Simulation of mining in non-homogeneous ground using the displacement discontinuity method

by J. A. C. DIERING*, B.Sc. Hons. (M.S.A.I.M.M. Grad.)

SYNOPSIS

The simulation of mining in faulted ground was recently described by Crouch. His method is extended here to enable a wide range of non-homogeneous mining problems to be modelled. Essentially, the boundary conditions at geological interfaces are incorporated into the overall iterative solution, which is necessary if faults and joints are to be modelled realistically. Two examples are given to demonstrate the method: the mining of coal a seam below a thick dolerite sill, and the mining of a flat reef in close proximity to a vertical fault where hangingwall and footwall rocks have different properties.

SAMEVATTING

Die nabootsing van mynbou in geskeurde grond is onlangs deur Crouch beskryf. Sy metode word hier verder uitgebrei sodat verskillende niehomegene mynbou probleme ondersoek kan word. Dit word hoofsaaklik moontlik gemaak deur die grensvoorwaardes by geologiese intervlakke in te sluit in die iteratiewe oplossing van die vergelykings, wat noodsaaklik is indien verskuiwings en nate getrou nageboots moet word. Twee voorbeelde word gegee om die metode te illustreer. Hulle is die afbou van 'n steenkoollaag onder 'n dik dolorietsoom en die afbou van 'n platrif in die opmiddellike omgewing van 'n vertikale verskuiwing waar die eienskappe van die rots in die dak verskil van die in die vloer.

Introduction

The simulation of the mining of tabular deposits in faulted ground was recently described by Crouch[1]. This description concerned a two-dimensional treatment by the displacement discontinuity method in the form of the computer programme MINAP. The analysis required that the host rock containing the ore-body and faults should be homogeneous, although the material properties of the tabular ore-body and fault infill could be specified arbitrarily. Typical applications of the method are the determination of stresses and displacements around service excavations, the delineation of tensile zones, and the estimation of energy-release rates and energy dissipation along faults. A major disadvantage of the approach described by Crouch[1] is its inability to take non-homogeneity into account. This paper describes an extension of Crouch's programme that permits the modelling of a wide range of non-homogeneous problems. The theoretical background is outlined, and two examples are described to illustrate the application of the method.

Theoretical Background

All the discontinuities in the region of interest are divided into straight displacement discontinuity elements. The discontinuities include faults, joints, reefs, seams, and interfaces between geological horizons with significantly different elastic properties. The normal, shear-stress, and displacement discontinuity components are assumed to be constant over each element, and a system of equations is written linking these components for each element with all the other elements. Stresses, displacements, or displacement discontinuities can be specified as boundary conditions. The unknown quantities to be determined are the displacement discontinuity components. Separate equations are written for the stress and displacement boundary conditions. Crouch's notation[1] is used here for convenience.

Let $\overset{i}{\underset{n}{\sigma}}$ and $\overset{i}{\underset{s}{\sigma}}$ be the induced normal and shear stresses, $\overset{i}{\underset{n}{d}}$ and $\overset{i}{\underset{s}{d}}$ the normal and shear displacement discontinuity components, and $\overset{i}{\underset{n}{u}}$ and $\overset{i}{\underset{s}{u}}$ the normal and shear displacements at one or other of the displacement discontinuity surfaces at the element i. The equations relating these quantities with the other elements are given[2] by

$$\overset{i}{\underset{s}{\sigma}}=\sum_{j}\overset{ij}{\underset{ss}{A}}\overset{j}{\underset{s}{d}}+\sum_{j}\overset{ij}{\underset{sn}{A}}\overset{j}{\underset{n}{d}} \quad \ldots\ldots\ldots\ldots (1)$$

$$\overset{i}{\underset{n}{\sigma}}=\sum_{j}\overset{ij}{\underset{ns}{A}}\overset{j}{\underset{s}{d}}+\sum_{j}\overset{ij}{\underset{nn}{A}}\overset{j}{\underset{n}{d}} \quad \ldots\ldots\ldots\ldots (2)$$

