A model of capital expenditure for gold-mine planning

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SYNOPSIS

An essential requirement for the planning of the global capacity of a mine is a knowledge of the functional relationship between the production level and the capital expenditure necessary to establish that level. An underlying mechanism connecting capital expenditure and mine production capacity is proposed in terms of a characteristic profile of the expenditure that both precedes and succeeds production expansion. By the assignment of a representative shape to this profile, the capital expenditure of individual mines can be modelled.

The empirical values describing the characteristic profile are computed for a number of gold mines, and an attempt is made to explain variations in the model coefficients from mine to mine in terms of physical parameters such as shaft depth, shaft concentration, and average rate of depth increase during the life of the mine.

SAMEVATTING

'n Noodsaaklike vereiste vir die beplanning van die globale vermoë van 'n myn, is kennis van die funksionele verband tussen die produksiepeil en die kapitaalbesteding wat nodig is om daardie peil te bereik. Daar word 'n grondliggende meganisme wat die kapitaalbesteding en 'n myn se produksievermoë met mekaar in verband bring, voorgestel in terme van 'n kenkromme van die besteding wat produksie-uitbreiding voorafgaan en daarop volg. Die kapitaalbesteding van individuele myne kan gemodelleer word deur 'n verteenwoordigende vorm aan hierdie kromme toe te ken.

Die empiriese waardes wat die kenkromme beskryf, word vir 'n aantal goudmyne bereken en daar word 'n poging aangewend om die variasies in die modelkoëffisiente van myn tot myn in terme van fisiese parameters soos skagdiepte, skagkonsentrasie en gemiddelde tempo van die toename in diepte gedurende die lewensduur van die myn, te verklaar.

Introduction

The appraisal of the attractiveness of investment opportunities in the mineral industry must be distinguished from the design of a mining venture to achieve an economic objective. This paper deals with the latter problem, with particular emphasis on the optimum choice of mine production capacity for a new gold-mining venture. In practice, the capacity decision is made in a series of steps starting with initial feasibility studies and culminating in detailed design specifications for the proposed mine. The early stages of this decision chain can be explored, at low cost, by the formulation of a simple financial model of the enterprise.

An essential component of such a model is a functional relationship between the production capacity and the capital expenditure required to establish that capacity. This relationship should allow for dynamic effects such as the duration of construction, as well as for the possibility of time-varying production rates. A theoretical capital model of this kind is developed in this paper, and the model parameters are estimated empirically by the use of published data on the cost and production of a number of gold mines that were established recently.

Data on Capital Expenditure and Production Rate

Since historical data on capital expenditure are subject to general economic inflation trends, all the capital amounts have to be corrected to a constant money base in terms of a representative inflation index. In the present study, the index chosen for the normalization of capital expenditure is the average working cost in the gold-mining industry (rands per ton milled) as published in the Annual Report of the Chamber of Mines of South Africa[1] with 1970 as the base year. Data on capital expenditure and production rates were extracted from the annual reports of twenty recently established gold-mining companies, which are situated in the Evander, Far West Rand, Klerksdorp, and Orange Free State goldfields. Data on capital expenditure were taken from the *Fixed Assets* section of the balance sheets given in the annual reports, usually under the sub-headings 'mining lease, freehold property and mineral rights' and 'shaft sinking, plant, development and equipment'.

Secondary items considered to fall beyond the scope of the capital model and recorded under the following or similar headings were excluded:

- general capital expenditure (income and expenditure account),
- preliminary expenses,
- share commission and share issue,
- shares in cooperative concerns, and
- trade investments.

In addition, no expenditure attributed to uranium plant or uranium projects was included.

The production level of each mine was regarded as the tonnage milled per annum. This was noted, together with the tonnage of waste rock hoisted and reef tons sorted on surface, when these were recorded in the annual reports. In addition to this information, the lengths of the major shafts and, in some instances, the breakdown of capital expenditure into individual categories were noted. In the subsequent analyses of the data (Tables I to IV), the following symbols denote the four areas in which the mines are situated:

E Evander
F Far West Rand
K Klerksdorp
O Orange Free State.

* Chamber of Mines Research Laboratories (Mining Operations), Carlow Road, Melville, Johannesburg 2092.

Plots of Capital and Production Rates

Two typical time-dependent profiles of annual capital expenditure and production rate are given in Figs. 1a and 1b for mines 1 and 11 respectively. From Fig. 1a it is evident that the level of capital expenditure falls as the rate of the production build-up diminishes, but it does not fall to zero when no further expansion occurs. This behaviour can be associated with the 'expansion' and 'replacement' components of capital expenditure.

