Beta coefficients and the discount rate in project valuation

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SYNOPSIS

This paper presents a method that enables one to estimate the risk-adjusted discount rate that should be used in project discounted cash-flow analysis. The risk-adjustment process is carried out via the capital asset pricing model of modern finance theory, in which the appropriate measure of risk is the beta coefficient of the project concerned. The resulting discount rate thus depends on the properties of the project itself, and not on the nature or cost of the capital of the firm considering the investment.

SAMEVATTING

Hierdie referaat beskryf 'n metode om die risiko-aangepaste diskontokoers wat in projekverdiskonteerde kontantvloeisontleding gebruik behoort te word, te beraam. Die risikoaanpasproses geskied via die kapitaalbateprysbepalingsmodel van die moderne finansiele teorie, waar die toepaslike mate van risiko die betakoëffisiënt van die betrokke projek is. Die resulterende diskontokoers hang dus van die eienskappe van die projek self af, en nie van die aard of koste van die kapitaal van die firma wat die belegging oorweeg nie.

Introduction

It has been stated that decisions regarding capital investment represent the most exacting task of the management of any company. Excellence in activities such as manufacturing operations, research, distribution, sales organization, raw-material acquisition, and industrial relations are absolutely necessary for the profitability and the survival of an enterprise. However, excellence in these operational areas can rarely do more than postpone the ultimate collapse of a corporation that consistently lacks excellence in the development of a soundly based capital expenditure strategy[1].

These comments have particular relevance for the decision-makers of the mining industry. The capital investment required to establish a new mine is now so large that a single bad decision can be ruinous. To further complicate an already difficult decision, one has the impression that all the major variables that affect the cash flows — metal prices, working costs, capital expenditures, inflation, exchange rates, and the like— are today subject to even greater uncertainty than they were a decade ago. It is therefore not surprising that the mining industry has traditionally shown a remarkable willingness to try any new tool or technique that might facilitate its capital budgeting decisions.

It is probably true to say that the mining industry pioneered the use of the discounted cash flow (DCF) technique in South Africa[2]. Today this technique is so pervasive that it is hard to conceive of any major mining house embarking on a new venture without the final formal decision being based on a DCF analysis. However, the technique is not without shortcomings; perhaps the greatest of these is the difficulty of choosing the 'right' discount rate.

Fig. 1 illustrates the impact of the discount rate on the net present value (NPV) of a real-life investment oppurtunity. This particular analysis sought to value a then unexploited mineral deposit that was potentially up for sale. It can be seen that the NPV falls by more than R65 million as the discount rate is raised from 10 per cent p.a. to 15 per cent p.a. Clearly, an imprecisely defined discount rate can result in a great deal of uncertainty regarding the 'value' of a project.

The existence of this problem is, of course, well-known, and indeed Janisch[3] has suggested certain procedures for overcoming it in gold-mine valuations. He recognizes that a guide to the choice of the discount rate is given by the stock market and that 'the discount rate apparently being applied by the market to each share . . . will differ from mine to mine'. The only additional factor that needs to be introduced into his analysis is the idea of risk. Fortunately, modern finance theory provides formal guidelines in this regard.

The theory of finance has undergone a revolution over the past two decades. The seminal development was Markowitz's quantification of the concept of risk in the early 1960s. This development led ultimately to the formulation of a rigorous theory of asset pricing, which establishes the price of any capital asset in terms of its risk. The purpose of this paper is to show how these concepts of modern finance might be applied to investment decisions in the mining industry.

Theory[4,5]

It is generally accepted among finance theorists that the objective of a firm is to maximize the wealth of its shareholders. In a classic analysis, Hirshleifer[6] showed that this objective is achieved if the firm accepts only projects having positive NPVs. The discount rate that should be used to calculate the NPV is not something internal to the firm, but is rather a rate that is established in capital markets. The net contribution of a particular project to the market value of the firm is then the value that the market would place on that project if it could be operated as a mini-firm, i.e., if its incremental cash flows could be 'bundled up' and offered to the market as a special and distinct security. This project value is clearly independent of the type or amount of other assets held by the parent firm.