$$\overset{i}{\underset{s}{u}}=\sum_{j}\overset{ij}{\underset{ss}{B}}\overset{j}{\underset{s}{d}}+\sum_{j}\overset{ij}{\underset{sn}{B}}\overset{j}{\underset{n}{d}} \quad \ldots\ldots\ldots\ldots (3)$$

$$\overset{i}{\underset{n}{u}}=\sum_{j}\overset{ij}{\underset{ns}{B}}\overset{j}{\underset{s}{d}}+\sum_{j}\overset{ij}{\underset{nn}{B}}\overset{j}{\underset{n}{d}} \quad \ldots\ldots\ldots\ldots (4)$$

where the A and B terms are the influence coefficients derived by Crouch[2]. As an example, $\overset{ij}{\underset{sn}{A}}$ gives the shear stress induced at element i by the normal displacement discontinuity component of element j. The A and B coefficients are therefore termed stress and displacement coefficients respectively. Equations (1) to (4) can be rewritten in matrix form as

$$[\sigma]=[A]\ [d] \quad \ldots\ldots\ldots\ldots (5)$$

$$[u]=[B]\ [d] \quad \ldots\ldots\ldots\ldots (6)$$

If the behaviour of joints or faults is governed by a Mohr–Coulomb failure criterion, equations (5) and (6) become non-linear and must be solved iteratively. An efficient method of solution is that of successive over-relaxation[3]. A sufficient but not necessary condition for the convergence of the equations is that the matrices $[A]$ and $[B]$ should be diagonally dominant; that is to say, the diagonal elements of any row must be greater

*Steffen, Robertson and Kirsten Inc., Johannesburg.

in magnitude than the sum of the magnitudes of the off-diagonal elements in the same row. The question of diagonal dominance is important in what follows.

Consider the hypothetical problem of Fig. 1 in which the two elastic subregions 1 and 2 have different elastic properties. Equations (5) and (6) can be written for this problem as follows: for subregion 1,

$$\left.\begin{aligned}\begin{bmatrix}\overset{1}{\sigma}\\ \overset{2}{\sigma}\end{bmatrix} &= \begin{bmatrix}\overset{11}{A} & \overset{12}{A}\\ \overset{21}{A} & \overset{22}{A}\end{bmatrix}\begin{bmatrix}\overset{1}{d}\\ \overset{2}{d}\end{bmatrix}\\ \begin{bmatrix}\overset{1}{u}\\ \overset{2}{u}\end{bmatrix} &= \begin{bmatrix}\overset{11}{B} & \overset{12}{B}\\ \overset{21}{B} & \overset{22}{B}\end{bmatrix}\begin{bmatrix}\overset{1}{d}\\ \overset{2}{d}\end{bmatrix}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (7)$$

and for subregion 2,

$$\left.\begin{aligned}\begin{bmatrix}\overset{3}{\sigma}\\ \overset{4}{\sigma}\end{bmatrix} &= \begin{bmatrix}\overset{33}{A} & \overset{34}{A}\\ \overset{43}{A} & \overset{44}{A}\end{bmatrix}\begin{bmatrix}\overset{3}{d}\\ \overset{4}{d}\end{bmatrix}\\ \begin{bmatrix}\overset{3}{u}\\ \overset{4}{u}\end{bmatrix} &= \begin{bmatrix}\overset{33}{B} & \overset{34}{B}\\ \overset{43}{B} & \overset{44}{B}\end{bmatrix}\begin{bmatrix}\overset{3}{d}\\ \overset{4}{d}\end{bmatrix}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (8)$$

where

$$\overset{ij}{A} = \begin{bmatrix}\overset{ij}{A}_{ss} & \overset{ij}{A}_{sn}\\ \overset{ij}{A}_{ns} & \overset{ij}{A}_{nn}\end{bmatrix}$$

and similarly for the $\overset{i}{\sigma}$, $\overset{i}{u}$, $\overset{i}{d}$, and $\overset{ij}{B}$ terms.