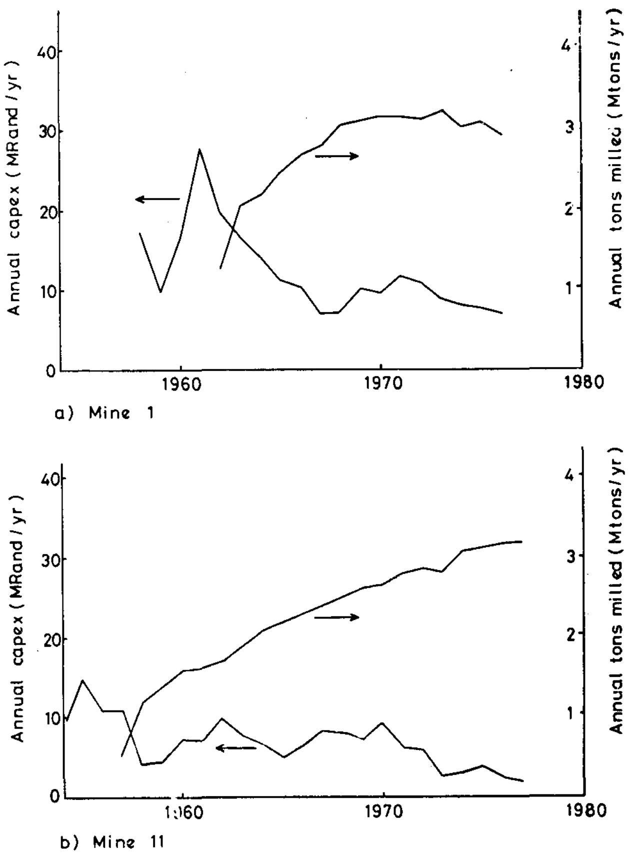

Fig. 1—Annual capital expenditure (1970 basis) and tons milled as functions of time for two mines

These components may also be exhibited when cumulative capital is plotted against achieved production rate, as in Fig. 2a. However, this distinction is not always clear, as can be seen from Fig. 2b. It should also be noted that the initial slope of the curve for actual cumulative capital versus production (Fig. 2a or 2b) cannot be directly related to the proportionality between capital and capacity. The reason is that, at any particular point on the graph, the cumulative capital is made up of amounts contributing both to the current level of production and to expenditures on construction programmes already initiated for future expansion. This overlap must be resolved before the correct proportionality can be deduced.

Before proceeding to the theoretical formulation of a model relating production capacity to capital expenditure, it is useful to note that, for the set of mines

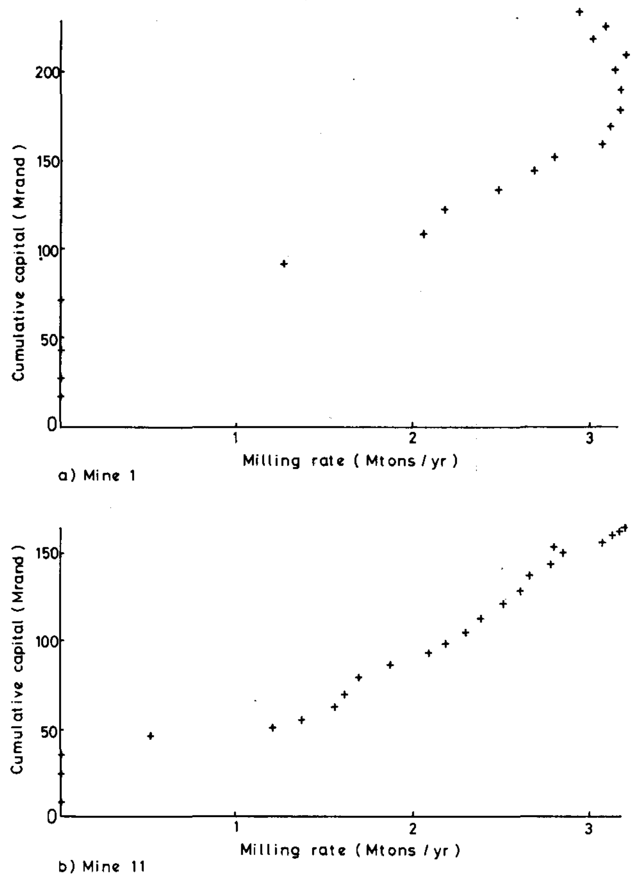

Fig. 2—Cumulative capital (1970 basis) plotted against current production level for two mines

considered in this study, the total length of major shaft installations appears to be a strong determinant of cumulative capital expenditure. Specifically, for the twenty mines chosen for the study, it was possible to estimate the total length of shafts installed at various times during the life of each mine. The total shaft length for each mine at a specified year is given in Table I, together with the cumulative normalized capital expenditure up to that year. This cumulative capital includes expenditure on shaft sinking and underground development, as well as all surface expenditures on reduction plant, buildings, and housing. The corresponding cumulative tonnage milled is also included in the table.

In Fig. 3, the cumulative capital expenditure is plotted against the total length of shafts. This discloses that the cumulative capital and total shaft length correlate remarkably well, with a correlation coefficient of 0,92 according to the regression equation

$$K = 11{,}2\, L_{\mathrm{T}} - 0{,}6, \quad \ldots\ldots \quad (1)$$

where K = cumulative gross capital expenditure (MRand)

L_{T} = total length of major shafts (km).

The negligible value of the constant term in the regression equation shows that, for the twenty mines considered, capital expenditure (arising from both surface and underground works) is directly proportional to the total extent of shaft sinking. An explanation for this

