Investment criteria and choice of production rate in the planning of gold-mine production

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SYNOPSIS

By use of a functional model of capital expenditure developed earlier, a prototype cash-flow model is formulated as a basis for the investigation of the economic consequences when the production rate of a mine is manipulated as a fundamental design variable. The optimum production capacities that maximize the common economic criteria of net present value, present-value ratio, and wealth growth rate are determined. It is shown that these choices depend on certain key groupings of the economic parameters, and that they have a fixed interrelationship.

The basic cash-flow model is extended further to include the effects of a constrained rate of production buildup and the imposition of State taxation on generated income. An approximation to the initial tax-free period of a new mining venture is developed, and, by use of these results, the optimum choice of production level is analysed for when both the buildup rate is constrained and tax is imposed. This extended model permits a comparison between the performance of existing mines and the predicted theoretical production levels for these mines.

SAMEVATTING

Daar word met behulp van 'n funksionele model van kapitaalbesteding wat vroeër ontwikkel is, 'n prototipe van 'n kontantvloeimodel geformuleer as grondslag vir die ondersoek van die ekonomiese gevolge wanneer die produksietempo van 'n myn as 'n fundamentele ontwerpveranderlike gemanipuleer word. Die optimale produksievermoeë wat die gewone ekonomiese maatstawwe van netto huidige waarde, huidige waardeverhouding en die tempo van inkomstegroei maksimeer, word bepaal. Daar word getoon dat hierdie keuses van sekere sleutelgroeperings van die ekonomiese parameters afhang en dat daar 'n vaste onderlinge verband tussen hulle is.

Die basiese kontantvloeimodel word verder uitgebrei om die uitwerking van 'n begrensde tempo van produksieopbouing en die oplegging van staatsbelasting op gekweekte inkomste in te sluit. Daar word 'n benadering van die aanvanklike belastingvrye tydperk van 'n nuwe mynonderneming ontwikkel en die optimale keuse wat die produksiepeil betref, word met behulp van hierdie resultate ontleed vir wanneer die opboutempo begrens word en belasting gehef word. Hierdie uitgebreide model maak dit moontlik om die werkverrigting van bestaande myne en die voorspelde teoretiese produksiepeile vir hierdie myne te vergelyk.

Introduction

The mathematical expression of a model for the overall cash flow in a mine hinges on a number of assumptions. The first of these depends on whether the components of the model are to be represented as deterministic quantities, or whether they are to be treated as random variables to represent 'real life' uncertainties. Clearly, a thorough understanding of the deterministic representation is necessary before a more involved probabilistic analysis is undertaken. The formulation of a deterministic model is of sufficient complexity to warrant detailed exploration, and this paper is confined to that aspect.

The second essential requirement is a definition of the measure of investment worth that is to be used as a criterion of the relative profitability of design alternatives. This is connected to the objectives of the mining company (there may be several as pointed out by Jordi and Currin[1]) and to the spectrum of investment opportunities available to the company (Ashton and Atkins[2], Chambers[3], Nemhauser and Ullman[4]). In the present study it is posited that the firm has a single objective, and that the net effect of assigning available funds to competing alternatives can be approximated by a single opportunity growth curve.

It must be emphasized that both the opportunity curve, and the cost and price parameters of the cash-flow model, should be deflated and expressed in terms of purchasing-power equivalents. Thus, for exponential growth, the discount rate, s, applied to the deflated cash flows is the re-investment rate of the mining company over and above the general level of monetary inflation. This investment rate, s, is referred to below as the 'opportunity interest rate' of the mining company, and represents the net growth rate of the best alternative to the contemplated venture; s is assumed to be expressed as a fraction per year, the percentage equivalent being $100s$.

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Formulation of a Cash-flow Model

A highly simplified model of the construction and operating cash flows of a gold mine is developed in this section. The hypothetical model is defined to be such that the mine becomes productive at the full design rate after an elapsed construction time of D years. The mine continues to operate at that level until the entire ore reserve is depleted, when operations cease immediately.

If y = production level (Mton milled/yr),
D = construction time (yr),
L = ultimate life of the mine as measured from the start of construction (yr), and
T = total reserve tonnage available for milling (Mton),

it is evident that, for a constant production rate,

$$T = y(L-D) \quad . . . . \quad (1)$$

From the capital–capacity model estimated earlier[5], the rate of capital expenditure is

$$K'(t) = Ay/D \text{ for } 0 \leqslant t \leqslant D \text{ and} \quad . . . . \quad (2)$$

$$K'(t) = By \quad \text{for } t > D, \quad . . . . \quad (3)$$

where A = coefficient of capital cost for expansion (MRand/(Mton/yr))
B = coefficient of capital cost for replacement (MRand/Mton).

