# The reduction in amplitude and change in phase of the diurnal temperature variation of ventilation air

by K. R. VOST*, M.Sc., A.M. Aus. I.M.M. (Visitor)

**SYNOPSIS**

Calculations are given for the reduction in amplitude and change in phase of the diurnal variation in air temperature as the air flows through a mine. The concrete lining of a typical shaft was found to have a much greater effect than the steelwork. An increase in the conductivity of the concrete, or filling of the steelwork with water, did not have enough effect on the rise in air temperatures underground during the day to justify the costs involved. The steel temperature followed the air temperature much more closely than the concrete temperature did. Calculations are given for the heat stored in steel and concrete during the day.

**SAMEVATTING**

Daar word berekenings gegee vir die vermindering in amplitude en faseverandering van die daaglikse variasie in lugtemperatuur namate die lug deur 'n myn vloei. Daar is gevind dat die betonvoering van 'n tipiese skag 'n veel groter uitwerking as die staalwerk. 'n Verhoging van die geleivermoë van die beton, of die vul van die staalwerk met water, het nie 'n groot genoeg uitwerking op die styging van lugtemperature gedurende die dag ondergronds gehad om die koste daaraan verbonde te regverdig nie. Die staaltemperatuur het die lugtemperatuur baie noukeuriger as die betontemperatuur gevolg. Daar word berekenings gegee vir die hitte wat gedurende die dag in staal en beton opgegaar word.

## Introduction

The air temperature at the surface of a mine exhibits annual and diurnal variations, and also variations in average temperature from one day to the next caused by changes in atmospheric conditions. These variations are reduced in magnitude (i.e., damped) and changed in phase as the air flows through the mine. Possible causes are the large masses of steelwork in the shafts, and the concrete and rock surrounding the airways. There is no published information on the relative importance of the steelwork, and the first objective of the project described in this paper was to determine this theoretically and also calculate the heat stored in the steelwork, concrete, and rock during the variations in air temperature. The second objective was to increase the damping and phase change of the diurnal variation by changing the physical conditions in the airways, for example by increasing the conductivity of the concrete lining or filling the buntons with water. The rise in air temperature at the surface during the main production shift might then be delayed underground until after the shift was over, and also reduced significantly in magnitude.

The effect of the diurnal variation was analysed in preference to the annual variation since it was of greater practical interest. A sinusoidal variation was assumed. Changes in atmospheric conditions cause the diurnal variation to differ significantly from a sinusoid, but a good approximation can be obtained by the combination of several sinusoids.

## Nomenclature

$k$=thermal conductivity, W/m . °C
$\rho$=density, kg/m$^3$
$c$=thermal capacity, J/kg . °C
$\alpha$=thermal diffusivity, m$^2$/s
$\theta$=temperature, °C
$t$=time, s
$r$=radius of airway, m
$m$=mass of steel or water per unit length of airway, kg/m
$A$=area of contact with air per unit length of airway, m$^2$/m
$H$=surface heat-transfer coefficient, W/m$^2$ . °C
$G$=mass flow-rate of air, kg/s
$\Omega=\frac{2\pi}{T}$, where $T$ is the period of the oscillation, rad/s

*Subscripts*

a indicates air, e.g. $c_a$ is the thermal capacity of air (at constant pressure)
s indicates steel, e.g. $\theta_s$ is the temperature of steel
w indicates water, e.g. $m_w$ is the mass of water per unit length of airway
(No subscript is used for rock or concrete).

## Data Used in Calculations

Typical values of various quantities used in the calculations are listed in Tables I to III. Reasons for their selection are given in Addendum A.

## Penetration of Temperature Variations into Shaft Walls

Shafts are usually lined with concrete, and an average thickness of 0,3 m is typical. To determine whether the rock behind the concrete takes part in the diurnal heat exchange, the depth of penetration into a continuous mass of concrete was first calculated. The concrete can be taken as a semi-infinite solid since the depth of penetration is small compared with the shaft dimensions. A shaft that has been ventilated for a few months will have only a small average temperature gradient in the concrete. The effect of the diurnal variation will be superimposed on this since the equation for heat con-

*School of Physics, W.S. & L.B. Robinson University College, University of New South Wales, Broken Hill, Australia.

duction is linear. The theory[1] used assumes that the periodic variation in air temperature has been maintained long enough for the transient disturbance caused by starting up the periodic variation to have died away, leaving a periodic variation in the concrete of the same frequency as that in the air.

