

The prediction of air temperatures in intake haulages in mines

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SYNOPSIS

This paper reviews the theoretical methods used in South Africa for the prediction of heat pick-up and air-temperature gradients along main haulages. These methods include those of Starfield, Ramsden, McPherson, and Goch and Patterson. The differences in predictions based on typical airway conditions are explained as being due to differences in the mathematical models on which the predictions are based. The largest discrepancies were found for dry airways and for airways with bulk air coolers.

The errors in predictions of the effect of machines and coolers that have been running for only a few months are calculated, and the limitations of various simplified methods are discussed.

SAMEVATTING

Hierdie referaat gee 'n oorsig oor die teoretiese metodes wat in Suid-Afrika gebruik word vir die voorspelling van hitteopname en lugtemperatuurgradiënte langs hoofvervoerweë. Hierdie metodes sluit dié van Starfield, Ramsden, McPherson, en Goch en Patterson in. Die verskille tussen voorspellings wat op tipiese luggangtoestande gebaseer is, word verklaar as synde toe te skryf aan verskille in die wiskundige modelle waarop die voorspellings gebaseer word. Die grootste verskille is gevind in die geval van droë luggange en luggange met massalugverkoelers.

Die foute in voorspellings van die uitwerking van masjiene en verkoelers wat net 'n paar maande lank geloop het, word bereken en die beperkings van verskeie vereenvoudigde metodes word bespreek.

Introduction

This paper reviews theoretical methods for the prediction of heat pick-up in main haulages, emphasis being given to the methods currently in use or under consideration in South Africa. These include a new interactive program prepared by McPherson¹. The heat-flow predictions of the various methods are compared.

Main intake airways are generally older than two years, and they have cooled sufficiently for the heat transfer per metre length of airway to have been reduced to values that are low compared with those in production zones. However, there could be over 100 km of tunnels in an established gold mine for which a typical² heat output would be 16 MW.

The magnitude of the heat load is not the only reason for requiring that the methods of prediction should be accurate. Many of the theoretical assumptions are also used in the production areas, where experimental verification is difficult because of the additional sources of heat such as machinery. Rock is the biggest source of heat in gold mines, and it is therefore important that the methods of prediction should be accurate.

Nomenclature

The symbols that are most commonly used in this paper are listed below. Other symbols are defined as required.

- θ_a = dry-bulb temperature of the air (°C)
- θ_w = wet-bulb temperature of the air (°C)
- θ_s = temperature of the rock surface (°C)
- θ_{sd} = temperature of the dry portion of the rock surface (°C)
- θ_{sw} = temperature of the wet portion of the rock surface (°C)
- θ_v = virgin-rock temperature as used in equations (°C)
- VRT = virgin-rock temperature as used in tables (°C)
- h = surface-heat transfer coefficient ($W/m^2 \cdot ^\circ C$).

Heat Transfer in Airways

The types of heat transfer associated with airways are shown in Fig. 1. The assumed conditions, i.e. a damp footwall but dry sidewalls and hangingwall, result in a variation in temperature around the perimeter. A net heat transfer by radiation to the footwall from the dry surfaces will occur. There will be heat transfer by convection from the dry surfaces to the air, and from the air to the damp footwall.

The heat flux, i.e. the rate at which heat is transferred across unit area at the air-rock interface, is given³ by

$$\text{Flux} = h(\theta_s - \theta_a) + fLE[p_s - p] + K(\theta_s - \bar{\theta}), \dots \quad (1)$$

where θ = temperature of rock (°C)

θ_a = dry-bulb temperature of air (°C)

θ_s = surface temperature of rock (°C)

r = distance from centre of airway (m)

h = surface heat-transfer coefficient, being the

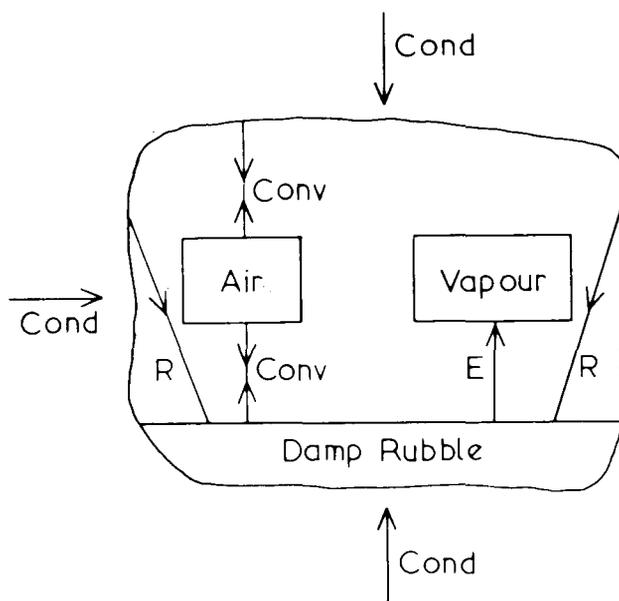


Fig. 1—Heat transfer in a mine airway
Cond = Conduction **E** = Evaporation
Conv = Convection **R** = Radiation

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- sum of the convective and radiative transfer coefficients ($W/m^2 \cdot ^\circ C$)
- f = wetness factor, which varies from zero for a perfectly dry footwall to unity for a thoroughly wet footwall, and therefore allows for a damp footwall that has no free water on it
- L = latent heat of evaporation of water (J/kg).
- E = coefficient of mass transfer ($kg/m^2 \cdot s \cdot kPa$)
- p_s = saturated vapour pressure at temperature θ_{sw} (kPa)
- p = partial pressure of water vapour in the air (kPa)
- K = overall transfer coefficient for radiation from hangingwall and sidewalls to footwall; this coefficient is discussed further below equation (11) ($W/m^2 \cdot ^\circ C$).
- $\bar{\theta}$ = average temperature of surface to which θ_s is radiating; if the unit area under consideration is in the hangingwalls or sidewalls, then $\bar{\theta}$ is the average surface temperature of the footwall and θ_s equals θ_{sd} ; if the unit area is in the footwall, $\bar{\theta}$ is the average surface temperature of the hangingwalls and sidewalls ($^\circ C$).

With the typical approximately square cross-section of mine airways, the shape of the airway ceases to have any appreciable effect after about two diameters from the centre of the airway⁴. The isotherms in the rock are then approximately circular, and radial flow occurs. Close to the airway and near the centre of the approximately plane surfaces that form the perimeter, the flow is linear rather than radial. This affects the interpretation of temperature measurements in the rock⁵, but the error in heat pick-up is small⁴ if radial flow is assumed.

Calculations are given in the Addendum for the heat flux with linear and radial flow using finite and infinite heat-transfer coefficients. Linear flow gives increasingly smaller fluxes than radial flow, decreasing to 12 per cent after ten years.

A simple explanation for this difference is that the volume of rock from which heat flows to a given rectangular area on the rock surface is a segment of a wedge with radial flow, as opposed to the parallel-sided slab that is the case with linear flow. As the rock cools, the volume with radial flow exceeds that with linear flow by increasing amounts.

The Addendum also shows that, if air temperatures are constant, a varying heat-transfer coefficient has a minimal effect after a few months with radial flow.