TABLE I

CUMULATIVE SHAFT LENGTHS, ESTIMATED LENGTH REQUIRED FOR CUMULATIVE PRODUCTION, AND CUMULATIVE CAPITAL EXPENDITURE

Mine	Area symbol	Year	Total shaft length km	Length required for cumulative production km	Cumulative production Mton milled	Cumulative capital (MRand-1970)	Shaft density, $\bar{d}$ m/t milled
1	F	76	16,9	11,8	41,4	232,3	284
2	F	73	3,3	3,3	24,9	58,6	133
3	F	75	8,1	6,7	32,3	63,3	207
4	E	76	3,0	3,0	21,1	31,1	144
5	E	76	2,9	2,9	14,4	20,3	199
6	F	78	7,7	4,5	15,9	104,0	285
7	E	76	5,3	3,4	13,5	40,7	250
8	O	72	3,8	3,8	11,0	48,7	341
9	O	76	8,8	8,8	31,5	124,0	281
10	K	75	18,5	16,3	48,5	212,4	336
11	K	72	17,3	9,0	32,3	149,4	279
12	F	76	3,7	—	2,2	39,9	—
13	F	69	8,2	6,1	17,7	70,0	348
14	F	76	7,9	—	5,9	99,4	—
15	F	76	12,8	10,6	45,6	158,0	232
16	O	69	5,2	4,8	26,2	88,5	182
17	K	70	8,9	6,4	38,8	68,3	164
18	O	73	9,4	9,1	36,2	107,3	251
19	O	73	9,8	9,1	35,0	111,4	259
20	O	72	8,5	6,3	36,6	72,5	171
						Average	241

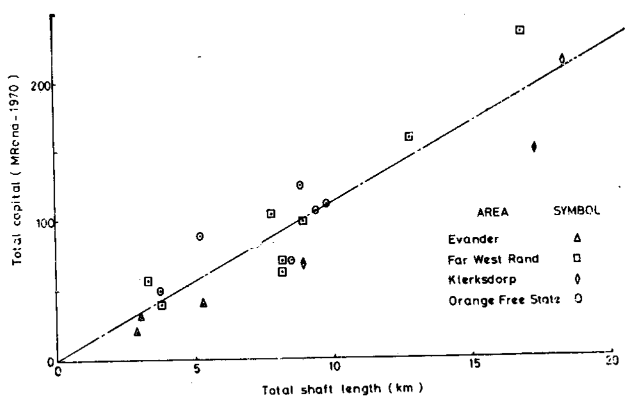

Fig. 3—Cumulative gross capital (1970 basis) plotted against length of shafts for each of twenty mines

direct proportionality is that a mine can be considered to be an aggregate of 'shaft-like' building blocks each of which comprises a shaft facility associated with fixed segments of the surface infrastructure, including reduction plant, shaft buildings, housing, and offices. If it is postulated that each unit has a certain life production profile, a general relationship can, in principle, be developed between the number of currently operating units and the cumulative number of units installed. Based on this concept, it is shown in the Addendum that the relationship between capital expenditure and production capacity can be expressed in the following general form:

$$K'(t) = (1/H)\int_0^{D+t} G^*(D-r+t)\, y'(r)\, dr \quad \text{(MRand/yr)}, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $K'(t)$ = rate of capital expenditure at time t (MRand/yr)

H = production rate of a shaft unit (Mton/yr)

$G^*(x)$ = rate of capital expenditure associated with a basic production unit at time x (MRand/yr) (x is the time relative to the start of production of the unit, i.e,. for $-D < x < 0$, the unit is being constructed, and, for $x > 0$, the unit is in production)

D = duration of the construction period (yr)

r = dummy integration variable

$y'(r)$ = rate of change of production rate at time r (Mton/yr^2).

Development of the Model

Some guidance as to an appropriate form for the function $G^*(x)$ can be obtained from an analysis of the records of capital expenditure and production (milling) rates published in the annual reports of gold-mining companies. If the planned production level is the primary motivating force that demands a programme of capital expenditure, then the mining venture or its planners can be thought of as a 'system' that converts an input time series of the production history of a mine into a corresponding time series of required capital expenditures. This is shown diagrammatically in Fig. 4.

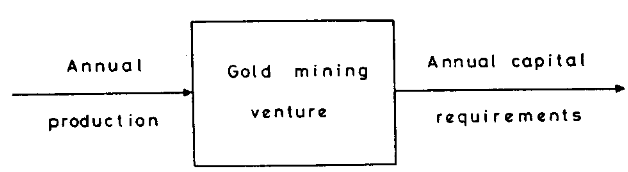

Fig. 4—'System' representation of the link between capital and production rates

Many methods have been proposed[2–7] for the development of systems of process control, industrial plant design, communication theory, and econometrics. This paper requires the estimation of the function $G^*(x)$, which characterizes the capital expenditure associated with a production unit. Since the data on capital and production rates usually comprise 10 to 30 pairs of observations for each gold mine, a modified direct least-squares estimation procedure was employed in a model in which a single weight function, $h(t)$, is estimated for all the mines simultaneously. The result is thus an average description of the behaviour of all the mines concerned.

If $(1/H)\ G^*(x) = h(x)$, $h(0) = 0$, and $y'(0) = 0$, equation (2) can be expressed in the discrete form

$$K't = \sum_{i=1}^{D+t-1} h_i y'_{D+t-i};\ t \leqslant L-D+1, \quad . \quad . \quad . \qquad (3)$$

where $h_1, h_2, \ldots, h_D$ represent construction profile weights and $h_{D+1}, \ldots, h_{D+t-1}$ represent replacement profile weights.