The boundary conditions across the interface defined by elements 2 and 3 in Fig. 1 are

$$\left.\begin{aligned}\overset{2}{u} &= \overset{3}{u}\\ \overset{2}{\sigma} &= \overset{3}{\sigma}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (9)$$

Substituting (9) into (7) and (8) and rearranging (without the equations for $\overset{1}{u}$ and $\overset{4}{u}$, which are not specified) gives

$$\begin{bmatrix}\overset{11}{A} & \overset{12}{A} & 0 & 0\\ \overset{21}{A} & \overset{22}{A} & -\overset{33}{A} & -\overset{34}{A}\\ \overset{21}{B} & \overset{22}{B} & -\overset{33}{B} & -\overset{34}{B}\\ 0 & 0 & \overset{43}{A} & \overset{44}{A}\end{bmatrix}\begin{bmatrix}\overset{1}{d}\\ \overset{2}{d}\\ \overset{3}{d}\\ \overset{4}{d}\end{bmatrix} = \begin{bmatrix}\overset{1}{\sigma}\\ 0\\ 0\\ \overset{4}{\sigma}\end{bmatrix} \quad \ldots\ldots (10)$$

As the $\overset{22}{B}$ and $\overset{33}{B}$ terms are equal in magnitude, and as the $\overset{22}{A}$ and $\overset{33}{A}$ terms are approximately equal depending on the values of the shear moduli[2] in the two subregions, equation (10) is not diagonally dominant and cannot in general be solved by the method of successive over-relaxation. They can also not be solved efficiently by any elimination scheme such as Gaussian elimination because of the non-linearities introduced by the Mohr–Coulomb failure criterion.

It has been found that equations (10) can be solved iteratively provided that the diagonal terms in $\overset{22}{A}$ are greater in magnitude than those of $\overset{33}{A}$. The following algorithm assures this condition for a wide range of problems when applied at each iteration for the solution.

(i) Calculate normal and shear displacements at the interface of the stiff material.