During the production phase of the mine, $D \leqslant t \leqslant L$, R denotes the net revenue per ton milled and C_V denotes the working cost per ton milled. It is assumed that the average grade of ore mined is fixed, and that C_V is independent of the production level, y. (For the gold-mining industry as a whole, the annual working costs do not exhibit any clear-cut economy-of-scale effects[6]). When expressed in terms of equivalent purchasing power (i.e., after deflation), R, C_V, B, and A are assumed to be constant. Hence, for the hypothetical cash-flow model, the net rate of cash flow, $f(t)$, is given by

$$f(t) = -Ay/D \text{ MRand/yr for } 0 \leqslant t \leqslant D \text{ and} \quad (4)$$

$$f(t) = ry \quad \text{MRand/yr for } t > D, \quad (5)$$

where $r = R - C_V - B$ R/ton milled. . . . . (6)

In addition to these expenditures, an anomalous capital amount, K_0, which is not related to the production capacity of the mine, may be incurred at the start of operations. The net present value of the mining venture is defined as

$$NPV = -K_0 + \int_0^L f(t)/V(t)\,\mathrm{d}t, \quad (7)$$

where $f(t)$ is the net cash flow rate and $V(t)$ is the postulated opportunity growth curve. For exponential growth, $V(t)$ is characterized by an interst rate, s, and

$$V(t) = e^{st}. \quad (8)$$

Investment Criteria

If the rate of cash flow in equation (7) is considered to be composed of expenditures required to establish the mine (capital) followed by generated returns, the net present value can be expressed as

$$NPV = -NK + NF, \quad (9)$$

where

NK = net present value of expansion capital (MRand), and

NF = net present value of generated returns (MRand).

The present value ratio, PVR, is defined by

$$PVR = NF / NK \quad (10)$$

The wealth growth rate or external rate of return, w, is defined as

$$NK\, e^{wL} = NF\, e^{sL} \quad (11)$$

Investment criteria are discussed elsewhere[7, 8] in greater detail.

Investment Efficiency

It is useful for the net present value derived from equation (7) to be expressed in the form of an efficiency. The most convenient form for this efficiency is the ratio of the total net present value of the venture to the total net value of the extractable reserve. The net value of the reserve is the total realizable revenue of the mineral content minus the total operating cost required to mine and beneficiate the reserve. In terms of the parameters of the basic cash-flow model, this efficiency can be expressed as

$$n = NPV/rT = (NF - NK)/rT. \quad (12)$$

When capital costs and the opportunity interest rate become vanishingly small, n approaches unity. The substitution of equations (1), (4), and (5) into equation (7), and the assumption of an exponential discount function, $V(t) = e^{st}$, yield the following expressions for the net present value components, NK and NF:

$$NK = K_0 + Ay\,(1 - e^{-sD})/sD \text{ and} \quad (13)$$

$$NF = ry\,(1 - e^{-sT/y})\, e^{-sD}/s. \quad (14)$$

These relationships can be expressed succinctly by the use of the following dimensionless groupings of the basic parameters, and the representation of the fixed or 'setup' capital component by

$$k = K_0/rT. \quad (15)$$

The production rate of the mine can be represented implicitly by the ratio of the total operative life of the mine, T/y, to the payback period of the capital for expansion, A/r. (This payback period excludes the construction period, D.) If this life ratio is denoted by b,

$$b = (T/y)/(A/r) = rT/Ay. \quad (16)$$

The opportunity interest rate, s, is represented by the dimensionless quantity

$$q = sA/r. \quad (17)$$

Finally, the dimensionless preproduction period, d, is defined as the ratio of D to the undiscounted payback period of expansion capital, A/r, i.e.,

$$d = D/(A/r). \quad (18)$$

Hence, it can be seen that $sD = qd$, $sT/y = qb$, and equation (12) becomes

$$n = e^{-qd}[1 - e^{-qb} - (e^{qd} - 1)/d]/qb - k. \quad (19)$$

It can be shown that the present-value efficiency, n, is a maximum when the life ratio, b, satisfies the equation

$$G(qb) = (e^{qd} - 1)/d, \quad (20)$$

where $G(x) = 1 - e^{-x}(1 + x)$ with $x = qb$ . . . (21)

The optimum life ratio, b^*_{NPV}, is thus a function of only two dimensionless parameters, namely $q = sA/r$ and $d = D/(A/r)$. From this it is clear that the ratio A/r, which is the undiscounted payback period of expansion capital expenditure excluding the preproduction period D, is a fundamental determinant of the optimum life of the mine.