To obtain an average estimate of the construction-replacement function, h_i, a set of equations similar to equation (3) is drawn up for each mine, but the construction-replacement function is restricted to the same value in each case. This results in an overdetermined set of equations in the unknown profile values, h_i, which can be estimated according to a least-squares criterion. The necessary calculations were programmed on a minicomputer, and the construction-replacement 'response' profile was computed for several values of the construction profile time span. These results are plotted for the particular case of $D = 6$ years in Fig. 5, together with the standard deviation level corresponding to each weight. The estimated construction-replacement profile shows that a drop in the rate of expenditure occurs in the final year of the construction period (Weight 6). This corresponds to the transient phase in the production build-up rate after the construction phase has been completed. Most of the construction capital appears to be spent over a period of four years; this period is determined mainly by the initial expansion characteristics of the set of mines analysed. Another feature of the profile is that capital expenditure continues at a significant level after the construction phase has been completed. This component of capital is related to the extension and replacement expenditures required to

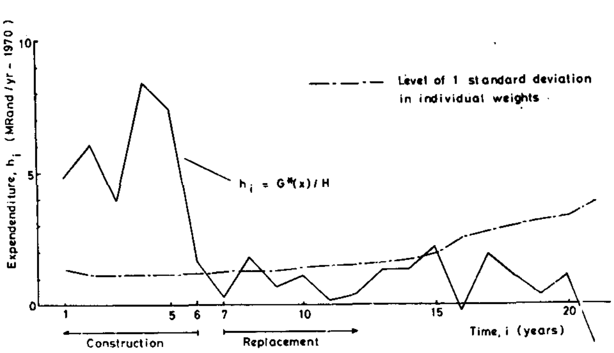

Fig. 5—Average construction/replacement profile estimated for twenty gold mines

sustain the operating unit.

The average capital costs of expansion and replacement can be calculated from the profile weights as follows:

$$\text{Expansion cost} = \sum_{i=1}^{D} h_i \text{ MRand/(Mton/yr)}$$
$$= 32{,}4 \text{ MRand/(Mton/yr) in 1970-based costs } (D = 6 \text{ yr}).$$

$$\text{Replacement cost} = 1/(T-D) \sum_{i=D+1}^{T} h_i \text{ MRand/ Mton}$$
$$= 0{,}77 \text{ MRand/Mton in 1970-based costs and } T-D = 15.$$

With regard to the choice of a suitable capital-production model, it would appear that, within the limits of accuracy of the data, a constant level of construction expenditure followed by a constant but lower level of replacement expenditure can be used to approximate the average construction-replacement mechanism. The application of this specific form of model to the data for individual mines is discussed below. The constant construction expenditure is referred to as a 'rectangular' construction profile.

Construction–Replacement Model with Rectangular Construction and Constant Replacement Profile

The rectangular construction and constant replacement model can be derived from equation (2) by the assignment of the following values to $G^*(x)$:

$(1/H)\ G^*(x) = A/D$ for $0 \leqslant x \leqslant D$,
$(1/H)\ G^*(x) = B$ for $x > D$.

When these expressions are substituted in equation (2), it can be shown by integrating this expression that

$$K(t) = A(Y(t+D)-Y(t))/D+BY(t)+K_0 \text{ MRand};$$
$$0 \leqslant t \leqslant L-D, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

where

$K(t)$ = cumulative capital up to year t (MRand)
$Y(t)$ = cumulative production up to year t (Mton)
= 0 for $0 \leqslant t \leqslant D$
D = length of construction period (yr)
A = expansion cost coefficient (MRand/(Mton/yr))
B = replacement cost coefficient (MRand/Mton)
K_0 = anomalous capital not directly related to capacity incurred as an initial set-up cost (MRand)
L = ultimate life of the mining venture (yr).

The coefficients estimated for the rectangular profile model are presented in Table II. The model was applied by the specification of the start year and end year of the capital–production rate sequences, as well as a search grid for the construction length, D. For each mine, the start year was chosen initially as the first year of recorded capital expenditure, and was then increased if the fitted values showed gross distortions. This corresponds to the assumption that anomalous capital expenditure, K_0, which does not relate directly to increasing production rate, is incurred in the first years of the preproduction period. This assumption was found to be satisfactory for all the mines, except mine 18, which was found to be fitted more accurately if the anomalous capital com-

TABLE II

RECTANGULAR CONSTRUCTION PROFILE MODEL

Mine and area	Constant period $\hat{D}$	s_D	Expansion cost $\hat{A}$	s_A	Replacement $\hat{B}$	s_B	Anomalous capital $\hat{K}_0$	s_{K_0}	Sum of squared errors	Time origin shift
1 F	2,4	0,2	31,7	1,5	2,7	0,1	18,7	2,7	211	0
2 F	1,6	0,2	11,9	0,7	0,9	0,9	7,0	1,0	19	0
3 F	2,3	0,1	21,4	0,8	0,5	0,0	14,2	0,7	30	7
4 E	3,1	0,2	15,5	0,4	0,6	0,0	0,6	0,5	3	0
5 E	2,5	0,1	19,0	0,7	0,1	0,1	0,5	0,6	4	0
6 F	3,2	0,1	30,3	1,0	4,1	0,1	1,6	1,2	13	0
7 E	3,1	0,9	16,7	0,9	0,2	0,2	4,1	0,9	4	0
8 O	4,3	0,1	44,3	1,3	−0,2	0,2	3,9	0,8	11	0
9 O	3,3	0,2	31,5	2,2	0,3	0,1	42,8	2,2	38	8
10 K	3,4	0,2	37,4	1,2	−0,8	0,2	2,1	0,6	80	0
11 K	4,1	0,5	28,5	2,2	1,9	0,1	1,9	3,4	23	2
12 F	—	—	—	—	—	—	—	—	—	—
13 F	5,1	0,9	31,2	2,1	0,6	0,3	3,8	1,3	61	0
14 F	—	—	—	—	—	—	—	—	—	—
15 F	2,3	0,3	31,0	2,2	0,8	0,2	24,5	2,5	489	4
16 O	3,9	0,3	20,9	1,5	0,6	0,2	24,0	1,9	16	3
17 K	2,1	0,7	12,9	2,3	0,6	0,1	14,0	1,7	133	0
18 O	5,7	0,7	24,9	2,5	−0,1	0,2	25,0*	3,1	208	0
19 O	3,1	0,4	22,7	2,7	0,1	0,3	31,6	2,6	123	4
20 O	4,6	0,5	21,6	1,3	0,4	0,1	7,8	1,6	101	2
With interposed lag:										
8	3,3	0,1	39,5	2,0	0,2	0,3	6,4	1,3	39	0
10	2,6	0,3	32,8	1,9	0,0	0,3	3,8	1,4	156	5
18	5,5	0,6	23,8	2,4	0,1	0,2	24,1*	3,3	221	0