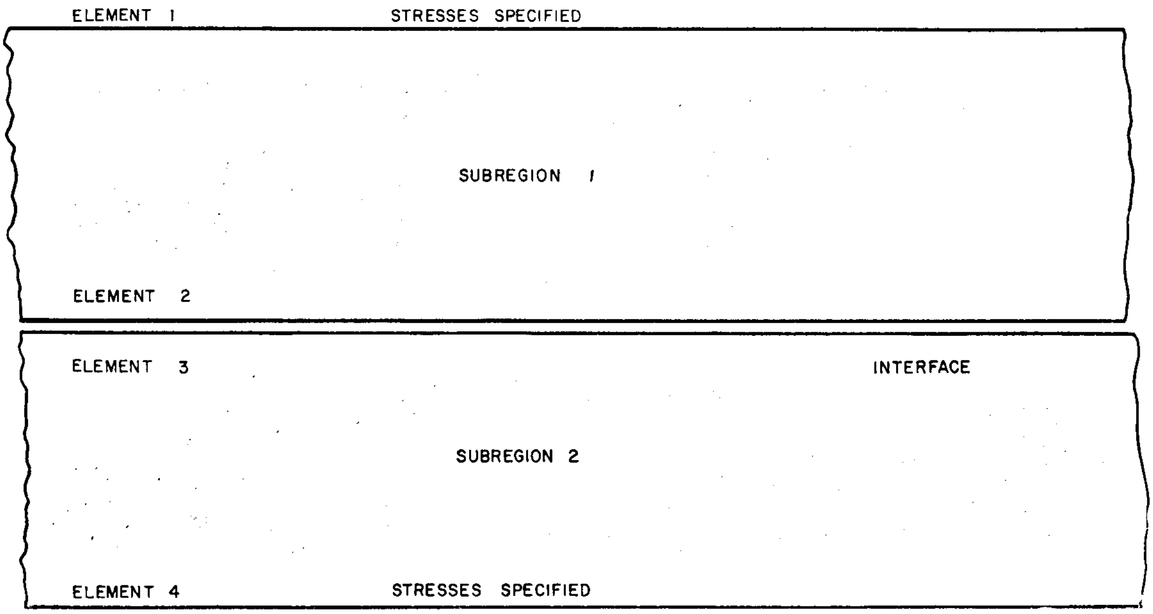

Fig. 1—Hypothetical problem showing the interface between two subregions

TABLE I

COMPARISON OF STRESSES AND DISCLACEMENTS FOR HOMOGENEOUS AND NON-HOMOGENEOUS ANALYSES

Reference point	Non-homogeneous model				Homogeneous model			
	u_y m	σ_x	σ_y MPa	τ_{xy}	u_y m	σ_x	σ_y MPa	τ_{xy}
1	−0,043	0,6	1,1	0	−0,039	1,0	0,4	0
2	−0,044	0,5	1,4	0	−0,040	0,8	0,1	0
3	−0,044	−6,4	0,3	0	−0,042	0,3	−0,1	0
4	−0,044	−2,1	0,0	0	−0,044	−1,0	−0,1	0
5	0,056*	0	0	0	0,059*	0	0	0
6	+0,011	−1,4	0,3	0	+0,013	−0,2	0,2	0
7	−0,035	0,5	1,3	−0,1	−0,032	0,4	1,6	−0,3
8	−0,034	−5,9	2,8	0,6	−0,030	0,0	2,9	0,0
9	−0,034	−5,0	6,4	1,6	−0,028	−0,4	4,6	0,5
10	−0,029	0,3	10,7	1,8	−0,023	0,8	9,4	2,0
11	−0,011	—	12,9	−0,1	0,010	—	12,0	−0,4
12	−0,006	−0,5	9,2	−2,0	−0,003	0,7	8,6	−1,9

*Convergence

(ii) Use these displacements as boundary conditions for the softer material.

(iii) Calculate induced normal and shear stresses at the interface of the soft material.

(iv) Use these stresses as boundary conditions for the stiffer material.

This algorithm has been successfully incorporated into Crouch's MINAP programme to allow two-dimensional, non-homogeneous mining problems to be modelled.

A disadvantage of the above algorithm, however, is that it is not possible to prevent rigid body motion of a stiff subregion completely enclosed within a softer subregion unless displacements in the stiff subregion are specified separately. This restriction can, however, be overcome for many problems, as demonstrated by the examples that follow.

Mining of a Coal Seam below a Dolerite Sill

Many South African coal mines have coal seams that are overlain by thick dolerite sills. The stresses in these sills and their effect on the stresses in and around the coal seams are therefore of considerable interest. The geometry used in the modelling of such a situation is shown in Fig. 2. The geometry and elastic properties were chosen more to demonstrate the effects of non-homogeneity than to model any particular situation. The stresses and displacements for 12 reference points are shown in Table 1, the locations of these reference points being shown in Fig.2.

