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Energy considerations in rock mechanics: fundamental results

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SYNOPSIS

If a transition from one equilibrium state to another occurs during the course of mining, energy transfer takes place in the rock mass, which is assumed to be an elastic continuum. The energy components are defined as follows: the work done by external and body forces, W; the increase in stored energy in the mass, U_c; the strain energy in the rock mined during the transition, U_m; the work done on mine support or backfill, W_s; and the released energy, W_r. The energy balance is defined as $(W + U_m) - (U_c + W_s) = W_r > 0$, where the first set of components in parentheses represents the sources of energy, while that of the second corresponds to the known modes of energy expenditure and the component on the right is the unaccounted-for energy surplus, which must be released and dissipated in some form. The inequality $W_r \geqslant U_m > 0$ applies.

Expressions for the energy components are derived first for the general case and then for the 'slit' model used to simulate tabular excavations.

Through specific examples, the importance of the progressive nature of mining is emphasized. The size of the steps used to enlarge the excavations appears to play a significant role in the mode of energy transfer. This observation forms the basis of the second paper in this series of two papers.

SAMEVATTING

As daar in die loop van ontginning 'n oorgang van een ewewigstoestand na 'n ander voorkom, vind daar energie-oordrag plaas in die rotsmassa wat veronderstel word 'n elastiese kontinuum te wees. Die energiekomponente wat omskryf word, is soos volg: die arbeid verrig deur uitwendige en liggamskragte, W; die toename in die opgebergde energie in die massa, U_c; die vervormingsenergie in die rots wat ontgin word tydens die oorgang, U_m; die arbeid verrig op die mynbestutting of terugvulsel, W_s; en die vrygestelde energie, W_r. Die energiebalans word omskryf as $(W + U_m) - (U_c + W_s) = W_r > 0$, waar die eerste stel komponente tussen hakies die energiebronne voorstel, terwyl dié van die tweede stel ooreenstem met die bekende wyses van energiebesteding en die komponent aan die regterkant die energie-oorskot is waarvan daar nie rekenskap gegee word nie en wat vrygestel en in die een of ander vorm gedissipeer *moet* word. Die ongelykheid $W_r \geqslant U_m > 0$ is van toepassing.

Uitdrukkings vir die energiekomponente word eers vir die algemene geval afgelei en dan vir die 'gesplete' model wat gebruik word om tafelvormige uitgrawings na te boots.

Die belangrikheid van die progressiewe aard van ontginning word met behulp van spesifieke voorbeelde benadruk. Die grootte van die trappe wat gebruik word om die uitgrawings te vergroot, speel blykbaar 'n belangrike rol in die wyse van energieoordrag. Hierdie waarneming dien as grondslag vir die tweede referaat in hierdie reeks van twee referate.

Introduction

More than twenty years ago, Cook[1] called attention to the significant energy changes that take place during mining. In the synopsis of his paper, he stated that 'the excess potential energy causes the damage noticed as rockburst'. During the intervening two decades, various energy quantities, especially the energy release rate, have become important tools of those engineers and scientists who strive to combat the rockburst hazard.

In view of the importance of energy considerations, it is surprising that so far no comprehensive treatment of the subject has appeared in print. The first reasonably rigorous discussion, although it concentrated only on tabular excavations, was given by Cook *et al*[2]. in their synthesis of the results of rockburst research in South Africa up to 1965. Soon after the appearance of that paper, Cook[3] published a new exposition of the subject, which is somewhat less than rigorous, but widens significantly the application of energy principles.

A reasonably comprehensive discourse on the energy changes in mining was prepared in 1973, in which new light was thrown on several aspects of the subject. However, for various reasons, only a much abbreviated version of this study was published in the following year[4]. In more recent times, others have made further contributions[5,6], giving exposure to other features of the topic.

The purpose of the present study is to examine the energy problem rigorously: firstly, to provide a sound foundation for the energy approach to mine design and, secondly, to furnish rockburst research with an acceptable theoretical basis. It is postulated throughout the paper that the behaviour of the rock mass surrounding mining excavations is linearly elastic, but not necessarily homogeneous or isotropic. It is recognized that rock in the close vicinity of cavities is frequently fractured and is therefore no longer an elastic continuum. Nevertheless, an elastic study does provide a limiting case of behaviour and reveals trends that have important practical applications. The work presented here incorporates the results of recent research.

Mining is a progressive activity. Excavations in mines are changed in shape and made to grow in size with time. It is logical, therefore, for an investigator to examine the energy changes resulting from a specific change in the mining geometry, rather than to analyse, as others have done, the energy transformations that would occur if mining from the virgin state to the current geometry were to take place in one step.

The study is presented in two parts. In the first paper,

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general results are derived and discussed. The second paper will be devoted to the analysis of energy changes in such instances where mining progresses in small steps. An attempt will be made to emphasize the practical significance of energy considerations in the study of problems involving tabular excavations.

Definitions

Enlargement of Excavations

Two arbitrary states of mining are defined, with the aim of clarifying energy changes resulting from the additional mining that transfers the geometry from state I to state II.

Consider a part of the semi-infinite rock mass enveloped by surface S and enclosing all mining excavations in states I and II (Fig. 1). The volume enclosed by S is $V_o + V_m + V$, where V_o is the volume of all existing excavations in state I, V_m is the volume of rock to be mined in order to cause the transition from state I to state II, and V is the volume of rock within S that remains unmined in state II. It is postulated that, both in state I and in state II, the forces present in the system are in *equilibrium*.

In general, the stress or traction vector acting on a surface, T_i, is defined in terms of the unit interior normal to the surface, μ_j, and the components of the stress tensor, say τ_{ij}, that $T_i = \mu_j \tau_{ij}$ $(i,j = 1,2,3)$. Here and in the sequel, the summation convention of tensor notation is used. In this paper, compressive stresses are taken to be positive. A consequence of this convention is that the positive displacement represents movement towards the negative direction of the co-ordinate axes. The sign convention employed is explained in some detail in Addendum I.

Now, consider state I first (Fig. 1a). The components of the displacement vector in this state are denoted by $u_i^{(p)}$ and those of the stress tensor by $\tau_{ij}^{(p)}$. In these notations superscript p stands for 'primitive' or 'of the earlier state'. In state I the volume of excavations totals V_o and the surface of these cavities is S_o. A part of this surface, say S_o^*, is supported by some material with a volume of V_o^* and the remainder is traction-free. The effect of the support is the traction vector of the support reaction, denoted by $R_i^{(p)}$. Assume that the body force per unit volume in the rock is X_i and that in the support is X_i^*. Having defined the forces affecting volume V in state I, the equation of equilibrium of this body takes the following form:

$$\int_{S+S_m} T_i^{(p)}\,dS + \int_{S_o^*} R_i^{(p)}\,dS + \int_V X_i\,dV = 0, \quad \ldots\ldots\ldots \quad (1)$$

where S_m is the unexposed part of the bounding surface of V_m. If a part or all of volume V_m is connected to and therefore forms part of the extension of existing cavities, then the already exposed part of the bounding surface of V_m is assumed to be traction-free.

Next, consider state II (Fig. 1b). The difference between this and the earlier state is brought about by the removal of the rock from volume V_m and by the introduction of the additional support of volume V_m^*. Symbols of displacement, stress, and traction components lack superscript in state II. Now the equation of equilibrium of volume V can be put into the following form:

$$\int_S T_i\,dS + \int_{S^*} R_i\,dS + \int_V X_i\,dV = 0, \quad \ldots\ldots\ldots\ldots \quad (2)$$

where $S^* = S_o^* + S_m^*$, and S_m^* is the additional supported area. Note that, on the freshly exposed rock surface, that is on S_m, traction $T_i = 0$. This does not necessarily mean that surface S_m is traction-free, as part of the new surface might be supported and thus form part of S_m^*, in which case the support reaction R_i would apply to it.

