A note on rapid determinations of present values in the presence of growth and inflation

by O. L. PAPENDORF*, Pr. Eng., D.Sc. (Eng.)

SYNOPSIS

The method presented in this paper can be used, instead of lengthy cash-flow determinations, for the calculation of present values in the face of physical growth and inflation.

SAMEVATTING

Die metode wat in hierdie referaat behandel word, kan in plaas van omslagtige kontant-vloeibepalings gebruik word vir die berekening van huidige waardes met die oog op fisiese groei en inflasie.

Introduction

Lengthy cash flows have contemptuously, perhaps unfairly so, been described as sterile attempts at vindicating exceptionally vague conceptions of value by a mere manipulation of strings of figures – just a poor show of false proficiency in attempts at impressing more enlightened people. As an example, the doughty mining engineer doing his utmost to prove a case in a court of law may profit by a rapid and straightforward method of determining present values for a large number of variables 'thrown' at him by opposing counsel. This is extremely important since the ability to argue convincingly in the face of determined onslaughts is much more important than a mere attempt to defend rigidly presented schedules of calculations. The same applies where the mining engineer is called upon for an expert opinion by the board of directors of a company or by a firm of solicitors on behalf of their clients.

TABLE I

CALCULATED PRESENT VALUES

Item		n_1	n_2	n_3	g_1	g_2	g_3	r_1	r_2	r_3	i	k $\times 10^4$	d	r_0	PV $\times 10^4$	Cum PV $\times 10^4$
Income		7	5	8	20	21	14	16	13	10	15	42	1	15	5531,891	5531,891
Costs	(i) Salaries and wages	6	10	4	6	5	4	14	10	6	15	20	1	15	490,819	490,819
	(ii) Stores	5	10	5	4	6	5	17	15	10	15	30	1	17	1071,247	1562,066
	(iii) Replacements	10	6	5	4	4	4	20	10	10	15	25	1	25	996,319	2558,385
	(iv) Power and fuel	6	10	4	10	8	8	14	12	10	15	15	1	14	577,765	3136,150
	(v) General charges	10	5	5	6	5	4	14	10	6	15	10	1	15	273,376	3409,525
	(vi) Administration	10	5	5	6	5	4	12	8	6	15	6	1	15	137,050	3546,575
Working profit																R1985,316

Flexibility of the Computer

The advent of powerful minicomputers – the pocket-size alpha numerical scientific calculators with more than two thousand bytes – for the first time satisfies the practising mining engineer's ardent wish for an effective yet perfectly simple tool that he can use to illustrate the effects of changes in physical production, costs, and income, etc. on the value of a proposition. It affords him an opportunity of demonstrating his skills – that is, of 'weighing up' a position as seen from his angle, which is after all his main function in such a situation. In the example given in Table I, for instance, the present value of one of the cost items can be determined in less than three-quarters of a minute. Provided then that he 'keeps his cool' in the witness box or boardroom, he should be capable of presenting his case effectively. This he cannot hope to achieve only with sets of prepared schedules.

The Value of Mineral Rights

Well-presented cash flows find their rightful place in fully fledged feasibility studies, and the discounting procedures that accompany these are correct when used by accountants, and industrial and mining engineers. In mineral-valuation procedure, however, the issue at stake is the value of mineral rights, which is invariably reflected in comparable sales, that is, in the operations of the open market. The issue is not merely the net present value of a cash flow as is so often thought; that is only part of the issue. The question may then well be asked 'Why perform cash flows at all when market values invariably apply?'. The answer to this seeming paradox lies in the ultimate and acid test of court procedures, where only an apparent 'weight' attaches to calculated values[1]. This is the case not only in South Africa but overseas as well. The all-important 'weight' attaching to the engineer's calculated value assists the other party in arriving at an equitable market value of the rights at stake.

As an illustration of this point, the case of L. Loubser versus the South African Railways and Harbours concerning clay rights on the farm Babsfontein, north of Johannesburg, can be cited. Judge A. S. Botha remarked that discounting cash flow techniques amounted to 'little more than playing with figures'. *The Financial Mail*, in referring to this case in its issue of 12th November, 1976, had the follwing to say: 'DCF valuing appeals to mathematicians and company accountants.