*Distributed over preproduction period

ponent was considered to be distributed uniformly over the preproduction period. Mines 12 and 14 had too few data points to yield meaningful estimates of these parameters.

From Table II, it can be seen that for three mines (8, 10, and 18) the replacement coefficient, B, was estimated to be a negative quantity. This appears to be due to an abnormally large time lag before production is built up at the end of the construction phase. These three cases were therefore re-estimated with a slightly modified form of equation (4), in which the characteristic construction profile has an interposed time lag of 1 year at the end of the construction phase. The appropriate expression is

$$K_t = (A/(D-1))\ (Y(t+D)-Y(t+1))+BY_t+K_0 \quad (5)$$

The revised estimates of D, A, B, K_0, based on equation (5) are presented at the foot of Table II for mines 8, 10, and 18. (Tests on other mines indicated that the model using an interposed time lag was generally inferior to the straightforward model with a rectangular profile.)

The importance of performing the model estimation from a time origin that is as close as possible to the first recorded observation must be emphasized. In all cases, the richest 'information content' of the capital–capacity data is contained in the initial build-up phase of the mine, and should be included as far as possible in the range over which the proposed model is fitted. An example illustrating the accuracy of the rectangular-profile model is plotted in Fig. 6. It appears that the model can provide an adequate description of mine capital expenditure in terms of mine production rate.

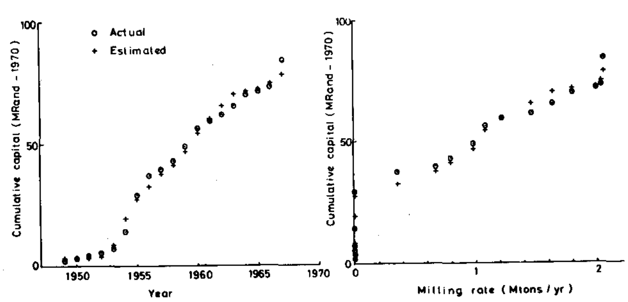

Fig. 6—Comparison of estimated and actual capital expenditure for Mine 10

Review of Estimated Coefficients

In this section, the values of the coefficients estimated for each mine by use of the rectangular construction profile model are reviewed on an industry basis. The values of the coefficients used are given in Table II. Values for mines 8, 10, and 18 were taken from the last three lines of that table.

Mines 1 and 6 operate a longwall system, and mines 13 and 15 have been converted from scattered to longwall systems. It is observed that the longwall mines tend to have relatively high expansion and replacement costs. This is certainly plausible when it is considered that the extent of shaft sinking on longwall mines tends to be high, since this mining method is necessitated by deep workings. Also, at increasing depth, the replacement capital costs include large-scale items such as underground refrigeration plants, high primary development costs, and increased expenditure on support equipment.

High values of the fixed-capital component also appear to be associated with a surplus design capacity of the initial infrastructure. In the absence of clear-cut evidence that large K_0 values are associated with capacity-independent cost items of significant magnitude, and in particular since the property costs of the set of mines considered in this study are small, it is proposed that, for the purposes of developing an overall capital estimation relationship, the gross expansion costs should be modelled as a combination of the K_0 and A components. The overall expansion coefficient, A_0, is defined as

$$A_0 = A + K_0/y_m, \quad \quad (6)$$

where y_m is the ultimate designed production capacity of the mine. It must be stressed that, if the costs of property and mineral rights become more significant for new mines, they should not be absorbed into the overall expansion coefficient, A_0. Thus the estimates of A_0 made below should be treated as being exclusive of the fixed costs associated with the purchase of property and mineral rights.

A_0 can be expected to be related to the difficulty of gaining initial access to the reef area. This is, in turn, determined by factors such as working depth, H, and the number of ancillary shaft facilities required for ventilation. The extent of these major development works can be estimated by the definition of a shaft-density (or shaft-concentration) parameter, $\bar{d}$. (At the end of the life of the mine, $\bar{d}$ represents the total length of major shafts divided by the total reserve area.) Values of A_0 given by equation (6) are presented in Table III, together with the initial depth, H, and the shaft concentration, $\bar{d}$, estimated for the mines considered. The correlation coefficients of A_0 versus H and A_0 versus $\bar{d}$ are 0,61 and 0,73 respectively. A_0 is plotted against initial depth, and against average shaft concentration, in Figs. 7 and 8 respectively.