Comparison of Theoretical Methods

Methods That Include Radiation

Wiles^{6,7} developed the most realistic physical model of the heat-transfer processes occurring in airways. Since the calculations were made before computing facilities were adequate, he was obliged to make a few rather arbitrary assumptions in solving his equations³, and the method is no longer in use.

The most realistic model of those currently in use is that of Starfield and Dickson³, which is based on the work of Wiles. The heat flux at the rock surface is given

by equation (1), and θ_s is calculated at intervals around the airway perimeter. Each run takes 50 minutes on the IBM 360 Model 30, and a rapid method⁸ was therefore developed. In this method, wet-bulb and dry-bulb gradients were obtained by linear interpolation between previously calculated graphs in which wet-bulb and dry-bulb gradients were plotted for different airway conditions against the wet-bulb temperature of the air from different values of the wet-bulb depression (i.e. $\theta_d - \theta_w$).

The interpolation was justified by the linear relations found between both the wet-bulb and the dry-bulb gradients and the wet-bulb depression, virgin-rock temperature, thermal conductivity, and the reciprocals of air quantity and airway perimeter. The insensitivity of the wet-bulb gradient to footwall wetness also simplified the calculations.

The graphs of wet-bulb gradient against wet-bulb temperature were approximated as parabolae. The graphs of dry-bulb gradient against wet-bulb temperature were approximated as straight lines, apart from the case of dry airways in which the dry-bulb gradient was calculated from the wet-bulb gradient on the assumption of constant absolute humidity along the airway.

The procedure may have introduced errors in the predictions as will be seen later.

Models That Ignore Radiation

The other models^{1,9} discussed in this paper make a number of assumptions that are often unjustified but that simplify the calculation considerably. These assumptions are as follows.

- (i) An airway with a footwall that is damp with, for example, a wetness factor of 0.2 has the same heat pick-up as one with 20 per cent of the footwall completely wet and the rest completely dry.
- (ii) The net radiation from the dry sidewalls and hangingwall to the wet portion of the footwall can be ignored.
- (iii) The surface temperature, θ_{sw} , of the wet portion of the footwall is uniform, and the surface temperature, θ_{sd} , of the rest of the perimeter, which is dry, is also uniform.
- (iv) There is no flow of heat across the radial boundaries between the rock below the wet and dry portions of the perimeter. The flow is then one-dimensional, whereas Starfield and Dickson³ assumed two-dimensional flow, which is more realistic.

The errors caused by these assumptions will be discussed later.

According to these assumptions, the boundary condition for the dry portion simplified from equation (1) to

$$\text{Flux} = h(\theta_{sd} - \theta_d) \dots \dots \dots (2)$$

Solutions at the rock surface with this boundary condition and $\theta = \theta_v$, the virgin-rock temperature, at time $t = 0$ are given^{10,11} in the form

$$\frac{\theta_{sd} - \theta_d}{\theta_v - \theta_d} = \phi(\beta, \tau) \dots \dots \dots (3)$$

where β = Biot number = ah/k , where a is the radius of the airway (m) and k the thermal conductivity of the rock ($W/m \cdot ^\circ C$)

$\tau = \text{Fourier number} = at/2$, where a is diffusivity

$\phi(\beta, \tau)$ is a tabulated integral.

The most extensive tabulation of $\phi(\beta, \tau)$ is by Jaeger and Chamalaun¹⁰, and Starfield¹¹ also gives values. There is a useful subroutine in the McPherson program¹ written by K. Gibson, which was obtained by the fitting of curves to tabulated values. When the present author compared predictions by this subroutine with tables given by Jaeger and Chamalaun over a range of τ from 0,1 to 500 (which, for Rand quartzite, corresponds to 35 hours to 20 years) and of β from 0,1 to 15, where a 10 m² haulage with 45 m³/s airflow would have a β of 6, the results agreed within 2 per cent.

The heat flux in W/m² at the dry surface then becomes $h(\theta_{sd} - \theta_d) = h(\theta_v - \theta_d)\phi(\beta, \tau)$ (4)

Models That Ignore Radiation between Dry and Wet Surfaces

McPherson's and Ramsden's methods differ in the determination of θ_{sw} . McPherson's method will be considered first.

The boundary condition at the wet surface from equation (1) without radiation and with $f = 1$ is

Flux = $h(\theta_{sw} - \theta_d) + LE[p_s - p]$ (5)

If a fictitious air temperature, θ'_d , is introduced, where

$\theta'_d = \theta_d - \frac{LE}{h} [p_s - p]$, (6)

then equation (5) can be rewritten as

Flux = $h(\theta_{sw} - \theta'_d)$ (7)

This allows the tabulated integral $\phi(\beta, \tau)$ in equation (3) to be used.

This fictitious air temperature is used by several authors^{1,4,12} in McPherson's program, θ_{sw} and θ'_d are adjusted by an iterative technique until the total heat flux from the wet portion given by equation (7) equals the sum of the convective and evaporative fluxes given by equation (5); θ_{sw} is then known, and the air temperature gradients can be predicted¹.

Ramsden⁹ obtained θ_{sw} by an alternative method. In this method, the footwall is still taken as either completely wet or dry, but it is assumed that

$\theta_{sd} - \theta_{sw} = \theta_d - \theta_w$ (8)

This assumption is only approximately true, as will be discussed, but allows simplified equations to be produced for air-temperature gradients and heat pick-up. *Formulae for Surface-heat Transfer Coefficient*

McPherson's method differs from that of Starfield in the way in which surface-heat transfer coefficients (h) are determined. McPherson uses an equation based on underground measurements:

$h = \frac{3540}{3,6}$ (friction factor) (air velocity). . . (9)

Starfield uses a more generally accepted relationship for the convective component, h_{con} , obtained from laboratory measurements on pipes:

$h_{con} = 0,023 F \frac{k_a}{D} (R_e)^{0,8} (P_r)^{0,4}$, (10)

where F is a factor based on the roughness of the airway k_a is the conductivity of air at bulk temperature (W/m·°C)

D is the hydraulic diameter of the airway (m)

R_e is the Reynold number

P_r is the Prandtl number.

Equation (9) gives appreciably higher values than equation (10). For example, in an airway with an air velocity of 2,54 m/s, an airflow of 23,5 m³/s, and a friction factor of 0,010 kg/m³, Starfield and Dickson³ quote a convective component of h of 10,62 W/m²·°C and a total h of 13,12 W/m²·°C when the radiative component was added. From McPherson's formula, the value would be 24,98 W/m²·°C.

Experimental measurements by Hemp¹³ suggest that equation (10) may overestimate the convective component of h , although no great accuracy was claimed for the experimental data. Experimental measurements by Vost¹⁴ gave good agreement with equation (10).

Ramsden followed Starfield's procedure.

Variation in Temperature of Inlet Air

All the prediction methods known to be in use in South Africa assume that the temperature of the inlet air is constant or slowly varying. This simplifies computation considerably since the effects of previous changes in air temperature do not have to be computed.