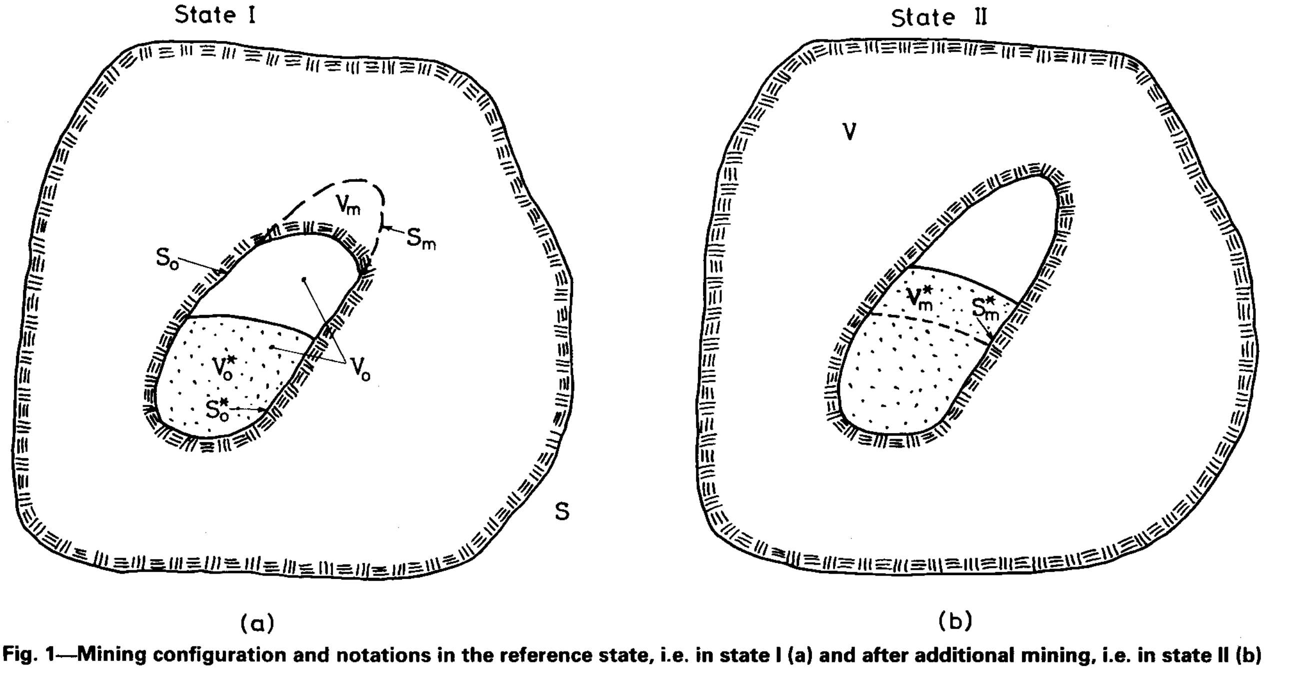

Fig. 1—Mining configuration and notations in the reference state, i.e. in state I (a) and after additional mining, i.e. in state II (b)

At this stage it will be advantageous to introduce the notion of *induced* components. Observe that it is always possible to write

$$u_i = u_i^{(p)} + u_i^{(i)}, \quad \tau_{ij} = \tau_{ij}^{(p)} + \tau_{ij}^{(i)}, \quad T_i = T_i^{(p)} + T_i^{(i)}, \; . \; . \quad (3)$$

where the components with superscript i are changes *induced* by additional mining.

If the equation in (1) is subtracted from the equation of equilibrium in (2), the result is

$$\int_S T_i^{(i)}\, dS - \int_{S_m} T_i^{(p)}\, dS + \int_{S^*} R_i^{(i)}\, dS = 0. \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

This relationship is the equation of equilibrium of volume V in terms of the *induced system of forces*. The second term, however, contains a primitive component, suggesting perhaps some inconsistency. This term comes about because, as was noted earlier, $T_i = T_i^{(p)} + T_i^{(i)} = 0$ on S_m. It follows from this relationship that

$$T_i^{(i)} = -T_i^{(p)} \quad \text{on } S_m. \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

Thus, there is no inconsistency. On the newly exposed surface $-T_i^{(p)}$ is, in fact, the induced traction.

Note that, in practical applications, the enveloping surface S usually consists of a part of the ground surface and some remote surfaces in the rock mass. It is postulated that traction $T_i^{(p)}$ does not change on S owing to some active loading mechanism, but merely alters as a result of the excavation of rock from V_m and the introduction of support into V_m^*. However, equilibrium considerations require that

$$\int_S T_i^{(i)}\, dS = \int_{V_m} X_i\, dV - \int_{V_m^*} X_i^*\, dV, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

which means that the integral of $T_i^{(i)}$ on S cannot be zero regardless of how far S is removed from the excavations. There are two exceptions to this rule: the integral of $T_i^{(i)}$, or in fact the traction vector itself, can be zero provided that either $V_m = V_m^* = 0$ or the body forces are zero.

In this section mine support is treated as some form of continuum that resists deformation. Continuous linings of tunnels and backfill in stopes satisfy this description, but some other support systems do not. The equations could, of course, be modified to accommodate the behaviour of any type of support system, for example that of individual props. However, no significant advantage would be gained by introducing such refinements.

It is postulated that a support, once introduced, is never removed; that is, the increment S_m^* is always positive. Naturally, an analogous statement applies to volume V_m since the mining of a volume of rock is an irreversible process.

States I and II can be chosen arbitrarily. Moreover, state II logically becomes state I for the next step of mining. A particular instance of state I is the 'virgin' state. This choice of state I leads to the computation of the components of *total* energy change. In the sequel, whenever this seems necessary, the virgin state will be indicated by the subscript v.

It is important to record a number of practical conclusions that emerge from the discussion in this section:

(i) it follows from (4) that the induced system is free of body forces,
(ii) field observations initiated in state I can yield only the induced components, that is $u_i^{(i)}$ and $\tau_{ij}^{(i)}$,
(iii) stress components $\tau_{ij}^{(i)}$ can be obtained through measurement of the induced strain component, and
(iv) stress components $\tau_{ij}^{(p)}$ and τ_{ij} can be obtained only through stress-relief methods, and these measurements must be made in state I for the determination of $\tau_{ij}^{(p)}$ and in state II for the acquisition of τ_{ij}.

In the sequel it will be convenient to refer to symbols without superscript, to those with superscript i, and to those with superscript p as *resultant* (or total), *induced*, and *primitive* components of stress or of displacement, etc. respectively. To avoid confusion it should be noted that the terms 'primitive' and 'induced' in the literature usually apply to quantities in the virgin state and to the stress or other changes relative to the virgin state, respectively. The terminology suggested here seems more logical and fits the progressive nature of mining more closely.

Significant Energy Components

The purpose of this section is to identify sources of energy during the transition from state I to state II, to elucidate the mechanisms through which energy is expended, and to define the quantity of energy, if any, which is not accounted for.

The most obvious source of energy is the *work done by the external and body forces when acting through the induced displacements.* This work, which is denoted by W, can be expressed in the following form:

$$W = -\Big[\int_S \Big(T_i^{(p)} + \tfrac{1}{2}T_i^{(i)}\Big) u_i^{(i)}\, dS + \int_V X_i u_i^{(i)}\, dV +$$

$$\int_{V_o^*} X_i^*\, u_i^{(i)}\, dV + \tfrac{1}{2}\int_{V_m^*} X_i^* u_i^{(i)}\, dV\Big], \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

where the negative sign is a consequence of the sign convention mentioned earlier. Here, the first term represents the work done by the external forces, which are *gravitational* and/or *tectonic* in origin. The remaining terms symbolize the contributions by body forces.

When an elastic body is strained, energy in the form of 'strain energy' is stored in it. The strain energy can be expressed as the integral of the strain energy density function Φ. This function defines the strain energy stored in a unit volume of the stressed material, and it can be expressed in the following form:

$$\Phi = \tfrac{1}{2}\,\tau_{ij} e_{ij} > 0 \qquad i,j = 1,2,3, \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

which is always positive and where e_{ij} are the components of the strain tensor:

$$e_{ij} = \tfrac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big). \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

The *change* caused in the strain energy stored in volume V by the extraction of the rock from volume V_m equals the difference between the energy content of V in state II, which is U, and its energy content in state I, which is $U^{(pp)}$. The change in strain energy in V is denoted by U_c.

In state II the density function of the strain energy is given as

$$\begin{aligned}\Phi &= \tfrac{1}{2}\tau_{ij}\, e_{ij} = \tfrac{1}{2}\left(\tau_{ij}^{(p)} + \tau_{ij}^{(i)}\right)\left(e_{ij}^{(p)} + e_{ij}^{(i)}\right)\\ &= \tfrac{1}{2}\left(\tau_{ij}^{(p)} e_{ij}^{(p)} + \tau_{ij}^{(i)} e_{ij}^{(p)} + \tau_{ij}^{(p)} e_{ij}^{(i)} + \tau_{ij}^{(i)} e_{ij}^{(i)}\right)\\ &= \Phi^{(pp)} + \Phi^{(ip)} + \Phi^{(pi)} + \Phi^{(ii)} > 0. \quad \ldots\ldots (10)\end{aligned}$$

On the basis of the definitions, it is clear that

$$U = \int_V \Phi\, dV, \qquad U^{(pp)} = \int_V \Phi^{(pp)}\, dV, \quad \ldots\ldots\ldots (11)$$

and from these results

$$U_c = U - U^{(pp)} = \int_V \left(\Phi^{(ip)} + \Phi^{(pi)} + \Phi^{(ii)}\right) dV. \quad .\ (12)$$

Another useful energy quantity is $U^{(ii)}$. This is the strain energy that would be stored in V if the induced stresses were to act on originally unstrained body. From the definition in (10),

$$U^{(ii)} = \int_V \Phi^{(ii)}\, dV = -\tfrac{1}{2}\left(\int_S T_i^{(i)} u_i^{(i)}\, dS - \int_{S_m} T_i^{(p)} u_i^{(i)}\, dS + \int_{S^*} R_i^{(i)} u_i^{(i)}\, dS\right). \quad \ldots\ldots\ldots (13)$$

The second part of this relationship follows from Clapeyron's theorem, which states[7] that 'the total strain energy in a body which is in equilibrium under the action of given surface and body forces is equal to half the work done by those forces acting through their displacements from the unstrained state to the position of the equilibrium'.