If the variation in air temperature in a given cross-section of the shaft is $\theta_1 \sin \omega t$, then, at a distance $x$ into concrete, the temperature variation is given[1] by

$$\theta=\theta_1 \eta \exp(-cx) \sin(\omega t - cx - \epsilon) \quad \ldots\ldots \quad (1)$$

$$\eta=\sqrt{\frac{1}{1+\frac{2c}{b}+2\frac{c^2}{b^2}}}$$

$$b=\frac{H}{k}$$

$$c=\sqrt{\frac{\omega}{2a}}$$

$$\epsilon=\tan^{-1}\frac{1}{1+\frac{b}{c}}.$$

If $H$, $k$, and $a$ for a concrete-lined shaft (Tables I and II) and $\omega$ for the diurnal variation are substituted, equation (1) becomes

$$\theta=0{,}57\theta_1 \exp(-7{,}6x)$$
$$\sin\times(7{,}3\times10^{-5}t-7{,}6x-0{,}37). \quad \ldots\ldots \quad (2)$$

As $x$ increases, the amplitude decays exponentially and the phase lags further behind the air temperature. The amplitude and time lag for various values of $x$ are shown in Table IV.

The amplitude at 0,3 m is only 6 per cent of the amplitude of the variation in air temperature. It can be assumed that the rock in an actual shaft will take very little part in the diurnal heat exchange, and will not need to be considered in an analysis of the damping of diurnal variations.

TABLE I

ROCK (QUARTZITE) AND CONCRETE

| Medium | $k$ W/m.°C | $\rho$ kg/m³ | $c$ kJ/kg.°C | $a$ $10^{-6}$m²/s |
|---|---|---|---|---|
| Concrete | 1,5 | 2 400 | 1,0 | 0,63 |
| Rock | 5,2 | 2 670 | 0,83 | 2,36 |

TABLE II

AIRWAYS

| Type of airway | Diameter m | Air velocity m/s | Mass flow-rate ($G$) kg/s | Heat-transfer coefficient ($H$) W/m².°C |
|---|---|---|---|---|
| Shaft | 9,6 | 10 | 796 | 18 |
| Main intake | 3,4 | 2,5 | 25 | 13 |

$c_a=1{,}014$ kJ/kg.°C.

TABLE III

STEELWORK IN THE SHAFT

| Type of steelwork | $m_s$ kg/m | $m_w$ kg/m | $A_s$ m²/m | $H$ |
|---|---|---|---|---|
| Guides | 736 | — | 7,1 | 38 |
| Buntons | 647 | 274 | 8,6 | 38 |
| Total of guides and buntons | 1 383 | 274 | 15,7 | — |

$c_s=0{,}49$ kJ/kg.°C
$c_w=4{,}19$ kJ/kg.°C

TABLE IV

TEMPERATURE VARIATION IN CONCRETE

| Distance into concrete ($x$) m | Amplitude °C | Time lag h |
|---|---|---|
| 0,0 | 0,57 $\theta_1$ | 1,4 |
| 0,1 | 0,27 $\theta_1$ | 4,3 |
| 0,2 | 0,13 $\theta_1$ | 7,2 |
| 0,3 | 0,06 $\theta_1$ | 10,1 |

## Effect of Rock or Concrete

The theory[2] assumes a dry airway with a circular cross-section. If the periodic component of the air temperature at the entrance to the airway is $\theta_1 \sin \omega t$, then the periodic component of the air temperature at $y$ metres along the airway is given by

$$\theta_a=\theta_1 \exp\left(-\frac{2\pi r y k \lambda \gamma_1}{C_a G}\right) \sin\left(\omega t-\frac{2\pi r y k \lambda \gamma_2}{C_a G}\right), \quad (2)$$

where $\lambda=\sqrt{\frac{\omega}{a}}$ (the unit of $\lambda$ is m$^{-1}$)

and $\gamma_1$ and $\gamma_2$ are dimensionless quantities that are given in terms of Kelvin functions (ker, etc.) as given in Addendum B.

(In reference 2, the minus sign is missing between $\omega t$ and the phase lag in this equation.)

*Shafts*

Calculations were made, at a depth of 2 km, of the time lag and the exponential factor by which the amplitude of the diurnal variation is damped. It was assumed that only the concrete lining takes part in the heat exchange. To determine the effect of an increase in conductivity, the calculation was repeated with rock instead of concrete without changing the heat-transfer coefficient. The results are given in Table V.