Two factors — uncertainty and gearing — greatly

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complicate the preceding scenario; indeed, they generate important theoretical problems that have not yet been fully solved. If gearing is ignored and a number of important simplifying assumptions are made, it can be shown that the conventional procedure of discounting the expected cash flows at a single discount rate may in *some* cases still be used in the determination of the present value of a long-lived asset. However, the rate used in the discounting procedure needs first to be 'adjusted' for risk. The remainder of this paper is devoted to the estimation of this risk-adjusted discount rate†.

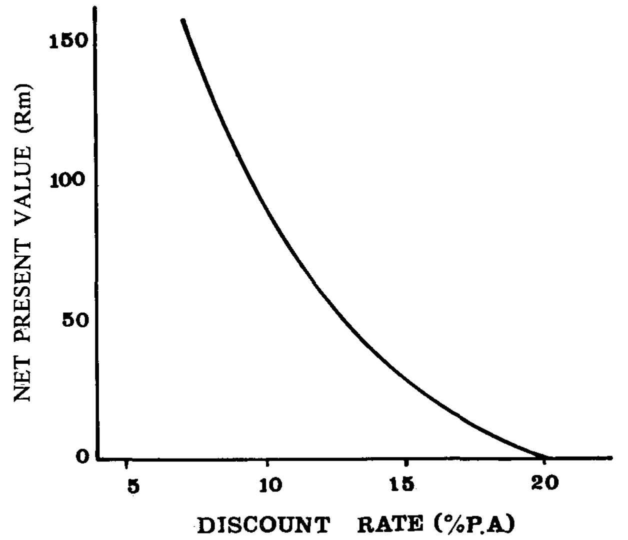

Fig. 1—Project NPV versus discount rate

The prime tool to be used in the establishment of the risk-adjusted discount rate is the capital-asset pricing model (CAPM), which states that

$$k_j = R_F + \beta_j[E(R_m) - R_F], \quad \text{. . . . .} \quad (1)$$

where k_j=appropriate risk-adjusted discount rate for project j,

R_F=rate or return on 'risk-free' securities,

$E(R_m)$=*expected* rate or return on the market portfolio of risky assets, and

β_j=the beta coefficient for project j, which provides a measure of the 'systematic' risk of the project, i.e., that part of the risk which is 'market related' and hence needs to be taken into account when the cash flows are discounted.

The original derivation of this model is generally attributed to Sharpe[9], Lintner[10], and Mossin[11], and it can now be found in virtually every introductory

†Needless to say, the ensuing discussion is limited strictly to those special cases where the simplifying assumptions apply. A discussion of these assumptions is beyond the scope of the present paper. The interested reader is referred to Treynor and Black[7], and also Robichek and Myers[8].