Since the function $G(x)$ is never greater than unity when $x > 0$, equation (20) possesses a real solution only if the economic parameters satisfy

$$(e^{qd} - 1)/d < 1, \quad (22)$$

which can be expressed in the equivalent form

$$q < q_c, \text{ where} \quad (23)$$

$$q_c = (1/d)\ \ln\,(1 + d). \quad (24)$$

For example, when $d = 0$, $q_c = 1$ but, when $d = 1$, $q_c = 0{,}693$. (The expression $d = 1$ implies that the preproduction period is equal to the undiscounted payback period.)

Equation (20) enables curves of the optimum life ratio, b^*, to be plotted against q with d as a parameter. Fig. 1 illustrates the two cases $d = 0$ and $d = 1$, and shows that, in the interval $0{,}1 \leqslant q \leqslant 0{,}6$, the optimum life ratio falls roughly between 3,5 and 5. As given in another paper[5], an estimate of the average expansion cost, A/C_V, is approximately 4 years. For a typical value of $r/C_V = 1$, $A/r = 4$, and it is apparent from equation (17) that, when $q = 0{,}1$, the interest rate, s, is 2,5 per cent per annum and, when $q = 0{,}6$, s is 15 per cent per annum. This indicates that, within a broad range of interest levels, the optimum life ratio is dependent mainly on the prevailing value of A/r. In the two extreme cases of zero discount rate, $q = 0$ or, if q tends to q_c, the optimum life ratio becomes infinite. The limiting curve $d = 0$ also indicates that the criterion of net present value yields optimum operating lives that are never less than 3,35 times the undiscounted capital payback period, A/r.

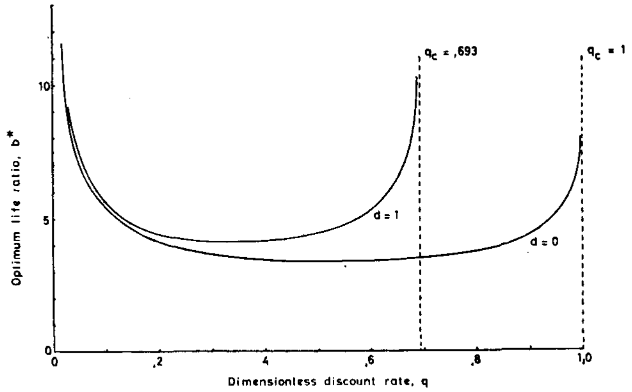

Fig. 1—Optimum life ratio, b^*, plotted against the quantity q for different preproduction periods, d

Comparison of Investment Criteria

Some analogous results are now developed for the criteria of present-value ratio (PVR) and wealth growth rate (WGR). In this analysis only the limiting case of a small preproduction period is treated (i.e., $d = 0$). The limiting case simplifies the analysis but retains the essential behaviour of these criteria in relation to net present value. From equation (10) for PVR, and expressions (13) and (14) for NK and NF, it can be shown that the necessary condition that the life ratio must satisfy for maximization of PVR is given by

$$q/k = e^{qb} - qb - 1. \quad \ldots \ldots \ldots (25)$$

This relationship between b^*_{PVR} and q is shown in Fig. 2 for $k = 0{,}1$ and $k = 0{,}2$. The broken portions of these curves correspond to present-value ratios of less than unity. It is apparent that, as the setup capital component, k, increases, the maximum PVR condition corresponds more closely to the maximum NPV condition. As $k \rightarrow 0$, the PVR objective requires $b^*_{PVR} \rightarrow \infty$ or the production rate to be very small. Thus, the maximization of the present-value ratio is sensitive to the magnitude of the setup capital, K_0. An examination of the definitions of k, q, and b shows that the maximizing condition (25) for PVR is independent of the net income.

From equations (11), (13), and (14), the criterion of wealth growth rate can be formulated in a similar manner in terms of the dimensionless parameters q, b, and k. The optimum choice of b for maximization of the wealth growth rate can be shown to be given by

$$qb\{1/(e^{qb}-1) - k/(q(1+kb))\} = \ln\{(1-e^{-qb})/(q(1+kb))\}. \quad \ldots \ldots \ldots (26)$$

This implicit relationship between b^*_{WGR} and q is also shown for the values $k = 0$, $k = 0{,}1$, and $k = 0{,}2$ in Fig. 2. It is interesting to note that, as k decreases, the b^*_{WGR} curves become closer to the optimum NPV curve for large values of q but lie on the opposite side to the b^*_{PVR} curves. The common point of intersection between b^*_{WGR}, b^*_{NPV}, and b^*_{PVR}, representing the maximum internal rate of return, is clearly shown in Fig. 2. It can be proved[6] that this intersection occurs whatever the functional forms assumed by the present value of capital and net revenue, including the case of a significant preproduction period, D.

