A new method for the computation of heat and moisture transfer in a partly wet airway

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SYNOPSIS

This paper is concerned with heat flow in the plane of a typical cross-section of a deep underground airway.

A new algorithm, called the quasi-steady method, is presented for the calculation of rock temperatures in that plane. The algorithm is used in the computation of heat and moisture transfer rates from the rock to the ventilating air for an airway that is wet on part of its circumference and dry elsewhere. The speed and accuracy of the new method are demonstrated on a microcomputer, and it is argued that this method offers a considerably more versatile and physically meaningful alternative to previous methods for the prediction of increases in air temperature along mine airways.

The algorithm is also used to explore the concept of a uniformly wet airway that is equivalent to a partly wet airway in terms of its effect on the ventilating air. The concept is shown to be reasonable, but more complex than one might have anticipated.

Finally, several useful extensions and applications of the new method are suggested.

SAMEVATTING

Hierdie referaat handel oor hittevloei op die vlak van 'n tipiese dwarssnit van 'n diep ondergrondse luggang.

Daar word 'n nuwe algoritme, bekend as die kwasi-bestendige metode, voorgestel vir die berekening van rotstemperature op daardie vlak. Die algoritme word gebruik in die berekening van die hitte- en vogoordragtempo's van die rots na die ventilasielug, vir 'n luggang wat vir 'n gedeelte van sy omtrek nat is en elders droog. Die spoed en akkuraatheid van die nuwe metode word op 'n mikrorekenaar gedemonstreer en daar word aangevoer dat hierdie metode 'n heelwat meer veelsydige en fisies betekenisvolle alternatief vir vorige metodes vir die voorspelling van die styging van lugtemperature in mynluggange langs bied.

Die algoritme word ook gebruik om die konsep van 'n eenvormig nat luggang wat in terme van sy uitwerking op die ventilasielug, gelyk is aan 'n luggang wat gedeeltelik nat is, te ondersoek. Daar word getoon dat die idee redelik is, maar baie ingewikkelder as wat 'n mens sou verwag.

Ten slotte word daar verskeie nuttige uitbreidings en toepassings van die nuwe metode aan die hand gedoen.

Introduction

Heat flow through the rock surrounding a deep underground airway in the direction parallel to the length of the airway can be neglected in comparison with the heat flow in the plane of any cross-section of the airway. If, therefore, one can compute the heat flow in the plane of a typical cross-section, it is relatively easy for one to repeat the calculation and integrate the results numerically along the length of the airway. This paper is concerned exclusively with heat flow in a typical cross-sectional plane.

The problem of heat flow in the plane is characterized by the following.

(1) An internal boundary (the surface of the airway) in an infinite rock mass. If we represent the surface of the airway by a circle of radius a, then the rock is represented by the region $a \leqslant r < \infty$.

(2) Usually, part of the airway surface is wet and part is dry. Thus, in general, we do not have a symmetrical flow of radial heat into the airway, and the temperature $v(r, \theta, t)$ at any point (r, θ) in the rock will be a function of both the polar coordinates as well as time t since the airway was first ventilated.

(3) At the surface of the airway, sensible (dry) heat is convected from the rock to the ventilating air, moisture is evaporated from wet portions of the surface, and heat may be radiated from dry portions to the cooler, wet portions of the surface and the air itself. The equations describing these transfer processes are complex and non-linear.

These characteristics make it impossible for a formal mathematical solution to be found for the temperature pattern in the rock. The solving of the heat-flow equation (in two space variables and time) numerically, while conceptually straight-forward, is a slow and expensive process. This conflicts with what ventilation engineers really need for predicting temperature and humidity increases along mine airways: a fast and inexpensive computer program that can be used interactively on a microcomputer.

Up to now this conflict has been resolved in one of two ways.

(a) Starfield[1], Gibson[2], Amano *et al.*[3], and others reduced the problem to one of radial symmetry by using a uniformly damp airway to represent one that is in reality partly wet and partly dry.

(b) Starfield[4] used finite-difference techniques to solve a large set of partly wet cross-section problems numerically, 'massaged' the results into a fairly compact data base, and then developed a rapid program for the interpolation between those results and integration of the solution over the length of the airway.

Unfortunately, both these methods mask the physical processes that are actually taking place, For example, if one disagrees with the heat-transfer coefficients used by

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Starfield in his rapid program, there is no way of correcting this short of recomputation of the entire data base. Ideally, one would like to have a fast, approximate, but physically meaningful solution of the heat-flow problem in the plane of a cross-section. This paper introduces a new approach, called the quasi-steady algorithm, which has just those properties.