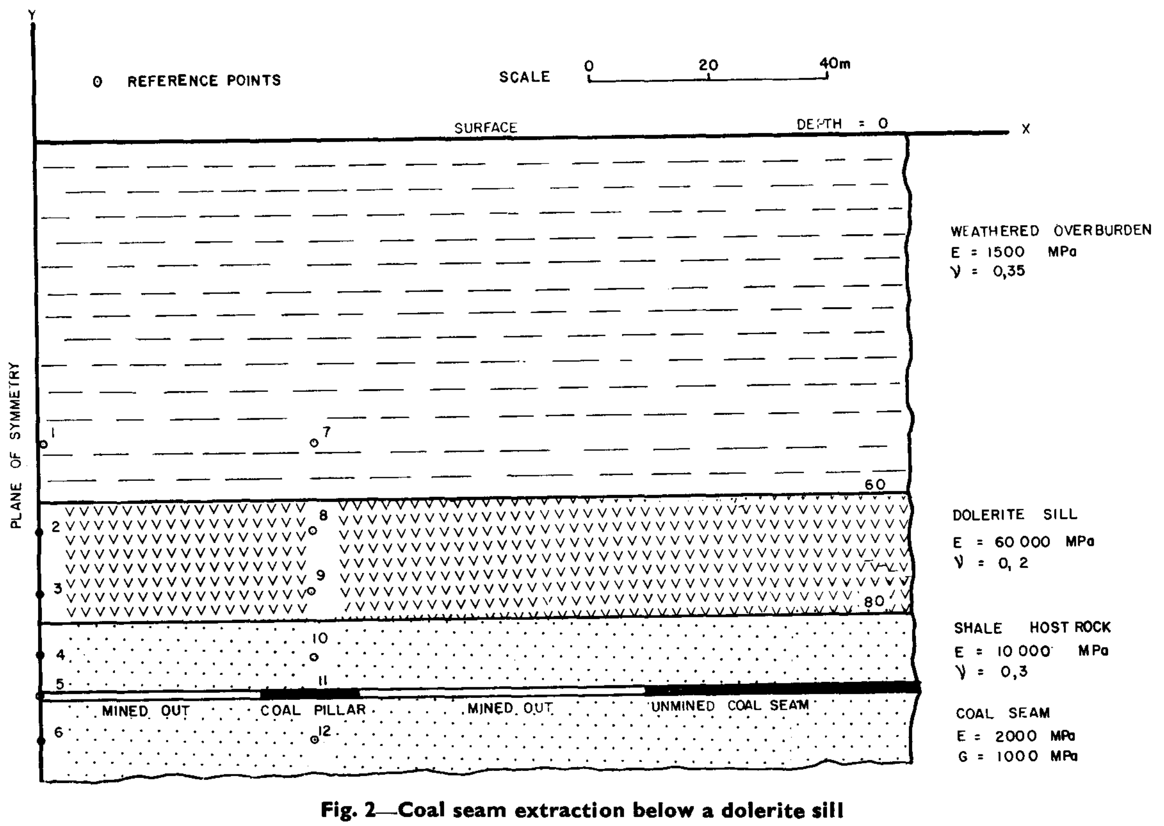

Fig. 2—Coal seam extraction below a dolerite sill

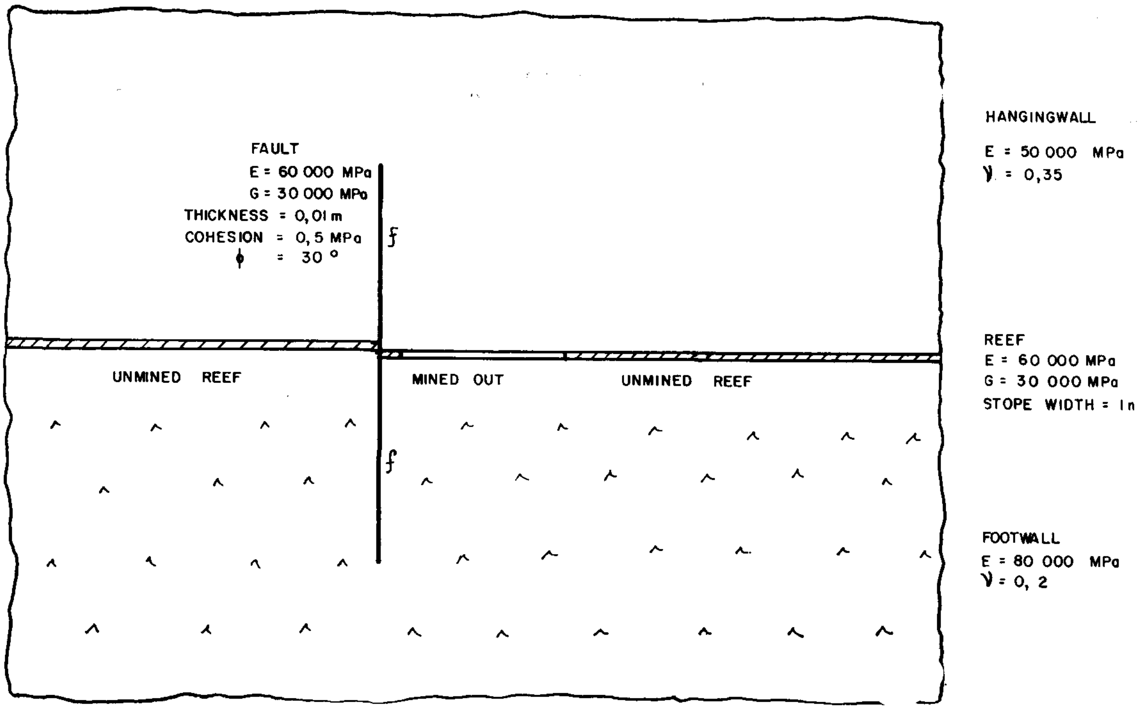

Fig. 3—Tabular excavation approaching a fault

Although the displacement distributions are similar, the homogeneous model fails to detect the very high horizontal tension in the sill and overestimates the zones of induced tension above the excavation.

Application to Gold Mining

The problem of mining through a dyke can be handled in much the same way as the above problem, provided that the displacements in the dyke are specified remote from the mining. A second problem of interest is the effect of different hangingwall and footwall rocks on stress distributions and the behaviour of faults. Fig. 3 shows the idealized geometry for such a problem. Constant field stresses of 30 MPa and 60 MPa were applied in the horizontal and vertical directions respectively. Elastic parameters for the hangingwall and footwall were as follows:

Hangingwall: $E_1=50\ 000$ MPa
$\nu_1=0{,}35$
Footwall: $E_2=80\ 000$ MPa
$\nu_2=0{,}20$

The analysis showed that the normal stresses, and hence the shear strengths, along the fault were the same, within a few per cent, in both the hangingwall and the footwall rocks. The shear displacements along the fault were about 1,5 times greater in the hangingwall than in the footwall. Consequently, to a first degree of approximation, some 60 per cent of the energy dissipated along the fault would be in the hangingwall.

Discussion