In practice, the temperatures of the inlet air in main haulages can vary significantly in a relatively short time, unless bulk air cooling is practised. The main reason is a change in atmospheric conditions on the surface. The variation in air temperatures can have large effects on heat flow¹⁵. This makes it difficult to verify the theoretical methods by underground measurement. Comparison should be made only when weather conditions on the surface have been stable for several days. Other causes of rapid variation in air temperature, such as the watering-down of shaft stations, should be discontinued before and during the measurements.

Standard Airways for Which Predictions Were Made

To allow comparison between the different methods, predictions were made of air temperatures for a typical condition in a main haulage as summarized in Table I.

The inlet-air temperatures most often used were wet-bulb and dry-bulb temperatures of 20/20°C for air emerging from an underground bulk air cooler and 29/37°C for air from a wet shaft that was not subjected to bulk air cooling. A few calculations were made with inlet-air temperatures of 28/42°C from a dry shaft, and 24/24°C as an alternative higher temperature for air from a bulk air cooler.

The friction factor used by Starfield was 0,010 kg/m³.

TABLE I
AIRWAY CONDITIONS FOR CALCULATIONS

Cross-sectional area	= 10,0 m ²
Perimeter	= 12,65 m
Air quantity	= 45 m ³ /s
Barometric pressure	= 100 kPa
Conductivity of rock	= 5,54 W/m·°C
Specific heat of rock	= 830 J/kg·°C
Density of rock	= 2670 kg/m ³
Virgin-rock temperature	= 50°C, 60°C
Age	= 4 years

Values of 0,010 kg/m³, 0,015 kg/m³, and 0,018 kg/m³ were chosen for the McPherson program to cover the possible range in main intake airways.

The Starfield program prints out results for footwall wetnesses of 0,0 (dry), 0,2 (damp), and 1,0 (wet). The corresponding wetness factors in McPherson's program are a quarter of these since the wetness is given as a fraction of the entire airway surface. Wetness factors in the present paper are quoted according to Starfield's definition. The values of 0,2, 0,3, and 0,4 were used in damp airways with McPherson's program (i.e. values of 0,05, 0,075, and 0,10 with McPherson's definition) to find when the best agreement with Starfield was obtained.

Values of the surface-heat transfer coefficient are required later in the paper for the DRYAIR program for calculations in the section on machines and coolers and the Addendum. A value of 18,63 W/m²·°C was taken at the standard air velocity of 4,5 m/s. This agreed with the value predicted from equation (10) and also gave a Biot number of 6,0, which simplified the consultation of tables in these calculations.

The extent of the agreement between wet-bulb temperatures predicted by the different methods is the most important point for investigation, but a lack of agreement between dry-bulb temperatures and moisture contents would indicate fundamental differences in the models.

The predictions by McPherson's program and by the rapid method⁸ of Starfield are compared in the next three sections.

Predictions for Dry Airways

The accuracy of the different numerical methods can be assessed by comparison either with underground measurements or with analytical solutions. As underground airways do not generally have slowly varying inlet-air temperatures, the validation of theoretical

methods by underground measurements is difficult, as already discussed. Analytical solutions have been achieved only for dry airways¹⁶. Although such airways are not common in practice, they provide a useful test of the various methods. As Van Heerden's analytical solution¹⁶ predicts temperatures for only a few parameters, a program was written for dry airways and tested against his predictions.

In the program, the heat flow for each 50 m element of airway was calculated from equation (4) and from the sub-routine already mentioned for $\phi(\beta, \tau)$ in McPherson's program¹. The increase in dry-bulb temperature in each element was obtained from the mass flowrate and the specific heat of air. The increase in wet-bulb temperature was calculated from another sub-routine in McPherson's program. Predictions of dry-bulb temperature by the resulting program, DRYAIR, agreed well with those by Van Heerden's method¹⁶. Predictions by DRYAIR are now compared with those by the programs of Starfield and McPherson.

Hemp¹⁷ has pointed out an anomaly in Starfield's predictions for dry airways. The flowrate of heat into each unit length of airway, expressed per unit of temperature difference between virgin-rock temperature and dry-bulb temperature, should be constant in view of the linearity of the equations for heat transfer and conduction; in fact, it varies as will be shown.

Hemp did not calculate the resulting error in air temperatures. This was achieved by the DRYAIR program, as shown in Table II. The error in dry-bulb temperature in 2 km of airway using Starfield's program varied from an over-estimate of 2,6°C with bulk cooled air to an under-estimate of 1,4°C for air coming off a dry shaft. The corresponding errors in wet-bulb temperature were an over-estimate of 1,2°C with bulk cooled air and an under-estimate of 0,3°C for air coming off a dry shaft.

The DRYAIR program gave a constant value for

TABLE II
TEMPERATURES PREDICTED FOR A DRY AIRWAY

Distance along airway m	Starfield					DRYAIR				
	Wet bulb °C	Dry bulb °C	Moisture content g/kg	Heat flow		Wet bulb °C	Dry bulb °C	Moisture content g/kg	Heat flow	
				kW	kW/100 m·°C				kW	kW/100 m·°C
0	20,00	20,00	14,89			20,00	20,00	14,89		
100	20,34	20,94	14,96	61,25	2,07	20,20	20,68		37,88	1,28
900	22,53	27,38	15,41			21,66	25,59			
1000	22,75	28,06	15,46	42,70	1,92	21,82	26,14		30,82	1,28
1900	24,30	33,12	15,75			23,08	30,58			
2000	24,43	33,58	15,77	28,53	1,71	23,20	31,02		24,52	1,28
0	29,00	37,00	22,49			29,00	37,00	22,49		
100	29,08	37,33	22,52	21,45	1,67	29,07	37,29		16,41	1,28
900	29,64	39,53	22,68			29,53	39,39			
1000	29,69	39,76	22,69	14,90	1,44	29,58	39,63		13,40	1,28
1900	30,11	41,44	22,82			29,99	41,53			
2000	30,14	41,59	22,83	9,88	1,16	30,03	41,72		10,68	1,28
0	28,00	42,00	18,43			28,00	42,00	18,43		
100	28,03	42,11	18,44	7,12	0,90	28,07	42,18		10,10	1,28
900	29,22	42,87	18,48			28,36	43,45			
1000	28,24	42,94	18,48	4,88	0,68	28,39	43,63		8,23	1,28
1900	28,38	43,51	18,51			28,66	44,81			
2000	28,40	43,56	18,52	3,20	0,50	28,69	44,92		6,56	1,28

heat pick-up in kW/100 m·°C, whereas Starfield's values decreased along the airway. A consistent rise in moisture content was predicted by the Starfield program, which should not occur in a dry airway. This rise was greatest when there was bulk air cooling.

These errors were presumably caused by the method of interpolation discussed earlier. The dry-bulb gradient in dry airways was obtained less directly than other gradients, and is likely to be the least accurate part of the rapid program. It was calculated from the wet-bulb gradient for dry airways on the assumption that the absolute humidity must remain constant. The wet-bulb gradient for dry airways was assumed to be equal to that for damp airways.

The McPherson program gave much better agreement with DRYAIR.

The simplifying assumptions made by McPherson, which were discussed earlier, are not necessarily valid for damp and wet airways, but are valid for dry airways. Examples are the omission of radiation and the assumption of one-dimensional flow. The two programs were therefore based on a similar theory.