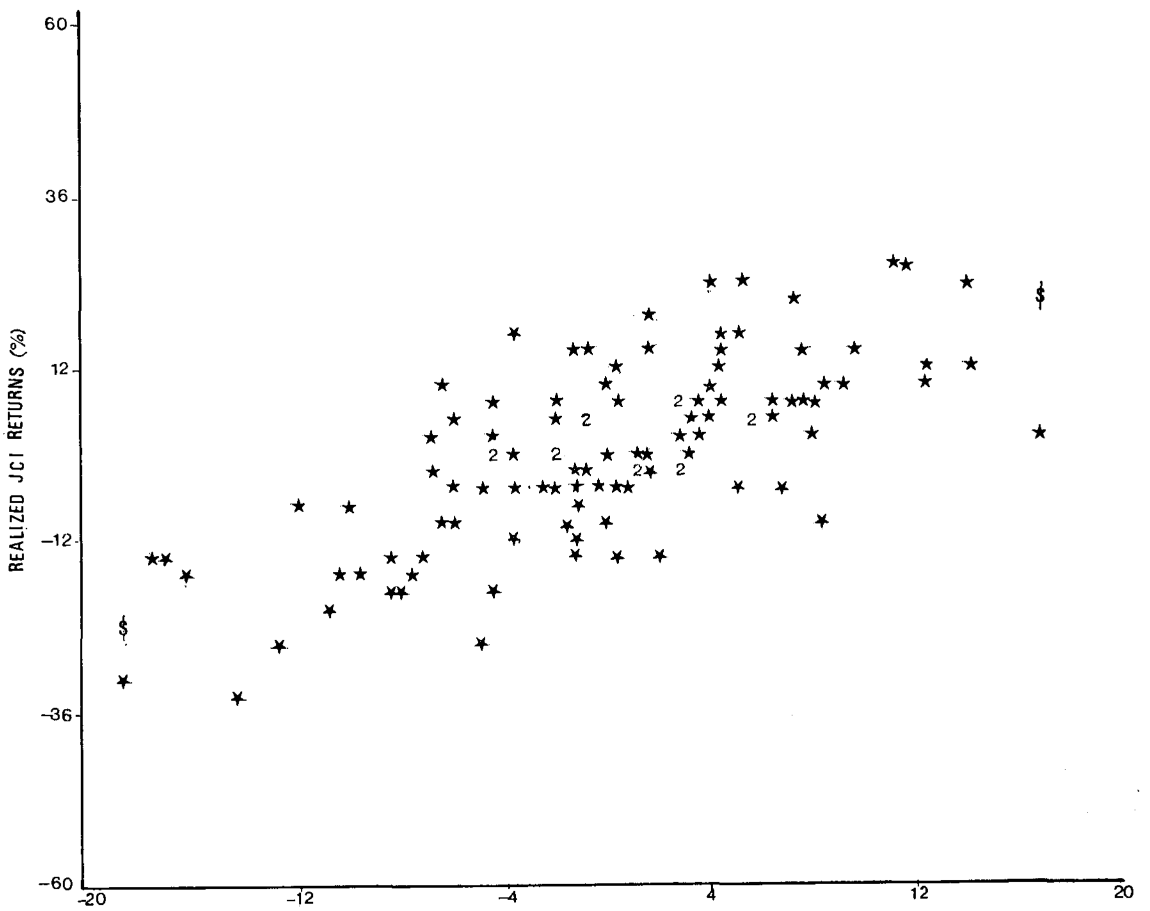

Fig. 2—Scatter diagram for the regression of JCI returns on the JSE Actuaries All Market returns

finance text (see for example Van Horne[12]). A simplified discussion with illustrations drawn from the Johannesburg Stock Exchange (JSE) is given by Gilbertson[13].

To make this model useful in a practical sense, one requires estimates of the three parameters β, R_F, and $E(R_m)$. Typical estimation procedures are described below.

Estimation of the Beta Coefficient

Beta coefficients are generally estimated via the 'market model', which seeks to explain *realized* security returns by postulating a linear relationship between such returns and an 'underlying market factor'. Specifically, the model states that

$$R_i = a_i + \beta_i R_m + e_i, \quad \quad (2)$$

where R_i = realized rate of return on security i,

R_m = realized rate of return on the market,

a_i and β_i are constants,

e_i = residual random 'disturbance' term having a Gaussian distribution and an expected value of zero.

Under certain simplifying assumptions[14], it can be shown that the linear regression coefficient, β_i, that minimizes the variance of the residual terms, e_i, in equation (2) is identical to the risk measure, β_j, that was utilized in equation (1). Hence, the market model provides a direct method for estimating, via regression analysis, the beta coefficient, β_j, which is required to make the CAPM useful in a practical sense. It is customary in empirical work to convert equation (2) to 'risk premium' form so as to remove 'noise' arising from fluctuations in the general level of interest rates. This is achieved by subtracting R_F from R_i and R_m in equation (2).

A typical regression analysis is illustrated in Fig. 2. It shows the scatter diagram that results when the realized weekly returns on the ordinary shares of Johannesburg Consolidated Investment Company Limited (JCI) — capital gains plus dividends expressed as a percentage of the opening price — are regressed against the corresponding realized returns on the JSE Actuaries All Market Index. Table I compares the resulting regression statistics with those of some other important mining and mining-finance shares.

