Theories of ball wear and the results of a marked-ball test in ball milling

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SYNOPSIS

Formulas expressing the dependence of the mass of a grinding ball on the amount of material milled by a ball mill are derived. It is shown that, with the aid of these formulas, it is relatively simple for the results of a marked-ball test to be analysed and for the relative performance and cost-effectiveness of the various types of ball under test to be evaluated. Predictions based on theory and the results obtained in a marked-ball test in a ball mill are compared in detail and shown to be in close agreement.

The difficulties associated with marked-ball tests are considered, and an explanation for the decrease in the calculated consumption of the balls, which follows naturally from the theory, is presented. The marked-ball anomaly is explained, and it is demonstrated that the ball consumption in a ball mill can be predicted with fair accuracy from the results of a marked-ball test.

SAMEVATTING

Formules wat die afhanklikheid uitdruk van die massa van 'n maalbal op die hoeveelheid materiaal wat deur 'n balmeule gemal word, word afgelei. Daar word aangetoon dat, met behulp van hierdie formules, dit betreklik maklik is om die resultate van 'n gemerkte-baltoets te analiseer en die relatiewe werkverrigting en bestedings-geregeverdighed van die verskillende soorte balle te beweer. Voorstelings wat op teorie gegrond is en die resultate verkry uit 'n gemerkte-baltoets in 'n balmeule word in besonderhede vergeleik en toon 'n groot ooreenkom. Die probleme verbonden aan gemerkte-baltoets word oorwoeg, en 'n verdeelde vir die afname in die berekenings van die ball, wat vanaf gemaal word en vervang word, is verduidelik. Die gemerkte-bal anomalie word uiteegselt en dit word gedemonstreer dat die balmeul met redelye akkuraatheid uit die resultate van 'n gemerkte-baltoets voorspel kan word.

Introduction

The consumption of balls in the fine grinding of ores is a major item in the costs of milling operations. A wide variety of grinding balls are available from various manufacturers, but one experiences substantial difficulties in selecting the ball that will be most cost-effective for grinding a particular ore under particular milling conditions.

Plant tests involving the gradual introduction of a full charge of balls of a new type are long and expensive. An attractive alternative is a test in which several types of balls carrying distinguishing marks (e.g. line holes) are introduced simultaneously into a mill and are removed for examination at intervals. However, the validity of the results obtained by the latter procedure (referred to as a marked-ball test) has frequently been disputed.

In a closely monitored test conducted at the Markikana Mine of Western Platinum Limited, six grades of marked balls, one set of which was identical to the 'background' charge of balls, were introduced simultaneously into a mill. This paper analyses and discusses the results of this test.

Evaluation of Grinding Balls

The conventional procedure employed in ball mills illustrates the scale and cost of a test aimed at measuring the durability and cost-effectiveness of a particular type of ball. In this procedure, the entire charge of a mill is gradually replaced by an alternative type of ball. The replacement is done gradually because it is considered desirable for the graded size distribution of the balls within the mill to be preserved. The technique used is as follows. The addition of the 'old' type of ball is discontinued, and the largest (top) size of the alternative type of ball is added at the rate required to maintain the charge at the desired level.

If \( M \) is the total mass of the charge within a mill and \( dM/dt \) is the rate at which the balls are consumed, the time constant (as it refers to ball consumption) of a mill is

\[
\theta = -\frac{Mt}{dM/dt}
\]

In large mills such as those used on South African gold and platinum mines, the ball charge can vary from 20 t to more than 40 t, and the time constants can range from a few weeks to several months. After the old type of ball has been consumed, a period of at least a few time constants is required before the new rate of addition can be established with certainty. In view of the large masses of grinding media and the long time constants involved, it is not surprising that these experiments are costly and time-consuming.

In milling practice, it is usual for ball consumption to be considered with reference to the mass of material milled, \( T \). In terms of \( T \), the ball consumption in kilograms per ton milled becomes

\[
\frac{dM}{dt} = \frac{dM}{dT} \cdot \frac{dT}{dt}
\]

A 'tonnage constant' for the mill, \( \tau_m \), can be defined as follows:

\[
\tau_m \equiv \frac{dM}{dT} \cdot \frac{dT}{dt}
\]
\[ \tau_m = - \frac{M}{dM/dT} \]  

(3)

(Because \( dM/dt \) is negative, \( \tau_m \) is positive, i.e. the mass of the charge decreases with increasing tons milled.)