The thermal conductivity, $k$, of rock is 3,5 times that of concrete. The product, $\rho c$, is approximately the same for both, and the diffusivity, $a$, of rock is 3,7 times that of concrete. Despite the big difference in $k$, the time lags were approximately equal. This occurred because the phase lag in equation (2) was proportional to $k\lambda\gamma_2$. $\gamma_2$ was unaffected by $\lambda r$ but decreased as $(\lambda k)/H$ increased, and the value in rock was approximately half that in concrete. $\lambda$ is inversely proportional to $\sqrt{a}$ and, although $k$ for rock was 3,5 times that of concrete, $k\lambda$ was only about 1,8 times that of concrete, and the values of $k\lambda\gamma_2$, and hence the time lags, were approximately equal.

An increase of $k$ by a factor of 3,5 reduced the amplitude damping factor from 0,53 to 0,43. The damping factor decreases as $k\lambda\gamma_1$ increases, but the increase in $k$ was offset by the reduction in $\lambda$ and $\gamma_1$.

The large increase in conductivity had less beneficial effects than expected, and, although the conductivity of concrete could be increased, for example by the addition of copper wire to the concrete, the costs would outweigh the benefits.

*Main Intake Airways*

The mass flow-rate of air was 25 kg/s for the main intake airway, which is 3 per cent of $G$ for the shaft (796 kg/s). Table VI shows that 200 m of intake airway caused approximately the same time lag and damping as 2 km of shaft (Table V). Equation (2) shows that the main reason is the much lower $G$ in the intake airway. The length of shaft required for a given damping and phase change of a given mass flow-rate was about one-third of the total length of intake airway required.

### Effect of Steelwork

A theory is now developed for the phase change and damping of air in a shaft that would be produced by the steel in the buntons and guides alone if there was a heat exchange of zero between the air and the concrete lining. Solutions are not given for when the heat exchange occurs with concrete and steelwork simultaneously. From the solutions for concrete and steelwork separately, the relative importance of each and the relative effects of changes in physical conditions can still be found accurately.

The details of the steelwork in a typical shaft are given in Table III and Addendum A. The steel is thin enough for the temperature of the steel in a given cross-section of the shaft to be regarded as uniform. The flow of heat vertically in the guides and buntons can be neglected. The effect of the air inside the buntons can also be neglected, since the product of mass and thermal capacity is much smaller than for the steel (by a factor of 883). The heat-balance equation for the steel would then be

$$-c_s m_s \frac{\delta\theta_s}{\delta t} = HA_s(\theta_s - \theta_a), \quad \dots \dots \dots \quad (3)$$

where the units of both terms are watts per metre length of shaft.

Heat energy is added or removed from the airstream by the steelwork, and is added by autocompression. In obtaining the heat balance for the air, a term involving the rate of increase in heat content of the air, which is very small in practice[3], is neglected. The resulting equation is

$$c_a G \frac{\delta\theta_a}{\delta y} = HA_s(\theta_s - \theta_a) + Gg, \quad \dots \dots \dots \quad (4)$$

where $y$ is the depth in the shaft.

If $\theta_s$ is eliminated from equations (3) and (4), the following equation is given for $\theta_a$:

$$c_a G \frac{\delta\theta_a}{\delta y} + c_s m_s \frac{\delta\theta_a}{\delta t} + \frac{c_s m_s c_a G}{HA_s} \frac{\delta}{\delta t}\left(\frac{\delta\theta_a}{\delta y}\right) = Gg. \quad (5)$$

If $\theta_a = \theta_2 + \theta_1 \sin \omega t$ at $y=0$, where $\theta_2$ and $\theta_1$ are constant, the solution of equation (5) is the sum of two components:

(a) the average air temperature

$$\theta_2 = \frac{g}{c_a} y,$$

whose second term is the increase caused by autocompression, and

(b) the periodic component, which is the one of interest in this paper, given by

$$\theta_a = \theta_1 \exp\left\{ -\frac{(\omega c_s m_s)^2 y}{HA_s c_a G\left[1+\left(\frac{\omega c_s m_s}{HA_s}\right)^2\right]} \right\}$$

$$\sin\left\{ \omega t - \frac{\omega c_s m_s y}{c_a G\left[1+\left(\frac{\omega c_s m_s}{HA_s}\right)^2\right]} \right\} \quad \dots \dots \quad (6)$$

Simplification occurs in the case of buntons and guides since, from Table III,

$$\left(\frac{\omega c_s m_s}{HA_s}\right)^2 = 0{,}0069$$

and equation (6) becomes

$$\theta_a = \theta_1 \exp\left[-\frac{(\omega c_s m_s)^2 y}{HA_s c_a G}\right] \sin\left[\omega t - \frac{\omega c_s m_s y}{c_a G}\right]. \quad . \quad (7)$$

At a depth of 2 km, the time lag is 28 minutes, and the amplitude damping factor is exp $(-1{,}02 \times 10^{-2})$, i.e. 0,99. The steelwork had a much smaller effect than the concrete, which produced a time lag of 1,05 hours and an amplitude damping factor of 0,53 (Table V).