The Quasi-steady Algorithm

Heat conduction through isotropic rock is described by the diffusion equation

$$a\nabla^2 v = \partial v/\partial t, \quad (1)$$

where v is the temperature and a is the thermal diffusivity. In almost all airways, the termperature v changes slowly after a few months of the opening of the airway. If one could then neglect $\partial v/\partial t$, equation (1) would reduce to the much simpler Laplace's equation

$$\nabla^2 v = 0, \quad (2)$$

which leads to a steady-state solution, i.e. a solution that no longer varies with time. Unfortunately, there are two problems with this. In the first place, while $\partial v/\partial t$ is small, temperature changes with time are nevertheless significant. In the second place, the appropriate steady-state solution in the infinite plane $a \leqslant r < \infty$ does not exist. To obtain a steady-state solution, we must choose some boundary at $r = R$, say where R is presumably large, and postulate that the temperature at that boundary is constant and equal to the temperature of the virgin rock. The problem then is that the solution to equation (2) depends explicitly on the value we choose for R, and up to now there has been no rational method for its choice. We shall now develop an algorithm for finding R in such a way that it changes with time, thereby mimicking the slow temperature changes in the rock.

Consider the following problem in dimensionless form. The surface of a circular hole of unit radius is kept at unit temperature for all time $t > 0$. Heat flows radially into the surrounding rock, which has unit thermal conductivity and diffusivity, and is initially at zero temperature throughout. The mathematical formulation for the temperature $v\ (r, t)$ in the rock at radius r and time t leads to the equation

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\,\frac{\partial v}{\partial r} = \frac{\partial v}{\partial t}. \quad (3)$$

with initial condition $v(r, 0) = 0$ for all $r > 1$

and boundary conditions $v(1, t) = 1$ for all $t > 0$

and $v(r, t) \to 0$ as $r \to \infty$.

Let $G(t)$ be the thermal gradient that we obtain at the boundary $r = 1$ after we have solved this problem, i.e. $G(t) = -\partial v/\partial r$ at $r = 1$. This function has been tabulated by Goch and Patterson[5].

Suppose now that we neglect $\partial v/\partial t$ in equation (3). It then reduces to

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\,\frac{\partial v}{\partial r} = 0$$

with boundary conditions $v = 1$ at $r = 1$

and $v = 0$ at some far boundary $r = R$.

The solution is

$$v(r) = [\ln (R) - \ln (r)]/\ln (R), \quad (4)$$

and the thermal gradient at $r = 1$ is

$$-\partial v/\partial r = 1/\ln (R).$$

We can now choose R such that the thermal gradient in this case exactly matches the true thermal gradient $G(t)$ at time t, i.e. we put

$$1/[\ln R] = G(t) \text{ or } R = \exp [1/G(t)], \quad (5)$$

and then equation (4) will give us the correct value of the heat flux into the circular hole at any time t. We call this the quasi-steady solution because we have solved Laplace's equation instead of the time-dependent diffusion equation, but by (5) our far boundary $r = R$ is a function of time. The solution through time is thus like a movie film: each frame is in itself a steady-state solution, but the boundary R moves into the rock from one frame to the next. This is akin to the intuitive concept of a 'depth of cooling'.

The next step is to assume that, for a large class of problems, the position of the far boundary $R(t)$ is insensitive to the precise form of the boundary condition at the surface of the hole. If this assumption is correct, then we can find the quasi-steady solution to more complex problems at any time t merely by choosing $R(t)$ to satisfy equation (5) and solving Laplace's equation with $v = 0$ at $r = R(t)$ and the appropriate boundary condition at $r = 1$. This assumption is intuitive: we can, however, try to substantiate it by numerical testing.

Suppose, for example, that we have forced convection into air at unit temperature, leading to a boundary condition of the form

$$\partial v/\partial r = \eta(v-1) \text{ on } r = 1 \text{ for all } t > 0.$$

The solution of Laplace's equation that satisfies this as well as $v = 0$ on $r = R$ is

$$v = -\eta\ln (r/R)/[1 + \eta\ln (R)],$$

and hence the flux at the boundary $r = 1$ is

$$F = -\partial v/\partial r = \eta/[1 + \eta\ln (R)].$$

By substitution of $\ln (R) = 1/G(t)$ as in equation (5), this becomes

$$F = \eta G(t)/[\eta + G(t)]. \quad (6)$$

(Notice that as $\eta \to \infty$, $F \to G(t)$, which is correct.)