The relation between \( \tau_m \) and \( \theta \) is

\[ \tau_m = \theta \frac{dT}{dt} \]  

(4)

where \( dT/dt \) is the mass of material milled per unit of time; \( \tau_m \) is measured in tons or kilotons.

The cost-effectiveness of balls is measured by the use of an index, \( E \), given by

\[ E = C \left| \frac{dM}{dT} \right| \]  

(5)

where \( C \) is the cost per ton of balls to the mine. The more cost-effective the balls, the smaller is \( E \).

Because of the large financial benefits that can be secured if the most cost-effective balls are used, it is always desirable that a variety of balls should be evaluated. Clearly, several years of labour and many hundreds of tons of balls would be required before the conventional procedure could show which of the ball types available would be most cost-effective in the conditions pertaining to any given milling installation.

Norman and Loeb\(^2\) have pointed out that the relative merits of several types of ball can be determined simultaneously in a marked-ball test. The use of such a test could therefore result in a saving of several years of labour and several hundreds of tons of steel, but it is well known that many mill operators regard the results of marked-ball tests with some reservations.

Certain difficulties are associated with the interpretation of the results of marked-ball tests. These difficulties are as follows:

(i) ball consumption decreases systematically with time (or mass of material milled) in a manner suggesting that equilibrium has not been reached, and

(ii) marked balls that are of the same type as the ‘background’ of balls within the mill appear to be more durable than the background balls — an effect referred to as the marked-ball anomaly.

The results of a marked-ball test are of interest in relation to theories of ball wear. The work done at Western Platinum Limited\(^7\) shows that, with the aid of a theory of ball wear, it is easy for one to analyse the results of a marked-ball test, to explain the dependence of ball consumption on the tonnage milled, to understand the marked-ball anomaly, and to formulate an index that describes the most cost-effective ball.

### Theories of Ball Wear

**The Volume Theory**

The volume theory postulates that the rate of ball wear is proportional to the mass (or volume) of a ball. It has its origin in the idea that the majority of comminution events occur in the ‘too’ of the charge, where impactive events predominate. In these events, the work done by a ball in stressing ore particles is thought to be proportional to the kinetic energy that catacting and tassuing balls receive from the rotary action of the mill, and therefore to the mass or volume of a ball. Davis\(^3\) has advanced evidence in support of this theory.

If \(-dm/dT\) is proportional to \( m \), where \( T \) is the amount of material milled (in tons or kilotons), we can write

\[ \frac{dm}{dT} = \frac{m}{\tau} \]  

(6)

where the proportionality constant, \( \tau \), will be called the ‘tonnage constant’ of a ball; \( \tau \) is measured in the same units as \( T \).

From equation (6),

\[ m(T) = m_o \exp \left(-\frac{T}{\tau} \right) \]  

(7)

where \( m_o \) is the original mass of a ball. It can be seen from equation (7) that, after \( \tau \) tons of material have been milled, the mass of a ball will be reduced by a factor of \( e^{-1} \) (\( e \) being the ‘base’ of natural logarithms), so that

\[ m(\tau) = m_o e^{-1} \] (approximately 37 per cent of \( m_o \)). Also, if \( m_i \) and \( m_o \) are the respective ball masses before and after \( T \) tons of material have been milled, then \( \tau \), the tonnage constant, is given by

\[ \tau = \frac{T}{\ln(m_i/m_o)} \]

**The Surface Theory**

The surface theory has its origin in the idea that the vast majority of comminution events are due to three-body abrasive grinding of the particles of ore by the balls, and therefore suggests that the rate of wear is proportional to the surface area of a ball. Norman and Loeb\(^2\), Prentice\(^4\), and several other authors, have advanced evidence in support of this theory.

If \( m \) is the mass of a ball, its surface area is proportional to \( m^{2/3} \). If the wear rate is proportional to the surface area, we can write

\[ \frac{dm}{dT} = bm^{2/3} \]  

(8)

where \( b \) is a constant.

From equation (8), we obtain

\[ \frac{m^{1/3}}{m_o^{1/3}} = 1 - 3bT \]  

(9)

or

\[ m(T) = (m_o^{1/3} - 3bT)^{3} \]  

(10)
Equations (9) and (10) can be valid only if the shapes of the balls remain constant (i.e. spherical) and are valid only when

\[ T < \frac{3m^2}{b}. \]

In this paper, the constant \( b \) is referred to as the surface-wear constant.

