An improved method for the determination, by a MINSIM type of analysis, of stresses and displacements around tabular excavations

by J. A. C. DIERING*, B.Sc. Hons., Grad. M.S.A.I.M.M.

SYNOPSIS

The determination of stresses and displacements close to tabular excavations is of considerable importance in the planning of tunnels and of remnant and pillar layouts. This paper gives a description of a simple yet effective method for the achievement of improved values from an analogue or MINSIM type of analysis. Essentially, closures and rides (the movement of hanging-wall relative to footwall) in mined-out areas are assumed to vary in a piecewise linear fashion as opposed to a piecewise constant fashion. As the former fashion more closely resembles the true variation, it is possible for the stresses and displacements at points close to the excavations to be calculated with much better accuracy.

SAMEVATTING

Spannings en verplasings na aan tabulêre uitgrawings is van uiterste belang by die beplanning van tonnels, restante en pilaar-uitlegte. Hierdie referaat beskryf 'n eenvoudige dog effektiewe metode om verbeterde akkuraatheid met 'n MINSIM of analoog tipe oplossing te verkry. Die variasie van rotssluiting en skuifbeweging (die beweging van die dak relatief tot die vloer) in uitgemynde gebiede word as stuksgewys linieêr beskou teenoor stuksgewys konstant. Gevolglik word die werklike variasie beter nageboots en is dit moontlik om die verplasings en spannings na aan die uitgrawing met veel groter akkuraatheid te bepaal.

Introduction

The face element principle of Salamon[1–4], the displacement discontinuity method of Starfield and Crouch[5], and the analogue computers used to model tabular excavations in coal, gold, and platinum mines in South Africa have a number of features in common. These analyses are often collectively referred to as a MINSIM type of analysis, and this notation is used throughout this paper. A MINSIM analysis requires the ore-body to be divided into a number of square elements that are either mined or unmined. The interaction of mined elements with one another is then either calculated numerically or determined by the network of resistors making up an analogue computer. In both cases, the movement of the hangingwall relative to the footwall is determined. The normal component of movement is usually termed *closure* or *convergence*, while the shear components are termed *rides*. The closure and rides are assumed to be constant over each square element and, provided a large number of elements is used, the errors resulting from this assumption are small.

Once closure and rides are determined at each element, stresses and displacements can be determined at points in the rock mass by the summing of the effects of each element. Most formulations for this type of problem have the restriction that stresses cannot be determined reliably at points closer than the width of one element from any mined element. In addition, these points have to be directly above or below the centre of an element.

A simple example is given to demonstrate the severity of these restrictions in practical mining problems. For the modelling of a region in which a number of 30 m pillars are left intact, if a block size of 30 m is selected, the stresses and displacements at points within 30 m of the excavation or directly above or below the pillar edges cannot be determined. These inaccessible regions are often of considerable interest. The use of a smaller block size requires either a much larger number of elements or the implementation of what is commonly referred to as a windowing mechanism. Both these options result in substantial increases in computer time and data preparation time. This paper describes a simple yet effective method of obtaining stresses and displacements anywhere within one-third the width of an element of mined areas. The method requires little additional computer time and no extra data preparation.

A good estimate of the closure and rides at the corner or node of any element is given by the average of the values at the four elements adjacent to that node. A knowledge of these nodal values enables values anywhere within any mined element to be determined by linear interpolation between the nodes. As these interpolated values of closure and rides are more accurate than the assumption that closures and rides are constant over each element, it is possible for stresses and displacements around the excavation to be determined with greater accuracy.

The improved accuracy obtained by a linear variation of boundary unknowns for similar boundary element formulations was reported by Cruse[6], and Lachat and Watson[7] have developed even more sophisticated types of elements. As the method described here is based on the simplest type of element, it is termed a *pseudo-linear* element, which is intermediate between the constant and the linear type of element.

It has been found that the pseudo-linear element gives an accuracy similar to that given by constant elements that are one-half to one-third smaller, provided that the geometry is reasonably well represented by the larger size. This pseudo-linear element was developed for a displacement discontinuity formulation of Starfield and Crouch[5], but should be equally applicable to the analogue computers and MINSIM family of programs[8] (i.e., Salamon's face elements [1–4]).

*Steffen, Robertson and Kirsten Inc., P.O. Box 8856, Johannesburg 2000.