This example is an important one; it is, in fact, the dimensionless form of the problem of radially symmetric heat transfer into a circular airway. The flux is a function of the dimensionless heat-transfer coefficient η and the dimensionless time $\tau (= at/a^2)$, and has been tabulated by Starfield[6]. Equation (6) was compared with Starfield's table and the percentage error is plotted in Fig. 1 as a function of η and τ. The results are very good indeed for sufficiently large times, although, for small times, the error introduced by the assumption $\partial v/\partial t = 0$ cannot be ignored. However, for established airways, τ is nearly always greater than 20, in which case, as is apparent from Fig. 1, the quasi-steady solution is correct to within 3 per cent. Equation (6) thus provides an attractive solution for radially symmetric heat flow into a circular airway, replacing Starfield's table by a simple formula in terms of the Goch–Patterson flux G. For computational purposes, the function G itself can be approximated by an equally simple algorithm, which is given in Addendum I, or else by a polynomial approximation[7].

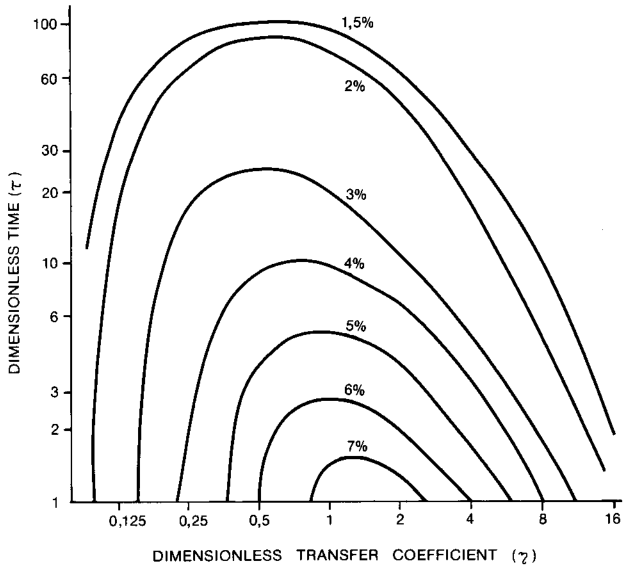

Fig. 1—Percentage error introduced by the use of the quasi-steady method in the calculation of the heat flux into a circular airway

This success encourages one to develop and test the quasi-steady algorithm for the more complex problem of a partially wet airway.

Quasi-steady Solution for a Partially Wet Airway

We consider an airway that is circular in cross-section. The bottom portion, which subtends an angle 2β at the centre of the circle, is wet, with wetness factor f, while the remainder of the surface is perfectly dry (Fig. 2). The wetness factor is defined by Starfield[1] and can vary from 0 (dry) to 1 (thoroughly wet).

At a dry segment of the surface, there is convective transfer of heat from the rock to the ventilating air, and also radiative heat transfer to the cooler, wet surface and, to a lesser extent, to the air as well. The heat flux at the surface $r = a$ for $|\theta| > \beta$ is thus of the form

$$k \frac{\partial v}{\partial r} = H(v_s - v_D) + K(v_s - v_{wet}) , \quad \quad (7)$$

where the symbols are as defined in Addendum II.

At a wet segment of the surface, radiative heat is absorbed, moisture is evaporated, and sensible (dry) heat is convected to (or, in some cases, from) the ventilating air. The heat flux at $r = a$ for $|\theta| \leqslant \beta$ is thus

$$k \frac{\partial v}{\partial r} = H(v_s - v_D) + K'(v_s - v_{dry}) + fLE[p_{sat}(v_s) - p]. \quad \quad (8)$$

To obtain the quasi-steady solution, we must solve Laplace's equation for the rock temperature $v(r, \theta)$ subject to the boundary conditions (7) and (8), plus the additional condition that v is equal to v_R, the temperature of virgin rock, at the outer boundary $r = R$. By analogy with equation (5), at time t the position of R is given in terms of the Goch–Patterson flux by