**Methods of Analysis**

Obvious methods for the analysis of the results of a marked-ball test are suggested by equations (7) and (9). From equation (7),

\[ \ln m = \ln m_0 - T/T', \]

i.e. the \( \ln \) (ball mass) varies linearly with the mass of material milled, and the tonnage constant of the ball is simply the reciprocal of the slope of the graph. On the other hand, equation (9) shows that the cube root of the ball mass varies linearly with the mass of material milled. The slope of this graph is equal to the surface-wear constant.

**Results of the Marked-ball Test**

In the marked-ball test mentioned earlier, which was conducted in the course of a full-scale test at Western Platinum Limited\(^1\), the entire charge of the No. 3 A-stream ball mill was replaced with balls from the Fonderies Magotteaux in Belgium. The mill is an 8ft by

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### TABLE I

<table>
<thead>
<tr>
<th>Ball grade</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.7.81 Original mass</td>
<td>850.8</td>
<td>850.8</td>
<td>3.02</td>
<td>850.8</td>
<td>850.8</td>
<td>3.02</td>
<td>850.8</td>
<td>850.8</td>
</tr>
<tr>
<td>9.8.81</td>
<td>734</td>
<td>21151</td>
<td>21151</td>
<td>0.143</td>
<td>734</td>
<td>21151</td>
<td>21151</td>
<td>0.143</td>
<td>734</td>
</tr>
<tr>
<td>7.9.81</td>
<td>619</td>
<td>18169</td>
<td>18169</td>
<td>0.122</td>
<td>619</td>
<td>18169</td>
<td>18169</td>
<td>0.122</td>
<td>619</td>
</tr>
<tr>
<td>12.10.81</td>
<td>537</td>
<td>22458</td>
<td>22458</td>
<td>0.131</td>
<td>537</td>
<td>22458</td>
<td>22458</td>
<td>0.131</td>
<td>537</td>
</tr>
<tr>
<td>9.11.81</td>
<td>480</td>
<td>18092</td>
<td>18092</td>
<td>0.113</td>
<td>480</td>
<td>18092</td>
<td>18092</td>
<td>0.113</td>
<td>480</td>
</tr>
<tr>
<td>21.12.81</td>
<td>364</td>
<td>79870</td>
<td>79870</td>
<td>0.113</td>
<td>364</td>
<td>79870</td>
<td>79870</td>
<td>0.113</td>
<td>364</td>
</tr>
<tr>
<td>15.2.82</td>
<td>253</td>
<td>141026</td>
<td>141026</td>
<td>0.101</td>
<td>253</td>
<td>141026</td>
<td>141026</td>
<td>0.101</td>
<td>253</td>
</tr>
</tbody>
</table>

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\(^1\)Average of a sample of nominally 10 balls withdrawn from the mill

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8 ft rubber-lined ball mill running at 85 per cent of the critical speed with a ball charge of 22 t. The daily throughput is about 650 t of ore from the Mesoeresk Reef. Six grades of Magotteaux balls were tested simultaneously. One of the grades, namely grade 4, was identical to the background of balls within the mill. The mill was recharged once a fortnight with 60 mm balls to restore the level of the charge to the bottom of the inlet trunnion, and the masses of marked balls were monitored at monthly intervals. The mine records of the test results are given in Tables I and II.

In Table I, the values given in column 6 were calculated from the formula

$$K = \frac{22(m_o - m(T))}{m_oT},$$

where \(m(T)\) is the mass of a ball after \(T\) tons of new feed have been milled, and 22 t is the mass of the balls in the mill. The calculated ball consumptions (column 9 of Table I) are obtained by the formula

$$K = \frac{22(m_o - m(T))}{m_oT}.$$

It is interesting that these figures generally decrease systematically with increasing tons milled (although there are some exceptions) in a manner suggesting that equilibrium has not yet been reached. This effect is explained below.

The marked-ball anomaly can be ascertained by comparison of the values for the grade 4 balls in column 9 of Table I with those in column f of Table II, which represent the total ball consumption of the operating mill. It can be seen that, after a total of 149.6 kt had been milled, the total ball consumption by the operating mill was approximately 0.14 kg/t, but for grade 4 marked balls, the calculated ball consumption was 0.049 kg/t, i.e. the grade 4 marked balls, even though they are identical to the background, appear to be much more durable.

**Analysis of the Results of the Marked-ball Test**

Because grade 4 balls constituted the background of balls within the mill, the results obtained from these balls will be discussed in some detail as an illustration of the theories.