The temperature of the steel also closely followed the air temperature. If the variation in air temperature in a given cross-section of shaft is $\theta_1 \sin \omega t$, the solution of equation (3) when the initial transient has died down is

$$\theta_s = \frac{\theta_1}{\sqrt{1+\left(\frac{\omega c_s m_s}{HA_s}\right)^2}} \sin\left(\omega t - \tan^{-1}\frac{\omega c_s m_s}{HA_s}\right). \quad . \quad . \quad (8)$$

TABLE V

TIME LAG AND DAMPING IN THE SHAFT

| Medium | $k$ W/m.°C | $\rho$ kg/m³ | $c$ kJ/kg.°C | $a$ $10^{-6}$m²/s | $\lambda$ m⁻¹ | $(\lambda k)/H$ | $\lambda r$ | $\gamma_1$ | $\gamma_2$ | Time lag h | Amplitude damping factor |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Concrete | 1,5 | 2400 | 1,00 | 0,63 | 10,79 | 0,90 | 51,8 | 0,52 | 0,22 | 1,05 | 0,53 |
| Rock | 5,2 | 2670 | 0,83 | 2,36 | 5,55 | 1,60 | 26,6 | 0,40 | 0,12 | 0,98 | 0,43 |

TABLE VI

TIME LAG AND DAMPING IN THE MAIN INTAKE AIRWAY

| Medium | $\lambda$ m⁻¹ | $\lambda r$ | $(\lambda k)/H$ | $\gamma_1$ | $\gamma_2$ | Length of airway m | Time lag h | Amplitude damping factor |
|---|---|---|---|---|---|---|---|---|
| Rock | 5,55 | 9,5 | 2,22 | 0,33 | 0,12 | 1000 | 5,4 | 0,02 |
| | | | | | | 500 | 2,7 | 0,14 |
| | | | | | | 200 | 1,1 | 0,45 |
| | | | | | | 100 | 0,5 | 0,67 |

The time lag is 19 minutes, and the amplitude damping factor 0,997. At the surface of the concrete, the time lag was 1,4 hours and the amplitude damping factor was 0,57 (Table IV); therefore the time lag and damping were much greater in the concrete than in the steelwork.

If the steel temperature lags behind the air temperature, damping must occur (and *vice versa*). Suppose, for example, that the air temperature is above its average value. If the steel temperature lags being the air temperature, then the sinusoids intersect when $\theta_a$ has fallen below its maximum. The maximum $\theta_s$ must occur at the point of intersection since, for greater times, $\theta_a$ is less than $\theta_s$ and heat flows from the steel into the air. The maximum $\theta_s$ must therefore be less than the maximum $\theta_a$ if there is a time lag.

The effect of variations in $m_s$, $A_s$, and $G$ can be determined by the approximate equation (7). If $m_s$ is doubled without any changes in the other parameters, the $m_s^2$ factor in the exponential gives an amplitude damping factor exp $(-4\times 1{,}02\times 10^{-2})$, i.e. 0,96, and the time lag is doubled to 56 minutes. If $G$ or $A_s$ are halved without any changes in the other parameters, the amplitude damping factor becomes exp $(-2\times 1{,}02\times 10^{-2})$, i.e. 0,98. When $G$ is halved, the time lag is doubled to 56 minutes but is unchanged when $A_s$ is halved.

## Buntons Filled With Water

A possible way to increase the damping and phase change of the air in a shaft would be to fill the buntons with water; the effect is now calculated.

The vibration caused by the cages running in the guides would produce some mixing of the water. To simplify the analysis, it was assumed that the temperature of the water was uniform in any given cross-section of the shaft and that $\theta_w$ equalled $\theta_s$ in the same cross-section.