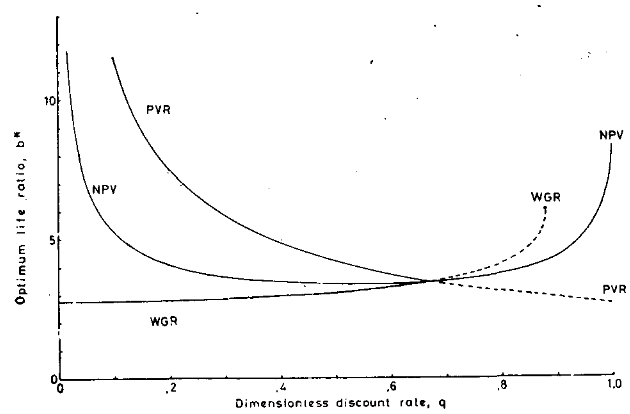

Fig. 2—Optimum choice of life ratio, b^*, according to *NPV*, *PVR* and *WGR* criteria, plotted against discount quantity, q

Choice of Economic Objective

The optimum production capacity depends on the economic criterion designated as the required objective. It has been demonstrated above that, for a fixed set of parameters, the optimum capacity increases in magnitude if the present-value ratio, the net present value, or the wealth growth rate is adopted as the economic goal.

Maximization of the present-value ratio is vulnerable to the structure of the relationship between capital and capacity. In particular, if the setup capital, K_0, is small (or equivalently if there are negligible economies of scale in force), then the most efficient utilization of capital resources cannot be achieved. In addition, PVR takes no direct cognizance of the grade of the ore reserve. Alternatively, the wealth growth rate incorporates all the economic parameters of the simple model, but yields results similar to those yielded by the net present value if the opportunity interest rate is not negligible. The criterion of net present value is in itself a rigorously definable concept and maximizes the total wealth. Other considerations must be included if negligible opportunity interest rates are available (i.e., $q \rightarrow 0$). In the latter case, redefinition of the fundamental objectives and priorities of a company operating in this type of economic environment is clearly necessary.

In view of the logical and rigorous definition of net present value, it appears that it is the most suitable elementary objective to be maximized in the estimation of the economically optimum production capacity. It is, however, desirable that the assessment of the proposed venture should be supplemented by the measures of wealth growth, payback period, and payback time lag. In particular, wealth growth is the preferable measure of the true yield of the investment as opposed to the internal rate of return, which is generally inconsistent with reinvestment assumptions.

Generalized Cash-flow Model

In practice, the cash-flow model developed above is distorted by a number of factors, the most important of which is the imposition of State taxation. Before tax and lease payments can be analysed, consideration must be given to the inclusion in the cash-flow model of an extended period of production buildup. The net present value of cash flows corresponding to a time-dependent production rate of $y(t)$ Mtons per year over the life, L, of the mine, is obtained by the substitution of equations (4), (5), and (8) into equation (7). This yields

$$NPV = -K_0 - \int_0^L K'(t)\, e^{-st}\, dt + \int_0^L ry(t)\, e^{-st}\, dt \quad \text{MRand} \quad \ldots (27)$$

$K'(t)$ is the annual rate of capital expenditure used to expand production capacity. The net revenue per ton milled, r, includes the annual replacement component of capital and is defined by equation (6). From the empirical model for the expenditure of expansion capital,

$$K'(t) = (A/D)(y(t+D) - y(t)) \text{ MRand/yr.} \quad \ldots (28)$$

If the milling rate, $y(t)$, is a non-decreasing function of time, equation (28) can be substituted into equation (27), and the integration limits can be re-arranged to yield

$$NPV = -K_0 - (A/D)(e^{sD}-1)\{y(L)e^{-sL}/s + \int_D^L y(t)e^{-st}\, dt\} + \int_D^L ry(t)e^{-st}\, dt \text{ MRand} \quad \ldots (29)$$

$y(L)$ is the ultimate production level at the end of the mine life, L, which is constrained by the available milling reserve, T, according to the relationship

$$\int_D^L y(t)dt = T \text{ Mton.} \quad \ldots (30)$$

A question that arises in connection with the model allowing for unrestricted production rate is what shape function $y(t)$ should assume for the net present value defined by equation (29) to be maximized, subject to the total reserve constraint (equation (30)). From the structure of these equations it can be shown that the best profile, $y^*(t)$, represents a constant level of production throughout the operating life of the mine, and corresponds to the simple cash-flow model discussed in detail above. If the allowed class of optimum profiles includes the option of shutting the mine during unfavourable price or cost conditions, a more robust technique such as dynamic programming has to be employed to yield the most favourable segmented production profile. However, this refinement is of little practical value since intermittent mine closures are almost certainly unacceptable as an operating policy.