Denote by U_m the *strain energy content of* V_m in state I. This volume of rock is destressed in state II as a result of mining; therefore, U_m becomes available and must be accounted for. Note that

$$U_m = \int_{V_m} \Phi^{(pp)}\, dV = \tfrac{1}{2}\left(\int_{S_m} T_i^{(p)} u_i^{(p)}\, dS - \int_{V_m} X_i\, u_i^{(p)}\, dV\right). \ (14)$$

Finally, it is necessary to define the *work done on the supports*, W_s, in the course of the transition from state I to state II. In Fig. 2 a typical load–deformation curve of some backfill material is depicted. If the body forces are momentarily neglected, the work done by the contact forces in deforming the support over the elementary area dS may be given by

$$\begin{aligned} dW_s &= \left(\int_{u_i^{(p)}}^{u_i} R_i\, du_i\right) dS\\ &= \left[R_i^{(p)} u_i^{(i)} + \int_{u_i^{(p)}}^{u_i} (R_i - R_i^{(p)})\, du_i\right] dS\\ &= \left(R_i^{(p)} u_i^{(i)} + \tfrac{1}{2}\alpha_i R_i^{(i)} u_i^{(i)}\right) dS, \quad \ldots\ldots\ldots (15)\end{aligned}$$

where in the last step the numerical value of the parameter α_i is chosen so that the expression $\frac{1}{2}\alpha_i R_i^{(i)} u_i^{(i)}$ equals the cross-hatched area in Fig. 2. The practical range of α_i is $0 \leq \alpha_i \leq 2$. Its numerical value depends on the shape of the deformation curve of the support. For example, if the curve is linear, then $\alpha_i = 1$.

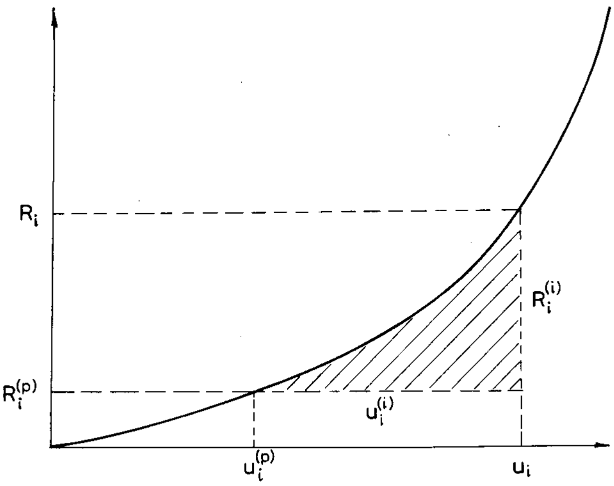

Fig. 2—The definition of the work done in deforming the support. The shaded area equals $\frac{1}{2}\alpha_i R_i^{(i)} u_i^{(i)}$

It is now simple to obtain the total work done by the contact and body forces on the support during additional mining:

$$W_s = \int_{S_o^*} R_i^{(p)} u_i^{(i)}\, dS - \int_{V_o^*} X_i^* u_i^{(i)} dV + \tfrac{1}{2}\left(\int_{S^*} \alpha_i R_i^{(i)} u_i^{(i)}\, dS - \int_{V_m^*} X_i^*\, u_i^{(i)}\, dV\right). \quad \ldots\ldots (16)$$

Energy Balance

Basic Considerations

In this section an attempt is made to establish the energy balance of the transition from state I to state II.

There are two sources of energy in the system. These are, firstly, the work done by the external and body forces, that is W, and, secondly, the strain energy stored in volume V_m in state I, that is U_m. Thus, the total energy source is $(W + U_m)$.

Energy is expended in two obvious ways: firstly, by straining further the rock mass in volume V, this strain energy change being denoted by U_c. If the cavities are supported, the second mode of dissipating energy is through the loading and deforming of supports. This energy quantity is denoted by W_s. Hence, the total energy expended in a known manner is $(U_c + W_s)$.

Clearly, the amount of energy consumed in deforming rock and support cannot be greater than the total energies available. Furthermore, because the energy stored in the rock to be extracted, that is U_m, is *not* available to strain rock or support, the following inequality must apply:–

$$W \geqslant U_c + W_s \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (17a)$$

and, because $U_m > 0$, it follows that

$$(W + U_m) > (U_c + W_s). \quad \ldots\ldots\ldots\ldots \quad (17b)$$

It has become customary to refer to the excess energy that needs to be expended during the transition from one state to another as 'released energy' or W_r. On the basis of (17), it follows that

$$W_r = (W + U_m) - (U_c + W_s) > 0 \quad \ldots\ldots\ldots \quad (18a)$$

and that

$$W_r \geqslant U_m > 0. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (18b)$$

It is obvious from this result that *additional mining is always associated with some released energy, which must be dissipated in some manner.*

If the next configuration of mine openings were to be reached suddenly by the instantaneous removal of all the rock in V_m, there would be oscillations in the rock mass. The new equilibrium corresponding to state II would be attained through the damping resulting from slight inelasticity in the rock. Denote the kinetic energy that would be dissipated in this damping by W_k. Since in an elastic rock no other mechanism for energy dissipation exists, it can be concluded from (18a) and (18b) that

$$W_r = U_m + W_k, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (18c)$$

where

$$W_k = W - (U_c + W_s) \geqslant 0. \quad \ldots\ldots\ldots\ldots \quad (18d)$$

When dealing with tabular excavations later, it will be advantageous to use the expression for the released energy in a somewhat different form.

Assume that the total strain energy stored in state I in the unmined rock mass, that is in volume $V + V_m$, is U'. In state II the total strain energy stored in volume V, in accordance with (11), is U. The difference between U and U' is given by the energy change in volume V resulting from the transition from state I to state II, which is U_c according to (12), minus the energy content of V_m in state I, which is U_m in terms of (14). Thus,

$$U - U' = U_c - U_m. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

Now take (18a) and rearrange it slightly:

$$W_r = W - [(U_c - U_m) + W_s],$$

which can now be put into the following form:

$$W_r = W - [(U - U') + W_s]. \quad \ldots\ldots\ldots\ldots \quad (20)$$

Relationships between Energy Components

An important result of the theory of elasticity is the reciprocal theorem[7], which states that 'if an elastic body is subjected to two systems of body and surface forces, then the work that would be done by the first system in acting through the displacements due to the second system of forces is equal to the work that would be done by the second system in acting through the displacements due to the first system of forces'.

Postulate that the first system of forces acting on volume V is that corresponding to state I and that the second system of forces is that *induced* by the change from state I into state II.

The work done by the first system of forces on the displacements of the second set is

$$2U^{(pi)} = -\Big(\int_{S+S_m} T_i^{(p)} u_i^{(i)} \, dS + \int_{S_o^*} R_i^{(p)} u_i^{(i)} \, dS + \int_{S^*} X_i u_i^{(i)} \, dV\Big) = 2\int_V \Phi^{(pi)} \, dV \quad \ldots\ldots \quad (21)$$

and work done by the second system of forces on the displacements of the first set is given by

$$2U^{(ip)} = -\Big(\int_S T_i^{(i)} u_i^{(p)} \, dS - \int_{S_m} T_i^{(p)} u_i^{(p)} \, dS + \int_{S^*} R_i^{(i)} u_i^{(p)} \, dS\Big) = 2\int_V \Phi^{(pi)} \, dV. \quad \ldots\ldots \quad (22)$$

An examination of the proof of the reciprocal theorem shows that $U^{(pi)}$ and $U^{(ip)}$ can in fact be expressed as volume integrals of $\Phi^{(pi)}$ and $\Phi^{(ip)}$, respectively [8]. According to the reciprocal theorem,

$$U^{(pi)} = U^{(ip)}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$

It follows immediately from this result and from (13) that U_c in (12) can be simplified:

$$U_c = U^{(ii)} + 2U^{(pi)}. \quad \ldots\ldots\ldots\ldots\ldots \quad (24)$$

It is now possible to seek a relationship between W, that is the work done by the external and body forces, and the other energy components. To do this, note from (13) that

$$-\tfrac{1}{2}\int_S T_i^{(i)} u_i^{(i)} \, dS = u^{(ii)} - \tfrac{1}{2}\int_{S_m} T_i^{(p)} u_i^{(i)} \, dS + \tfrac{1}{2}\int_{S^*} R_i^{(i)} u_i^{(i)} \, dS;$$

also from (16) that

$$-\Big(\int_{V_o^*} X_i^* u_i^{(i)} \, dV + \tfrac{1}{2}\int_{V_m^*} X_i^* u_i^{(i)} \, dV\Big) = W_s - \int_{S_o^*} R_i^{(p)} u_i^{(i)} \, dS - \tfrac{1}{2}\int_{S_m^*} \alpha_i R_i^{(i)} u_i^{(i)} \, dS,$$

and similarly from (21) that

$$-\Big(\int_S T_i^{(p)} u_i^{(i)}\,dS + \int_V X_i u_i\,dV\Big) =$$

$$2U^{(pi)} + \int_{S_m^*} T_i^{(p)} u_i^{(i)}\,dS + \int_{S_o^*} R_i^{(p)} u_i^{(i)}\,dS.$$