The solution for the periodic component of $\theta_a$ will be equation (6) with $c_s m_s$ replaced by $(c_s m_s + c_w m_w)$. From Table III,

$$\left[\frac{\omega(c_s m_s + c_w m_w)}{HA_s}\right]^2 = 0{,}11.$$

Therefore, the approximation to equation (6) cannot be used. At a depth of 2 km, the time lag is 55 minutes and the amplitude damping factor 0,93. When the calculation was repeated without water in the buntons, the time lag was 13 minutes, and the amplitude damping factor 1,00. The addition of water to the buntons therefore would increase the time lag by 42 minutes and would produce slight damping, but this would probably not justify the costs involved.

## Heat Storage in Concrete and Steelwork

Periodic changes in air temperature will cause heat to be stored in the concrete and steel during one half of a cycle and to be returned to the air in the other.

*Concrete*

The heat stored can be found by first calculating the rate at which heat enters the concrete per unit length of shaft, which equals

$$-kA\left(\frac{\delta\theta}{\delta x}\right), \qquad (10)$$

where $A$ is the shaft perimeter and the concrete is taken as a semi-infinite solid whose surface is the plane $x=0$.

The relation between $\theta$ and $x$ is given by equation (1). If expression (10) is integrated with respect to time over the appropriate half cycle[1], then the heat stored in the concrete per unit length of shaft is

$$2\sqrt{\frac{\rho c k}{\omega}}\, A\eta\theta_1, \qquad (11)$$

where $\eta$ and $\theta_1$ appear in equation (1).

When $\theta_1$ is 1 °C, the heat stored in the concrete per metre of shaft is $7{,}7\times 10^6$ J/m.

*Steelwork*

In Addendum C it is shown that the heat energy stored in steel per unit length of shaft is

$$\frac{2c_s m_s\,\theta_1}{\sqrt{1+\left(\frac{\omega c_s m_s}{HA_s}\right)^2}}. \qquad (12)$$

When $\theta_1$ is 1 °C, the value for the buntons and guides is $1{,}4\times 10^6$ J/m.

The ratio of the heat stored in the concrete and in the buntons and guides is 5,5 to 1. An estimate of the relative capabilities of storing heat cannot be obtained from the ratio of the products of thermal capacity and mass per unit length for concrete and steel. This ratio would be 33 to 1.

The instantaneous rates at which heat is stored in the concrete and steelwork over the 2 km of shaft can be calculated by use of the amplitude damping factors and time lags of the variation in air temperature at the foot of the shaft. If the diurnal variation of air temperature at the surface of the mine has an amplitude of 10 °C, the heat stored in the concrete provides 4000 kW of cooling during the 2 hours before the peak temperature is reached at the surface. There is no net flow back into the airstream until 5 hours after this peak temperature. Heat stored in the steelwork with the buntons filled with water provides a maximum of 1900 kW of cooling. Heat begins to be returned to the airstream 95 minutes after the peak temperature has been reached at the surface.

## Conclusions

Diurnal variations do not penetrate to any appreciable extent into the rock behind the concrete lining of the shaft, and the rock does not need to be considered in calculations of the damping of diurnal variations.

The concrete lining of a 2 km shaft produced a time lag of 1 hour and a damping of 50 per cent. An increase in the conductivity of the concrete had little effect on the phase change caused by the shaft, and the increase in damping would not justify the costs involved in achieving a higher conductivity of the wall concrete.

The buntons and guides in 2 km of shaft produced a time lag of 23 minutes and virtually no damping, which was a much smaller effect than in the concrete. The steel temperature followed the air temperature much more closely than the concrete temperature did. The addition of water to the hollow buntons increased the time lag by 42 minutes, but this would probably not justify the costs involved.

The heat stored in the concrete during the day was six times that stored in the buntons and guides.

## Acknowledgements

The work described in this paper was carried out as part of the research programme of the Research Organisation of the Chamber of Mines of South Africa while the author was on sabbatical leave from the Broken Hill Division of the University of New South Wales, Australia. The author thanks Dr A. Whillier, who proposed the project and made many valuable suggestions as it progressed. The hospitality of the Chamber of Mines is also gratefully acknowledged.


## References


1. Jakob, M. *Heat transfer*, vol. 1. New York, Wiley. 1949.
2. Hiramatsu, Y., and Amano, K. Calculation of the rate of flow, temperature and humidity of air currents in a mine. *Int. J. Rock. Mech. Min. Sci.*, vol. 9. 1972. pp. 713-727.
3. Jordan, D. W. The numerical solution of underground heat transfer problems, Part 1. *Int. J. Rock Mech. Min. Sci.*, vol. 2. 1965. pp. 247-270.