The observation that the constant-rate profile is the shape that yields the most efficient economic performance of the venture if production is unrestricted has already been made by Noren[9]. It must be emphasized that the constant level arises as a consequence of the integrand in equation (29) being a linear function of production rate, $y(t)$. Time-varying profiles can arise only if the integrand is a non-linear function of the rate of change in production rate as, for example, in Cullingford and Prideaux[10], and Hwang *et al.*[11].

Constrained Rate of Production Buildup

The rate of production in the initial years of the productive life of a mine is generally constrained by technical, financial, and risk-aversion considerations. It is assumed that the actual production rate achieved in this early period is described by function $\Psi(t)$, which is continuous and non-decreasing but otherwise arbitrary.

In the face of this constraint, it must be decided at which point in time, u, that expansion should be discontinued and the mine should be operated at the achieved rate, $\Psi(u)$, for the remaining life of the ore reserve. In the determination of the optimum point, u^*, it is convenient for the general present-value relationship given by equation (29) to be expressed in terms of u only. This can be done as follows:

$$y(L) = \Psi(u) \quad \ldots (31)$$

$$\int_D^L y(t)e^{-st}dt = \int_D^u \Psi(t)e^{-st}dt + \Psi(u)(e^{-su} - e^{-sL})/s \quad (32)$$

$$\int_D^L y(t)dt = \int_D^u \Psi(t)dt + (L-u)\Psi(u) = T \quad \ldots (33)$$

If these equations are substituted into equations (29) and (30), the necessary condition, $\delta(NPV)/\delta u = 0$, for maximum NPV, can be demonstrated to be given by

$$G(s(L-u)) = (e^{sD}-1)A/rD, \quad \ldots (34)$$

where $G(x) = 1 - e^{-x} - xe^{-x}$, as defined by equation (21).

The structure of equation (34) is identical to the necessary condition (20), which was derived above for the simple cash-flow model. This interesting result asserts that, subject to a criterion of net present value, the optimum steady-state operating life of the mine is independent of the detailed shape of the profile defining production buildup. The actual production capacity and present value are of course dependent on this profile.

Taxation of Gold Mines

The South African gold-mining industry is subject to unique tax considerations, and the formulation of an analytical cash-flow model of current taxation is the purpose of the present section. This will provide a basis for the determination of the economic optimum production rate after tax. A summary of South African mine taxation is given by Storrar[12], and an example of the detailed computation of annual tax payments for a new gold mine has been given by Wells[13].

For a new gold mine, no lease or tax payments are made for certain periods after the start of production. The lengths of these tax-free periods are determined individually for lease and tax by the redemption of allowed cumulative balances of capital expenditure against generated income. A convenient approximation is the representation of the recurrence relationship for the unredeemed tax capital balance by a continuous differential relationship, and the assumption that both lease and tax payments are incurred in full from the time that this balance falls to zero. If the unredeemed balance at time t is denoted by the function $S(t)$, it can be shown[6] that a suitable description of the dynamic behaviour of $S(t)$ is made regarding the rate of general inflation and the computation of the tax-free period. The general expression for the tax-free period, based on the continuous model of the unredeemed balance of capital expenditure, was given by equation (36). If it is assumed that the special rate of capital allowance is close to the inflation rate, and if the quantity ma is neglected, an approximate expression for the tax-free period, t^*, is

$$\int_0^{t^*} (K'(t)-I(t))\mathrm{d}t+K_0 = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

where $K'(t)$ and $I(t)$, the cash-flow rates of capital and net income, are now expressed in terms of constant purchasing power. If equation (28) is used as the cash-flow model for capital expenditure, and if the net mining income from equation (5) is given by $I(t) = ry(t)$ for $t > D$, equation (37) can be expressed as

$$K_0+(A/D)\int_{t^*}^{t^*+D} y(t)\ \mathrm{d}t = \int_D^{t^*} ry(t)\ \mathrm{d}t. \quad . \quad . \quad (38)$$

If the production rate, $y(t)$, follows a general buildup profile, $\Psi(t)$, for a certain period of time, $D \leqslant t \leqslant u$, after which the mine is operated at a constant production rate, $y = \Psi(u)$, it is apparent from equation (38) that t^* is influenced by the time, u, at which steady operation commences. Three possibilities arise:

1. $t^*+D \leqslant u$
2. $t^* < u < t^*+D$
3. $t^* \geqslant u$.

The time of steady operation, u, is contained in one of these three intervals, and the integration ranges in equation (38) must be resolved for each interval.

Optimum After-tax Production Rate

The inclusion of the taxation cash flows in conjunction with a finite rate of production buildup complicates the analysis of the optimum choice of production rate. the values $t^*/D = 1$, 2, and 3 are illustrated in Fig. 3.