* Practising Mining Consultant, P.O. Box 1497, Krugersdorp 1740, Transvaal.

It is rigorous, elegant and accurate within the assumptions made about future cash flows. The trouble is, it all hangs on those assumptions. If they are out, so is the DCF'.

These remarks are equally applicable to the case where an expert opinion is sought by business men or a firm of solicitors.

In summary, then, if an engineer giving evidence or advice is to construct his valuation within the tried and accepted framework of the rules of law and legal decisions (which means in effect that he may be 'bombarded' at will under cross-examination or by, say, a board of directors) and his opinion is to be considered solely as a 'weighting' factor, then he would act wisely in this dilemma by using a tool such as that presented by equation (2). This is flexible enough to permit the input variables to be changed rapidly during an argument on the pros and cons of the different effects, say, of cost structures, escalation rates, spans, investment rates, etc. – something quite impossible with pre-constructed cash flow schedules.

Development of a Formula

If the first year's total cost or income, k (i.e., tonnage $\times$ unit cost, or income per ton in the case of a mine), increases at the rate g per cent per annum, the total amount will be

at the end of the first year	k
at the end of the second year	$k(1+g)$
at the end of the third year	$k(1+g)^2$
at the end of the nth year	$k(1+g)^{n-1}$

This growth is related to the tonnage increase and not inflation.

If the inflation rate per annum is r per cent, the above amounts will be

at the end of the first year	$k(1+r)$
at the end of the second year	$k(1+g)(1+r)^2$
at the end of the third year	$k(1+g)^2(1+r)^3$
at the end of the nth year	$k(1+g)^{n-1}(1+r)^n$.

Discounting this cash flow at an interest rate i per annum, the present value

of the first year's cash flow $= \dfrac{k(1+r)}{(1+i)}$

of the second year's cash flow $= \dfrac{k(1+g)(1+r)^2}{(1+i)^2}$

and of the nth year's cash flow $= \dfrac{k(1+g)^{n-1}(1+r)^n}{(1+i)^n}$

The term

$$\frac{(1+g)(1+r)}{(1+i)}$$

can be called the *price-growth relative*.

If $\dfrac{(1+g)(1+r)}{(1+i)} = 1+\lambda$,

then the present value

$$PV = \frac{k(1+r)}{(1+i)} + \frac{k(1+g)(1+r)^2}{(1+i)^2} + \cdot \frac{k(1+g)^{n-1}(1+r)^n}{(1+i)^n}$$

$$= k\left[\frac{(1+\lambda)}{(1+g)} + \frac{(1+\lambda)^2}{(1+g)} + \frac{(1+\lambda)^3}{(1+g)} + \cdot \frac{(1+\lambda)^n}{(1+g)}\right]$$

$$= \frac{k(1+\lambda)}{(1+g)} \times \frac{1-(+\lambda)^n}{1-(1+\lambda)}$$

$$= \frac{k(1+\lambda)\{1-(1+\lambda)^n\}}{-\lambda(1+g)}$$

If $\dfrac{1-(1+\lambda)^n}{-\lambda} = \sigma$, the general equation becomes

$$PV = \frac{k(1+\lambda)\sigma}{(1+g)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In the limiting case, when $(1+\lambda)^n = 1$, $\sigma = n$.

A case can now be presented of three spans where n_1 equals the number of periods in the first span, n_2 the number in the second, and n_3 the number in the third. The three spans will then equal the total life N of the investment:

$$PV = \frac{k(1+\lambda_1)\sigma_1}{(1+g_1)} + \frac{k(1+\lambda_1)^{n1}}{(1+g_1)} \left[\left\{(1+\lambda_2)\sigma_2 + (1+\lambda_2)^{n2}(1+\lambda_3)\sigma_3\right\}\right]$$

For a deferred period d at an inflation rate r_0 and simplifying further,

$$PV = \frac{k(1+r_0)^d}{(1+i)^d(1+g_1)}$$
$$\left[(1+\lambda_1)\sigma_1 + (1+\lambda_1)^{b1}\left\{(1+\lambda_2)\sigma_2 + (1+\lambda_2)^{b2}(1+\lambda_3)\sigma_3\right\}\right] n \quad (2)$$

In the limiting cases, where $(1+\lambda_{1,2,3})^{n1,2,3} = 1$, $\sigma_{1,2,3} = n_{1,2,3}$.