## Addendum A: Data Used in the Calculations

The reasons for the selection of some of the typical values in Tables II and III are given here.

*Table II*

$H$ was calculated from Starfield and Dickson's[1] equation (5) using a friction factor of 1,7 for the main intake airway and 1,05 for the concrete-lined shaft.

$G$ was calculated using a density of air of 1,1 kg/m³.

*Table III*

The data were based partly on No. 1 shaft[2] of Kloof G.M. Co. Ltd. The following assumptions were made:

| | | |
|---|---|---|
| Number of compartments | —men and material | =6 |
| | —rock | =4 |
| Buntons | —dimensions | =381 mm × 102 mm aerofoil section |
| | —thickness | =10 mm |
| | —total length in a given cross-section of shaft | =44,5 m |
| | —vertical spacing | =4,57 m |
| Guides | —mass/unit length | =36,8 kg/m |
| | —dimensions | =152 mm × 102 mm 'top hat' |
| Density of steel | — | $=7{,}85\times10^3$ kg/m³. |

The value of $H$ for the buntons was obtained from Whillier's[3] Fig. 8,8 for flow outside pipes, and the same value was assumed for the guides.

*References*


1. Starfield, A. M., and Dickson, A. J. A study of heat transfer and moisture pick-up in mine airways. *J. S. Afr. Inst. Min. Metall.*, vol. 68. 1967. pp. 211-234.
2. Moll, C. Modern shaft systems in the Republic of South Africa. International Conference on Hoisting — Men, Materials, Minerals. Johannesburg, 16th to 24th Oct., 1973.
3. Whillier, A. *The ventilation of South African gold mines.* Johannesburg, The Mine Ventilation Society of South Africa, 1974.


## Addendum B: Kelvin Functions

The dimensionless quantities $\gamma_1$ and $\gamma_2$ in equation (2) are evaluated from

$$\gamma_1=-\frac{R_3 ker'\lambda r+R_4 kei'\lambda r}{R_3^2+R_4^2}$$

$$\gamma_2=\frac{R_4 ker'\lambda r-R_3 kei'\lambda r}{R_3^2+R_4^2}$$

$$R_3=ker\ \lambda r-\frac{\lambda k}{H}ker'\ \lambda r$$

$$R_4=kei\ \lambda r-\frac{\lambda k}{H}kei'\ \lambda r.$$

McLachlan[1] gives tables of Kelvin functions (*ker, ker', kei, kei'*) up to $\lambda r=10$ and formulae for $\lambda r>10$.

*Reference*


1. McLachlan, N. W. *Bessel functions for engineers.* London, Oxford University Press, 1955.


## Addendum C: Heat Storage in Steelwork

If the variation in air temperature in a given cross-section of shaft is $\theta_1 \sin \omega t$, by equation (8)

$$\theta_S=\frac{\theta_1}{\sqrt{1+\frac{\omega^2}{d^2}}}\sin(\omega t-\delta) \quad . . . . . . . (12)$$

where $d=\frac{HA_S}{c_S m_S}$

and $\tan\delta=\frac{\omega}{d}$.

The rate of heat gain per unit length of shaft $=c_S m_S\frac{\delta\theta_S}{\theta t}$

$$=\frac{c_S m_S\omega\theta_1}{\sqrt{1+\frac{\omega^2}{d^2}}}\cos(\omega t-\delta).$$

Heat is gained when $-\frac{\pi}{2}<\omega t-\delta<\frac{\pi}{2}$,

i.e. when $\frac{2\delta-\pi}{2\omega}<t<\frac{2\delta+\pi}{2\omega}$.

The heat stored per unit length of shaft $=\int\frac{c_S m_S\omega\theta_1}{\sqrt{1+\frac{\omega^2}{d^2}}}\cos(\omega t-\delta)dt$

$$=\frac{c_S m_S\omega\theta_1}{\sqrt{1+\frac{\omega^2}{d^2}}}\left[\frac{\sin(\omega t-\delta)}{\omega}\right]\left(\frac{2\delta+\pi}{2\omega}\right)/\left(\frac{2\delta-\pi}{2\omega}\right)$$

$$=\frac{2c_S m_S\theta_1}{\sqrt{1+\frac{\omega^2}{d^2}}}.$$

When $\theta_1$ is 1 °C, the values are

$1{,}4\times10^6$ J/m for buntons and guides

$0{,}6\times10^6$ J/m for buntons alone

$2{,}8\times10^6$ J/m for buntons filled with water.