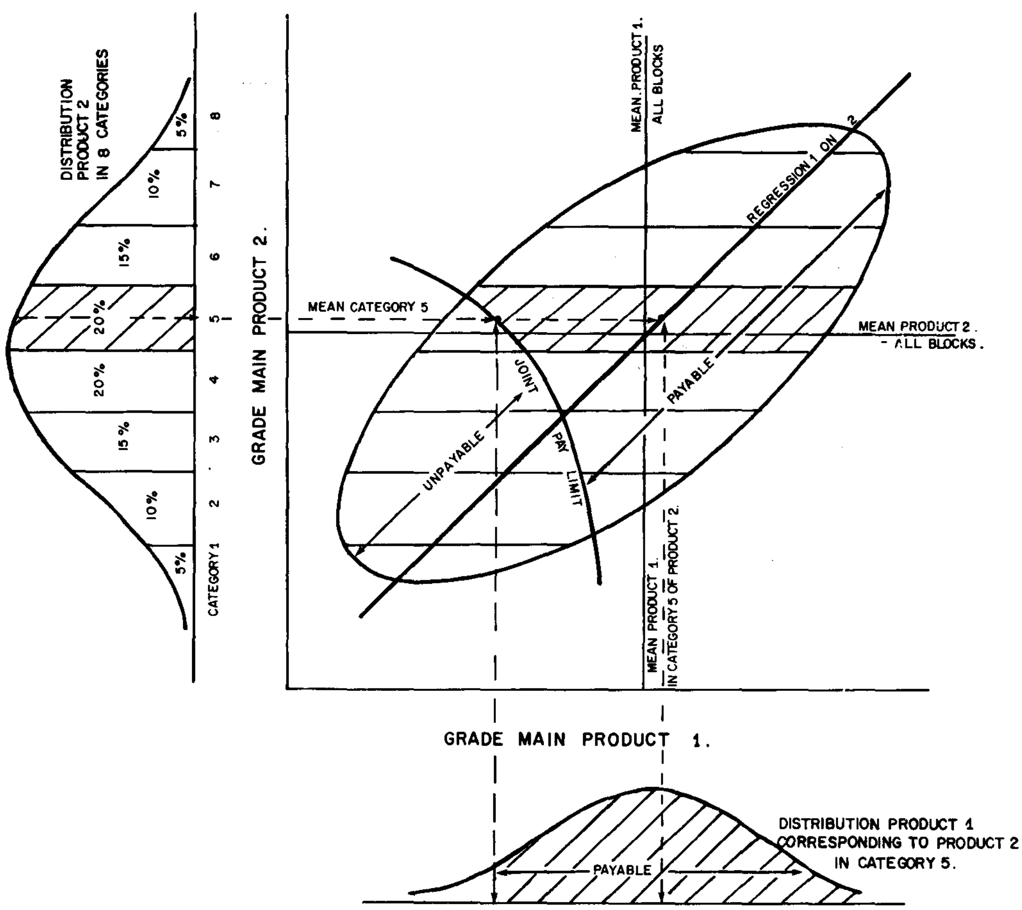

Fig. 5—Bi-variate lognormal model for mining selectively to a joint pay limit based on two main products

tions of the main product 1, each with log variance $= r^2 \times$ log variance of main product 1.

For each of these 8 distributions, the joint pay-limit value for main product 1 corresponding to the mean value for main product 2 in that category is now determined, and hence the pay fraction and pay grade (product 1) for each category, and for the ore-body as a whole when the 8 pairs of values are combined on the appropriate weighted basis of the frequencies 5, 10 . . . 10, 5 per cent as set originally.

As in Section 2, the joint pay limit is now calculated for the final year of production, and an average joint pay limit is determined, and hence the adjusted pay fraction, tons, and grade for main product 1. The pay grade for main product 2 is calculated direct from the average grades of the 8 original categories weighted by the percentages of the total ore-body to be selected in each category, i.e. by the products of the original percentages 5, 10, 15, 20 . . . 5 per cent and the pay fraction in each category.

(*b*) *Byproducts*

For products not correlated with main product 1, the mean ore-body grades as selected are accepted.

For a byproduct that is correlated with the main product, a first approximation is accepted using the distribution of the means along the regression line of the byproduct on the main product 1 (see Section 2 above) and the overall percentage pay fraction as determined in Section 3(a). These pay values for the byproducts are now used to revise the joint pay limit in Section 3(a) and, if changed by more than $2\frac{1}{2}$ per cent the calculations in 3(a) and 3(b) are repeated using the revised pay limit.

Addendum 2: Summary of General Risk-analysis Model

General Sequence of Calculations

Section A:
Details of input requirements.

Section B:
Selection of values for risk variables — omitted if only one run is required on likely values of variables.

Section C:
Calculation of effects of selective mining; mine life; and of production, sales, stocks tons, and contents year by year to the end of the mine life.

Section D:
Calculation of revenue from sales, year by year; and of gross minerals in course of realization.

Section E:
Calculation of all escalated costs of production and of stocks year by year.