$$R = a \exp[1/G(\alpha t/a^2)].$$

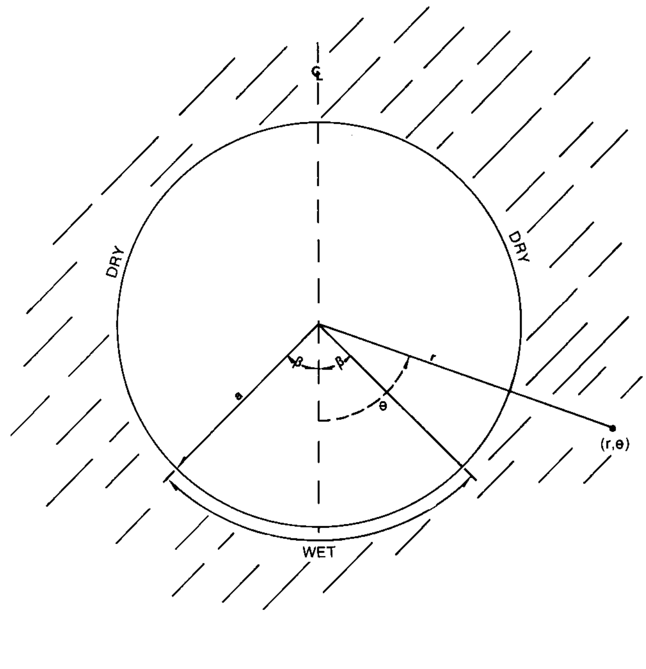

Fig. 2—Assumed geometry for the cross-section of an airway

The quasi-steady solution is derived in detail in Addendum III in the form of a Fourier series. In particular, the surface temperature $v_s = v(a, \theta)$ is given by

$$v_s = v_R + A_0 + \sum_{n=1}^{N} A_n \cos(n\theta), \quad . \ . \ . \ . \ . \ . \quad (9)$$

where the algorithm for the computation of the coefficients $A_0, A_1, \ldots\ldots, A_n$ is described in Addendum III.

As a test of this solution, results obtained from equation (9) were compared with an explicit finite difference solution of the fully time-dependent diffusion equation subject to boundary conditions (7) and (8) and the initial condition $v = v_R$ at $t = 0$ for all r and θ. As a test case, we chose input data that corresponded closely to an example given in Appendix B of the paper by Starfield and Dickson[8], although in their case they had obtained a finite difference solution for a square (in cross-section) airway. Specifically, the input data were as follows:

$k = 5{,}4$ W/mK $\alpha = 2{,}6 \times 10^{-6}$ m²/s

$K = 1{,}1$ W/m²K $E = 6{,}7 \times 10^{-8}$ kg/m²s Pa

$v_D = 35{,}0$ C $v_W = 23{,}9$ C

$L = 2{,}42$ kJ/g $a = 1{,}75$ m

$H = 13{,}1$ W/m²K $v_R = 43{,}3$ C

$P = 100$ kPa $t = 0{,}375$ year,

and it was assumed that the bottom quarter of the surface (i.e. $\beta = \pi/4$) representing the footwall was wet.

Fig. 3 shows how the surface temperature varies with angle θ for the two cases of a damp footwall (wetness factor $f = 0{,}2$) and a thoroughly wet footwall ($f = 1{,}0$). Superimposed on the finite difference solution are the results obtained by the quasi-steady method using 20 terms ($N = 19$) in equation (9). The comparison is excellent, and it should be noted that this test case is an exacting one: firstly, the large gap between the wet- and the dry-bulb air temperatures leads to a large temperature difference between the wet and dry segments of the surface; and, secondly, the dimensionless time parameter τ is equal only to 10,0.