The values in the first numerical column of Table I

TABLE I

REGRESSION STATISTICS FOR VARIOUS MINING SHARES

Share	JSE Code	Beta Estimate	Beta Std error	Correlation coefficient
Mining houses				
Anglo American	AAC	0,99	0,11	0,70
Johnnies	JCI	1,18	0,15	0,66
GFSA	GFS	1,38	0,16	0,69
Coal				
Trans Natal	TNC	0,54	0,15	0,39
Tavistock	TAV	0,63	0,17	0,39
Witcol	WIC	0,92	0,21	0,45
Gold and uranium				
Ergo	ERG	1,06	0,21	0,65
West Dries	WDR	1,16	0,13	0,71
Elandsrand	ELA	1,72	0,24	0,69
Other				
Palamin	PAM	0,58	0,14	0,43
Rusplat	RPT	1,15	0,19	0,57
Implat	IMP	1,23	0,17	0,63

TABLE II

ESTIMATES OF EX POST MARKET RETURNS AND RISK PREMIUMS

(Annualized from monthly returns on JSE Actuaries All Market Index; dividends are included. Interest rate on short-term government stock was used as the estimate of R_F)

Period	Historical market return % p.a.		Historical risk premiums % p.a.	
	Mean $\overline{R_m}$	Standard deviation	Mean $\overline{R_m - R_F}$	Standard deviation
Jan. 1960–Mar. 1979	14,3	20,5	*	*
Jun. 1973–Mar. 1979	12,6	25,9	5,2	25,8
Jan. 1978–Mar. 1979	39,9	19,5	28,7	19,5
Jan. 1979–Mar. 1979	24,9	11,1	17,5	11,1

*R_F data not readily available.

are the beta estimates that are required for use in the CAPM. The second numerical column provides an indication of the uncertainty that is associated with each estimate. Two comments are appropriate at this stage. Firstly, the underlying data on the share prices and the specific estimation procedures require meticulous attention if reliable beta estimates are to be obtained. Secondly, there is a great deal of empirical evidence that the beta coefficients of individual shares are not particularly stationary, i.e., these coefficients change with time, although gradually. Consequently, users of beta coefficients will want to update their estimates frequently, using, insofar as it is possible, only 'recent' share prices. In most countries, including South Africa, beta estimates are also provided on a commercial basis by specialized risk-measurement services.

Estimation of R_F and the Risk Premium

The remaining two parameters required for the practical utilization of the CAPM are R_F and $E(R_m)$. As proxy for the risk-free rate, R_F, one may well use the interest rate on short-term government bonds. The short-term rate, rather than the long-term rate, is chosen in order to match the time-horizon that will be used in the estimation of $E(R_m)$. (There is a remaining question regarding whether or not this rate is 'appropriate' for CAPM estimates. It can be argued that this interest rate is not determined by supply and demand in a competitive market. Government bonds are 'approved' assets in which institutions are obliged to invest part of their funds; under these circumstances, the resulting interest rate might well be artificially low.)

The estimation of $E(R_m)$ presents considerable difficulties. As a first guide, one might measure the historical returns and risk premiums actually realized on the JSE. Table II shows such an analysis.

It will be seen that both the average realized market return, $\overline{R_m}$, and the average realized risk premium, $\overline{R_m - R_F}$, depend greatly on the period over which they are measured. Furthermore, the standard deviations of the returns are so large as to accommodate virtually any reasonable estimate of $E(R)$ that might be generated. Consequently, we need a more specific procedure for estimating either $E(R)$ or the risk premium, $[E(R) - R_F]$.