Application of the Technique

Fig. 1 illustrates the concept. A span n of six periods can be represented by four items, the first one of which is, say, a capital item or items with a present value $PV1$; the second and third may be income and cost items having present values $PV2$ and $PV3$ respectively, and the fourth could be tax deductions with a present value $PV4$. The sum of these present values with their proper signs attached will then be the Net Present Value of the whole cash flow. The same result is obtained if the net cash flows $CF1$ to $CF6$ are discounted in the traditional way.

The advantage in the use of expression (2) for the determination of $PV2$ and $PV3$ lies in the opportunities afforded for the analysis of the effects that the different income and cost items have on the final outcome, namely the Net Present Value. The items represented by $PV1$ (capital) and $PV4$ (taxation) will, in most cases, have to be calculated separately since annual capital commitments are invariably discrete and thus not amenable to this type of 'continuous' formula. Since taxation is a function of capital allowed for redemption purposes, it

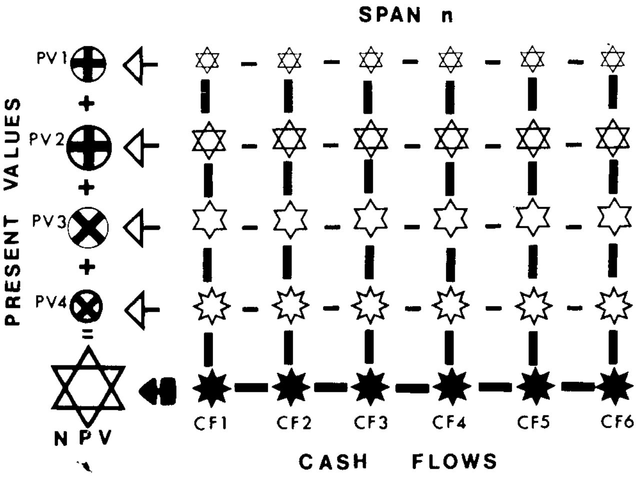

Fig. 1—Diagrammatic illustration of a cash flow

has to be determined in the same way. However, this variation from the general approach advocated here will be found to be slight since the cash flows in respect of these items are usually of short duration compared with the rest, which, in any case, constitute the main subject of argument.

A Program for a Minicomputer

Since a minicomputer must be employed – and computer inputs tend to be erroneous in the tense atmosphere resulting from argument — inputs must be unambiguous and clearly prompted by the machine. For this purpose, the Hewlett Packard HP 41C seems to meet the necessary requirements. However, there are others on the market that will do equally as well.

It is assumed that one of the cost items has the following parameters:
$n_1 = 6$, $n_2 = 10$, $n_3 = 4$, $g_1 = 6\%$ p.a., $g_2 = 5\%$ p.a., $g_3 = 4\%$ p.a., $r_1 = 14\%$ p.a., $r_2 = 10\%$ p.a., $r_3 = 6\%$ p.a., $i = 15\%$ p.a , $k = 20$ (being in R0 000), $d = 1$ year, and $r_0 = 14\%$ p.a. Then the procedure is as follows. The program, which appears in Table II, can be fed onto two magnetic cards for storage. After it has been loaded into the computer, the prompts will appear in the same order in which the parameters are presented above. The operator presses R/S after each entry, when the next prompt will appear in the display. The answer is the Present Value. If another run is to be made on a second item, the operator presses A, followed by the new set of parameters as before, and the second answer will be displayed. The computer will add this to the first one and so on for any number of determinations. When the operator depresses R/S after the answer, and before he depresses A, the cumulative total will appear in the display.