These results, when substituted into (7) and rearranged, yield

$$W = W_s + U^{(ii)} + 2U^{(pi)} + \tfrac{1}{2}\Big[\int_{S_m} T_i^{(p)} u_i^{(i)}\,dS + \int_{S^*}(1-\alpha_i)\,R_i^{(i)} u_i^{(i)}\,dS\Big]. \quad (25)$$

Finally, on account of (23) and (24), W can be put into the following form:

$$W = U_c + W_s + \tfrac{1}{2}\Big[\int_{S_m} T_i^{(p)} u_i^{(i)}\,dS + \int_{S^*}(1-\alpha_i)\,R_i^{(i)} u_i^{(i)}\,dS\Big]. \quad (26)$$

Released Energy and Other Results

All preparations have now been completed for the derivation of the expression for released energy. The definition in (18) and the result in (26) yield the following important formula:

$$\begin{aligned} W_r &= (W + U_m) - (U_c + W_s) \\ &= U_m + \tfrac{1}{2}\Big[\int_{S_m} T_i^{(p)} u_i^{(i)}\,dS + \int_{S^*}(1-\alpha_i)\,R_i^{(i)} u_i^{(i)}\,dS\Big], \end{aligned} \quad (27a)$$

which reinforces the earlier conclusion that, in the course of mining, *some energy must be released*. Furthermore, it follows from (18c) or (18d) that

$$W_k = \tfrac{1}{2}\Big[\int_{S_m} T_i^{(p)} u_i^{(i)}\,dS + \int_{S^*}(1-\alpha_i)\,R_i^{(i)} u_i^{(i)}\,dS\Big]. \quad (27b)$$

It has been known since the early 1960s that the process of mining is necessarily associated with the release of energy[1,2,9]. Cook[1,9] and others[2] argued that, in the case of an *unsupported* excavation, half the work done by the external and body forces must be released. Moreover, they suggested that, of necessity, the released energy was either transferred into kinetic energy, or it was consumed in the mass during fracturing and as a result of other non-elastic behaviour.

These long-held beliefs require revision. It is true that, under certain circumstances, the creation of an unsupported cavity does lead to the release of half the work done by the body and external forces. However, it is difficult, if not impossible, to gain good understanding of the mechanism involved in the dissipation of the released energy unless attention is given to the manner in which the excavation has been developed to its present shape and size.

Illustrative Examples

To gain a better insight into the energy changes associated with the development of mining excavations, two simple examples are analysed in this section. Both of these relate to *unsupported* cavities.

First, the *enlargement* of cavities in a *weightless* medium is examined. The results are derived by putting $X_i = X_i^* = 0$, $T_i^{(i)} = 0$ on the enveloping surface S and, as the cavities are unsupported, $R_i^{(p)} = R_i = 0$.

From (13) and (14), important results follow immediately:

$$U^{(ii)} = \tfrac{1}{2}\int_{S_m} T_i^{(p)}\,u_i^{(i)}\,dS, \quad U_m = \tfrac{1}{2}\int_{S_m} T_i^{(p)}\,u_i^{(p)}\,dS. \quad (28a)$$

The appropriate substitutions into (22) yield

$$U^{(ip)} = U_m > 0. \quad (28b)$$

Now, the equality in (23) and the relationship in (24) lead to

$$U_c = U^{(ii)} + 2U_m > 0. \quad (29a)$$

The substitution of this result and the first formula in (28a) into (26) produces

$$W = 2(U^{(ii)} + U_m) > 0. \quad (29b)$$

because $W_s = 0$. Finally, the expressions in (27a) and (27b), after employing the first relationship in (28a) again, give the formula for the released energy and for the potential kinetic energy:

$$W_r = U^{(ii)} + U_m > 0, \quad W_k = U^{(ii)}, \quad (29c)$$

confirming that in this example $W_r = \frac{1}{2}W$.

These results have already been reported elsewhere[4], and they do not appear to differ significantly from those published by Jaeger and Cook[8]. However, these authors obtained their results by considering the creation of a cavity in one step rather than by treating the more general case of increasing the size of the excavation in distinct steps.

By use of the formulae in (28) and (29), it is possible to deduce the energy components associated with an increase from a to c in the radii of a tunnel with a circular cross-section, and of a sphere. The relevant results were published some years ago without proof[4]. A summary of the original derivation is reproduced in Addendum II.

Here, the energy components for a circular tunnel are given in a form normalized with respect to U_m:

$$\frac{U^{(ii)}}{U_m} = \frac{1-(a/c)^2}{1-2\nu+(a/c)^2} = \beta = \frac{W_k}{U_m}, \text{ and} \quad (30a)$$

$$\frac{W}{U_m} = 2(1+\beta), \quad \frac{U_c}{U_m} = 2+\beta, \quad \frac{W_r}{U_m} = 1+\beta. \qquad (30b)$$

In (30a), ν denotes the Poisson's ratio. These functions are plotted in Fig. 3. It is interesting to note that, when the step in mining is large, that is when the ratio a/c is small, then the released energy, W_r, is considerably greater than the energy stored in the removed rock, U_m. Conversely, when the mining step is small, that is when the ratio a/c is close to unity, then the released energy is only slightly greater than U_m and consequently W_k is small. Thus, the curves in Fig. 3 suggest that, when mining is advanced in small steps, almost all the released energy is removed with the extracted rock. It is of considerable theoretical and practical interest whether this feature of energy release is the property of this example only, or whether it has wider validity.

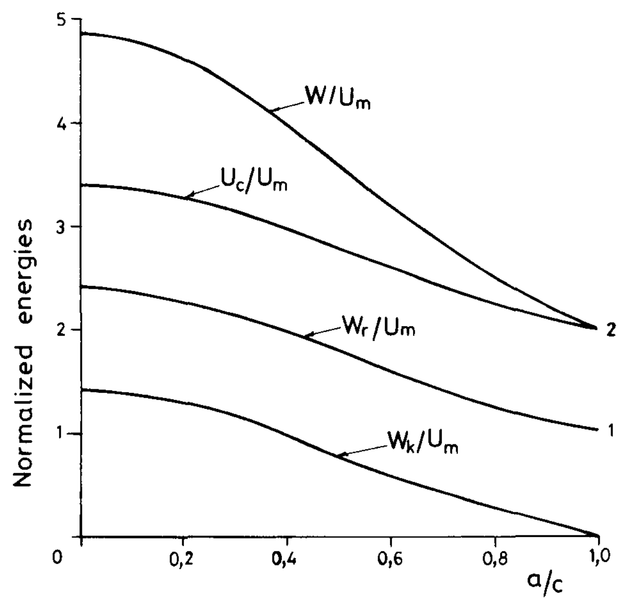

Fig. 3—Normalized energy changes due to an increase in the radius of a circular tunnel from *a* to *c*

The *second* example is related again to *unsupported* cavities, but on this occasion the body forces are *not neglected.* Postulate that all the excavated rock is transported to and piled on the ground surface in such a manner that the resultant induced forces on the boundaries of the cavities and the weight of the rock piled on the surface are in equilibrium. To explain the significance of this postulate, subdivide the enveloping surface S into two parts: the underground part, S_u, and the surface part, S_s, so as to have $S = S_u + S_s$. Assume that S_u is remote from the mine openings. Owing to the self-equilibrating feature of the system of forces, $T_i^{(i)}$ can be taken to be zero on S_u. Thus, the induced tractions on S will be restricted to the ground surface, that is to S_s.

Assume that state I corresponds to the *virgin* state and that the horizontal virgin displacement components are zero. Assume also that the ground surface is a horizontal plane and that there are no tectonic forces. It stems from these assumptions, provided that the rock is homogeneous and isotropic with Young's modulus E and Poisson's ratio ν, that the vertical and horizontal normal stresses are principal stresses. These are given by[8]

$$\tau_{33}^{(p)} = \varrho g x_3 = \gamma x_3, \quad \tau_{11}^{(p)} = \tau_{22}^{(p)} = \frac{\nu}{1-\nu}\gamma x_3 \quad . \; . \; . \; . \; . \quad (31)$$

where ϱ is the density of the rock, $\gamma = \varrho g$, and the vertical co-ordinate x_3 is measured downwards from the ground surface. The only non-zero strain and displacement components follow from (9), (31), and Hooke's law:

$$e_{33}^{(p)} = \frac{(1+\nu)(1-2\nu)\gamma x_3}{(1-\nu)E}, \quad u_3^{(p)} = \frac{(1+\nu)(1-2\nu)\gamma x_3^2}{2(1-\nu)E}. \quad (32)$$

The displacement component $u_3^{(p)}$ is fully defined apart from an additional rigid body movement, which, however, has no significance here.
From (10), (31), and (32),

$$\Phi^{(pp)} = \tfrac{1}{2}\tau_{ij}^{(p)}e_{ij}^{(p)} = \tfrac{1}{2}\tau_{33}^{(p)}e_{33}^{(p)} = \frac{(1+\nu)(1-2\nu)\gamma^2 x_3^2}{2(1-\nu)E},$$

which, when substituted into (14), leads to:

$$U_{mv} = \int_{V_m}\Phi^{(pp)}\mathrm{d}V = \frac{(1+\nu)(1-2\nu)\gamma^2}{2(1-\nu)E}\int_{V_m} x_3^2\mathrm{d}V. \quad . \; . \; . \quad (33)$$