Section F:
Calculation of annual sundry revenue and costs, creditors and debtors, stores, net minerals in course of realization, and escalated capital expenditure.

Sections G, H, I, J:
Details for long-term, building-society, and revolving-credit loans, and of the total loan position respectively.

Section K:
Calculation of equity issues, numbers of shares, issue expenses, etc.

Section L:
Provisional deposit interest, working profits, royalties, lease and tax payments, dividends, deposit balance, and final adjustments to the deposit interest dividends and the deposit balance.

Section M:
Calculation of factors for current to real cash flows and of the maximum shortfall of capital funds.

Section N:
Splitting of cash flows between individual investor groups to cover total equity and loan investments and the total corresponding receipts and returns for each group, i.e. D.C.F. return, pay backs, present values, etc.

Section O:
Index of all input and calculated variables, coding and computer-program section references.

Section P:
Details of output formats with coding of all output variables.

The following is a brief summary of the output from a typical risk analysis.

1. *Details of Each Investment Analysis, i.e. of Each of say* 50 *Iterations*

Set no. (from No. 1 to total, say 50)		
Pay and unpay head grade		Gold g/t
		Uranium g/t
Average head grade (after selective mining)		Gold g/t
		Uranium g/t
Product prices		Gold R/kg
		Uranium R/kg
Average payability		%
Mill tons		Millions
Cost deviations, %/100		Mining
		Transport
		Smelting
		Capital
Deviations of Rand inflation, %/100		Year 1
		Year 2
		Thereafter
Selected recoveries, %/100		Gold
		Uranium
Maximum shortfall capital funds		R million
		Year
Taxation, R million		Total
		Mining
Taxable income mining		R million
Investor Group 1:		
Current terms:	D.C.F. return	% p.a.
	Pay back	Years
		% p.a. thereafter
	Average pay back	Years
	Capital appreciation, %	At 8% p.a.
		At 12% p.a.
		At 18% p.a.
		At 25% p.a.
Real terms:	D.C.F. return	% p.a.
	Payback	Years
		% p.a. thereafter

Average Pay Back	Years
Capital	At 8% p.a.
Appreciation, %	At 12% p.a.
	At 18% p.a.
	At 25% p.a.

(Similarly for other Groups 2/4).

2. *Maximum Shortfall of Capital Funds*
Correlation table printed of years (8 categories) vs shortfall, R millions (8 categories).
3. *D.C.F. Returns*
8 frequency distributions listed in 8 categories of % p.a. return for current and real terms and for each of 4 investor groups
4. *Payback, Average Pay Back, and Returns % p.a. after Payback*
24 frequency distributions as for D.C.F. returns
5. *Capital Appreciations, %*
At 4 specified rates of discount for present values, in both current and real terms: 32 frequency distributions as for D.C.F. returns.

Acknowledgement

The opportunity and facilities offered by Anglo-Transvaal Consolidated Investment Company Limited for the development of the computer program, as well as the permission to publish this paper, are highly appreciated. Credit for a major programming effort is due to Messrs Jacot-Guillarmod and Jurek.

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Publications

Review

Mining annual review June 1979. London, Mining Journal, 1979. 624 pp. Available as part of the subscription to the *Mining Journal* or at £12,50 per copy.

This annual review is recognized internationally as the world leader in reporting on metals and minerals, and is eagerly awaited each year for its authoritative content, which is compiled by well-respected experts throughout the world. The gathering of all the data given in this book is an achievement of organization and compilation.

Metals and minerals are detailed under 60 headings from Aluminium to Zirconium. A review of the London Metal Exchange is included in this section and gives price movements and turnover of the more important metals. This section also includes details of the following:

Precious Metals
Platinum Metals
Older Major Metals
Light Metals
Steel Industrial Metals
Fuel Minerals
Nuclear Metals
Electronic Metals and Minerals
Chemical Metals and Minerals
Insulants and Refractories
Gemstones and Abrasives.

The section on exploitation covers mineral exploration, open-pit mining, underground mining, mineral processing, and extractive metallurgy.

The portion of the book devoted to various areas reviews activities in 128 countries of the world. The dependence of the U.S.A. on imports of metals and minerals is highlighted, and it is of interest to note that the U.S. trade in raw and processed minerals ran at a deficit of $19 billion in 1977. Details of the stockpiling of metals and minerals in the U.S.A. is of particular interest and significance to the economic world.

Other giants in the production of steel in the Western World are also dependent to a high degree on imports of minerals and metals. For instance, the only fossil fuel produced in Japan is coal, and owing to the great depths of their coal seams, the industry is assisted by the State, but production targets have not been achieved and Japan must still import large quantities.

The book emphasizes the significant role played by both Rhodesia and South Africa in the supply of chromium to world markets.

The strategic importance of cobalt supplies from Zaire is highlighted by the publication of figures to illustrate that 12 000 tons per annum of the world total production of 20 000 tons is derived from Zaire, and the U.S.A. imports 70 per cent of her requirements from this source.

This excellently produced book is a 'must' for all industrial and mining concerns, and should be readily available for reference purposes.

D. Malan