Background Theory

In what follows, the rides are assumed to be zero, so that only the closures need to be considered. The effects of rides are easily included in the overall scheme. The region of interest is assumed to be divided up into an N by N array of elements, and a local coordinate system is defined so that the X- and Y-axes are the directions of increasing row and column numbers respectively. It is required to determine the stresses $\sigma_{ij}(p)$ and displacements $u_i(p)$ at a point p = (x, y, z). The analytic solutions for $\sigma_{ij}(p)$ and $u_i(p)$ are given[9] by

$$\sigma_{ij}(p) = \int_S K_{3ij}(Q,p)\, d_3(Q)\, dS(Q) \qquad \ldots\ldots\ldots\ldots (1)$$
$$u_i(p) = \int_S T_{i3}(Q,p)\, d_3(Q)\, dS(Q),$$

where
S is the region of interest defined by the NXN elements,
Q is a point in the region,
$dS(Q)$ is an elemental part of the region about Q,
$d_3(Q)$ is the closure at the point Q, and
$K_{3ij}(Q,p)$ and $T_{i3}(Q,p)$ are influence coefficients given by Cruse[10].

The numerical approximation to equation (1) in most existing formulations can be given by

$$\sigma_{ij}(p) = \sum_{n=1}^{N} \sum_{n=1}^{N} a\, K_{3ij}(m,n,p)\, d_3(m,n) \qquad \ldots\ldots (2)$$
$$u_i(p) = \sum_{n=1}^{N} \sum_{n=1}^{N} a\, T_{i3}(m,n,p)\, d_3(m,n),$$

where
$d_3(m,n)$ is the constant closure over element m,n, and
a is the area of any element.

The coefficients T and K vary within each element so that some form of numerical integration may be required for the evaluation of the coefficients $K_{3ij}(m,n,p)$ and $T_{i3}(m,n,p)$, depending upon the rapidity of variation of T and K.

If r is the distance separating point p from point $\int$ within element m,n, then

$$K \propto \frac{1}{r^3} \text{ and } T \propto \frac{1}{r^2}.$$

This means that the integrated effect of K and T over each element becomes more difficult to evaluate numerically as the point p approaches element m,n. As the evaluation of K is more difficult than T, only the K terms are considered in what follows.

The use of Gauss Quadrature formulae[11] has been found to give adequate representations of the K terms in equation (2) provided that progressively more accurate formulae are applied as r decreases. An M point integration is given by

$$\int_{-1}^{1} \int_{-1}^{1} f(x,y)\, dx\, dy = \sum_{k=1}^{M} \sum_{l=1}^{M} f(x_k,y_l) W_k W_l \ldots (3)$$

where
k, y_l are specified points between −1 and 1, and
W_k, W_l are specified constants.

If h is half the width of element m,n, equation (3) can be applied to equation (2), giving

$$\sigma_{ij}(p) = h^2 \sum_{m=1}^{N} \sum_{n=1}^{N} \sum_{k=1}^{M} \sum_{l=1}^{M} K_{3ij}(hx_k, hy_l, p) d_3(m,n). \, .(4)$$

The use of equation (4) enables stresses and displacements to be determined as close to an element as desired, provided that the point p lies directly above or below an element. The restriction results from the fact that d_3 does not vary smoothly from element to element.

A good approximation to the closure at any node or element corner is given by the average of the closures of the four adjacent elements. If the four corners of an element (m,n) are given by $(m,n)_1$, $(m,n)_2$, $(m,n)_3$, and $(m,n)_4$, then a good approximation to the closure at the point hx_k, hy_j in element m,n is given by

$$\begin{aligned} d_3(a,b) = \; & 0{,}25(1-a)(1+b)d_3(m,n)_1 \\ & +(1-a)(1-b)d_3(m,n)_2 \\ & +(1+a)(1-b)d_3(m,n)_3 \\ & +(1+a)(1+b)d_3(m,n)_4), \ldots\ldots\ldots\ldots (5) \end{aligned}$$

where
a = hx_k and
b = hy_l.