Prediction for Airways without Bulk Air Coolers

In this section, predictions by the programs of McPherson and Starfield are compared. The effect of McPherson's simplification of the mathematical model of heat transfer in airways is demonstrated.

Wet Footwall

For a wet footwall with inlet-air temperatures of 29/37°C, Table III shows that the dry-bulb temperatures predicted by McPherson decreased initially much more rapidly than Starfield's temperatures, and remained well below them along the airway. The wet-bulb temperatures were also lower. An increase in friction factor caused a very small increase in the wet-bulb temperature, but decreased the dry-bulb temperature significantly. The differences in temperature between the Starfield and McPherson values using the smaller friction factor are also shown in Table III. The dry-bulb temperatures

differed on average by 2°C, but the discrepancy in wet-bulb temperatures did not exceed 1°C.

The heat transfers in the first 100 m of airway are now analysed to explain these differences in predicted temperatures. One reason for the faster initial reduction in dry-bulb temperature using the McPherson program is that the heat-transfer coefficient is more than twice that of Starfield, and the net loss of heat by convection from the air is double that of Starfield. There is a large heat flow from the air to the footwall, and a much smaller flow into the air from the dry surfaces. The evaporation rate is higher by a factor of 1.7, partly because of the higher coefficient of mass transfer, which is assumed proportional to h in McPherson's program (although in fact it is proportional to the convective component of h). The net gain of heat by the air, which is the difference between the gain from evaporation and the loss by convection, is similar in both cases, and the rise in wet-bulb temperature is the same.

Heat balance at the footwall is achieved in the McPherson program without radiation. The net radiation per 100 m of airway from the dry sidewalls and hanging-wall to the wet footwall, R in kilowatts, can be calculated³ from

$$R = 0,40 ab(1 - \epsilon) h_r(\theta_{sd} - \theta_{sw}), \quad \dots \dots (11)$$

where a is the shape factor (0,3)

b is the width of the airway (3,16 m)

ϵ is the emissivity of the humid air (0,2)

h_r is the equivalent radiation surface-transfer coefficient (6,2 W/m²·°C)

(K in equation (1) equals $a(1 - \epsilon)h_r$.)

At the entrance to an airway with a virgin-rock temperature of 50°C, McPherson's program predicted a θ_{sd} of 37,30°C and a θ_{sw} of 28,93°C, which would have given a net radiation of 16 kW per 100 m.

The evaporation rate from the footwall in McPherson's case is 118 kW per 100 m, and radiation would have constituted 14 per cent of the required flow to the footwall. In Starfield's case, the evaporation rate is 71 kW

TABLE III
AIRWAY WITH WET FOOTWALL AND INLET-AIR TEMPERATURE OF 29/37°C

Virgin-rock temp °C	Program	Friction factor kg/m ³	500 m along airway			1 km along airway			2 km along airway		
			Wet bulb °C	Dry bulb °C	Moisture content g/kg	Wet bulb °C	Dry bulb °C	Moisture content g/kg	Wet bulb °C	Dry bulb °C	Moisture content g/kg
50	Starfield	0,01	29,52	34,12	24,79	30,15	33,25	26,46	31,44	33,53	29,12
	McPherson	0,01	29,44	32,20	25,4	29,90	31,36	26,6	30,76	31,80	28,3
	McPherson	0,015	29,46	31,03	25,9	29,92	30,73	27,0	30,79	31,48	28,6
Differences in temperature with friction factor of 0,01 kg/m ³			0,08	1,92		0,25	1,89		0,68	1,73	
60	Starfield	0,01	29,85	34,94	25,09	30,78	34,60	27,20	32,61	35,55	30,89
	McPherson	0,01	29,68	32,82	25,6	30,36	32,29	27,2	31,64	33,15	29,5
	McPherson	0,015	29,70	31,58	26,2	30,39	31,54	27,6	31,67	32,69	29,9
Differences in temperature with friction factor of 0,01 kg/m ³			0,17	2,12		0,42	2,31		0,97	2,40	

per 100 m, and radiation of this amount would form 23 per cent of the required flow. The effect of the omission of radiation in the McPherson program is reduced because of the high values of evaporation rate and convective flow from air to footwall caused partly by the high surface-heat transfer coefficients.

By the McPherson program, θ_{sd} 500 m along the airway is 32,61°C, and θ_{sw} is 29,48°C, which gives a radiation of 5 kW per 100 m. The evaporation rate is 39 kW per 100 m, and radiation would have added 15 per cent to the flow to the footwall.

Comparisons were also made with the less common conditions of 28/42°C for a virgin-rock temperature of 50°C and 60°C. A similar pattern was found, but the discrepancy in dry-bulb temperatures was more pronounced, with a 3°C difference at 500 m for both values of virgin-rock temperatures. The difference in wet-bulb temperature was less than for inlet air of 29/37°C, with a maximum of 0,6°C at 2 km with a virgin-rock temperature of 60°C.

Damp Footwall

Table IV shows that, for damp footwalls with inlet-

air temperatures of 29/37°C, the agreement between the dry-bulb temperatures predicted by the two methods is better than for wet footwalls. With the same wetness factor and friction factor as Starfield's, the McPherson program predicts a higher dry-bulb and a lower wet-bulb temperature. When the wetness factor is increased to 0,3 or the friction factor to 0,015 kg/m³, the dry-bulb temperature falls below Starfield's value, and better agreement is obtained between the wet-bulb temperatures. The wet-bulb temperature 2 km along the airway is 0,5°C below Starfield's value, and the dry-bulb is 1°C lower, which gives better agreement than for wet airways.

A similar pattern was found with a virgin-rock temperature of 60°C. When either the wetness factor is increased to 0,3 or the friction factor is increased to 0,015 kg/m³, the wet-bulb temperature at 2 km by the McPherson program is 0,7°C below Starfield's value and the dry-bulb temperature is 1,2°C below.

With the less common inlet conditions of 28/42°C and virgin-rock temperatures of 50 and 60°C, the agreement was better. The wet-bulb temperature was only

TABLE IV

AIRWAY WITH DAMP FOOTWALL, INLET AIR TEMPERATURES OF 29/37°C, AND VIRGIN-ROCK TEMPERATURE OF 50°C

Program	Friction factor kg/m ³	Wetness factor*	500 m along airway			1 km along airway			2 km along airway		
			Wet bulb °C	Dry bulb °C	Moisture content g/kg	Wet bulb °C	Dry bulb °C	Moisture content g/kg	Wet bulb °C	Dry bulb °C	Moisture content g/kg
Starfield	0,01	0,2	29,44	36,16	23,73	29,91	35,79	24,85	30,87	35,77	26,90
McPherson	0,010	0,2	29,32	36,55	23,3	29,65	36,31	24,1	30,31	36,22	25,5
McPherson	0,010	0,3	29,34	35,77	23,7	29,70	35,12	24,7	30,43	34,77	26,4
McPherson	0,010	0,4	29,36	35,07	24,0	29,74	34,18	25,2	30,51	33,80	26,9
McPherson	0,015	0,2	29,33	35,80	23,7	29,68	35,17	24,7	30,39	34,83	26,3
McPherson	0,015	0,3	29,36	34,79	24,1	29,75	33,83	25,3	30,51	33,51	27,1
McPherson	0,015	0,4	29,37	33,94	24,5	29,78	32,88	25,8	30,59	32,76	27,6

* In all cases, Starfield's definition was used, with which a wetness factor of 0,2 corresponds to a wetness factor of 0,05 with McPherson's definition.