Now note that

$$\int_{V_m} X_i u_i^{(p)}\mathrm{d}V = \int_{V_m} X_3 u_3^{(p)}\mathrm{d}V =$$

$$\frac{(1+\nu)(1-2\nu)\gamma^2}{2(1-\nu)E}\int_{V_m} x_3^2\,\mathrm{d}V = U_{mv}. \quad . \; . \; . \; . \; . \; . \; . \; . \quad (34)$$

It can be deduced from the results in (34) and (14) that

$$\int_{S_m} T_i^{(p)}u_i^{(p)}\mathrm{d}S = 3U_{mv}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (35)$$

Next, observe that

$$\int_{S} T_i^{(i)}u_i^{(p)}\mathrm{d}S = 0,$$

because $T_i^{(i)} = 0$ on S_u and $u_i^{(p)} = 0$ on S_s. Thus (22), (23), and (35) yield the simple result

$$2U^{(ip)} = \int_{S_m} T_i^{(p)}u_i^{(p)}\mathrm{d}S = 3U_{mv}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (36)$$

Now introduce the notations

$$U_s^{(ii)} = -\tfrac{1}{2}\int_{S_s} T_i^{(i)}u_i^{(i)}\mathrm{d}S > 0, \quad U_u^{(ii)} = \tfrac{1}{2}\int_{S_m} T_i^{(p)}u_i^{(i)}\mathrm{d}S, \quad (37)$$

which as a result of (13) leads to

$$U_v^{(ii)} = U_s^{(ii)} + U_u^{(ii)}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (38)$$

It is now possible to summarize the results that follow from (24), (25), and (27) by use of the details in (36) to (38):

$$W_v = U_s^{(ii)} + 2U_u^{(ii)} + 3U_{mv} > 0,$$
$$U_{cv} = U_s^{(ii)} + U_u^{(ii)} + 3U_{mv} > 0,$$
$$W_{rv} = U_u^{(ii)} + U_{mv} > 0. \qquad (39)$$

These results can be compared directly with those given in (29). It is obvious that the body forces in this example exert a significant influence, and that the released energy is *not* equal to half the work done by the external and body forces.

In deep-level mining, the results in (39) can be simplified. On the basis of some well-known results in the theory of elasticity, an estimate of the work done by the rock that is stored on the surface, that is $U_s^{(ii)}$, is given by $\frac{1}{2}\gamma V_m w_a$, where w_a is the average downward vertical displacement of the ground surface under the stockpile[8]. It can be shown that $U_s^{(ii)}$ is negligible in comparison with U_{mv} whenever the ratio V_m/RH^2 is small. Here H is the distance from the ground surface to the centre of gravity of the excavations, and R is the approximate radius of the base of the rock pile on the surface. The details of this derivation are presented in Addendum III.

In most instances, when mining takes place at some considerable depth, the critical ratio V_m/RH^2 is small. Thus in deep-level mining (39) reduces to

$$W_v = 2U_u^{(ii)} + 3U_{mv}, \quad U_{cv} = U_u^{(ii)} + 3U_{mv},$$
$$W_{rv} = U_u^{(ii)} + U_{mv}, \qquad (40)$$

which again indicates that $W_v \neq \frac{1}{2}W_{rv}$.

In most mining applications, $U_u^{(ii)}$ and U_{mv} tend to be of similar magnitude. Perhaps the only notable exception to this observation is the tabular excavation, which has a negligible thickness, say h, in comparison with its lateral extent, typified by length L ($L \gg h$). It can be shown that $U_{mv}/U_u^{(ii)} \propto h/L$ for an open, unsupported tabular excavation. Thus, in this instance, the results in (40) simplify even further to

$$W_v = 2U_u^{(ii)}, \quad U_{cv} = U_u^{(ii)}, \quad W_{rv} = U_u^{(ii)}, \qquad (41)$$

which show that, under these conditions, $W_{rv} = \frac{1}{2}W_v$.

Tabular Excavations

General Concepts

Because of the unique importance of tabular excavations to South African mining, this paper would not be complete without a discussion of the main problem that arises from the shape of these mine openings.

It will be recalled that no restriction was placed on the shape of cavities in the derivation of the results given in the previous sections. Therefore, in principle, these results must be valid also for tabular excavations. However, the exact analysis of stresses and displacements around these excavations poses formidable difficulties, even if a powerful computer is used for the calculations. The problems arise, firstly, because one dimension of tabular excavations is orders of magnitude smaller than the others and, secondly, because the layout of an actual mine is complex and irregular.

In order to deal with rock mechanics problems pertaining to tabular excavations, numerical methods, based on the displacement discontinuity or 'slit' concept, have been developed to a high degree of sophistication[4]. It has been realized that significant advantages are to be gained by the neglect of the effects of those surfaces (bounding the excavation) that are normal to the plane of the deposit, in comparison with those surfaces that are parallel to this plane. Of course, in the case of tabular excavations, the area of the former surfaces is negligible in comparison with that of the latter.

There are a number of advantages in the viewing of tabular excavations as displacement discontinuities or as slits, the outlines of which coincide with the plan of the excavations. One of these is that the co-ordinates of two points, of which one is on the roof and the other positioned perpendicularly opposite on the floor, are the same, but that their displacements are different. Thus, the model neglects the thickness of the orebody and consequently does not simulate the extraction of volume V_m. The additional mining is reflected merely through the enlargement of the outline of the excavations. Such slit-like model geometry obviously results in local singularities in stresses at the boundaries of the displacement discontinuities.

In brief, the problems associated with obtaining some of the energy components for tabular excavations constitute not so much a property of the cavities themselves as a weakness of the mathematical model employed for handling such mine openings. Fortunately, it is possible to turn the shortcoming of the model to advantage. As a result, tabular deposits are handled today with almost the same adroitness and accuracy as any other type of mining excavation.

The advantages offered by the slit model in energy analysis are as follows.

(i) While the displacements are discontinuous when moving perpendicularly across, say from the roof to the floor of the excavation, the stresses remain continuous.
(ii) The weights of the extracted rock and of the backfill are neglected, and their volumes are taken to be zero.
(iii) As a result of (i) and (ii), the induced traction, $T_i^{(i)}$, can be taken to be zero on S, that is on the surface enveloping the whole system.
(iv) A further consequence of (i) and (ii) is that the work done by forces acting on the surfaces of the excavations can be expressed as an integral of the forces acting through the *relative* displacement of the roof and floor.

Items (i) to (iii) are self-evident, but the statement in (iv) requires some explanation. Conventionally[4], the x_3 axis of the co-ordinate system is normal to the plane of the deposit and points towards the floor. Denote the components of the relative displacement vector by s_i (i = 1,2,3) with the understanding that s_3 is the *convergence* and s_1 and s_2 are the *ride* components. These are defined as follows:

$$s_i = u_i^+ - u_i^-, \quad \ldots \quad (42)$$

where u_i^+ and u_i^- refer to the induced displacements of the floor and roof respectively. These definitions arise from the convention that s_3 should be positive when it denotes convergence and that u_3 is positive when it represents a motion towards the roof.

An example will be the best illustration of the method of computing the work done by forces on the surfaces of tabular excavations. For instance, compute the integral

$$I = \int_{S_m} T_i u_i \mathrm{d}S, \quad \ldots \quad (43)$$

where S_m corresponds to the surface of a tabular opening. The traction components on the walls of the excavation are continuous when moving from its roof to its floor. Consequently, the work done over a unit area of the floor by traction $T_i = \mu_j \tau_{ij}$ acting through the displacement u_i^+ is $\mu_j \tau_{ij} u_i^+$. Similarly, the work done over the corresponding area of the floor by traction $- \mu_j \tau_{ij}$ when acting through u_i is $-\mu_j \tau_{ij} u_i$. Here μ_j is the interior normal of the floor. Thus, the work done over a unit area of the roof *and* floor is

$$(\mu_j \tau_{ij} u_i^+) + (-\mu_j \tau_{ij} u_i^-) = \mu_j \tau_{ij} (u_i^+ - u_i^-) = T_i s_i, \quad (44)$$

where T_i on the right refers to the floor. The integral in (43) therefore takes the following form:

$$I = \int_{A_m} T_i s_i \mathrm{d}A, \quad \ldots \quad (45)$$

where A_m is the projection of the S_m surface onto the plane of the deposit and dA is an elementary area of this plane. An obvious corollary to the application of the slit model is that the work done by the forces on those projections of S_m that are perpendicular to the plane of the deposit are neglected.

It is useful to introduce some notations when dealing with tabular deposits. Assume that in state I the nomenclature for surfaces is as follows:

$$S_o \rightarrow A_o, \quad S_m \rightarrow A_m, \quad S_o^* \rightarrow B_o. \quad \ldots \quad (46)$$

In state II the surfaces are denoted by

$$(S_o + S_m) \rightarrow (A_o + A_m) = A,$$
$$(S_o^* + S_m^*) \rightarrow (B_o + B_m). \quad \ldots \quad (47)$$

A further consequence of the use of this model is that

$$s_i^{(p)} = 0 \text{ on } A_m. \quad \ldots \quad (48)$$

Energy Components

The energy changes associated with the extension of the mined-out area from A_o to A and the supported area from B_o to B can be obtained from the general results derived in the earlier sections.

