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A simple mathematical model for the calculation of pumping rate in an air-agitated pachuca tank

by P.F. SORENSEN*

SYNOPSIS

A simple mathematical model is presented that relates the 'head' developed at the surface of pulp in a tank to the power dissipated by the air in producing it. The value for the head is then used in the calculation of the superficial circulating velocity of the pulp and the area of the overflow, and the product of these two values gives the pumping rate of the pulp. Thus, the superficial circulating velocity, and hence pumping rate of the pulp, are related directly to the flow-rate of the air.

This hypothesis was tested with good results in an air-agitated pachuca tank (of 1 m in diameter) fitted with a draught tube. The method can also be used for the prediction of the pumping rates in production-scale vessels.

SAMEVATTING

Daar word 'n eenvoudige wiskundige model aangebied wat die kop wat op die oppervlak van 'n pulp in die tenk ontwikkel in verband bring met die krag wat deur die lug gedissipeer word met die vorming daarvan. Die waarde vir die kop word dan gebruik in die berekening van die oppervlaksirkuleersnelheid van die pulp en die oppervlakte van die oorloop, en die produk van hierdie twee waardes gee die pomptempo van die pulp. Die oppervlaksirkuleersnelheid, en vandaar die pomptempo van die pulp, hou dus regstreegs verband met die vloeitempo van die lug.

Hierdie hipotese is met goeie resultate getoets in 'n luggeroerde pachuca-tenk (diameter 1 m) wat met 'n trekbuis toegerus is. Die metode kan ook vir die voorspelling van die pomptempo in produksieskaalhouers gebruik word.

Introduction

South African gold mines normally use air to agitate conical-bottom (pachuca) slurry reactors. The best-known attempt to describe the flow patterns and to derive equations to express the circulation rate of the liquid in such reactors was made by Lamont[1] in 1958. Later work[2,3] describing the suspension of solids by bubble agitation owed more to a theoretical approach, based on fluid dynamics in various studies[4-6] on bubble columns, than to the more practical approach of Lamont[1].

The investigation described here was aimed at the formulation of a readily applicable method for the calculation of circulation rates, and hence the power consumed by the liquid, in a pachuca tank 1 m in diameter. The method would also provide an indirect means for the determination of the superficial velocity of a pulp.

Energy Balance

The energy dissipated by the air in its passage through a liquid induces a measurable differential 'head' at the surface, as shown in Fig. 1. This differential bears the following simple relation to the superficial velocity of the pulp if, in the computation of velocity, the mean differential $(Z_s - Z_b/2)$ is used:

$$V_b = \left[2\,(Z_s - Z_b)g\right]^{\frac{1}{2}} \quad \ldots \quad (7)$$

(The numbers of the equations refer to equations as derived in the Addendum. The nomenclature is explained at the end of the paper.)

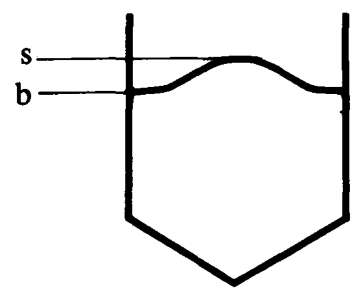

Fig. 1—Lifting of liquid by air

The power of the circulating liquid can be derived from equation (7) and is described by the following relation:

$$P_l = \tfrac{1}{2}(Z_s - Z_b)^{5/2} \cdot g^{3/2} \cdot \pi D \cdot \varrho_l^{*}. \quad \ldots \quad (11)$$

The power dissipated by the air on the other hand is

$$P_g = \text{mass flowrate of air} \times W_g, \quad \ldots \quad (3)$$

where

$$W_g = \frac{V_o^2}{2} + \frac{p_o}{\varrho_{go}} \ln \frac{p_o}{p_s} + (Z_o - Z_s)g.$$

It was shown by experiment that the power calculated from equation (11) was sufficiently close to that calculated from equation (3) to suggest that equation (7) gives a fair reflection of the superficial circulating velocity of the pulp.

Experimental

A pachuca tank fitted with a central draught tube (Fig. 2) was filled with a gold pulp having a relative density of 1,285. The height of the central plume (Z_s) and the surface height (Z_b) were measured at various flowrates of air with a metre stick below the top of the tank.