TABLE V

AIRWAY WITH INLET AIR TEMPERATURE OF 20/20°C AND MOISTURE CONTENT OF 14,9 g/kg

Virgin-rock temperature °C	Footwall wetness	Program	Friction factor kg/m ³	Wetness factor*	1 km along airway			2 km along airway		
					Wet bulb °C	Dry bulb °C	Moisture content g/kg	Wet bulb °C	Dry bulb °C	Moisture content g/kg
50	Wet	Starfield	0,01	1,0	23,05	24,90	17,25	25,40	27,57	19,93
		McPherson	0,01	1,0	21,81	23,02	16,2	23,38	24,64	17,9
		McPherson	0,015	1,0	21,81	22,69	16,4	23,39	24,26	18,2
50	Damp	Starfield	0,01	0,2	22,85	26,18	16,41	24,90	29,50	18,27
		McPherson	0,01	0,2	21,73	24,89	15,3	23,15	27,88	16,2
		McPherson	0,015	0,4	21,76	23,82	15,8	23,28	25,67	17,4
60	Wet	Starfield	0,01	1,0	24,02	26,59	18,05	26,97	29,99	21,66
		McPherson	0,01	1,0	22,39	24,02	16,6	24,45	26,15	19,0
		McPherson	0,015	1,0	22,41	23,58	16,9	24,47	25,65	19,3
60	Damp	Starfield	0,01	0,2	23,88	28,32	17,10	26,46	32,55	19,63
		McPherson	0,01	0,2	22,30	26,52	15,5	24,14	30,49	16,7
		McPherson	0,015	0,4	22,35	25,09	16,1	24,32	27,53	18,2

* In all cases, Starfield's definition of wetness factor was used.

0,2°C below Starfield's value at 2 km, and the dry-bulb was 0,6°C below.

Prediction for Airways with Bulk Cooling of Air

The difference between the predicted temperature in airways with bulk air coolers is higher than in those without coolers, as shown in Table V. At 2 km with a wet footwall and a virgin-rock temperature of 50°C, the McPherson program predicts a wet-bulb temperature that is 2°C lower than Starfield's, and a dry-bulb temperature of 3°C less. A higher friction factor made a negligible difference to the wet-bulb temperature, but increased the difference in the dry-bulb temperatures.

With a damp footwall, the differences in temperature are also large. The difference in wet-bulb temperature can be reduced by up to 0,2°C by the choice of higher friction and wetness factors, but the error in dry-bulb temperature is then increased greatly. At 2 km with a virgin-rock temperature of 60°C and the same friction and wetness factors as Starfield's, the McPherson program predicts a wet-bulb temperature that is 2,3°C lower than Starfield's, and a dry-bulb temperature of 2,1°C less.

One explanation for the large difference in predicted temperatures is now considered. Equation (1) shows that evaporation depends on the difference between the saturated vapour pressure at θ_{sw} and the vapour pressure of the air. The air leaving a bulk air cooler is saturated, and the difference in these pressures could be small. For example, McPherson's program predicts a θ_{sw} of 20,20°C if the air temperatures are 20/20°C, the virgin-rock temperature is 50°C, the footwall is wet, and the friction factor is 0,01 kg/m³. The difference in saturated vapour pressure at 20,20°C and 20,00°C is only 0,03 kPa, which is 1,2 per cent. A small difference in θ_{sw} makes a very large difference in evaporation. Further along the same airway, the differences in vapour pressure are still small. At 1 km, θ_{sw} is 21,95°C, for which the saturated vapour pressure is 2,64 kPa. The vapour pressure of the air is 2,54 kPa; so, again, a small change in θ_{sw} can make a large change in evaporation.

Values of θ_{sw} predicted by McPherson and Starfield can be compared by the use of information published by Starfield and Dickson³. In a cross-section with a wet

footwall and air temperatures of 23,89/35°C and a virgin-rock temperature of 43,33°C, Starfield's program predicts a θ_{sw} of 26°C, whereas the McPherson program predicts 23,80°C with the same friction factor of 0,010 kg/m³, and 23,73°C with a friction factor of 0,015 kg/m³.

Machinery and Coolers

The McPherson program, unlike Starfield's, allows the effect of machinery and coolers on air temperatures to be calculated. These can be extended sources such as conveyors, or localized sources such as motors, transformers, and air coolers. Cold-water pipes are treated as an extended source.

Because of the difficulty of quantifying evaporation from the wet surfaces of machines and coolers, McPherson assumed that the heat transfer was convective. In the calculation of the increase in dry-bulb temperature, this convective heat is added to other sensible heat sources in each element of an airway.

The theories for the prediction of air temperatures discussed earlier assume that air temperatures vary only slowly with time at a given point. Theoretically, the heat output from a machine must therefore have remained nearly constant from the establishment of the airway. The error in prediction caused by the later installation of a machine in an airway therefore deserves consideration.

Continuous Operation

The airway is assumed to be dry with a virgin-rock temperature of 50°C, and to have had a dry-bulb air temperature at the entrance of 30°C for the first three years. A cooler is then switched on at the entrance, which reduces the air temperature to 25°C. The other parameters are as given in Table I.

The surface temperature and heat flux at the entrance without memory, i.e. on the assumption that the air temperature has always been 25°C, were calculated from equations (3) and (4) and are shown in Table VI.

Memory of the previous air temperature of 30°C can be introduced by the recognition that the equation for heat conduction is linear and homogeneous. Each change in air temperature therefore produces its own effect at later times, and this effect is independent of other

TABLE VI
HEAT FLUX WITH AND WITHOUT MEMORY OF CHANGE IN AIR TEMPERATURE

Effect of machine on air temperature	Length of time machine on	Without memory		With memory	
		θ_s °C	Heat flux W/m ²	θ_s °C	Heat flux from rock W/m ²
Lowered to 25°C	Just on	26,43	26,6	31,15	114,5
	3,7 days	26,43	26,6	27,28	42,5
	2,8 months	26,41	26,4	26,60	29,8
	7,6 months	26,39	25,9	26,49	27,7
	1,0 year	26,37	25,5	26,43	26,6
Raised to 35°C	Just on	35,86	16,0	31,15	-71,8
	3,7 days	35,86	16,0	35,01	0,2
	2,8 months	35,85	15,8	35,66	12,4
	7,6 months	35,83	15,5	35,73	13,7
	1,0 year	35,82	15,3	35,75	14,0

changes that may occur. The surface temperature is given by

$$\theta_s = \theta_{a0} + (\theta_v - \theta_{a0})\phi + (\theta_{a1} - \theta_{a0})(1 - \phi_1), \quad (12)$$

where $\theta_v = 50^\circ\text{C}$, $\theta_{a0} = 30^\circ\text{C}$, and $\theta_{a1} = 25^\circ\text{C}$.

Then, ϕ is calculated for the full age of the airway and ϕ_1 for the time that the machine has been on. The results are given in Table VI. The effect of raising the air temperature to 35°C , instead of lowering it to 25°C , is also shown.