In the first example, the non-homogeneity of the problem had a marked effect on the stress distribution but not upon the displacements, whereas in the second problem the reverse was true. These examples demonstrate the ease with which the displacement discontinuity method can be applied to a wide range of non-homogeneous mining problems. The interfaces between geological subregions can also be subjected to a Mohr Coulomb yield condition in the same way as that described by Crouch[1], and can also contain an infill of finite thickness. Execution times for non-homogeneous analyses are about 2 to 4 times longer than for equivalent homogeneous analyses. The alterations required to the computer programme MINAP to permit the modelling of non-homogeneous problems are minimal, and it is believed that these changes constitute a useful improvement to the already well-known programme. The programme operates satisfactorily on a mini-computer, making it a very practical analytical tool available to small group users.

References

1. Crouch, S. L. Computer simulation of mining in faulted ground, *J. S. Afr. Inst. Min. Metall.*, vol. 79, no. 6. Jan. 1979. pp. 159-173.
2. Crouch, S. L. *Analysis of stresses and displacements around underground excavations; an application of the displacement discontinuity method.* University of Minnesota, Geomechanics Report to the National Science Foundation, 1976. 268 pp.
3. Fröberg, C. *Introduction to numerical analysis.* New York, Addison-Wesley Publishing Company, 1970. 433 pp.