We first consider the results from the point of view of the volume theory. In Fig. 1, which shows the ln (ball mass) versus the tons milled for grade 4 balls, it can be seen that the points are distributed linearly. The correlation coefficient is 0.9985. The line is a least-squares fit to the experimental points and yields a value for \(n\) of 6.741, corresponding to \(m_o = 846.4\) g. This value is in excellent agreement (0.4 per cent) with the known initial mass of 842.7 g. The reciprocal of the slope of the line is the tonnage constant. For grade 4 balls, we find that \(\tau_4 = 362\) kt.

![Data from Table I](https://example.com/data.png)

**Fig. 1—Fit of the volume theory to the wear data for grade 4 grinding balls by a plot of ln (ball mass) versus \(T\)**

**TABLE II**

<table>
<thead>
<tr>
<th>Period</th>
<th>Material milled, t</th>
<th>Balls added, t</th>
<th>Ball consumption</th>
<th>(\Delta M/\Delta T)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amount</td>
<td>Cumulative amount</td>
<td>Amount</td>
<td>Cumulative amount</td>
</tr>
<tr>
<td>1</td>
<td>22.6.81 to 5.7.81</td>
<td>8524</td>
<td>8524</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6.7.81 to 8.8.81</td>
<td>21 151</td>
<td>29 075</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>9.8.81 to 6.9.81</td>
<td>18 169</td>
<td>47 844</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>7.9.81 to 11.10.81</td>
<td>22 458</td>
<td>70 302</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>12.10.81 to 8.11.81</td>
<td>16 092</td>
<td>88 394</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>9.11.81 to 19.12.81</td>
<td>27 319</td>
<td>115 713</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>20.12.81 to 15.2.82</td>
<td>38 837</td>
<td>149 550</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>16.2.82 to 23.3.82</td>
<td>60 006</td>
<td>269 556</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>24.3.82 to 23.6.82</td>
<td>16 442</td>
<td>220 198</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>24.6.82 to 23.7.82</td>
<td>18 172</td>
<td>244 370</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>24.7.82 to 22.8.82</td>
<td>17 729</td>
<td>262 099</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>23.8.82 to 21.9.82</td>
<td>17 693</td>
<td>279 792</td>
<td>1</td>
</tr>
</tbody>
</table>

*Marked balls added to the mill
Column f = column e - column c
Column g = column d - column b

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It can be concluded that the function
\[ m(T) = 846,4 \exp(-T/362) \]  
(11)
is a good description of the way in which the mass of a grade 4 ball varies with the tonnage milled by the given mill.

The same data must also be scrutinized from the point of view of the surface theory, which predicts that, provided the shapes of the balls remain constant during the period of the test, the cube root of the ball mass will vary linearly with the tonnage milled. It is well known that the shapes of worn grinding elements can be distinctly non-spherical. The constant-shape approximation could therefore introduce an error. The magnitude of this error was investigated by measurement of the volumes, surface areas, and mean diameters of worn balls.

At the end of the test, the balls still had a well-rounded appearance. It was found that the surface area of a worn grade 4 ball was about 11 per cent greater than that of a sphere of equivalent volume, and that the diameter of the equivalent-volume sphere was within 4 per cent of the measured mean diameter of the ball. Since equation (9) shows that quantities proportional to \( m^{1/3} \), or the diameter, must be considered, we conclude that the constant-shape approximation introduces errors that vary from 0 at the beginning to almost 4 per cent at the end of the test. This error introduces an uncertainty of 4 per cent in the value of the surface-wear constant that will be obtained by the fitting of equation (9) to the data.

In Fig. 2, which shows the cube root of the ball mass versus the tonnage milled, it can be seen that the points are distributed linearly. The correlation coefficient is 0.9994. The line, which is the least-squares fit to the data, yields a value for \( m_0 \) of 837.4 g that is in excellent agreement (0.6 per cent) with the known initial mass. From the slope of this curve, we obtain the following value for the surface-wear constant of grade 4 balls:
\[ b = 2.33 \times 10^{-2} \text{g}^{1/3} \text{kt}^{-1}. \]

It can be concluded that the function
\[ m(T) = (9.425 - 7.77 \times 10^{-2}T)^2 \]  
(12)
is a good description of the way in which the mass of a grade 4 ball varies with the throughput of the given mill.

Equations (11) and (12), which represent the volume and surface theories respectively, are shown in Fig. 3 together with experimentally measured points. It can be seen that either theory is in excellent agreement with the measured ball masses.