Table VI shows that switching on the cooler causes a large increase in heat flow from the rock initially, which decreases fairly rapidly but after three months is still 13 per cent above the flow that was calculated by ignoring the previous air temperature of 30°C . If, instead, the air temperature is raised to 35°C , considerable heat flows back into the rock initially and reversal takes about 3,7 days. If the effect of the previous air temperature of 30°C is ignored, the heat flow from the rock is over-estimated by 28 per cent after 2,8 months and 14 per cent after 7,6 months. It takes a year for the fluxes to agree within 10 per cent. The changes in heat flux in a given time with memory of the change in air temperature are approximately the same in both cases.

Further along the airway, the fall of 5°C in air temperature caused by the cooler is considerably reduced initially owing to heat flow from the rock. It takes a considerable time before the air temperature approximates that given with a steady inlet-air temperature of 25°C . The air temperature can be found by calculation of the changes in air temperature along an airway that was originally at virgin-rock temperature and then subjected to a drop in air temperature of 5°C . Owing to the linearity of the equations involved, the magnitude of the changes due to the 5°C drop can be subtracted from the air temperatures produced by an inlet-air temperature of 30°C . The DRYAIR program described earlier was used for the calculations.

These calculations showed that 3,5 days after the cooler was switched on, the fall of 5°C at the entrance was reduced to $1,3^\circ\text{C}$ at 500 m and to $0,3^\circ\text{C}$ at 1 km. After 10 days, the fall in temperature at 1 km became

$2,4^\circ\text{C}$. Table VII shows the temperatures predicted for later times.

The under-prediction of air temperature if the effect of the previous air temperature of 30°C is ignored is $0,9^\circ\text{C}$ after three months, and $0,4^\circ\text{C}$ after a year.

Coolers would generally cause larger falls than 5°C , and the effect would be proportionally larger. Machines would generally cause much smaller rises than 5°C .

Intermittent Operation

Some machines are not operated continuously at constant power consumption, but are switched on and off or vary in power consumption. The theory discussed earlier^{1,8} assumes that air temperatures vary slowly with time, and cannot be used directly when machines vary in power consumption. Predictions by programs with memory of temperature changes involve long computing times unless the variation is cyclic, since the effects of all changes in power have to be found by Duhamel's theorem¹⁸. Instead, the McPherson program assumes that the average air temperature can be obtained from the average power consumption over a week. In practice, the air temperature varies as the power consumption varies, but this arbitrary assumption is a reasonable approach until full calculations are made on the effect of machinery. Starfield¹² has shown that a cooler of 1512 kW operating continuously gave virtually the same conditions at the end of a 610 m airway during shift as a 2145 kW cooler operating for 12 hours a day, i.e. of average power 1073 kW. However, Starfield used a model in which the wetness was distributed uniformly over the whole surface, which could have introduced errors in prediction.

Simplified Methods for the Prediction of Heat Flow

The programs of McPherson and Starfield involve too much computation to be conveniently included in the large programs required to predict the total cooling requirements of mines. However, simplified calculations can be produced for particular airway conditions.

Airways with Bulk Air Coolers

An example of a simplified calculation occurs in the

TABLE VII

AIR TEMPERATURES ALONG AN AIRWAY WITH AND WITHOUT MEMORY OF A CHANGE IN AIR TEMPERATURE AT ENTRANCE

Length of time machine on months	Distance along airway m	Air temp. with inlet of 30°C °C	Reduction in temp. due to 5°C fall °C	Air temp. with inlet of 25°C		Difference in temp. °C
				With memory °C	Without memory °C	
0,34	500	32,27	3,48	28,79	27,83	0,96
	1000	34,28	2,43	31,85	30,34	1,51
	2000	37,65	1,18	36,47	34,53	1,94
2,8	500	32,25	4,09	28,16	27,80	0,36
	1000	34,25	3,34	30,91	30,29	0,62
	2000	37,59	2,23	35,36	34,46	0,90
7,6	500	32,21	4,25	27,96	27,76	0,20
	1000	34,18	3,62	30,56	30,21	0,35
	2000	37,49	2,62	34,87	34,33	0,54
12,5	500	32,18	4,32	27,86	27,71	0,15
	1000	34,12	3,73	30,39	30,13	0,26
	2000	37,39	2,78	34,61	34,21	0,40

Chamber of Mines total mine program, which is based on the work of Van der Walt and Whillier¹⁹. In the first version, FRIDG4, heat from the rock in main intake airways was not taken into account specifically despite its being a significant source in many mines. The updated version, FRIDG5, incorporates a simplified calculation for main intake airways with bulk air coolers.

The airways are assumed to be dry and at least four years old. At this age, the difference between heat pick-up in dry airways with finite and infinite h is small, as shown in Table X. Use of infinite h , i.e. $\theta_s = \theta_d$, reduces the computation considerably since the heat flux as given by Goch and Patterson²⁰ is

$$\text{Flux} = \frac{k(\theta_v - \theta_d)T'}{a}, \dots \dots \dots (13)$$

where T' is a function of τ only. A simple relationship can be found between T' and τ , namely

$$T' = 0,182 \tau^{-0,182} \text{ for } 10 \leq \tau \leq 1000.$$

For the airway in Table I, the limits for τ correspond to 21 weeks to 40,5 years. If finite h were used, the evaluation of $\phi(\beta, \tau)$ in equation (4) would require a subroutine of eight lines.

The surface temperature of the rock just after the bulk air cooler is assumed to be equal to the temperature of the emerging saturated air.

The heat pick-up in the first 100 m, Q , is given by

$$Q = 200\pi kT'(\theta_v - \theta_s) \dots \dots \dots (14)$$

For every 100 m, a new value of θ_s is calculated from

$$\text{Increase in } \theta = \frac{Q}{Gc_a}, \dots \dots \dots (15)$$

where G is the mass flowrate of air (kg/s)

c_a is the specific heat of air at constant pressure, which was taken as 1,1 kJ/kg·°C.

By this simplified procedure, the values of heat pick-up at 1, 2, and 3 km along the airway were a maximum of 3 per cent less than the values given by the DRYAIR program, which uses finite h .

In practice, main intake airways are more likely to be damp than dry, which could increase the pick-up considerably. As a simple extension of equation (14), heat pick-ups for damp airways were calculated for the conditions in Table VIII by Starfield's rapid program, in which the upper allowable limits of virgin-rock temperature and volume flow are 60°C and 55 m³/s respectively. All 72 possible airway conditions were computed. The heat pick-ups from equation (14) with a virgin-rock temperature of 50 and 60°C ranged from 60 to 64 per cent of the pick-ups for damp airways. The percentage rose as the virgin-rock temperature decreased, until, at a virgin-rock temperature of 40°C, the range was 68 to 72 per cent. A regression analysis was used to find a relation between the percentages and the virgin-rock temperature. The resulting formula for heat pick-up was

$$Q = 200\pi kT'(\theta_v - \theta_s) 1,572 \left(\frac{\text{VRT}}{50}\right)^{0,32} \dots \dots (16)$$

The heat pick-ups from this equation agreed with Starfield's values within 4,5 per cent in 96 per cent of cases.