Gilbertson[13] has proposed a procedure for dealing with this situation. In Essence, the procedure

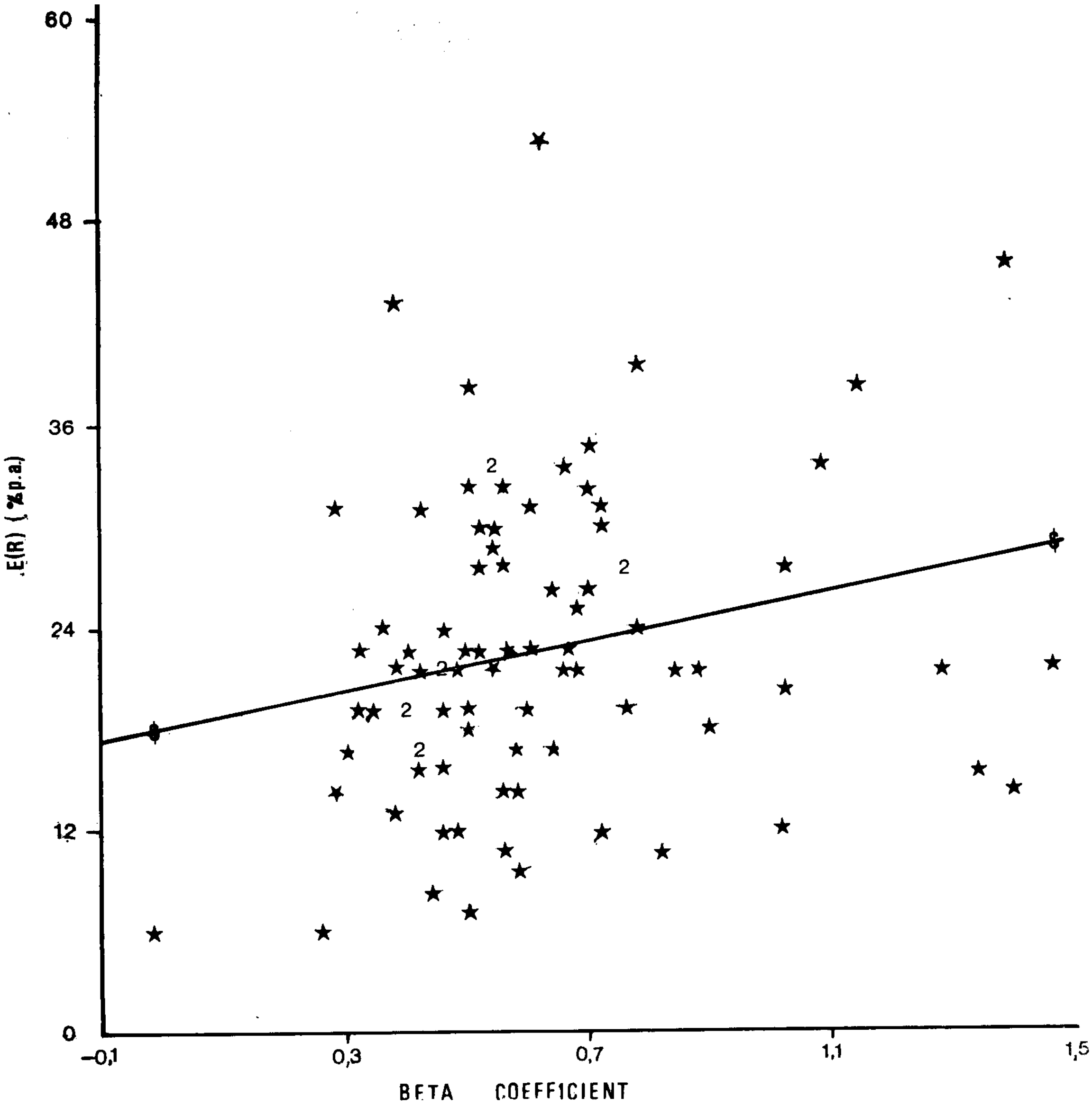

Fig. 3—Cross-sectional regression of $E(R)$ on beta for 87 individual shares (the regression line shown has been constrained to pass through the risk-free rate)

involves the use of forecasts of dividends and/or earnings for a large number of individual shares in the estimation of the expected returns for those shares. A cross-sectional regression of these expected returns on the corresponding beta coefficients then generate a least-squares estimate of the risk-premium, $[E(R_m) - R_F]$, in equation (1).