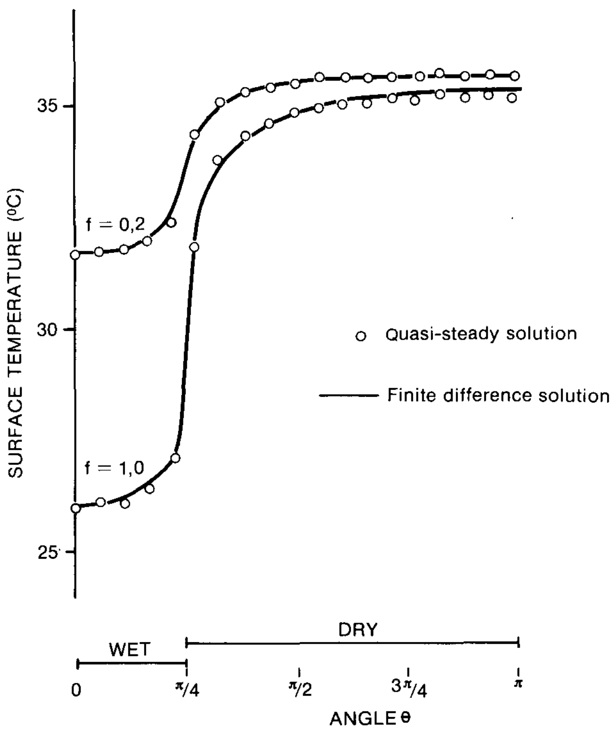

Fig. 3—Comparison between finite difference and quasi-steady solutions

It should also be noted that, while the finite-difference calculations for the graphs in Fig. 3 took about 20 hours each on an IBM personal computer, the 20-term quasi-steady solutions were obtained in under 2 minutes each. Moreover, the only purpose for the taking of 20 terms in the series was so that small fluctuations in the surface temperatures could be 'ironed' out; in practice, one's objective is to compute total heat- and moisture-transfer rates, and one needs at most 7 or 8 terms of the series to compute these accurately. The computation time then drops to less than 10 seconds. This makes it entirely feasible for one to divide a long airway into a number of segments and to repeat the quasi-steady solution for each segment as one integrates the effect of the heat and moisture transfer on the ventilating air as it flows from one segment to the next. This would lead to a new, improved, and considerably more versatile version of Starfield's rapid program[4] for the prediction of temperature increases in the ventilating air.

At this point it is useful to pause and consider the reliability of the quasi-steady algorithm. We have not actually proved that the method works, although we have shown that it gives excellent results in the test cases presented in this paper, as well as others not described here. The closest approach to the algorithm in the literature is to be found in the diffusion problems of moving boundaries[9], where a steady-state assumption is exploited and can be justified only by comparisons with accurate solutions. Intuitively, we can see why the quasi-steady algorithm should work: it does, indeed, satisfy the boundary conditions at the surface of the airway exactly, while the thermal resistance of the rock that has already cooled is approximated in a plausible way by the algorithm for specifying the position of $R(t)$ in terms of the Goch–Patterson flux. Intuitively, we can also see when the quasi-steady algorithm would be suspect. The assumption $\partial v/\partial t = 0$ implies that the flux in at the boundary $r = R$ exactly matches the flux out of the rock at $r = a$. Apart from being inaccurate for small times, the quasi-steady algorithm will therefore not be able to cope with fluctuating air temperatures or sudden changes in temperature with time (e.g. seasonal and daily fluctuations, or changes induced by the switching on and off of fans or refrigeration units). Starfield's rapid program[4] was not designed to cope with these conditions, either.

The Concept of Equivalent Wetness

In the Introduction it was mentioned that some investigators have approximated an airway that is wet on part of its circumference by one that is uniformly damp. Now that we have derived a fast and accurate method for solving the original problem, there would seem to be little need for investigating the uniform approximation. There are, however, two good reasons for doing this: (i) the quasi-steady algorithm cannot cope with fluctuating air temperatures, whereas a suitable model of a uniformly damp airway could, and (ii) to facilitate comparisons with work of those investigators[1–3] who have used a uniformly damp model.

Consider two circular airways that are identical in all regards except that the first is wet (with wetness factor f) only on that portion of the circumference subtending an angle 2β at the centre (as in Fig. 2), while the second is uniformly wet with wetness factor ϕ. If a value of ϕ can be found such that *both* the heat *and* the moisture transfer from rock to air in the two airways are identical, then the second airway can indeed be used as a model of the first. We then call ϕ the 'equivalent wetness factor' and, as a first guess, would anticipate that $\phi = \beta f/\pi$.

The quasi-steady algorithm presented in Addendum III enables one to test this idea numerically. First, it can be used to solve the real problem of a partly wet airway, and then solutions for a uniform airway can be obtained merely be specifying $\beta = \pi$ and changing f to ϕ. In the latter case, it is necessary only to take the first term in the Fourier series solution.

Comparisons were made for the test case presented in the previous section. Table I shows that, while an appropriate value of ϕ can indeed be found, it is considerably lower than the anticipated value of $\beta f/\pi$, which, if used, would predict the total heat transferred from rock to air (sensible heat plus the latent heat required for evaporation) fairly accurately, but would distort the partition between sensible and latent heat. In fact, the predicted moisture transfer could be out by nearly 100 per cent when the formula $\phi = \beta f/\pi$ is used.

A number of results are summarized in Fig. 4. In each case, a value of ϕ was found that matched both the sensible and the latent heat transfer for the real problem of a partly wet surface. Fig. 4 shows a plot of π/β times that value of ϕ versus f. If the formula $\phi = \beta f/\pi$ were valid, all the points would lie on the straight line for the case $\beta = \pi$. Fig. 4 thus highlights the inadequacies of this formula, although it does indicate that the formula may be adequate for small values of f.