It must be emphasized that equation (16) applies only to airways with bulk air coolers.

TABLE VIII

AIRWAY CONDITIONS WITH BULK AIR COOLING	
Inlet air temperatures	20/20°C, 24/24°C.
Volume airflow	35, 45, 55 m ³ /s.
VRT	40, 45, 50, 60°C.
Distances along airway	1, 2, 3 km.

Ramsden's Formulae for Air-temperature Gradients

As already mentioned, Ramsden⁹ obtained simplified equations for air-temperature gradients by assuming that

$$\theta_{sd} - \theta_{sw} = \theta_d - \theta_w \dots \dots \dots (17)$$

This was justified⁹ by the approach of dry rock surfaces in old, dry airways to the dry-bulb temperature, and the approach of wet surfaces to the wet-bulb temperature. Calculations by Starfield³ show that, although the θ_{sd} in airways with wet footwalls that are a few years old would typically be only a few tenths of a degree different from the θ_d , the θ_{sw} could be one or two degrees above the θ_w (e.g., Figure AII of reference 3). The value of θ_{sw} given by equation (17) would then be lower than Starfield's value. The footwall would therefore receive more heat by convection from the air, although none by radiation from the dry surfaces. The saturated vapour pressure at this lower θ_{sw} would be smaller than that predicted by Starfield, which would reduce evaporation. The agreement between the predictions of Starfield and those of Ramsden for wet airways was, in fact, poor.

For damp footwalls, the θ_{sw} in practice will be higher than for wet footwalls. Equation (17) predicts the same θ_{sw} , but the assumed completely wet area is smaller than the total footwall area. For damp airways, Ramsden⁹ obtained dry-bulb gradients of a much higher value than those of Starfield⁸. This was probably due to the assumed reduction in footwall area not being compensated fully by the lower θ_{sw} , which gave lower convective flow from the air to the footwall.

To obtain better agreement between the dry-bulb gradients, Ramsden had to multiply the wetness factor by 5. However, the original wetness factor was used for the calculation of wet-bulb gradients. Good agreement was obtained between Starfield's and Ramsden's predictions for damp airways by this arbitrary assumption.

Starfield's predictions for dry airways have already been shown to be in error, and Ramsden's predictions do not agree with them. It will be shown in the next section that Ramsden's formula for heat pick-up, which is derived from his predictions of air-temperature gradients, gives reasonable agreement with DRYAIR. The error in the θ_{sw} from equation (17) does not affect the calculation for a dry airway, and the air-temperature gradients in dry airways are therefore more accurate than for damp and wet airways.

Hiramatsu and Amano²⁰ also used equation (17) to determine θ_{sw} . They predicted dry-bulb temperatures only, and found large errors when these were compared with underground measurements (25 per cent of the temperatures differed by 2°C or more).

Ramsden's method for the temperature gradients is no longer used, but a simplified equation for heat pick-up based on gradient formulae is still used.

Ramsden's Formula for Heat Pick-up

Ramsden^{2,9} has given a simplified formula for heat pick-up in airways:

$$Q = 5,57 (WF + 0,255) (\theta_v - \theta_a) (CF), \quad \dots (18)$$

$$\text{with } CF = \left[\frac{PERIM}{12} \right]^{0,437} \left[\frac{AGE}{3} \right]^{-0,147} \left[\frac{k}{5,5} \right]^{0,853},$$

where Q = heat pick-up (kW/100 m length of airway)
 WF = wetness factor calculated as a fraction of the total area of the airway
 $PERIM$ = perimeter of airway (m)
 AGE = age of airway (years).

This formula was derived by least-squares fits to heat flows obtained from wet- and dry-bulb gradients calculated by Ramsden's⁹ method. As discussed, these gradients agreed with Starfield's predictions only for damp airways; therefore, agreement between equation (18) and Starfield's predictions cannot be expected for dry and wet airways.

The lack of dependence of Q on wet-bulb temperature contradicts Starfield and Dickson³ and Lambrechts²¹, who both found considerable dependence of the wet-bulb gradients on inlet wet-bulb temperature.

Table IX compares the heat pick-up from equation (18) with that from Starfield's rapid program and the DRYAIR program. For dry airways, much better agreement was obtained between Ramsden's formula and DRYAIR. The average difference was 11 per cent. Errors have already been found in Starfield's predictions for dry airways. For damp and wet footwalls, little agreement was found. The floor wetness has a much greater effect in Ramsden's formula. However, the agreement was good for damp airways with air temperatures of

29/37°C, which represent fairly typical conditions with no bulk cooling of air. Agreement was not expected with wet airways since the air-temperature gradients of Starfield and Ramsden did not agree.

Ramsden's work demonstrates the difficulty, and probably the impossibility, of obtaining simple formulae for airways with the full range of wetness, wet-bulb depression, etc. It was developed for use on a slide rule before the introduction of programmable calculators. The formula for heat pick-up could still be used for dry airways, and also for damp airways if bulk cooling of air is not practised and the shaft is not dry.

Discussion

In general, there is considerable disagreement between air temperatures and heat pick-up predicted by the different methods. The wet-bulb temperatures given by the McPherson program for damp and wet footwalls are all below those predicted by the Starfield program, and the dry-bulb temperatures are mainly lower. Ramsden's formula of heat pick-up does not generally agree with predictions based on the Starfield program.

However, for typical inlet-air temperatures of 29/37°C without bulk air cooling in the airway and with a typical damp footwall, the agreement is better. For example, the McPherson program predicts wet-bulb temperatures that are 0,5 to 0,7°C below Starfield's values at 2 km along an airway with inlet-air temperatures of 29/37°C and virgin-rock temperatures of 50°C and 60°C. Ramsden's formula for heat pick-up agrees with Starfield's values within 2 per cent in this case.

In the same airway but with a wet footwall, the differences between the predictions by McPherson and Starfield increase. Even less agreement is found when the air is bulk cooled and the footwall is damp or wet. Then

TABLE IX
HEAT PICK-UP BY METHODS OF RAMSDEN AND STARFIELD

VRT °C	Wet-bulb temp. °C	Dry-bulb temp. °C	Wetness factor	Heat pick-up, kW/100 m		
				Ramsden	Starfield	DRYAIR
50	20	20	0,0	42,1	61,3	37,9
			0,2	50,3	61,3	
			1,0	83,3	63,3	
60	20	20	0,0	56,1	87,2	50,5
			0,2	67,0	87,2	
			1,0	111,1	86,3	
50	29	37	0,0	18,2	21,5	16,4
			0,2	21,8	21,5	
			1,0	36,1	22,9	
60	29	37	0,0	32,2	38,8	29,0
			0,2	38,6	38,8	
			1,0	63,9	39,7	
50	28	42	0,0	11,2	7,1	10,1
			0,2	13,4	7,1	
			1,0	22,2	7,8	
60	28	42	0,0	25,2	19,0	22,7
			0,2	30,2	19,0	
			1,0	50,0	20,0	

there are differences of 2 or 3°C in wet- and dry-bulb temperatures.

These differences in predicted temperatures and heat pick-up are caused by over-simplification of the mathematical models of heat transfer in airways. The following are examples of the effects of over-simplification.