The results relevant to all six grades of ball were analysed in this way, and are summarized in Table III.

**TABLE III**

<table>
<thead>
<tr>
<th>Grade of ball</th>
<th>Tonnage constant ( r_v )</th>
<th>Correlation coefficient ( r_v )</th>
<th>Surface-wear constant ( b )</th>
<th>Correlation coefficient ( r_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>129</td>
<td>0.9986</td>
<td>0.065</td>
<td>0.9987</td>
</tr>
<tr>
<td>2</td>
<td>124</td>
<td>0.9995</td>
<td>0.064</td>
<td>0.9968</td>
</tr>
<tr>
<td>3</td>
<td>124</td>
<td>0.9993</td>
<td>0.003</td>
<td>0.9995</td>
</tr>
<tr>
<td>4</td>
<td>362</td>
<td>0.9985</td>
<td>0.023</td>
<td>0.9994</td>
</tr>
<tr>
<td>5</td>
<td>212</td>
<td>0.9988</td>
<td>0.039</td>
<td>0.9986</td>
</tr>
<tr>
<td>6</td>
<td>412</td>
<td>0.9993</td>
<td>0.018</td>
<td>0.9986</td>
</tr>
</tbody>
</table>

Fig. 2—Fit of the surface theory to the wear data for grade 4 grinding balls by a plot of \( m^{1/3} \) versus \( T \)

Fig. 3—Comparison of theoretical and experimental results for grade 4 balls

The results relevant to all six grades of ball were analysed in this way, and are summarized in Table III.

Two points of interest emerge from Table III. The tonnage constants and surface-wear constants that are associated with various grades of balls vary by a factor of more than 3. This variation must be related to the metallurgical properties of the various grades of balls. Secondly, the correlation coefficients, \( r_v \) and \( r_s \), are all close to unity, which shows that both theories provide good descriptions of the way in which the masses of the balls decrease as a function of the amount of material milled.

**Discussion**

**Agreement with Both Theories of Ball Wear**

It has been shown that both theories of ball wear are
good agreement with the actual wear of six grades of balls. For some grades the volume theory provides a marginally better fit (e.g. grades 2 and 6), but for other grades the surface theory offers a better description (e.g. grades 1, 4, and 5). This puzzling ambiguity is not due to the wear rate of a ball being simultaneously proportional to its mass and to its surface area. If it were, we should find that, according to the volume theory,

\[ \frac{dM}{dT} \propto m, \]

and, according to the surface theory,

\[ \frac{dM}{dT} \propto m^{1.3}. \]

Dividing these expressions by each other shows that \( m^{1/3} \), which is proportional to the diameter of a ball, is a constant, which is absurd. Therefore, the wear rate of a ball cannot be simultaneously proportional to its surface area and to its mass.

Part of the reason for the ambiguity is that the graphs for the two functions are of the same general shape, although one represents an exponential decay of the ball mass with the tons milled, \( m_0 \exp(-T/\tau) \), and the other a cubic decay of the ball mass with the tons milled, \( (m_0 - 1/3 \cdot 3m_0^{1.3})^3 \). These functions were brought into close correspondence with the data, and therefore with each other, by least-squares adjustment of the parameters.

The data extend over a limited range

\[ (m_0/m_f) < 3, \]

where \( m_0 \) is the original mass and \( m_f \) the final mass. One can always bring functions whose graphical representations are of the same general shape into close correspondence over a limited range by suitable adjustment of the parameters. This is partly why both functions are in good agreement with the data.

Another reason for the ambiguity might be that the balls are subjected to both abrasive and impactive wear.

Decrease in the Calculated Ball Consumption

The calculated ball consumption of marked balls (column 9 of Table 1) was obtained with the aid of the formula

\[ K = \frac{22}{T} \left[ \frac{m_0 - m(T)}{m_0} \right], \]

where the factor 22 is the tonnage of the grinding balls within the mill. Substituting for \( m(T) \) from equations (7) and (10), one obtains, by use of the volume theory,

\[ K(T) = \frac{22}{T} [1 - \exp(-T/\tau)], \]

and, by use of the surface theory,

\[ K(T) = \frac{22}{T} \left[ 1 - \left( \frac{bT}{3m_0^{1.3}} \right)^3 \right], \]

where equation (17) is valid for \( T < 3m_0^{1.3}/b \).