Fig. 4 does, however, establish the concept of an equivalent wetness factor, even if finding a value for it is more complex than one might at first imagine. Depending on how sensitive the graphs in Fig. 4 are to the parameters of a particular airway, this concept might or might not be practically useful. This is currently the subject of further research.

Concluding Remarks

The new algorithm presented here provides fast and accurate solutions for a general class of problems associated with heat flow in a plane. In particular, the algorithm (i) replaces tables for radially symmetrical heat flow into circular airways by a simple formula that is accurate for all but small times, and (ii) leads to an attractive computer program (again for all but small times) for solving the problem of a circular airway that is wet on only a part of its circumference. The latter solution can easily be repeated and hence incorporated into a larger program that will compute temperature and humidity increases from one segment to another along the entire length of an airway. This offers a more accurate and considerably more versatile and physically meaningful alternative to Starfield's rapid program[4].

The new algorithm can be used to explore the errors introduced by the use of a uniformly wet model to approximate an airway that is, in fact, wet on only a portion of its circumference. The concept of an equivalent wetness factor that was introduced is appropriate provided one is not naive in estimating it.

Several extensions of the work described in this paper are worthy of further investigation. It may, for example, be possible to extend the quasi-steady algorithm to airways that are not approximately circular in cross-section and to airways in anisotropic rock. The algorithm could definitely be used in a study of the effect of thermal insulation over part, but not all, of the perimeter of an airway. Finally, the concept of an equivalent wetness factor needs to be explored further with the objective of finding ways of computing the effects of fluctuations or sudden changes with time in inlet air temperatures and/or the velocity of the ventilating air on subsequent heat and moisture gradients.

Addendum I: Algorithm for Reproducing the Goch–Patterson Table

The following algorithm reproduces the Goch–Patterson[5] flux, which we have called $G(\tau)$, to within 1 per cent for $1{,}5 < \tau < 1000$, and 2 per cent for $\tau > 1000$.

If $1{,}5 < \tau < 10$, then

$$G(\tau) = 1{,}0/[0{,}979813 + 0{,}383760 \ln(\tau)];$$

if $10 < \tau < 100$, then

$$G(\tau) = 1{,}0/[0{,}839337 + 0{,}444718 \ln(\tau)];$$

if $100 < \tau < 1000$, then

$$G(\tau) = 1{,}0/[0{,}683043 + 0{,}479054 \ln(\tau)];$$

and if $\tau > 1000$, then

$$G(\tau) = 2z(1 - z - z^2 - z^3)/0{,}57722,$$

where $z = 0{,}57722/[\ln(4\tau) - 1{,}15444]$.

Addendum II: Notation

Geometry

a	Radius of the airway
r	Distance from the centre of the airway
θ	Angle as measured from the mid-point of the airway floor
β	Half the angle subtended at the centre by the wet or damp portion of the surface
R	Radius to the outer boundary where, according to the quasi-steady algorithm, the rock can be assumed to be at the temperature of virgin rock.

TABLE I

THE CONCEPT OF EQUIVALENT WETNESS

Description	β/π	Wetness factor for wet portion of circumference	Transfer rates per length of airway: Total heat W/m	Dry heat W/m	Moisture g/ms
Damp footwall, rest dry	0,25	0,2000	157	−63	0,091
Equivalent airway, $\phi = \beta f/\pi$	1,00	0,0500	164	−111	0,114
Equivalent airway, correct ϕ	1,00	0,0386	158	−62	0,091
Wet footwall, rest dry	0,25	1,0000	194	−346	0,223
Equivalent airway, $\phi = \beta f/\pi$	1,00	0,2500	236	−677	0,377
Equivalent airway, correct ϕ	1,00	0,1146	194	−346	0,223