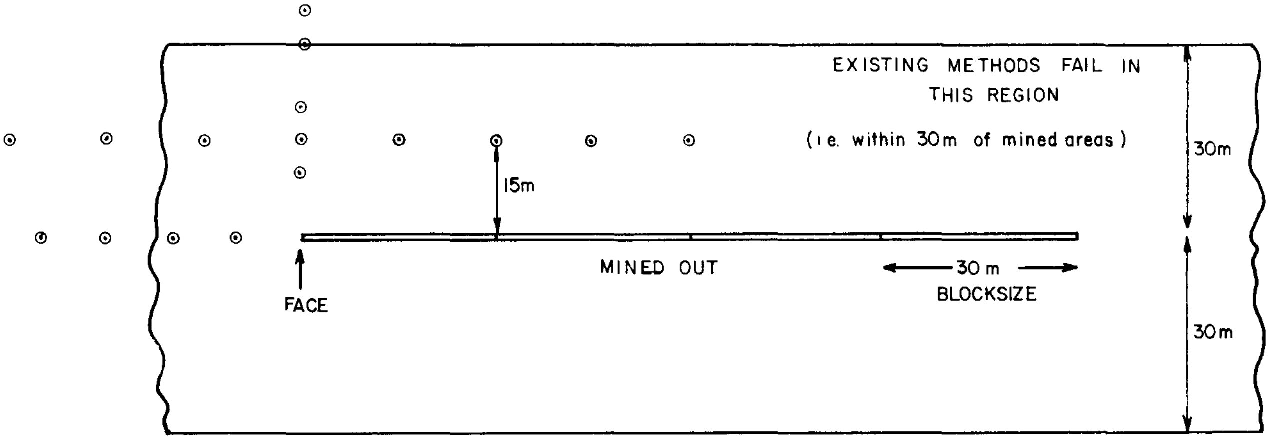

Fig. 1—Points at which stresses and displacements were calculated in an assumed stope 120 m wide with a blocksize of 30m