TABLE II

PROGRAM FOR HEWLETT 41C COMPUTER

01 LBL "Modpy"	53 /	105 +	157 STO 16
02†0	54 1	106 RCL 12	158 GTO 23
03 STO 19	55 +	107 1	159 LBL 24
04†LBL A	56 STO 08	108 –	160 RCL 02
05 FIX 3	57 "I=?"	109 CHS	161 STO 17
06 "N1=?"	58 PROMPT	110 /	162 GTO 25
07 PROMPT	59 100	111 STO 15	163†LBL 25
08 STO 00	60 /	112 XEQ 21	164 RCL 17
09 "N2=?"	61 1	113†LBL 21	165 RCL 14
10 PROMPT	62 +	114 RCL 13	166 *
11 STO 01	63 STO 09	115 1	167 RCL 13
12 "N3=?"	64 "K+?"	116 X( )Y	168 ENTER*
13 PROMPT	65 PROMPT	117 X=Y	169 RCL 10
14 STO 02	66 STO 10	118 XEQ 22	170 Y↑X
15 "G1=?"	67 "D=?"	119 ENTER↑	171 *
16 PROMPT	68 PROMPT	120 RCL 01	172 RCL 16
17 100	69 STO 11	121 Y↑X	173 RCL 13
18 /	70 'R0=?"	122 CHS	174 *
19 1	71 PROMPT	123 1	175 +
20 +	72 100	124 +	176 RCL 12
21 STO 03	73 /	125 RCL 13	177 Enter↑
22 "G2=?"	74 1	126 1	178 RCL 00
23 PROMPT	75 +	127 –	179 Y↑X
24 100	76 STO 26	128 CHS	180 *
25 /	77 RCL 03	129 /	181 RCL 15
26 1	78 RCL 06	130 STO 16	182 RCL 12
27 +	79 *	131 XEQ 23	183 *
28 STO 04	80 RCL 09	132†LBL 23	184 +
29 "G3=?"	81 /	133 RCL 14	185 RCL 10
30 PROMPT	82 STO 12	134 1	186 *
31 100	83 RCL 04	135 X( )Y	187 RCL 03
32 /	84 RCL 07	136 X=Y?	188 /
33 1	85 *	137 XEQ 24	189 RCL 09
34 +	86 RCL 09	138 ENTER↑	190 ENTER↑
35 STO 05	87 /	139 RCL 02	191 RCL 11
36 "R1=?"	88 STO 13	140 Y↑X	192 Y↑X
37 PROMPT	89 RCL 05	141 CHS	193 /
38 100	90 RCL 08	142 1	194 RCL 26
39 /	91 *	143 +	195 ENTER↑
40 1	92 RCL 09	144 RCL 14	196 RCL 11
41 +	93 /	145 1	197 Y↑X
42 STO 06	94 STO 14	146 –	198 *
43 "R2=?"	95 RCL 12	147 CHS	199 STO 18
44 PROMPT	96 1	148 /	200 ARCL X
45 100	97 X( )Y	149 STO 17	201 VIEW 18
46 /	98 X=Y?	150 XEQ 25	202 STOP
47 1	99 XEQ 20	151†LBL 20	203 RCL 18
48 +	100 ENTER↑	152 RCL 00	204 S† + 19
49 STO 07	101 RCL 00	153 STO 15	205 VIEW 19
50 "R3=?"	102 Y↑X	154 GTO 21	206 STOP
51 PROMPT	103 CHS	155†LBL 22	207 GTO A
52 100	104 1	156 RCL 01	208 END

Conclusion

The method presented can be of use to the practising mining engineer whose expert opinion is required by legal and business men in cases concerning mineral valuation. (A discussion of the concept *mineral value* is outside the scope of this note.).

References

1. SANDO, L., KUEHNLE J. G., and KUEBEL, O. F. *Condemnation appraisal practice*. Chicago, Amercian Institute of Real Estate Appraisers, 3rd edition, 1967.