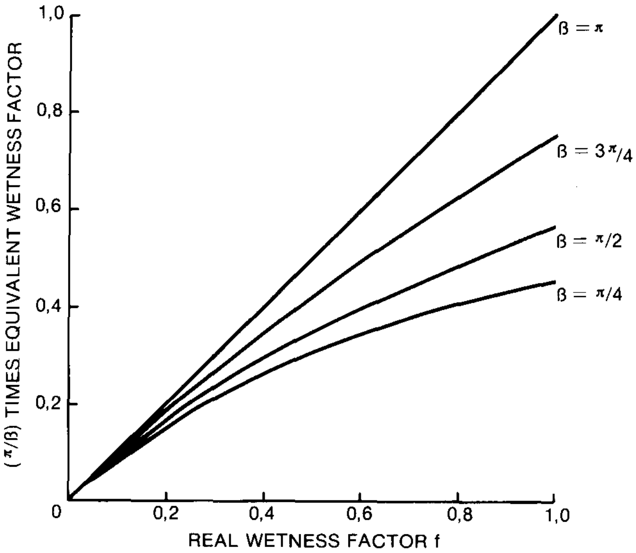

Fig. 4—Testing of the concept of equivalent wetness for different values of the angle β.

Psychrometry

v_D	Dry-bulb temperature of the ventilating air
v_W	Wet-bulb temperature of the ventilating air
P	Air pressure
$P_{sat}(x)$	Saturated vapour pressure at temperature x
$P'_{sat}(x)$	Slope of the curve for saturated vapour pressure at temperature x
p	Partial pressure of the vapour in the air
L	Latent heat of evaporation of water.

Rock properties and temperatures

a	Thermal diffusivity of the rock
k	Thermal conductivity of the rock
v_R	Temperature of the virgin rock
v	$v(r, \theta)$ Temperature at point (r, θ) in the rock
v_s	$v(a, \theta)$ Temperature of rock surface at angle θ
v_{dry}	Average temperature of the dry section of the rock surface
v_{wet}	Average temperature of the wet section of the rock surface.

Heat- and mass-transfer coefficients

H	Surface heat-transfer coefficient, including both convective and radiative heat transfer from the rock to the air; see, for example, Appendix A of Starfield and Dickson[8]
E	Coefficient of surface mass transfer
K	Overall coefficient for radiation from the dry to the wet portion of the rock surface
K'	Overall coefficient for radiation from the wet to the dry portion of the rock surface, $K' = K(\pi - \beta)/\beta$
η	Dimensionless heat-transfer coefficient $\eta = Ha/k$.

Miscellaneous

f	Wetness factor, varying from 0 for a perfectly dry surface segment to 1 for a thoroughly wet segment
ϕ	Equivalent wetness factor for a uniformly damp airway
t	Time, age of airway
τ	at/a^2 Dimensionless time
$G(\tau)$	The flux at dimensionless time τ as obtained from the tables of Goch and Patterson[5].

Addendum III: Quasi-steady Algorithm for a Partly Wet Airway

With reference to Fig. 2, we introduce a function $g(\theta)$, which has the value 1 on the wet portion of the circumference and is 0 on the dry portion, i.e.

$$g(\theta) = 1 \text{ for } |\theta| \leqslant \beta$$
$$= 0 \text{ for } |\theta| > \beta.$$

This function enables us to combine the boundary conditions, equations (7) and (8), into the single equation

$$k\frac{\partial v}{\partial r} = H(v_s - v_D) + g(\theta)\,[K'(v_s - v_{dry})] + fLE\{p_{sat}(v_s) - p)\} + [1-g(\theta)]K(v_s - v_{wet}), \quad \text{(A1)}$$

which holds on $r = a$ for all θ.

Since v_s at all points on the wet surface will be close to the average temperature of the wet surface v_{wet}, we can write

$$p_{sat}(v_s) = p_{sat}(v_{wet}) + (v_s - v_{wet})p'_{sat}(v_{wet}), \quad \text{(A2)}$$

where p'_{sat} is the slope of the curve for saturated vapour pressure.

Substituting (A2) in (A1), dividing throughout by k, and separating out terms in v_s, $g(\theta)$, and v_s times $g(\theta)$, we obtain an equation of the form

$$\left(\frac{\partial v}{\partial r}\right)_{r=a} = q_1 + q_2 v_s + q_3 g(\theta) + q_4 v_s g(\theta), \quad \text{(A3)}$$

where the q's are all independent of θ.

We shall find it convenient to expand $g(\theta)$ in a Fourier series in $[-\pi, \pi]$, i.e. we can write

$$g(\theta) = B_0 + \sum_{n=1}^{\infty} B_n \cos(n\theta)\,, \quad \ldots\ldots\ldots \quad \text{(A4)}$$

where $B_0 = \beta/\pi$

and $B_n = (2/n\pi)\sin(n\beta)$ for $n = 1,2,\ldots$ (A5)

Now it can be shown that

$$v(r, \theta) = v_R + c_0 \ln(r/R) + \sum_{n=1}^{\infty} c_n[(r/R)^n - (R/r^n)]\cos(n\theta), \quad \ldots \quad \text{(A6)}$$

satisfies Laplace's equation, as well as the condition $v = v_R$ at $r = R$ for all θ. It remains only to choose the c's so that equation (A3) is also satisfied.