One of the difficulties involved in the implementation of this procedure is that it requires an analysis of the financial prospects of a large number of companies; this is generally beyond the resources of the individual investor*. For most of the overseas stock markets, this type of $E(R)$, β analysis can be obtained on a routine basis from major stockbroking firms. At least one of the broking firms dealing on the JSE operates such a service.

Fig. 3 is the scatter diagram generated by the regression of a set of $E(R)$ estimates (prepared by the Johannesburg stockbroking firm) on the corresponding β estimates (derived by the regression of the share returns on the JSE Actuaries All Market Index returns). The $E(R)$ estimates were based on the dividend receipts and capital gains expected over the next twelve months from some 90 non-gold-mining companies included in the JSE Actuaries All Market Index. The least-squares best-fit line is given by the equation

$$E(R) = 18{,}5\% + \beta(6{,}8\%) \text{ p.a.} \qquad (3)$$

As might be expected from the wide scatter of the points in Fig. 3, this relationship is not statistically significant. Nevertheless, equation (3) represents our best estimate† of the risk-return trade-off on the JSE during August 1979. It might be noted that the imputed risk-free rate of 18,5 per cent is much higher than the rate on short-term gilts; this gives added significance to the bracketed comment in the first paragraph of this section.

*Additional difficulties arise in the estimation of the expected returns of gold-mining shares; in particular, the forecast dividend is sensitive to the assumed gold price, and this introduces an uncertainty that is not experienced with industrial shares.

†It is interesting to observe that the parameters in equation (3) are considerably higher than those estimated by Gilbertson[13] for industrial shares, viz

$$E(R) = 15{,}1\% + \beta(3{,}6\%) \text{ p.a.}$$

In Gilbertson's estimates, the beta coefficients of the industrial shares were determined by regression against the RDM-100 Index returns; the average of the resulting beta coefficients was close to the anticipated average of unity. In this present research, the average beta coefficients of the non-gold-mining shares is just above 0,6, so that, on average, the estimated required rates of return on these shares are not greatly different whether equation (3) or Gilbertson's results[13] are used in the CAPM. It is of interest, also, that the beta coefficients of gold-mining shares tend to be higher than those of industrial shares (when estimated against the Actuaries All Market Index). The CAPM would then imply that investors require higher returns from the former than they do from the latter.

Application of the CAPM to Project Valuation

The application of these concepts to project valuation is now straightforward. Consider a mining house that is investigating a possible new gold mine with properties similar to, say, Elandsrand. The discount rate that should be applied to the risky component of the project cash flows is then given by (see Table I)

$$k_{\text{ELA}}=18{,}5\%+(1{,}72\times 6{,}8\%)=30\% \text{ p.a.} \quad . \; . \; . \; (4)$$

This is, of course, a high rate of discount. Conversely, the risky component of the cash flows of a coal-mining venture similar to, say, Trans Natal should be discounted at the lower rate of

$$k_{\text{TNC}}=18{,}5\%+(0{,}54\times 6{,}8\%)=22\% \text{ p.a.} \quad . \; . \; . \; (5)$$

Note specifically that it is *not* the 'cost of capital' of the parent mining house that should be used in the discounting process; however, if desired, the 'cost of *equity* capital' of the parent could also be determined via the CAPM. For example, in the case of AAC, we have

$$k_{\text{AAC}}=18{,}5\%+(0{,}99\times 6{,}8\%)=25\% \text{ p.a.} \quad . \; . \; . \; (6)$$

This 'cost of capital' should be used as a discount rate only for those projects having risk properties that are 'homogeneous' with those of the (usually) well-diversified parent house; these would be rare projects indeed.