It can be seen that the correlation of A_0 and $\bar{d}$ is slightly better than A_0 and H, but the scatter of points prevents narrow confidence limits. Nevertheless, the analysis does demonstrate that the initial depth, or a related parameter such as $\bar{d}$, is a strong determinant of variations in capital expansion costs from mine to mine.

The replacement coefficient, B, can be expected to be related to the length of replacement shafts divided by the operating life of a major shaft area. One measure of this

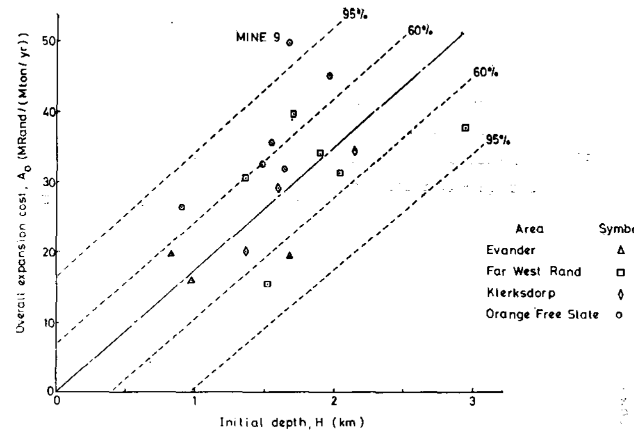

Fig. 7—Coefficient of overall expansion cost plotted against initial depth

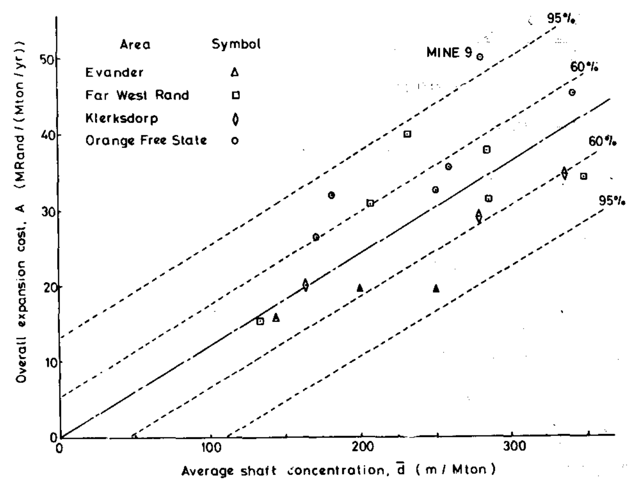

Fig. 8—Coefficient of overall expansion cost plotted against average shaft concentration

TABLE III

OVERALL EXPANSION COEFFICIENTS, INITIAL DEPTHS, AND SHAFT CONCENTRATIONS

Mine number	Area symbol	Expansion coefficient, A MRand/(Mton/yr)	Overall expansion coefficient, A_0 MRand/Mton/yr	Initial depth, H km	Shaft concentration, $\bar{d}$ M/Mton
1	F	31,7	37,5	2,95	284
2	F	11,9	15,4	1,52	133
3	F	21,4	30,6	1,37	207
4	E	15,5	15,9	0,97	144
5	E	19,0	19,5	0,82	199
6	F	30,3	31,1	2,04	285
7	E	16,7	19,3	1,68	250
8	O	39,5	45,0	1,97	341
9	O	31,5	49,8	1,68	281
10	K	32,8	34,4	2,15	336
11	K	28,5	29,2	1,60	279
12	F	—	—	0,94	—
13	F	31,2	33,9	1,90	348
14	F	—	—	1,91	—
15	F	31,0	39,6	1,71	232
16	O	20,9	31,8	1,64	182
17	K	12,9	20,1	1,37	164
18	O	23,8	32,7	1,49	251
19	O	22,7	35,5	1,55	259
20	O	21,6	26,3	0,90	171
		Average =	30,4		

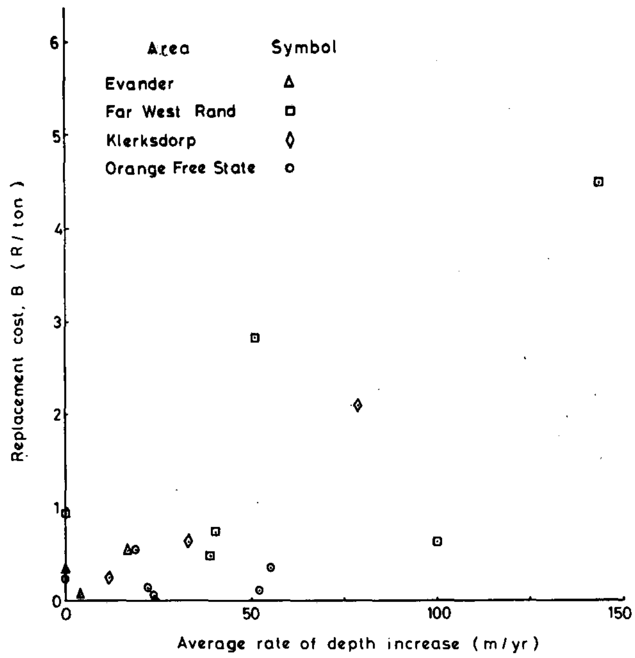

Fig. 9—Replacement coefficient plotted against average annual increase in depth

ratio is the average increase in mine working depth per year. B is plotted against the rate of depth increase in Fig. 9. Plots of B against total depth increase, maximum achieved depth, depth increase per million tons mined, and dip angle do not yield better correlations.