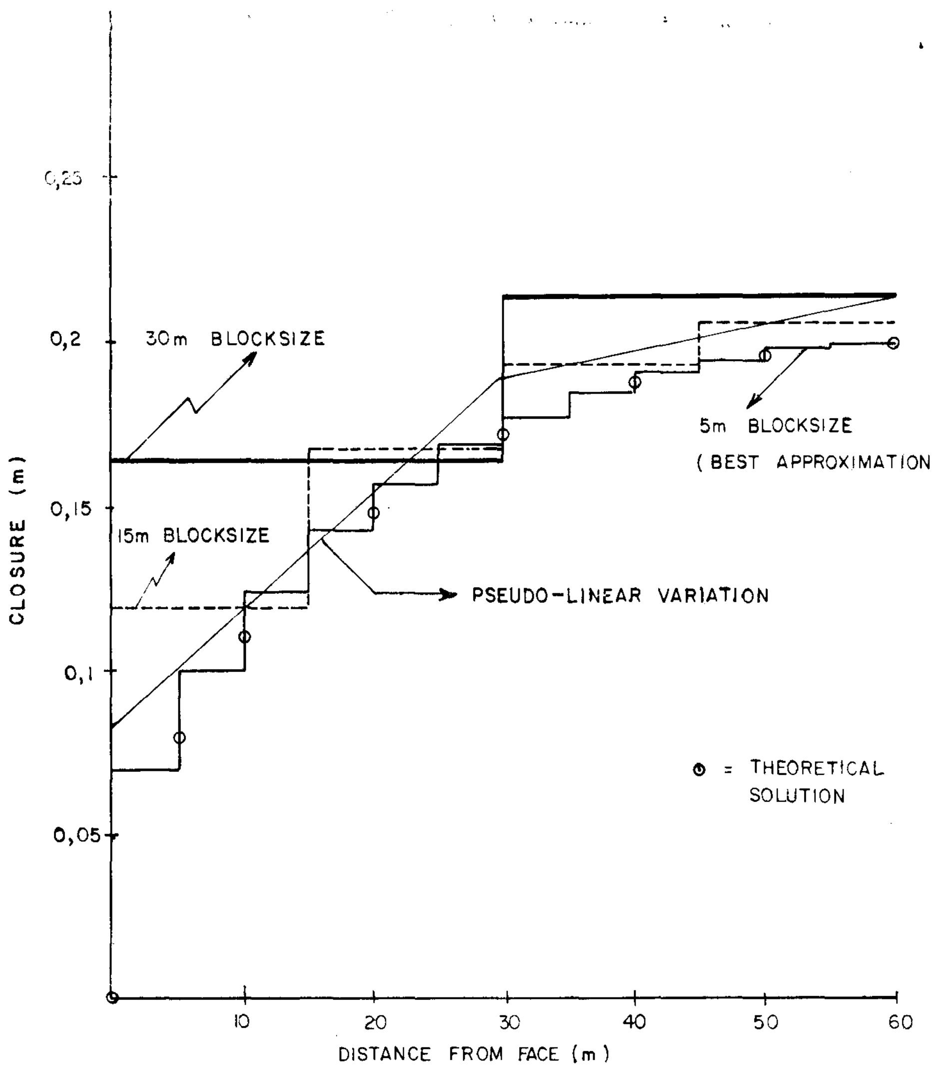

Fig. 2—Closure distribution for various blocksizes

The substitution of equation (5) into equation (4) gives the improved formula

$$\sigma_{ij}(p) = h^2 \sum_{m=1}^{N} \sum_{n=1}^{N} \sum_{k=1}^{M} \sum_{l=1}^{M} K_{3ij}(a,b,p)d_3(a,b). \quad (6)$$

It is not difficult to modify equation (6) further to allow for implementation of various lumping mechanisms[5, 8, 9].

Results

As a check of the pseudo-linear variation, the simple problem of a flat excavation 120 m by 1200 m subjected to vertical stress of 52 MPa and horizontal stress of 26 MPa was analysed. A Young's modulus of 60 GPa and Poisson's ratio of 0,2 were selected, and, as the length-to-breadth ratio of the excavation was 10 : 1, it was possible to check solutions with plane strain solutions obtained from Crouch's[12] MINAP program for displacement discontinuity elements. For the pseudo-linear variation, a blocksize of 30 m was chosen, while blocksizes of 30, 15, and 5 m were used in the MINAP analyses. Crouch has shown that the MINAP program produces more accurate solutions as the blocksize is decreased.

Fig. 1 shows a section across the excavation and various points at which stresses and displacements were calculated. Fig. 2 shows sections of the closure distributions for various blocksizes, and it can be seen that the pseudo-linear variation resembles the smallest blocksize more closely than a 15 m blocksize with constant closure elements. Fig. 2 also shows points plotted from the analytic solution[12]. Figs. 3 to 5 show variations in normal and shear stresses at the various points given in Fig. 1. In most cases, the results from the pseudo-linear elements are superior to the results obtained from constant closure elements of half the size.

Conclusions

The results of the test case above show that considerable improvement in the calculation of stresses and displacements at points close to a tabular excavation can be obtained from the use of the pseudo-linear type of element described. The accuracy obtained is approximately the same as that which would be obtained when six times as many elements of the constant closure type are used. As these stress calculations are of considerable importance when tunnels and remnant dna pillar layouts are being designed for tabular excavations, it is felt that the pseudo-linear variation of closures and rides described in this paper represents a useful improvement to the MINSIM family of programs, analogue computer systems, and displacement discontinuity types of programs. Its implementation requires only a relatively small modification to existing programs. The ideas described here should be easily adaptable to multi-reef formulations, as well as to those which take the earth's surface into account.

The improvements described here for the MINSIM type of program are complementary to the windowing mechanisms currently in use, since the pseudo-linear variation of closures and rides permits the accurate determination of stresses and displacements from the modelled geometry, while the windowing mechanism ensures that the true layout of a mine is represented accurately.

Acknowledgements

The author thanks Steffen, Robertson and Kirsten Inc. for the use of their computing facilities and for providing him time to prepare the paper. Thanks are also due to Dr T. R. Stacey for his valuable advice and assistance.

References

1. SALAMON, M. D. G. Elastic analysis of displacementsand stresses induced by the mining of seam or reef deposits, Part I. *J. S. Afr. Inst. Min. Metall.*, vol. 64, no. 4, 1963. pp. 129–149.
2. SALAMON, M. D. G. Elastic analysis of displacements and stresses induced by the mining of seam or reef deposits, Part II. *J. S. Afr. Inst. Min. Metall.*, vol. 64, no. 6. 1964. pp. 197–218.
3. SALAMON, M. D. G. Elastic analysis of displacements and stresses induced by the mining of seam or reef deposits, Part III. *J. S. Afr. Inst. Min. Metall.* vol. 64, no. 10. 1964. pp. 468–500.
4. SALAMON, M. D. G. Elastic analysis of displacements and stresses induced by the mining of seam or reef deposits, Part IV. *J. S. Afr. Inst. Min. Metall.* vol. 65, no. 5. 1964. pp. 319–338.
5. STARFIELD, A. M., and CROUCH, S. L. Elastic analysis of