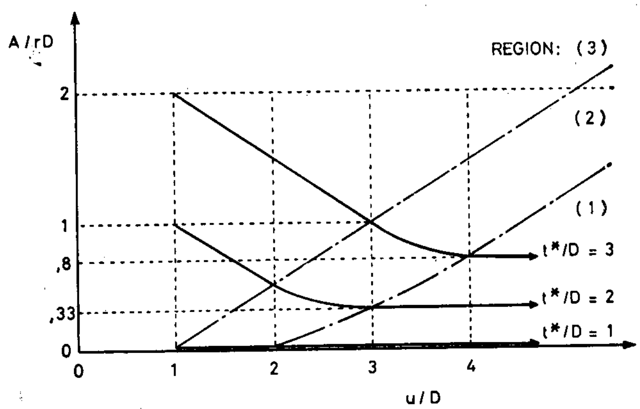

Fig. 3—Contours of tax-free period t^* as a two-dimensional function of A/rD and u/D

From Fig. 3 it can be inferred that, as u/D is increased above unity along a fixed value of A/rD, the tax-free period increases initially but tapers off to a constant maximum value when region 1 is entered. Typically, A/rD assumes values between 1 and 3. As the profitability of the venture increases, the net income, r, increases and A/rD decreases, yielding a shorter tax-free period for any given value of u/D. It can also be noted that, when the buildup rate is very rapid, $u/D \rightarrow 1$ and the tax-free period falls in region 3. In general, the foregoing analysis does not indicate *a priori* in which interval the optimum point of steady operation, u^*, will fall. However, it can be demonstrated[6] that the 'after tax' optimum falls in interval 3 if the following inequality holds:

$s^2T/m \leqslant 2(sA/r)G^{-1}\{(sA/r)\ (e^{sD}-1)/sD\}+2(sA/r)^2$, . . (43)
where G^{-1} denotes the inverse function of
$G(x) = 1-e^{-x}(1+x)$, (i.e., x expressed as a function of G).

Hence, for the linear buildup profile and with small K_0, it can be asserted that, if the three groups of dimensionless parameters s^2T/m, sA/r, and sD are such that relation (43) is true, then the optimum solution after tax, u^*, will definitely fall within interval 3. The equality in (43) is plotted in Fig. 4 as functions of sA/r with sD set to constant values of 0, 0,2, and 0,4. For example, if the buildup rate, m, is 0,5 Mton/yr², the discount rate, s, is 0,05 per year, the reserve, T, is 100 Mton, and the ratio A/r is 4 yr, then s^2T/m is 0,50 and sA/r is 0,20. For any practical value of the construction period, D, this falls above the critical curves in Fig. 4, indicating that the optimum solution, u^*, does not necessarily exist in interval 3 in this case.

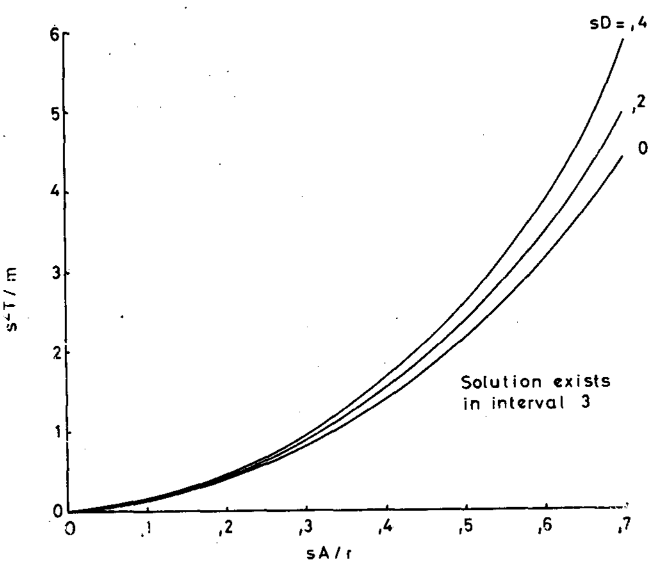

Fig. 4—Region in dimensionless parameter space in which the optimum time for steady operation starts in interval 3

From the linear buildup profile, the before-tax present value, PVM, can be obtained from equations (29), (31), (32), and (33) in the form

$$PVM = rm\, e^{-sD}\{-(A/rD)(e^{sD}-1)(1-e^{-s(u-D)})+1 -e^{-s(u-D)}-s(u-D)e^{-s(L-D)}\}/s^2, \quad (44)$$

where L is related to u by the combination of equations (33) and (39):

$$L = u+T/(m(u-D))-(u-D)/2. \quad (45)$$

Similarly, the present value of the tax payments corresponding to the linear buildup profile can be recovered for each of the three intervals by integration of the expression

$$PVX = f\int_{t^*}^{L} ry(t)\, e^{-st}\,dt, \quad (46)$$

where f is the tax fraction. The application of equation (46) to each of the three intervals yields the following expressions:

Interval 1: $(t^*+D \leqslant u)$

$$PVX = frm\, e^{-sD}((1+s(t^*-D))e^{-s(t^*-D)} -e^{-s(u-D)}-s(u-D)e^{-s(L-D)})/s^2, \quad (47)$$

t^* being given by equation (40).