Summary of Main Results

This section contains a brief summary of the suggested capital–capacity model, together with the recommended estimates of the model coefficients. The model should not be used for the inference of overall capital costs beyond a maximum cumulative production of about 60 Mtons milled. All the coefficient values are expressed as a ratio of the average working cost in the gold-mining industry in 1970, i.e., R7,34 per ton milled. This factor is denoted by W. It is suggested that current estimates should be obtained by the multiplication of the relevant coefficients by the current average working cost in the gold-mining industry. An example is presented in the next section.

The cumulative capital expenditure $K(t)$ at time t is related to the cumulative tonnage milled Y(t) by the relationship

$$K(t) = A_0\,(Y(t+D)-Y(t))/D+BY(t)+K_0 \text{ MRand} \quad (7)$$

It is suggested that, for the set of mines considered, A_0 can be estimated by

$$A_0 = a_1H;\ H = \text{initial shaft depth (km)}, \quad (8)$$

or $$A_0 = a_2\bar{d};\ \bar{d} = \text{shaft concentration (m/Mton)}, \quad (9)$$

with $a_1/W = 2{,}36 \pm 0{,}33$ at 95 per cent confidence

and $a_2/W = 0{,}0163 \pm 0{,}0018$ at 95 per cent confidence.

The mean value of coefficient B excluding longwall mines is

$$B_{avg}/W = 0{,}063. \quad (10)$$

The standard deviation/W = 0,067.

For the set of mines considered, the only significant capital expenditure that is independent of production, K_0, appears to be associated with the purchase of property and mineral rights. This expenditure was typically 1 per cent of the total capital expenditure. However, when the proposed model is used, it is recommended that all the costs of property and mineral rights should be estimated independently and added to the costs of expansion and replacement derived from the model.

From Table II, the construction duration, D, has mean value

D_{avg} = 3,2 yr.

Standard deviation = 1,1 yr. . . . . . . (11)

The total capital expenditure was also found to correlate

TABLE IV

REPLACEMENT COEFFICIENT, B, AND AVERAGE DEPTH INCREASE PER YEAR

Mine	Replacement coefficient B	First declaration	End year of fit	Approximate reef dip	Max. depth at first decl.	Max. depth at end year	Depth increase Total	Depth increase per year
1 L	2,746	62	73	21–22	2 948	3 507	559	50,8
2	0,931	61	71	10–15	1 518	1 518	0	0
3	0,467	49	72	30–35	1 368	2 254	886	38,5
4	0,561	62	72	15–19	969	1 138	169	16,9
5	0,075	62	73	20	817	861	44	4,0
6 L	4,125	68	73	36–40	2 042	2 760	718	143,6
7	0,227	67	73	20	1 682	1 682	0	0
8	0,232	61	67	5–10	1 973	1 973	0	0
9	0,315	56	72	20–25	1 682	2 041	359	22,4
10	0,042	56	66	8–10	2 149	2 267	118	11,8
11	1,926	57	72	20–30	1 598	2 780	1 182	78,8
12	—	73	—	10	935	—	—	—
13 S/L	0,553	54	63	22	1 899	2 796	897	99,7
14	—	72	—	18	1 911	—	—	—
15 S/L	0,845	52	73	21–22	1 712	2 563	851	40,5
16	0,556	55	65	5–10	1 640	1 826	186	18,6
17	0,640	41	68	10	1 372	2 270	898	33,3
18	0,101	54	69	20–25	1 487	2 270	783	52,2
19	0,071	54	69	20–25	1 547	1 907	360	24,0
20	0,359	52	70	25	896	1 888	992	55,1

with the total length of major shafts. The one-parameter relationship is

$$K/W = 1{,}53\ L_T/W\ \text{MRand/(R/ton)}, \quad . \quad . \ (12)$$

where K = total capital (MRand)

L_T = total length of major shafts (km).

In addition, the average shaft-length parameter, $\bar{d}$, was found to be related to the initial depth, H, by the single parameter correlation

$$\bar{d} = 143{,}5\ H\ \text{m/Mton}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \ (13)$$

where H is expressed in kilometres.

Adjustment of Cost Estimates to Current Values

As an example of how the cost estimates are adjusted to current values, consideration is given below to a hypothetical scattered gold mine that mills a total of 30 Mtons of ore over a period of 15 years at a uniform rate of production of 2 Mton/yr. If the total shaft concentration is 250 m/Mton, then, from equation (9),

$$A_0/W = 0{,}0163 \times 250\ \text{(R/(ton/yr))/(R/ton)}$$
$$= 4{,}08\ \text{yr.}$$

If

$B/W = 0{,}063$ for a scattered mine and K_0 is estimated independently, the capital amounts can be expressed in absolute terms by being multiplied by the current working cost. For example, capital components equivalent to the last quarter of 1979 must be multiplied by a factor of R30,2 per ton milled. Hence, for the hypothetical mine,

Expansion cost $= 2 \times 30{,}2 \times 4{,}08 = 246$ MRand

Replacement cost $= 0{,}063 \times 30{,}2 \times 2 = 3{,}8$ MRand/yr.

In the above calculation, the current working cost plays a dramatic role in the estimation of the gross capital expenditure. Although the working-cost index has shown a large upsurge since 1970, it should also be viewed in relation to the gold price increases during that period. It is suggested that, for the purposes of economic planning, the estimates of both mine revenue and capital should be expressed in relation to the expected working-cost level. This point is considered further in another paper[8].