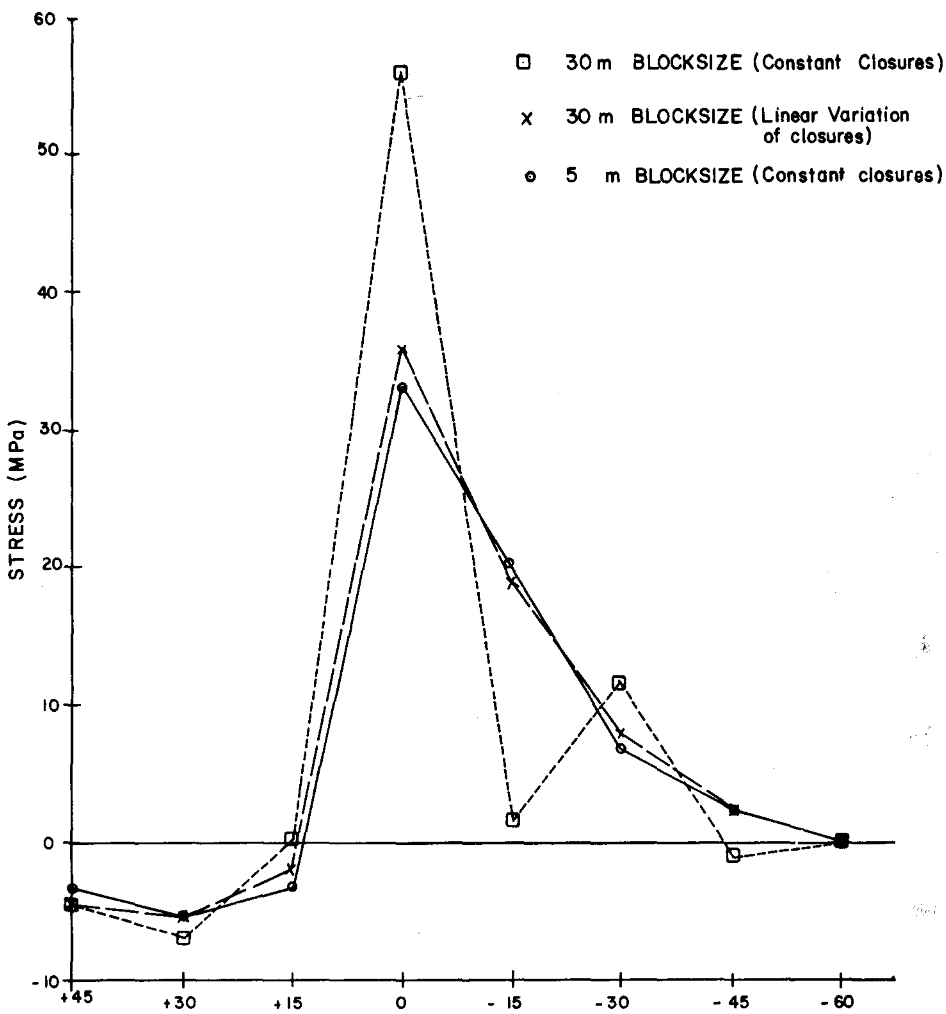

Fig. 3—Shear stresses 15m above the excavation

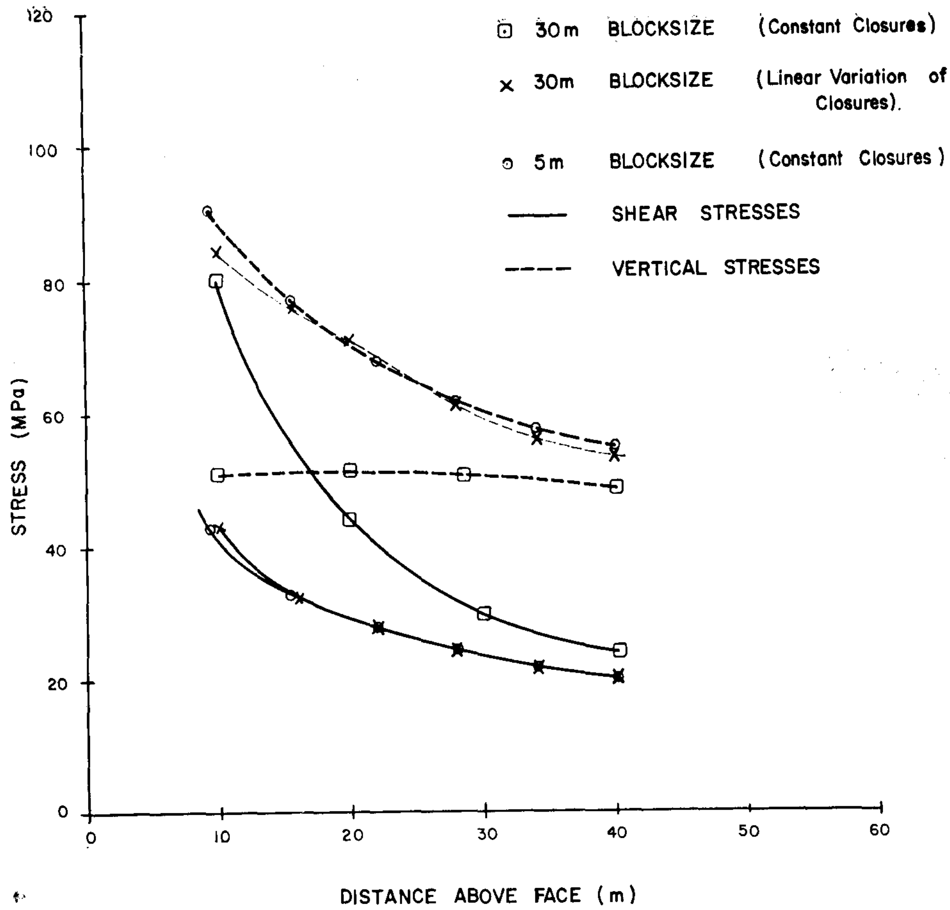

Fig. 4—Vertical and shear stresses directly above the face

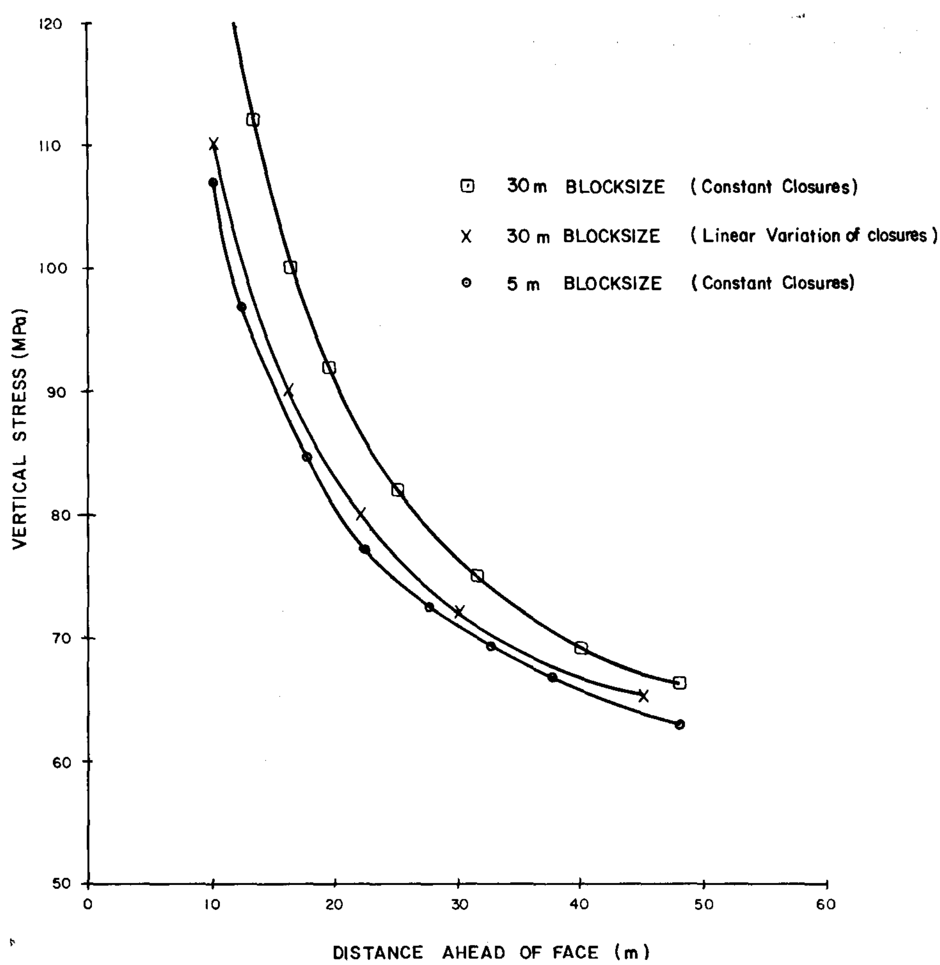

Fig. 5—Vertical stresses ahead of the face

single seam extraction. *New horizons in rock mechanics* (ed. H. R. Hardy, Jr and R. Stefhanko). New York, American Society of Civil Engineering, 1973. pp. 421–439.
6. Cruse, T. A. An improved boundary-integral equation method for three-dimensional stress analysis. *Computers and Structures*, vol. 3. 1973. pp. 509–527.
7. Lachat, J. C., and Watson, J. O. Effective numerical treatment of boundary integral equations: a formulation for three-dimensional elastostatics. *Int. J. Num. Methods in Enging.*, vol. 10. 1976. pp. 991–1005.
8. Morris, J. P. E. SHAMIP: a computer program for elastic analysis of large size, tabular, mono-planar, shallow, hard orebodies. Johannesburg, Chamber of Mines of South Africa, *Research Report* no. 51/75. 1975. 221 pp.
9. Diering, J. A. C. M.Sc. dissertation, University of the Witwatersrand. (In preparation.)
10. Cruse, T. A. Numerical solutions in three-dimensional elastostatics. *Int. J. Solids and Structures*, vol. 5, no. 1.1969. pp. 1259–1274.