- (i) The omission of radiation in the McPherson program has to be compensated for by higher heat-transfer coefficients than those used by Starfield. This causes a more rapid reduction in dry-bulb temperature along the airway when the air has a high wet-bulb depression.
- (ii) The surface temperature of the wet portion of the footwall appears to be considerably lower in the McPherson model than in the Starfield model. When air has been bulk cooled, this leads to much lower evaporation rates.
- (iii) Ramsden assumes that the difference between the surface temperatures of the wet and dry portions of the perimeter equals the difference between the wet- and dry-bulb air temperatures. This under-predicts θ_{sw} , and leads to large errors in air-temperature gradients with wet and damp footwalls. However, an arbitrary adjustment of the wetness factor for damp airways produced good agreement with Starfield's predictions.

So far in this section, only damp and wet airways have been discussed. With dry airways, Starfield's method of interpolation in the rapid program causes errors in dry-bulb temperatures. This part of the program should be replaced by a program similar to DRYAIR. Ramsden's formula for heat pick-up gives reasonable agreement with DRYAIR, as does McPherson's program.

The prediction method being used by the Chamber of Mines of South Africa for haulages with bulk air cooling is satisfactory if the airway is dry, but a correction factor is required in the more common situation of a damp airway.

An advantage of the McPherson program is that the effect of a machine or a cooler can be predicted. However, when a machine or cooler has been switched on during the previous few months, errors in prediction could occur because the McPherson program is unable to calculate the effect of changes in inlet-air temperatures. The heat-storage capacity of the rock surrounding an airway has considerable effects in these cases.

Areas where further research is required are listed below. The list is not restricted to intake haulages in South African gold mines.

- (i) *Varying air temperatures.* Machinery is increasingly used in mines, especially in the production area of coal mines. Many machines are switched on and off or operate at varying power settings. Duhamel's theorem in its existing form is not suitable for the calculation of the effects of changes in air temperature since the computation time is very long. A more readily usable method is needed. The important effects of changes in weather conditions could be calculated by this new method, as could the effects of running coolers for only part of the day. The new method would have

considerable applications in other areas of engineering and science.

- (ii) *Underground measurements.* Measurements of air temperature along intake airways with bulk air coolers would provide a useful check on the methods of prediction, especially if the rock temperatures were also measured.
- (iii) *Rapid program of Starfield.* The range of parameters needs to be extended, especially the upper limits of 60°C in virgin-rock temperature and 55 m³/s in airflow. As already discussed, the calculation for dry airways should be replaced. For wet and damp airways, computational techniques developed since Starfield and Dickson wrote the original program could possibly be used to produce a faster program.

Conclusions

The procedure that is recommended for the calculation of heat flow into intake haulages until further research is undertaken is as follows.

For haulages with a damp or wet footwall, Starfield's program should normally be used, even though it does not allow the direct calculation of the effect of machinery and coolers. However, the McPherson program, which has the advantage of allowing for machinery and coolers, can be used without large error in a frequently occurring situation. In this, air flows down a shaft that is not dry, and then passes along a haulageway with a damp footwall without the air being bulk cooled. Ramsden's formula for heat pick-up can also be used in this situation.

For dry haulages, a program similar to DRYAIR is the most convenient solution. Starfield's program should not be used, but the McPherson program or Ramsden's formula for heat pick-up could be used.

Errors of prediction can occur with McPherson's program if a machine or cooler has been switched on in the previous few months.

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Addendum: Heat Fluxes

Radial flow with a finite heat-transfer coefficient is the most realistic assumption for main intake airways, as already discussed. The change in heat flux produced by the assumption of linear flow or infinite h (i.e. θ_s equals θ_v) is now calculated.

The airway for the calculations was assumed to be dry with a virgin-rock temperature of 50°C and an inlet-air dry-bulb temperature of 30°C. The other parameters were as given in Table I.

The heat flux in W/m² for radial flow with finite h was

TABLE X

HEAT FLUXES FOR RADIAL AND LINEAR FLOW

Time of cooling	Radial flow			Linear flow		
	Heat flux, W/m ²		Surface temp. °C	Heat flux, W/m ²		Surface temp. °C
	Infinite <i>h</i>	Finite <i>h</i>		Infinite <i>h</i>	Finite <i>h</i>	
3,5 hours	380,6	216,4	41,6	350,4	212,0	41,4
10,6 hours	232,0	164,4	38,8	202,3	155,5	38,3
21,2 hours	172,3	133,7	37,2	143,0	122,0	36,5
7,4 days	76,6	67,6	33,6	49,6	48,3	32,6
13,3 days	63,2	56,5	33,0	36,9	36,4	32,0
29,5 days	49,7	45,2	32,4	24,8	24,6	31,3
147 days	33,2	30,8	31,7	11,1	11,1	30,6
1,0 year	27,4	25,7	31,4	7,01	7,00	30,4
4,0 years	21,5	20,4	31,1	3,50	3,50	30,2
10,0 years	18,7	17,8	31,0	2,22	2,22	30,1

calculated by use of equation (4) and the tables in reference 10.

For radial flow with an infinite *h*, the flux is given by Goch and Patterson²² as

$$k \frac{\delta\theta}{\delta r} = \frac{k(\theta_v - \theta_d) T'}{a}, \dots \dots \dots (19)$$

where *T'* is a function of τ only.

The flux for a semi-infinite solid (i.e. linear flow) with infinite *h* is¹⁸.

$$k \left(\frac{\delta\theta}{\delta x} \right)_{x=0} = \frac{k(\theta_v - \theta_d)}{\sqrt{\pi at}} \dots \dots \dots (20)$$

The flux for a semi-infinite solid with finite *h* is¹⁸

$$h(\theta_s - \theta_d) = h(\theta_v - \theta_d) e^{x^2} \operatorname{erfc} x, \dots \dots (21)$$

$$\text{where } x = \frac{h}{k} \sqrt{at}$$

and *erfc x* is the complementary error function.

For large values of time, equation (21) becomes¹⁸

$$h(\theta_s - \theta_d) = \frac{k(\theta_v - \theta_d)}{\sqrt{\pi at}} \left(1 - \frac{1}{2x^2} + \frac{3}{4x^4} \right) \dots \dots (22)$$

The heat fluxes in Table X show that infinite *h* over-predicts the heat flux initially by a factor of 1,7 in both cases, but the fluxes differ by less than 10 per cent within a few days for linear flow, and after a month for radial flow. The value of *h* would therefore be unimportant in dry airways after a few months of cooling if the air temperature remains constant, but in practice the air temperature varies as already discussed. In damp and wet airways, the value of *h* determines the convective flow including flow from the air to the footwall, and hence the change in dry-bulb temperature along the airway.

Table X also shows that the flux with linear flow falls increasingly below the flux with radial flow, which causes an increasingly large error if a linear-flow model is used for a tunnel, whether it has a square or a cylindrical cross-section. This point has been discussed by Hemp¹⁷.

The surface temperature with finite *h* falls rapidly

from 50 to 41°C in 4 hours with both linear and radial flow. After two weeks, θ_s is only 2°C above the dry-bulb air temperature with linear flow, and 3°C above with radial flow.

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