Interval 2: $(t^* < u < t^* + D)$

PVX is the same as in equation (47) except that t^* is given by equation (41).

Interval 3: $(t^* \geqslant u)$

$$PVX = frm\, e^{-sD}\, s(u-D)(e^{-s(t^*-D)}-e^{-s(L-D)})/s^2, \quad (48)$$

t^* being given by equation (42).

The net present value is obtained from

$$NPV = PVM-PVX. \quad (49)$$

Optimum Choice of Production Rate

Additional insight into the performance of the economic model developed above can be gained from the computation of the theoretical production levels of mines 13 and 18, which were used in the determination of the relationship between capital and capacity that was reported in an earlier paper[5]. The actual ore reserves for each mine were based on a study of the extent of extraction, which was carried out by Berger[14] for producing gold mines in 1976. The revenue, R, and working cost, C_V, were estimated from an average of the figures published in the annual reports of the two mines. All the values were normalized with respect to the same working-cost index that was used in the determination of the capital-expenditure coefficients A and B in that paper[5]. The basic data are shown in Table I.

A uniform rate of opportunity interest of 5 per cent ($s = 0{,}05$) over and above general inflation was assumed.

The lease and tax formulae were assumed to be as follows:

lease formula: $Y = 15-90/X$

tax formula: $Y = 60-360/X$.

Hence, the tax fraction, f, can be estimated[6] from

$$f = C_1-C_2(R/r), \quad (50)$$

with $C_1 \approx 0{,}7$ and $C_2 \approx 0{,}042$.

The optimum production rate for each mine was computed for a very rapid buildup of production, as well as a linear buildup rate corresponding approximately to the profile of the actual capacity of each mine. For a rapid buildup of production, the buildup period,

TABLE I

BASIC DATA FOR THE APPLICATION OF THE THEORETICAL MODEL OF CASH FLOW

Mine no.	Milling reserve T	Capital coefft A	Capital coefft B	Constr. time D	Revenue per ton R	Net profit per ton r
13	57,0	33,9	0,55	5,1	16,0	6,0
18	62,3	32,7	0,10	5,5	26,4	17,1

(r is given by equation (6): $r=R - C_V - B$)

$u-D$, is zero and the tax-free period is approximated by

$$t^*-D = A/r. \qquad (51)$$

In this case, equations (44) and (48) can be reduced to expressions in the production rate, y, which can then be varied to yield the optimum capacity, y^*. (The optimum value obeys an expression with a structure similar to that of equation (34)[6].)

For moderate values of production-buildup rate, m, the solution region containing the optimum buildup period, $u-D$, must be tested by the use of inequality (43). If this inequality shows that the solution is not necessarily in region 3, then equations (44) and (46) may have to be enumerated for both regions 1 and 2 so that $u-D$ can be determined. In the present case, it was found that the optimum buildup period for mine 13 was in region 3, whereas the optimum buildup period for mine 18 fell in region 2. The results are shown in Table II and are plotted in Fig. 5. It can be seen that, for mine 18, the optimum production level under constrained buildup conditions is considerably lower than that under unconstrained optimum conditions. This sensitivity is related to the high profitability[6] of mine 18.