The relative change in mass, i.e. the quantity in brackets in equation (15) as predicted by the use of the volume and surface theories, and the data for grade 4 balls are plotted in Fig. 4. \( K(T) \) is the slope of a chord from the origin to a point on either of the curves. Obviously, the slopes of chords from the origin decrease with increasing mass of material milled. This is confirmed by Fig. 5, which shows the graph of \( K(T) \) obtained by the use of equations (16) and (17) and the values for the calculated consumption of grade 4 balls obtained from column 9 in Table 1.

According to both theories, \( K(T) \) is a variable quantity that decreases monotonically with increasing throughput. This decrease is essentially due to the fact that the wear rates of balls decrease as they become smaller.

These considerations show that ball consumption should be related to constants associated with the test, i.e. the tonnage constant, \( \tau \), or the surface-wear constant, \( b \), rather than to a quantity dependent on tonnage such as the ball mass.

It is a simple matter for one to formulate a cost-effective index, \( E \), in terms of the constants of the test. In terms of the volume theory,

\[ E = C/\tau, \]

whereas, in terms of the surface theory,

\[ E = Ob, \]

where \( C \) is the cost per ton of the balls to the mine. The most cost-effective balls have the smallest value for \( E \).
**The Marked-ball Anomaly**

To the mine records given in Table II in columns a to f, we have added (in column g) the values \( \Delta M / \Delta T \), which represent the average ball consumption by the mill for each period of the test. It should be noted that these values represent the total ball consumption by the mill for the stated period and are not to be confused with the values for the calculated ball consumption as given in Table I, which refer only to marked balls. Fig. 6 gives the values for the average ball consumption by the mill as a function of the cumulative amount of material milled, and also summarizes the history of the test. While the discharge grates were regarded as still being serviceable, the mill was relined and charged with unmarked grade 4 balls as follows: 4 t of 40 mm balls, 5 t of 50 mm balls, and 14 t of 60 mm balls, and the plant test was initiated at \( T = 0 \). When 8,524 kt of material had been milled, the make-up addition of 2 t of balls included the six sets of marked balls. The marked balls were monitored until \( T = 149.6 \) kt, after which it became difficult for sufficient numbers of these balls to be found. Some balls of grades 4 and 6 were found in the mill at \( T = 257.9 \) kt. Their masses were measured, and these measurements were taken into account in the consideration of the theories.

Fig. 6 shows that the ball consumption \( \Delta M / \Delta T \) initially shows a pronounced decreasing trend, but that it tends towards a value of about 0.1 kg/t milled at the end of period 12, at which time the discharge grates were rather badly worn.

A ball consumption of 0.1 kg per ton milled implies a tonnage constant for the mill of \( \tau_M = 220 \) kt but, as shown earlier, the tonnage constant for grade 4 marked balls, \( \tau_4 = 362 \) kt. This is an illustration of the marked-ball anomaly: even though the marked balls are of the same type as the background ball charge, they appear to be more durable.

It has been shown that the volume and the surface theories are in very good agreement with the data available, and therefore either theory could be used in a theoretical discussion of the marked-ball anomaly. Nevertheless, we prefer to use the volume theory because the idea of a tonnage constant is useful.

We start by trying to calculate the ball consumption of the mill. The mass of the grinding balls within the mill can be written as

\[
M = N \bar{m},
\]

where \( N \) is the number of balls and \( \bar{m} \) is the average mass of the balls.

If it is assumed that the wear of balls of average mass is similar to that of the marked balls,

\[
\frac{\Delta M}{\Delta T} = \frac{\Delta (N \bar{m})}{\Delta T} = N \frac{\Delta \bar{m}}{\Delta T} = N \bar{m} \frac{M}{\tau_B} = \frac{M}{\tau_B}
\]

and

\[
\tau_m^{-1} = \frac{M}{\Delta M / \Delta T} = \frac{N \bar{m}}{\Delta (N \bar{m}) / \Delta T} = \frac{\Delta \bar{m}}{\Delta T} = \frac{\Delta \bar{m}}{\Delta T} = \tau_B^{-1}
\]

We have added the subscript \( B \) to emphasize that the tonnage constant appearing on the right-hand sides of these equations refers to a ball that is typical of the
background charge of grinding balls within the mill. Equation (20) suggests that the tonnage constant for the mill is equal to the tonnage constant for a ball. The data for grade 4 balls give

\[ T_B = \frac{22 \times 10^3}{362 \times 10^3} = 0.061 \text{ kg/t milled}, \]  

which is less than the actual ball consumption by the mill, and therefore just another indication of the marked-ball anomaly.