First deduce the strain energy that would be stored in V if the induced stresses were to act on unstressed rock. From (13)

$$U^{(ii)} = \tfrac{1}{2}\Big(\int_{A_m} T_i^{(p)} s_i^{(i)} \mathrm{d}A - \int_{B} R_i^{(i)} s_i^{(i)} \mathrm{d}A\Big). \quad \ldots \quad (49)$$

Two other important results are derived from (21) and (22):

$$2U^{(pi)} = -\Big(\int_{S} T_i^{(p)} u_i^{(i)} \mathrm{d}S + \int_{V} X_i u_i^{(i)} \mathrm{d}V +$$
$$\int_{A_m} T_i^{(p)} s_i^{(i)} \mathrm{d}A + \int_{B_o} R_i^{(p)} s_i^{(i)} \mathrm{d}A\Big), \quad \ldots \quad (50)$$

$$2U^{(ip)} = -\int_{B} R_i^{(i)} s_i^{(p)} \mathrm{d}A. \quad \ldots \quad (51)$$

The work done on the support or backfill can be obtained from (16):

$$W_s = \int_{B_o} R_i^{(p)} s_i^{(i)} \mathrm{d}A + \tfrac{1}{2}\int_{B} \alpha_i R_i^{(i)} s_i^{(i)} \mathrm{d}A. \quad \ldots \quad (52)$$

The determination of the change in stored energy must be done with care. The model does not recognize the *volume* of the rock that is to be mined, and therefore it assumes that $U_m = 0$. Thus, any formula derived to give the change in stored energy will yield the difference between the *total* energy stored in state II and that in state I. According to (19) this difference is $(U - U')$. Thus, when the slit model is used to simulate tabular excavations, the relationship in (24) takes the following form:

$$U_c = U - U' = U^{(ii)} + 2U^{(ip)}, \quad \ldots \quad (53)$$

which, on the basis of (49) and (51), yields

$$U_c = U - U' = -\int_{B} R_i^{(i)} s_i^{(p)} \mathrm{d}A +$$
$$\tfrac{1}{2}\Big(\int_{A_m} T_i^{(p)} s_i^{(i)} \mathrm{d}A - \int_{B} R_i^{(i)} s_i^{(i)} \mathrm{d}A\Big). \quad \ldots \quad (54)$$

Of course, for a real tabular excavation U_m is not zero, but it is difficult to estimate its magnitude if the slit model is used. However, it will be seen later that, in a particularly important instance, this can be done effectively.

The work done by external and body forces assumes a particularly simple form in this instance. Note that the relationship in (7) simplifies to

$$W = -\Big(\int_{S} T_i^{(p)} u_i^{(i)} \mathrm{d}S + \int_{V} X_i u_i^{(i)} \mathrm{d}V\Big). \quad \ldots \quad (55)$$

Now, observe on the basis of (50) that

$$-\Big(\int_{S} T_i^{(p)} u_i^{(i)} \mathrm{d}S + \int_{V} X_i u_i^{(i)} \mathrm{d}V\Big) = 2U^{(ip)} +$$
$$\int_{A_m} T_i^{(p)} s_i^{(i)} \mathrm{d}A + \int_{B_o} R_i^{(p)} s_i^{(i)} \mathrm{d}A$$

and, because $U^{(ip)} = U^{(pi)}$ according to the reciprocal theorem, a substitution from (51) leads to

$$W = \int_{A_m} T_i^{(p)} s_i^{(i)} dA + \int_{B_o} R_i^{(p)} s_i^{(i)} dA - \int_{B} R_i^{(i)} s_i^{(p)} dA. \quad (56)$$

A more apt form of W can be derived with the aid of (54) and (52). These yield

$$-\int_{B} R_i^{(i)} s_i^{(p)} dA = U - U' - \tfrac{1}{2}\int_{A_m} T_i^{(p)} s_i^{(i)} dA + \tfrac{1}{2}\int_{B} R_i^{(i)} s_i^{(i)} dA,$$

$$\int_{B_o} R_i^{(p)} s_i^{(i)} dA = W_s - \tfrac{1}{2}\int_{B} \alpha_i R_i^{(i)} s_i^{(i)} dA,$$

which, when substituted into (56), results in

$$W = (U - U') + W_s + \tfrac{1}{2}\Big[\int_{A_m} T_i^{(p)} s_i^{(i)} dA + \int_{B} (1-\alpha_i) R_i^{(i)} s_i^{(i)} dA\Big]. \quad (57)$$

The final step is to derive the released energy, which is appropriately defined for this situation in (20):

$$W_r = W - [(U - U') + W_s] = \tfrac{1}{2}\Big[\int_{A_m} T_i^{(p)} s_i^{(i)} dA + \int_{B} (1-\alpha_i) R_i^{(i)} s_i^{(i)} dA\Big]. \quad (58)$$

Total Energy Changes

A number of important results are obtained if state I is the *virgin* state. If this is so, then $A_o = B_o = 0$ and $s_i^{(p)} = 0$. Now introduce the notation

$$T_i^{(p)} = Q_i, \quad (59)$$

in the plane of the deposit, where

$$Q_i = \mu_j q_{ij} \qquad i, j = 1,2,3. \quad (60)$$

Here q_{ij} is the virgin stress tensor in the same plane and μ_j is the interior normal to the floor. In these circumstances, no confusion will arise if the following simplifications are employed:

$$s_i^{(i)} = s_i, \quad R_i^{(i)} = R_i, \quad A_m = A, \quad B_m = B. \quad (61)$$

It is now possible to deduce the required energy components from the corresponding expressions in the previous section. Thus, from (49),

$$U_v^{(ii)} = \tfrac{1}{2}\Big(\int_{A} Q_i s_i dA - \int_{B} R_i s_i dA\Big), \quad (62)$$

and, from (51),

$$U_v^{(ip)} = U_v^{(pi)} = 0. \quad (63)$$

This result, together with that in (53), leads to the following simple formula:

$$U - U' = U_v^{(ii)}. \quad (64)$$

Moreover, the relationship in (52) simplifies to

$$W_{sv} = \tfrac{1}{2}\int_{B} \alpha_i R_i s_i dA, \quad (65)$$

and (56) reduces to the fundamental result

$$W_v = \int_{A} Q_i s_i dA. \quad (66)$$

It is interesting that such a simple formula for tabular deposits defines the energy input of external and body forces. This result has been known since the 1960s for the case in which a horizontal reef (or seam) is a principal plane of the virgin stress system[2]. It is gratifying to see that the simple form applies even in the general case.

Finally, from (58) it is simple to obtain the formula for the total released energy:

$$W_{rv} = \tfrac{1}{2}\Big[\int_{A} Q_i s_i dA + \int_{B} (1-\alpha_i) R_i s_i dA\Big]. \quad (67)$$

This expression plays an important role in the computation of energy release rates in certain circumstances.

Summary

As a result of the increased size or number of mining excavations, displacements are induced in the surrounding rock. Acting through these displacements, the external and body forces do some work, W. This work is often referred to as 'gravitational' or 'potential' energy change. Also, a certain amount of strain energy is stored in the rock, which is removed during the process of mining, U_m. The sum $(W + U_m)$ can be regarded as the energy input that must be accounted for and has to be expended in some manner.

This energy is partially accounted for by an increase in the energy content of the surrounding rock mass. This increase is denoted by U_c. If some or all of the excavations are supported, or if backfill is employed, then some work is performed in deforming the support, W_s. It is assumed in this paper that the rock mass is an elastic continuum. No energy is therefore consumed in fracturing or in non-elastic deformation. Thus, the energy accountably expended during a step in mining is $(U_c + W_s)$.

It is concluded that these simple processes do not account for the full energy input, and some energy,

$$W_r = (W + U_m) - (U_c + W_s) > 0,$$

must be released and dissipated by other means. The lower limit of the released energy is defined as $W_r \geq U_m > 0$.

In the main body of the paper, expressions are derived to permit the computation of the various components of energy change. These results are general, and no restriction is posed on the size, shape, or number of excavations.

As an adaptation of the general results, formulae are derived to suit the treatment of tabular excavations through the employment of the displacement discontinuity or 'slit' model. All the formulae express the energy changes in terms of relative displacements of the roof and floor of the excavations and of the traction components in the plane(s) of the seam(s) or reef(s).

Two special examples are discussed in some detail. Both of them deal with unsupported cavities. In the first example, excavations are enlarged in a 'weightless' medium and, in the second, fresh mining openings are created in virgin ground that is loaded by body forces in such a manner that there are no horizontal virgin strains.

The first example provides an opportunity for the analysis of the energy changes due to an increase in the radius of a circular tunnel or a sphere. This study reveals that the pattern of energy release changes fundamentally if the total growth in the size of the cavity is achieved in many small steps instead of in a few large steps. This phenomenon has largely been ignored in the past, although it has great practical significance. The second paper in this series will be devoted mainly to the practical utilization of this observation.