In practical applications it must be borne in mind that the risk-adjusted discount rates given by equations (4) to (6) were estimated on the basis of a one-year time horizon and hence reflect *inter alia* the particular inflationary environment perceived by JSE investors for that period. This should be taken into account in the preparation of the project cash-flow stream that is to be discounted. Also, these discount rates are not immutable; they will change gradually with time as the beta coefficients vary. More important, they may undergo larger short-term changes as 'market sentiment'—inflation expectations, interest rates, and hence the risk premium — fluctuates; by the end of 1979 the risk premium may differ greatly from the 6,8 per cent p.a. estimated for August 1979*. Finally, and perhaps most important, there are significant uncertainties associated with the parameters estimated for equation (3). For example, presumably at present most users would prefer to use a 'risk-free' rate of around 10 to 12 per cent p.a., rather than the estimated value of 18,5 per cent p.a. They can incorporate such preferences by constraining the least-square line to pass through the desired point. (This particular constraint would, of course, result in a higher estimated risk premium, and hence in a greater spread in the discount rate estimates of equations (4) to (6).)

Conclusions

This paper has presented a methodology derived from modern finance theory that enables one to estimate the risk-adjusted discount rate that should be used in project DCF analysis. The risk-adjustment process is carried out via the CAPM, in which the appropriate measure of risk is the beta coefficient of the project concerned. Specifically, this rate depends on the properties of the project and not on the nature of the firm considering the investment opportunity.

It will no doubt appear to the reader that the use of these concepts involves a great deal of uncertainty; this is true, but it must nevertheless be affirmed that these procedures are the most precise that are currently available. Indeed, these concepts have been tested on several occasions in American courts, including the landmark Reserve Mining civil case and also the AT & T rate case before the Federal Communications Commission. In both these cases, the court accepted the beta-based cost of capital, while rejecting other estimates. The tools may still be blunt and crude, but as yet there are no others to equal them.

*'CAPM dynamics' is beyond the scope of this brief presentation, but the interested reader is referred to Litzenberger and Budd[15].

Acknowledgements

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References

1. THOMSON, W. C. (Chairman, Shell Chemicals U.K. Ltd, and Director, Shell International Chemical Co.). Public lecture.
2. LUTTRELL-WEST, L. W. Some aspects of mineral valuation in South Africa. *J. Chem. Metall. Min. Soc. S. Afr.*, vol. 48, no. 6. Dec. 1947. pp. 157-191.
3. JANISCH, P. R. The limitations of gold-mine valuation. *J. S. Afr. Inst. Min. Metall.*, Jun. 1976. pp. 439-444.
4. MYERS, S. C. (ed.). Modern developments in financial management. New York, Praeger Publishers Inc., 1976.
5. *Ibid.*, pp. 277 ff.
6. HIRSCHLEIFER, J. On the theory of optimal investment decision. *Ibid.*, pp. 329-352. (Also: *J. Polit. Econy*, vol. 66. Aug. 1958).
7. TREYNOR, J. L., and BLACK, F. Corporate investment decision. *Ibid.*, Chap. 16.
8. ROBICHEK, A. A., and MYERS, S. C. Conceptual problems in the use of risk-adjusted discount rates. *J. Finance*, vol. 21. Dec. 1966. pp. 727-730.
9. SHARPE, W. F. Capital asset prices: a theory of market equilibrium under conditions of risk. *J. Finance*, vol. 19. Sep. 1964. pp. 425-442.
10. LINTNER, J. Security prices, risk and the maximal gains from diversification *J. Finance*, vol. 20. Dec. 1965. pp. 587-616.
11. MOSSIN, J. Equilibrium in a capital asset market. *Econometrica*, Oct. 1966. pp. 768-783.
12. VAN HORNE, J. Financial management and policy. New York, Prentice/Hall International, 4th Edition, 1977.
13. GILBERTSON, B. The pricing of industrial shares on the Johannesburg Stock Exchange. *Investment Analysts Journal (S.A.)*, no. 14. pp. 21-36.
14. FAMA, E. F. A note on the market model and the two-parameter model *J. Finance*, vol. 28. Dec. 1973. pp. 1181-1185.
15. LITZENBERGER, R. H., and BUDD., A. P. Secular trends in risk premiums. *J. Finance*, vol. 27. Sep. 1972. pp. 857-864.