The calculations above do not resolve the marked-ball anomaly because they ascribe the ball consumption of the mill only to the rate at which finely divided material is ground off the ball surfaces. When the balls eventually become sufficiently small, they can pass through the holes in the discharge grates. Ball remnants can always be seen on the screens of discharge tummels. The calculations shown in equations (19) to (22) do not take into account the discharge of ball remnants from the mill. We take this factor into account by writing

\[ \frac{\Delta M}{\Delta T_{\text{mill}}} = \frac{\Delta M}{\Delta T_{\text{R}}} + \frac{M}{\tau_B}, \]  

where the remnant term, \( \frac{\Delta M}{\Delta T_{\text{R}}} \), is the total mass of the ball remnants that are rejected from the mill in unit time, and the term \( \frac{M}{\tau_B} \) represents the rate at which finely dispersed material is being ground off ball surfaces during milling.

For the determination of the remnant term, the discharge of ball remnants from the mill was monitored shortly before and after renewal of the discharge grates. The details of these measurements are recorded in Table IV, which shows that, with severely worn discharge grates, the mill discharged 111.3 kg of balls during the time in which 3644 t of ore were fed to the mill.

Theoretically, therefore, the ball consumption by the mill can be expressed as

\[ \frac{\Delta M}{\Delta T_{\text{mill}}} = \frac{111.3 \times 22 \times 10^3}{3,64 \times 10^4} = 0.091 \text{ kg/t milled}, \]  

Table II shows that, from 24th May to 21st September, 1982, when 70 226 t of ore were milled, 6 t of steel were consumed by the mill. This corresponds to a consumption of 0.085 kg of balls per ton of material milled, which is in pleasing agreement with the calculation based on theory.

Table IV also describes the discharge of remnants from the mill shortly after the discharge grates had been renewed, and it can be seen that the average mass of remnants decreased from 83 to 26 g, the average equivalent diameter from 28 to 19 mm, and the consumption of balls as remnants from 0.031 to 0.008 kg per ton milled. The condition of the discharge grates clearly has a profound influence on the total ball consumption.

According to our theory, we predict that, with new discharge grates fitted, the ball consumption of the mill should decrease to

\[ \frac{\Delta M}{\Delta T_{\text{New grates}}} = 0.061 + 0.008 = 0.069 \text{ kg/t milled}. \]

Reference to Table II shows that, during periods 13 to 15, a total mass of 54 401 t of ore were fed to the mill and a total mass of 4.3 t of balls were added. This corresponds to a ball consumption of 0.079 kg per ton milled, which is in reasonable agreement with our calculated value of 0.069.

Therefore, when the tonnage constant for a ball and the rate at which ball remnants are discharged from the mill are taken into account, the marked-ball anomaly can be explained, and the total consumption of balls by the mill can be predicted from a marked-ball test.

It is of interest that, whereas the addition rate of top-size balls is about 100 per kiloton of ore fed, the discharge rate is about three times that number. This

### Table IV

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Date</th>
<th>Ore milled</th>
<th>Ball remnants</th>
<th>Ball remnants</th>
<th>Consumption of balls as remnants per ton milled</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Number found</td>
<td>Mass</td>
<td>Average mass kg</td>
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<td></td>
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<tr>
<td>Discharge</td>
<td>6 Sep.</td>
<td>150</td>
<td>205</td>
<td>16.2</td>
<td>0.082</td>
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<td></td>
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<td>302</td>
<td>101</td>
<td>9.0</td>
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<tr>
<td></td>
<td>8 Sep.</td>
<td>656</td>
<td>277</td>
<td>23.8</td>
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<tr>
<td></td>
<td>9 Sep.</td>
<td>627</td>
<td>190</td>
<td>16.4</td>
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</tr>
<tr>
<td></td>
<td>10 Sep.</td>
<td>674</td>
<td>140</td>
<td>10.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>11 Sep.</td>
<td>668</td>
<td>203</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>12 Sep.</td>
<td>667</td>
<td>218</td>
<td>16.9</td>
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</tr>
<tr>
<td>TOTALS</td>
<td></td>
<td>3644</td>
<td>1309</td>
<td>111.3</td>
<td></td>
</tr>
<tr>
<td>Test 2</td>
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<td></td>
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<td></td>
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<tr>
<td>New</td>
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<td>646</td>
<td>267</td>
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<tr>
<td>discharge</td>
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<td>658</td>
<td>294</td>
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<td>grates</td>
<td>12 Nov.</td>
<td>616</td>
<td>270</td>
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<tr>
<td>fitted</td>
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<td>651</td>
<td>137</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>20 Sep., 1982</td>
<td>671</td>
<td>157</td>
<td>4.0</td>
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<tr>
<td></td>
<td>19 Nov.</td>
<td>606</td>
<td>133</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20 Nov.</td>
<td>606</td>
<td>133</td>
<td>3.2</td>
<td></td>
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<tr>
<td></td>
<td>21 Nov.</td>
<td>677</td>
<td>85</td>
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<td>169</td>
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</tr>
<tr>
<td>TOTALS</td>
<td></td>
<td>5603</td>
<td>1630</td>
<td>42.7</td>
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</table>
suggests that some breakage of balls is taking place in the mill. A recent sample of ball remnants discharged from the mill was found to contain about 25 per cent of clearly identifiable fragments, thus confirming that a substantial degree of breakage occurs in the mill.