The second example is the simplest application of the general energy formulae to a rock mass loaded by body forces. Even this relatively elementary example suffices to prove the incorrectness of the early publications that suggested that, for unsupported excavations, the released energy equals half the work done by body and external forces. The earlier conclusion is valid only in restricted situations. A particularly good example of this is a medium that is free of body forces.

Addendum I: Sign Conventions

In texts on the theory of elasticity, normal tensile stresses are taken to be positive. In rock and soil mechanics, it is often advantageous and, in fact, customary to reckon compressive stresses as positive[8]. This convention is adopted here. However, for this to be done consistently, three changes in the usually accepted definitions have to be introduced.

(i) The convention[7] with regard to the signs of the elements of the stress tensor, τ_{ij}, requires modification. Take a parallelepiped with faces parallel to the planes defined by the co-ordinate axes. If an *interior* normal to a given face of this body is drawn, then one of two things can happen: the normal either has the same sense as the positive direction of the axis to which the face is perpendicular, or it does not. In the former case, the components τ_{ij} are reckoned positive if the corresponding components of force act in the directions of increasing x_1, x_2, x_3. If, however, the direction of the interior normal points in the direction opposite to that of the positive co-ordinate axis, then the positive values of the components τ_{ij} are associated with forces directed oppositely to the positive directions of the co-ordinate axes. This modification in convention involves the replacement of the *exterior* normal with its *interior* counterpart.

(ii) To have positive compressive normal strains, displacements must be reckoned positive when pointing in the negative direction of the axes[8]. Thus, if u_i are the displacements of a point initially at x_1^0, x_2^0, x_3^0, its final position will be at

$$x_i = x_i^0 - u_i. \quad \ldots\ldots\ldots\ldots\ldots \quad (I.1)$$

It is obvious from this definition that, strictly speaking, the displacement vector is $-u_i$. However, to avoid confusion, this terminology is not used in the main body of the paper except in mathematical formulae, e.g. the energy formula in (7), where not to do so would lead to incorrect results.

(iii) To be consistent with the other alterations, the signs of the components of the strain tensor, e_{ij}, must also be changed to obtain positive compressive normal strains. To achieve this, the following definition[8] of normal strain is introduced:

$$e = \frac{L^{\circ} - L}{L^{\circ}} = -\frac{L - L^{\circ}}{L^{\circ}}, \quad \ldots\ldots\ldots \quad (I.2)$$

where L° is the original and L is the deformed distance between two points.

It should be noted that all equations of the theory of elasticity can be converted to the sign convention defined in this annexure by changing of the sign of τ_{ij}, e_{ij}, and u_i wherever they occur, and by the use of interior instead of exterior normals to surfaces.

Addendum II: Computation of Energy for a Weightless Medium

The goal here is to demonstrate the validity of some of the expressions presented in the main body of the paper. To this end, the energy changes resulting from the enlargement of the radius of a circular tunnel and a spherical cavity from a to c are derived. In both instances, the excavations are assumed to be unsupported, the virgin stress distribution is taken to be hydrostatic, and the body forces are neglected.

The relationships in (29) apply to these problems:

$$W = 2(U^{(ii)} + U_m), \quad U_c = U^{(ii)} + 2U_m,$$
$$W_r = U^{(ii)} + U_m, \quad \ldots\ldots\ldots\ldots\ldots \quad (II.1)$$

where $U^{(ii)}$ and U_m can be obtained from (28a) or from (13) and (14) respectively. The latter approach is chosen in this annexure. Thus

$$U^{(ii)} = \int_V \Phi^{(ii)} dV, \qquad U_m = \int_{V_m} \Phi^{(pp)} dV. \quad \ldots\ldots \quad (II.2)$$

Circular Tunnel

The volume of rock to be mined is $V_m = \pi(c^2 - a^2)$ per unit length of the tunnel. The stress distribution around the tunnel is well known[8]. The radial, $\sigma_r^{(p)}$, and tangential, $\sigma_t^{(p)}$, principal stresses at a point defined by radius r are given by

$$\sigma_r^{(p)} = p\left(1 - \frac{a^2}{r^2}\right), \qquad \sigma_t^{(p)} = p\left(1 + \frac{a^2}{r^2}\right), \quad \ldots\ldots \quad (II.3)$$

where p is the virgin hydrostatic compression and a is the radius of the tunnel. If plane strain conditions are assumed, the radial, $e_r^{(p)}$, and tangential, $e_t^{(p)}$, strain components are as follows:

$$e_r^{(p)} = \frac{(1+\nu)p}{E}\left[(1-2\nu) - \frac{a^2}{r^2}\right],$$

$$e_t^{(p)} = \frac{(1+\nu)p}{E}\left[(1-2\nu) + \frac{a^2}{r^2}\right]. \quad \text{(II.4)}$$

On the basis of the expressions in (II.3) and (II.4), it is possible to obtain, by use of the definition in (8), the density function of the primitive strain energy:

$$\Phi^{(pp)} = \tfrac{1}{2}\left(\sigma_r^{(p)}e_r^{(p)} + \sigma_t^{(p)}e_t^{(p)}\right)$$

$$= \frac{(1+\nu)p^2}{E}\left[(1-2\nu) + \frac{a^4}{r^4}\right]. \quad \text{(II.5)}$$

Now, if the second expression in (II.2) is employed, the energy content of the rock in volume V_m follows immediately:

$$U_m = \int_0^{2\pi}\int_a^c r\Phi^{(pp)}\,dr\,d\varphi$$

$$= \frac{(1+\nu)p^2}{E}\left[(1-2\nu) + \frac{a^2}{r^2}\right]V_m. \quad \text{(II.6)}$$

The next problem is the deduction of $U^{(ii)}$, which would be the strain energy content of the surrounding rock mass if the induced stresses were to act on unstrained rock. The induced stress and strain components are as follows:

$$\sigma_r^{(i)} = -q\frac{c^2}{r^2}, \quad \sigma_t^{(i)} = q\frac{c^2}{r^2}, \quad \text{(II.7)}$$

and

$$e_r^{(i)} = -\frac{(1+\nu)q}{E}\frac{c^2}{r^2}, \quad e_t^{(i)} = \frac{(1+\nu)q}{E}\frac{c^2}{r^2}. \quad \text{(II.8)}$$

In these forumalae, q is the primitive radial stress in state I at $r = c > a$, that is at the surface that is to be exposed. From $\sigma_r^{(p)}$ in (II.3), q can be deduced by the substitution of $r = c$:

$$q = p\left(1-\frac{a^2}{c^2}\right) = p\frac{c^2-a^2}{c^2}. \quad \text{(II.9)}$$

It is important to note that the magnitude of q is strongly influenced by the difference in radii, that is by $(c - a)$.

The induced density function of the strain energy follows from (II.7) and (II.8):

$$\Phi^{(ii)} = \tfrac{1}{2}\left(\sigma_r^{(i)}e_r^{(i)} + \sigma_t^{(i)}e_t^{(i)}\right) = \frac{(1+\nu)q^2}{E}\frac{c^4}{r^4}, \quad \text{(II.10)}$$

and then

$$U^{(ii)} = \int_0^{2\pi}\int_c^{\infty} r\Phi^{(ii)}\,dr\,d\varphi = \frac{\pi(1+\nu)q^2c^2}{E},$$

which, after the substitution for q from (II.9), becomes

$$U^{(ii)} = \frac{(1+\nu)p^2}{E}\left(1-\frac{a^2}{c^2}\right)V_m. \quad \text{(II.11)}$$

Finally, the substitution of U_m in (II.6) and $U^{(ii)}$ above into the formulae in (II.1) leads to

$$W = \frac{4(1-\nu^2)p^2}{E}V_m, \quad U_c = \frac{(1+\nu)p^2}{E}\left(3-4\nu+\frac{a^2}{c^2}\right),$$

$$W_r = \frac{2(1-\nu^2)p^2}{E}V_m, \quad \text{(II.12)}$$

which are identical to the results that were presented earlier[4] but that were published without proof at the time.

Spherical Cavity

The derivation of the results corresponding to a spherical cavity parallels that used in conjunction with a tunnel. Thus, a much condensed presentation will suffice here. Note that $V_m = 4\pi(c^3-a^3)/3$ in this instance.

The primitive stress and strain components are given in spherical co-ordinates[7]:

$$\sigma_r^{(p)} = p\left(1-\frac{a^3}{r^3}\right), \quad \sigma_t^{(p)} = \sigma_\varphi^{(p)} = p\left(1+\frac{a^3}{2r^3}\right), \quad \text{(II.13)}$$

$$e_r^{(p)} = \frac{p}{E}\left[(1-2\nu) - (1+\nu)\frac{a^3}{r^2}\right],$$

$$e_t^{(p)} = e_\varphi^{(p)} = \frac{p}{E}\left[(1-2\nu) + \tfrac{1}{2}(1+\nu)\frac{a^3}{r^3}\right], \quad \text{(II.14)}$$

from which the strain energy density function follows:

$$\Phi^{(pp)} = \tfrac{1}{2}\left(\sigma_r^{(p)}e_r^{(p)} + 2\sigma_t^{(p)}e_t^{(p)}\right)$$

$$= \frac{3p^2}{2E}\left[(1-2\nu) + \tfrac{1}{2}(1+\nu)\frac{a^6}{r^6}\right]V_m, \quad \text{(II.15)}$$

and ultimately

$$U_m = 4\pi\int_a^c r^2\Phi^{(pp)}\,dr$$

$$= \frac{3p^2}{4E}\left[2(1-2\nu) + (1+\nu)\frac{a^3}{c^3}\right]V_m, \quad \text{(II.16)}$$

is obtained.