11. Zienkiewicz, O. C. *The finite element method in engineering science*. New York, McGraw-Hill, 1971.
12. Crouch, S. L. Computer simulation of mining in faulted ground. *J. S. Afr. Inst. Min. Metall.*, vol. 79, no. 6. 1979. pp. 159–173.

Coal exploration

Coal, never more important, never more needed on a global basis than now, means that coal explorationists face a tremendous challenge in the decade of the 80s. Coal must be found and mined in locations that minimize transportation costs to markets, and must be of a quality that meets the stricter coal-burning regulations.

These are the main reasons why the Third International Coal Exploration Symposium is being convened in Calgary (Alberta, Canada) from 23rd to 26th August, 1981. Calgary was picked as the logical site for this Symposium by the delegates who attended the first two Coal Exploration Symposia – one in London and one in Denver (Colorado).

You don't have to be a geologist to attend. If you burn coal, if you mine coal, if you transport coal, or if you are participating in the financing of coal projects anywhere in the world you are welcome to attend.

For further information on the Symposium, please communicate with George O. Argall Jr, Symposium Chairman, *World Mining/World Coal*, 500 Howard Street, San Francisco, California, 94105 U.S.A. Telephone: (415) 397–1881. Telex: 278273. Cable: MILFREEPUB, San Francisco.

S.A.I.M.M. diary

26th-30th January 1981 **2nd-6th February 1981** **9th-13th February 1981** University of the Witwatersrand	Vacation Schools on Increased Underground Extraction of Coal
23rd April 1981 Kelvin House	Colloquium on Wear and Abrasion in Industry in collaboration with the Institution of Metallurgists (South African Branch)
5th June, 1981 National Institute for Metallurgy	Colloquium on 'The influence of a high gold price on a low-grade mining area
29th June-3rd July 1981 University of Pretoria	Vacation School on The Heat Treatment of Steel — Theory and Practice (organized by the SAIMM Material Engineering Division)
27th-31st July, 1981 National Institute for Metallurgy	Vacation School on Uranium Ore Processing — Extraction of Uranium
26th April-2nd May, 1982 Pre-Congress Tours	
3rd-7th May, 1982 Carlton Hotel, Johannesburg	Twelfth Mining and Metallurgical Congress in collaboration with the Geological Society of South Africa
10th-23rd May, 1982 Post-Congress Tours	