Conclusions

The underlying structure evolved can be used to develop general relationships between the rate of capital expenditure and the production capacity of a mine. The important distinction is recognized between 'static' models linking overall capital and capacity requirements and so-called 'dynamic' models that allow for the time-dependent nature of capital cash flows and production rates.

The general capital–capacity model constructed within this framework can easily be extended to include more complex situations such as trends in the duration of construction arising from anticipated improvements in technology. More important, the theory is readily amenable to modification if sufficient insight or data are available. The analysis for a model that would be appropriate for several recently established gold mines indicates that a characteristic pattern of capital expenditure is associated with these mines. By the assignment of a fixed functional form to this pattern, the capital expenditure for individual mines can be predicted satisfactorily from production rate. At the industry level, the model coefficients display wide variations. These can be ascribed to differences in the extent of overdesign of the initial installations, as well as to physical differences such as depth of workings and rate of increase in mining depth as exploitation proceeds.

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Addendum

It is assumed that large-scale facilities, which can be considered to be shaft-like units with a self-contained surface infrastructure, are required for the operation of a mine. The pattern of capital expenditure will depend on the rate at which these units are built, the expected lifetime of the units, and the extent to which the units that have ceased production are replaced. In parallel, the overall production rate of the mine will depend on the instantaneous aggregate of operating units.

It is further assumed that

$a(t)$ = the cumulative number of large facilities, including replacements, that have been built and commissioned up to time t, and

$n(t)$ = the number of these large facilities that are currently operating and hence sustaining the overall production rate at time t.

The time schedule of capital expenditure required during the construction of the unit is represented by a function $G(x)$ with the time axis, x, reversed so that $G(x)$ is the rate of expenditure x years before the date of completion. The number of new units that become operative between time r and $r+dr$ is $a'(r)dr$. The capital rate of expenditure required during the construction of these units at a specified time, $t \leqslant r$, before the units are commissioned is therefore given by $G(r-t)a'(r)dr$. The total rate of capital expenditure at time t is consequently

$$K'(t) = \int_t^L G(r-t)\, a'(r)dr, \quad . \quad . \quad . \quad . \quad \text{(A1)}$$

where L = ultimate life of the venture

$K'(t)$ = rate of capital expenditure (MRand/yr).

Additional capital expenditures that are incurred to replace sub-components of the representative unit, or that are by their nature distributed over the producing life of the unit (for example main haulage development), can be modelled if the function $G(x)$ is allowed to have positive values when $x < 0$. The total capital expenditure can be obtained if the range of integration of the right-hand side of equation (A1) is extended to the entire interval $(0, L)$.

The cumulative number of installed units, $a(t)$, can be related to the current production level, $y(t)$, as follows.

If a 'mortality' function, $H(x)$, represents the production rate generated by one unit x years after it has been commissioned, the overall production rate, $y(t)$, at any instant in time, t, is given by

$$y(t) = \int_0^t H(t-r)a'(r)dr \text{ Mton/yr.} \quad . \quad . \quad . \quad \text{(A2)}$$

Equations (A1) and (A2) now represent an indirect relationship between capital expenditure rate, $K'(t)$, and production rate, $y(t)$. For a more compact relationship, it is convenient to adopt the view that the representative production units maintain a constant production level of H Mton/yr indefinitely but that these units are continually refurbished according to the characteristic replacement profile $G(x)$ for $x < 0$. Equation (A2) becomes $y(t) = H\ a(t)$ if $a(0) = 0$. Substitution of this relationship into equation (A1) yields

$$K'(t) = (1/H) \int_0^L G(r-t)y'(r)dr \text{ MRand/yr.} \quad . \quad \text{(A3)}$$

Equation (A3) is a direct relationship between capital expenditure and production level. In practice, the construction profile will be such that the duration of the construction period is a finite length of time, D years, and therefore $G(x) = 0$ for $x > D$. This structure of the function $G(x)$ can be exploited to simplify equation (A3). If $G(x)$ is transformed into a new function $G^*(x)$, defined as

$$G^*(x) = G(D-x), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(A4)}$$

then equation (A3) can be written as

$$K'(t-D) = (1/H) \int_0^t G^*(t-r)\, y'(r)dr \text{ MRand/yr} \quad \text{(A5)}$$

$$\text{for } D \leqslant t \leqslant L.$$

Equation (A5) has the form of the standard convolution integral and can be solved for $y'(t)$ with operational transform techniques[3]. For example, if the production rate, $y(r)$, is a step function and $y'(r) = y.\delta(r-D)$, equation (A5) becomes

$$K'(t-D) = (y/H)\ G^*(t-D) = (y/H)\ G(2D-t). \quad \text{(A6)}$$

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Underwater mining

The 12th Underwater Mining Institute will be held from 20th to 22nd October, 1981, in Madison, Wisconsin. The 12th UMI programme will include presentations on the mineralogy of marine sulphide deposits, tectonic setting for spreading centre sulphide deposits, seafloor sulphides in the Galapagos and other Pacific areas, new developments in Southeast Asia offshore tin operations, geophysical techniques for finding underwater copper lodes, new geochemical techniques for marine minerals exploration, changes in the international mining trade of relevance to marine mining, and the impact for Sea Grant minerals research on industry. The programme will also include tours to local research laboratories, and an annual banquet and award presentation.

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