Next the component $U^{(ii)}$ is derived. The induced stress and strain components are

$$\sigma_r^{(i)} = -q\frac{c^3}{r^3}, \quad \sigma_t^{(i)} = \sigma_\varphi^{(i)} = \frac{q}{2}\frac{c^3}{r^3},$$

$$q = p\left(1-\frac{a^3}{c^3}\right), \quad \ldots \ldots \ldots \ldots \ldots \quad \text{(II.17)}$$

$$e_r^{(i)} = -\frac{(1+\nu)q}{E}\frac{c^3}{r^3},$$

$$e_t^{(i)} = e_\varphi^{(i)} = \frac{(1+\nu)q}{2E}\frac{c^3}{r^3}. \quad \ldots \ldots \ldots \ldots \quad \text{(II.18)}$$

These results lead to:

$$\Phi^{(ii)} = \frac{3(1+\nu)q^2}{4E}\frac{c^6}{r^6}, \quad \ldots \ldots \ldots \ldots \quad \text{(II.19)}$$

which, in turn, yields

$$U^{(ii)} = 4\pi\int_c^\infty r^2\Phi^{(ii)}\mathrm{d}r = \frac{3(1+\nu)p^2}{4E}\left(1-\frac{a^3}{c^3}\right)V_m. \quad \text{(II.20)}$$

The final steps involve the derivation of the other energy change components on the basis of the formulae in (II.1):

$$W = \frac{9(1-\nu)p^2}{2E}V_m,$$

$$U_c = \frac{3p^2}{4E}\left[5-7\nu+(1+\nu)\frac{a^3}{c^3}\right]V_m,$$

$$W_r = \frac{9(1-\nu)p^2}{4E}V_m. \quad \ldots \ldots \ldots \ldots \ldots \quad \text{(II.21)}$$

Addendum III: A Simple Example Involving Body Forces

The second example introduced in the paper involves the derivation of energy changes due to the development of unsupported excavations in virgin ground. It is stated in the course of the discussion that $U_s^{(ii)}$ is negligible in comparison with U_{mv} in deep-level mining, that is whenever V_m/RH^2 is small. It will be recalled that $U_s^{(ii)}$ is the strain energy induced by the displacement of the rock pile on the surface, R is the approximate radius of the pile, and H the distance from ground surface to the centre of gravity of V_m.

From (33)

$$U_{mv} = \frac{(1+\nu)(1-2\nu)\gamma^2}{2(1-\nu)E}\int_{V_m} x_3^2\mathrm{d}V, \quad \ldots \ldots \ldots \quad \text{(III.1)}$$

where the integral

$$I = \int_{V_m} x_3^2 dV, \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad \text{(III.2)}$$

is the moment of inertia of volume V_m with respect to the plane of the ground surface. It is well known[10] and easily proved that

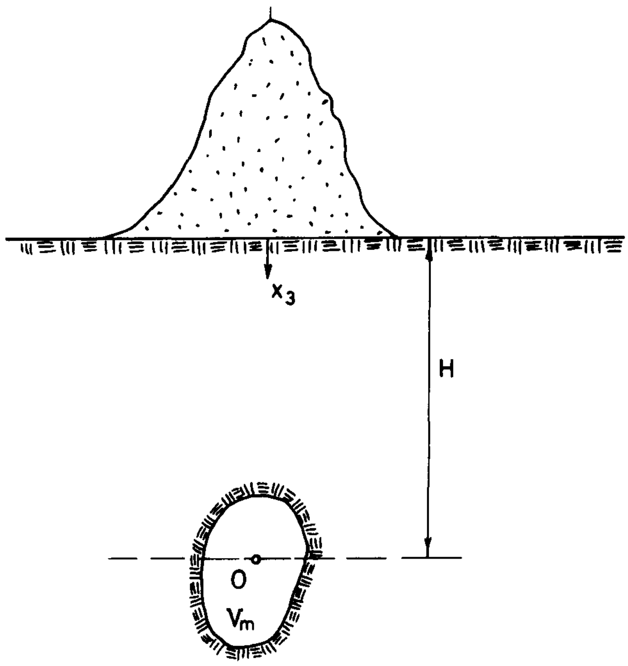

Fig. 4—Notations concerning energy computations for an open cavity in gravitational field

$$I = I_o + H^2V_m, \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad \text{(III.3)}$$

where I_o is the moment of inertia of V_m with respect to a horizontal plane across its centre of gravity, point O in Fig. 4. As I_o is positive, clearly $I > H^2V_m$. Thus,

$$U_{mv} > \frac{(1+\nu)(1-2\nu)\gamma^2H^2V_m}{2(1-\nu)E}. \quad \ldots \ldots \ldots \ldots \quad \text{(III.4)}$$

It is mentioned earlier that $U_s^{(ii)} \simeq \frac{1}{2}\gamma V_m w_a$, where w_a is the average depression of the ground surface beneath the rock pile. A good estimate of w_a is given[8] by

$$w_a \simeq \frac{0{,}54\pi(1-\nu^2)Rp}{E}. \quad \ldots \ldots \ldots \ldots \ldots \quad \text{(III.5)}$$

Here $p = \gamma V_m/\pi R^2$; thus,

$$U_s^{(ii)} = \frac{0{,}27(1-\nu^2)\gamma^2V_m^2}{RE}. \quad \ldots \ldots \ldots \ldots \ldots \quad \text{(III.6)}$$

From this and from (III.4) a reasonable estimate of the upper bound of the ratio $U_s^{(ii)}/U_{mv}$ follows:

$$\frac{U_s^{(ii)}}{U_{mv}} \leqslant \frac{0{,}54(1-\nu)^2}{1-2\nu}\frac{V_m}{RH^2}. \quad \ldots \ldots \ldots \ldots \ldots \quad \text{(III.7)}$$

The value of the co-efficient of V_m/RH^2 remains less than unity as long as $\nu \leqslant 0{,}4$. It is therefore reasonable to conclude that the upper limit of $U^{(ii)}/U_{mv}$ is determined by the value of the ratio V_m/RH^2, and that it will be small as long as this ratio remains small.

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Metallurgy of basic and ultrabasic rocks

The metallogenesis and associated mineral deposits of ophiolites, layered intrusives, and other igneous complexes, greenstone belts and modern ocean crust, in the context of IGCP Project 197 ('Metallogeny of ophiolites'), IGCP Project 161 ('Sulphide deposits in mafic and ultramafic rocks'), and the work of the European Commission Chromite Research Group, form the theme of the international conference that is to be held at the University of Edinburgh, Scotland, from 10th to 12th April, 1985. The conference will convene at the British Geological Survey in Edinburgh on 9th April, 1985, directly after the Easter holiday in the United Kingdom.

Original research and exploration papers, including experimental, isotopic, and chemical studies, and geochemical and geophysical exploration methods, are requested. Particular attention will be paid to deposits of platinum-group metals, chromite, and nickel and copper sulphides. Invited international authorities will review the principal topics of the conference.

An excursion to the Ballantrae and Unst ophiolite complexes will be made after the conference, and further tours can be arranged to meet registrants' interests.

A volume of proceedings (papers, discussion, and authors' replies) will be published by the Institution of Mining and Metallurgy in 1985–86.

The conference has been organized to coincide with the meeting of the European Geological Societies, 'Evolution of the European lithosphere' which will take place in the same lecture complex.

All enquiries should be addressed to the Conference Office, The Institution of Mining and Metallurgy, 44 Portland Place, London W1N 4BR, England (telephone: 01-580 3802; telex: 261410 IMM G).

Mine surveying

The ISM is an association of specialists in mine surveying and related fields. It is a non-government organization of C category (Mutual Information Relationships) according to the regulations of UNESCO and a permanent member of the World Mining Congress.

The ISM was set up to foster international co-operation in the field of mine surveying by means of

- organization of scientific symposia;
- constitution of commissions for special fields in mine surveying;
- promotion of publications, especially transactions of symposia and commissions;
- co-operation with other international societies in the field of mining, geodesy, geology, geophysics, rock mechanics, etc;
- promotion of co-operation in mine surveying by exchanging students, scientists, and technicians between countries.

In September 1985 The Royal Institution of Chartered Surveyors is hosting the VIth International Congress and Exhibition of the ISM at Harrogate. Previous Symposia have been held in Prague (1969), Budapest (1972), Leoben (1976), Aachen (1979), and Varna, Bulgaria (1982).

If you would like further information about any aspect of ISM and the VIth Congress, please contact

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Corrigendum

In the July issue of this *Journal* (vol. 84, no. 7), the last paragraph on page 220 should have appeared as the first paragraph on page 221.