If we write $v_s = v(a, \theta) = v_R + A_o + \sum_{n=1}^{\infty} A_n \cos(n\theta)$ . . . (A7)

and $\left(\frac{\partial v}{\partial r}\right)_{r=a} = A_o \gamma_o + \sum_{n=1}^{\infty} A_n \gamma_n \cos(n\theta)$, (A8)

then from (A6) it can be shown that

$$\gamma_o = 1/[a \ln(a/R)]$$

and $\gamma_n = (n/a)[(a/R)^{2n} + 1]/[(a/R)^{2n} - 1]$
for $n = 1, 2,$ . . . . (A9)

Unless we want to actually compute the temperature profiles in the rock, it will suffice to find values for $A_o, A_1, \ldots$ without ever evaluating $c_o, c_1, \ldots$

Substituting (A4), (A7), and (A8) in equation (A3) gives

$$A_o\gamma_o + \Sigma A_n\gamma_n \cos(n\theta) = q_1 + q_2\{v_R + A_o + \Sigma A_n \cos(n\theta)\}$$
$$+ q_3\{B_o + \Sigma B_n \cos(n\theta)\} + q_4\{B_o + \Sigma B_n \cos(n\theta)\}$$
$$\{v_R + A_o + \Sigma A_n \cos(n\theta)\}$$

and matching like terms in $\cos(n\theta)$ for $n = 0, 1, 2, \ldots, N$ leads to a set of linear equations that can be solved for $A_o, A_1, \ldots, A_N$. We can write these equations

$$\sum_{n=0}^{N} G_{mn} A_n = C_m \qquad m = 0, 1, 2, \ldots N, \quad \text{(A10)}$$

where $C_o = q_1 + q_2 v_R + B_o(q_3 + q_4 v_R)$

$G_{oo} = \gamma_o - q_2 - q_4 B_o$.

If $n \neq 0$,

$C_n = B_n(q_3 + q_4 v_R)$

$G_{nn} = \gamma_n - q_2 - q_4 B_o - \frac{1}{2} q_4 B_{2n}$,

and, if both $n \neq 0$ and $m \neq 0$,

$G_{mn} = -\frac{1}{2} q_4 (B_{n+m} + B_{|n-m|})$

$G_{mo} = -q_4 B_m$ . . . . (A11)

and $G_{on} = -\frac{1}{2} q_4 B_n$.

Our algorithm for solving the problem thus proceeds as follows:

Step 1: Choose a value for N

Step 2: Compute $\gamma_o, \gamma_1, \ldots, \gamma_N$ from equation (A9) and $B_o, B_1, \ldots, B_{2N}$ from (A5).

Step 3: Guess at values for v_{dry} and v_{wet}

Step 4: Compute q_1, q_2, q_3, and q_4

Step 5: Compute all the C's and G's in equation (A11)

Step 6: Solve the $N + 1$ linear equations in (A10) for $A_o, A_1, A_2, \ldots, A_N$

Step 7: Compute better values of v_{dry} and v_{wet} from the formulae:

$$v_{dry} = v_R + A_o - \sum_{n=1}^{N} [A_n/n(\pi - \beta)] \sin(n\beta)$$

$$v_{wet} = v_R + A_o + \sum_{n=1}^{N} (A_n/n\beta) \sin(n\beta)$$

Step 8: Repeat steps 4 to 7 until the values of v_{dry} and v_{wet} emerging from step 7 are close to those used in step 4. In practice, 'close' means to within about half a degree, and this can usually be achieved in one or two iterations.

Once the A's are known, the surface temperatures can be computed from equation (A7) if one wants to find them explicitly. The heat and moisture transfer from rock to air can, however, be calculated directly from the final values for v_{dry} and v_{wet}, equation (A12), as follows: The sensible (dry) heat transfer per unit time and unit length of airway is

$2H(\pi - \beta)a(v_{dry} - v_D)$ from the dry surface

and $2H\ \beta a(v_{wet} - v_D)$ from the wet surface,

and the mass of moisture evaporated per unit time and unit length of the airway is

$2\beta a f E\{p_{sat}(v_{wet}) - p\}$.

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