The pronounced initial decrease in the total ball consumption shown in Fig. 6 can be explained easily. The differential equations (6) and (8) show that the wear rate of a ball decreases with its mass. In the test, the initial mill charge comprised balls of only three different sizes: 60 mm, 50 mm, and 40 mm. The average ball mass was initially about 570 g, and a sample drawn from the mill at a later stage in the test gave an average mass of 333 g, which showed that, although only top-size balls were added, the average ball mass decreased as a function of the tonnage milled. Because the average ball size decreased, the average wear rate also decreased, and the consumption of balls per ton milled therefore decreased with increasing tonnage milled. This effect has its origin in the "artificial" size distribution of the balls comprising the initial charge to the mill. It is generally believed that a mill produces a graded size distribution of balls and that, when equilibrium is reached, the ball consumption becomes constant.

As already shown, the consumption of the background balls in the charge can be calculated with fair accuracy from the results of a marked-ball test. However, the question arises as to what the ball consumption would be if the mill were charged with balls of one of the other grades. It can be shown that the remnant term will be inversely proportional to the tonnage constant and that the total ball consumption will therefore also be inversely proportional to the tonnage constant. For example, if the consumption of grade 4 balls is 0.09 kg per ton milled, then the consumption of grade 6 balls will be 0.09 × 362/424 = 0.077 kg per ton milled, where \( \tau_6 = 424 \) is the tonnage constant for grade 6 balls. If, in addition, the cost-effective index, \( E_6 = C_6/\tau_6 \), where \( C_6 \) is the cost to the mine of grade 6 balls, is lower than that for grade 4 balls, it will be advantageous for the mine to change to grade 6 balls.

These calculations emphasize that the value of a marked-ball test is greatly enhanced when it includes marked balls that are of the same type as the charge.

Summary and Conclusions

In this work, the kinetics of ball wear were considered, and formulae for the dependence of the mass of a ball on the amount of material milled by a ball mill were derived from the volume and surface theories of ball wear. The theoretical expressions were compared in detail with the results obtained from a marked-ball test, and it was found that the values obtained by the use of both theories are in remarkably good agreement with the experimental data. This ambiguity can be explained if it is noted that the volume theory predicts an exponential decay, and the surface theory a cubic decay, of the mass of a ball with the amount of material milled, and that the graphical representation of such functions, which are of the same general shape, can always be brought into close correspondence over a limited range by suitable adjustment of the parameters.

One may be able to use ball-wear data to distinguish between the two theories if these data extend over a wider range, say \( m/m_4 \) of 10 or 20, and if the ball masses are monitored at more frequent intervals. However, it should be noted that, in large ball mills, which operate at speeds higher than 70 per cent of the critical speed, the balls are undoubtedly being subjected to the mechanisms of both abrasive and impactive wear.

The idea of a tonnage constant was found to be useful in the study of the relative performance of balls and their cost-effectiveness.

The marked-ball anomaly was found to arise from the fact that the measurements of ball consumption by a mill include the scrap that is discharged from the mill, whereas the consumption of marked balls relates only to the wear of balls that are still too large to pass through the discharge grates. The dependence of the calculated consumption of marked balls on time or tonnage milled is due merely to the fact that, as the balls become smaller, their wear rate decreases. Predictions of the ball consumption in a mill were made with fair accuracy from the results of a marked-ball test. It is emphasized that the usefulness of a marked-ball test is greatly enhanced when a set of marked balls identical to the background ball charge of the mill is included in the various sets of balls under test.

Acknowledgements

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References