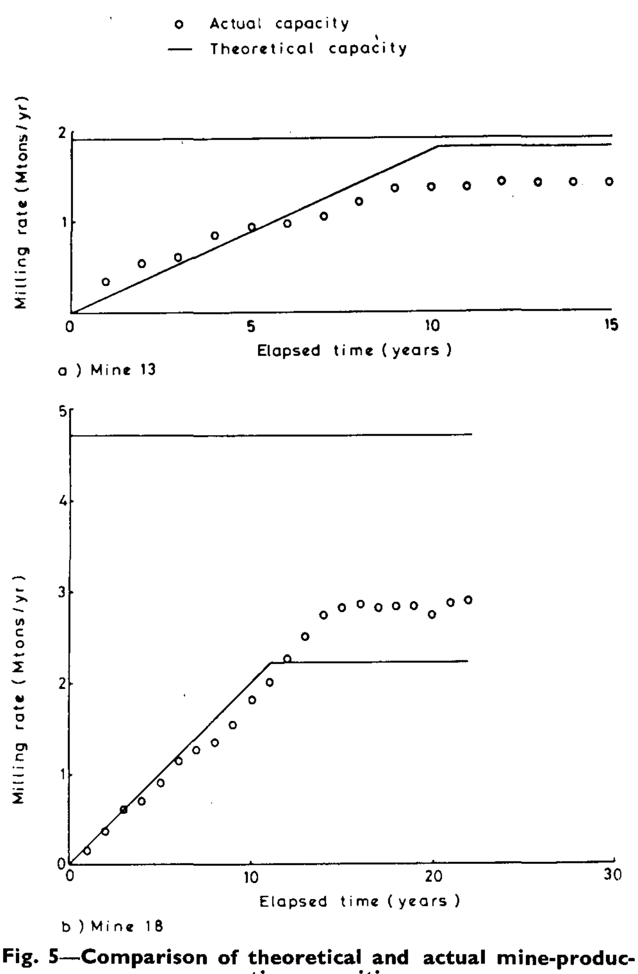

Fig. 5—Comparison of theoretical and actual mine-production capacities

Conclusions

The mathematical model formulated by use of the capital–capacity model developed earlier[5] can be used for the identification of the critical relationships that have to be satisfied for an optimum mine life, and for the determination of the bounds on the optimum mine life that provides a maximum net present value. The mine capacities yielding the maximum present value ratio, net present value, and wealth growth rate increase when the net present value is positive, but intersect at the point of maximum internal rate of return. The net present value appears to be the most suitable elementary objective.

The cash-flow model proposed allows an arbitrary buildup of production, but a rapid buildup to a constant level is economically the best. The optimum steady-state operating life of a mine (before tax) is independent of the detailed shape of the buildup profile.

The optimum buildup period is shown to fall in one of three regions, and the model is sufficiently realistic to be used in the choice of optimum production capacity of a mine.

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TABLE II

OPTIMUM PRODUCTION LEVELS FOR TWO MINES

Mine no.	Tax fraction f	Buildup rate m	Solution region	Tax-free period t^*-D	Optimum buildup period $u-D$	Optimum production rate y^*
13	0,588	∞	3	5,7	0	1,9
		0,18	3	10,6	9,9	1,8
18	0,635	∞	3	1,9	0	4,7
		0,20	2	5,7	11,2	2,2

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Conference on tin

A Fifth World Conference on Tin, organized jointly by the International Tin Council and the Ministry of Primary Industries of the Government of Malaysia, is to be held in Kuala Lumpur from 19th to 23rd October, 1981.

Little was said in the four previous conferences on tin about the financial and economic aspects of the tin-producing and tin-consuming industries. The broad aim of the Fifth World Conference on Tin is to review new production resources and consumption potential, not simply in the light of the technological means required for their exploitation or development, but also from the financial and economic viewpoints, with particular emphasis being placed on the investment needed to realize that potential.

The provisional list of papers to be presented has been prepared with the intention of covering the medium-term outlook both of the tin-producing and the tin-consuming industries.

On the production side, an examination is to be made of new resource discoveries of significant interest that are expected to come on stream over the next five to ten years, the new technology required to exploit them, and the role of current research into the improvement of production and recovery techniques.

Consumption potential will be appraised, not only from the viewpoint of the traditional tin-using industries in the industrialized countries, but also from that of the developing tin-consuming industries of Third World countries. The question of substitution will be assessed, as well as the recycling of packaging materials; research into new areas of tin consumption will also be considered.

A further Section of the Conference will look at aspects of the marketing of tin and its outlook, seen from a number of different viewpoints.

The final Section will review the question of investment and the cost of its financing both in the production and in tin consumption, not only in the broader context of the tin industry as a whole, but also drawing on specific project examples designed to illustrate, as appropriate, the pattern of possible future developments within a given geographical area. The Conference will conclude with a Panel on Investment, selected from appropriate participants, to answer questions on investment raised during the course of the discussions.

Further information can be obtained from either of the following organizers:

The Secretary,
The International Tin Council,
Haymarket House,
1 Oxendon Street,
London SW1Y 4EQ,
England.
Tel: 01–930 0321

The Secretary,
Steering Committee for the Fifth World Conference on Tin, Ministry of Primary Industries,
Gurney Road,
Kuala Lumpur, Malaysia.
Tel: 986133

Extraction metallurgy

An international symposium on extraction metallurgy, organized by the Institution of Mining and Metallurgy in cooperation with Gesellschaft Deutscher Metallhütten- und Bergleute, Benelux Métallurgie, and the Société Française de Métallurgie, will be held in London from 21st to 23rd September, 1981.

Papers will be presented to show developments in the technology of extracting various ferrous and non-ferrous metals to reduce production costs by

- Lower investment and operating costs for plant and equipment
- Energy cost savings
- Improved control and modelling for design
- Treating new feedstocks
- Environmental protection
- New process routes

All enquiries should be directed to the Meetings Secretary, The Institution of Mining and Metallurgy, 44 Portland Place, London W1N 4BR, England.