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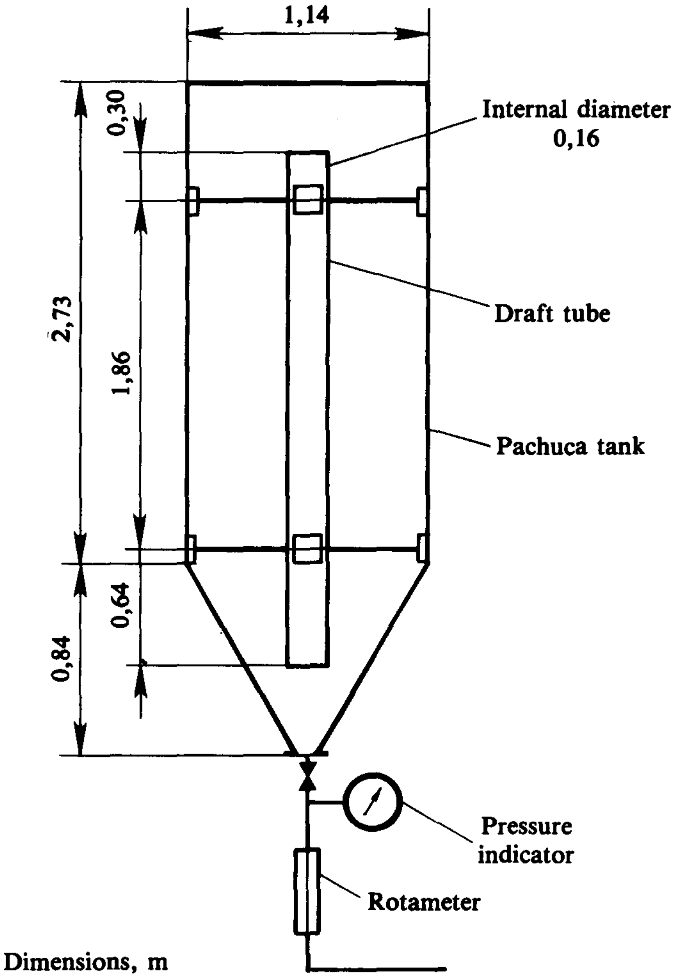


Fig. 2—The experimental equipment

The power was computed at various flowrates of air, and the results are given in Table I. It can be seen that there is good agreement between the power values derived from the measured air flow and those derived from corresponding measurements of the head. This implies that the derived value V_b is a fair reflection of the superficial pulp velocity at the place indicated. Attempts to measure a superficial velocity with an ultrasonic meter for purposes of comparison proved futile. Fig. 3 is a graph of the calculated superficial circulating velocity of the pulp as a function of the measured normal volumetric flowrate of the gas.

Comparison of Plant-scale Mechanical and Air Agitators

The bubble column in the draught tube can be compared with a mechanical stirrer in a draught tube in a cylindrical tank. A comparison of pumping rates may be instructive.

'Adequate' mixing can be induced by an aeration rate at normal temperature and pressure of about 1,5 m^3/h per ton of solids in a pachuca tank with a 60° conical bottom[7]. For a pachuca 10 m in diameter with a draught tube of 3 m diameter, the induced head ($Z_s - Z_b$) can be derived from equation (11), and the superficial circulating velocity from equation (7). The exposed overflow area can then be computed from ($Z_s - Z_b$). The product of this and the superficial circulating velocity gives the pumping rate.

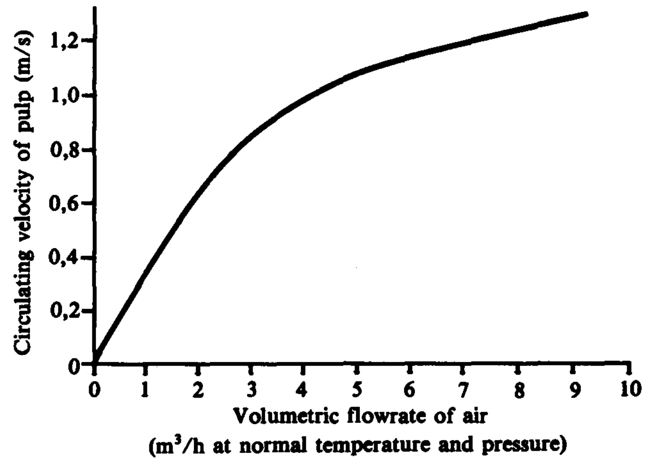


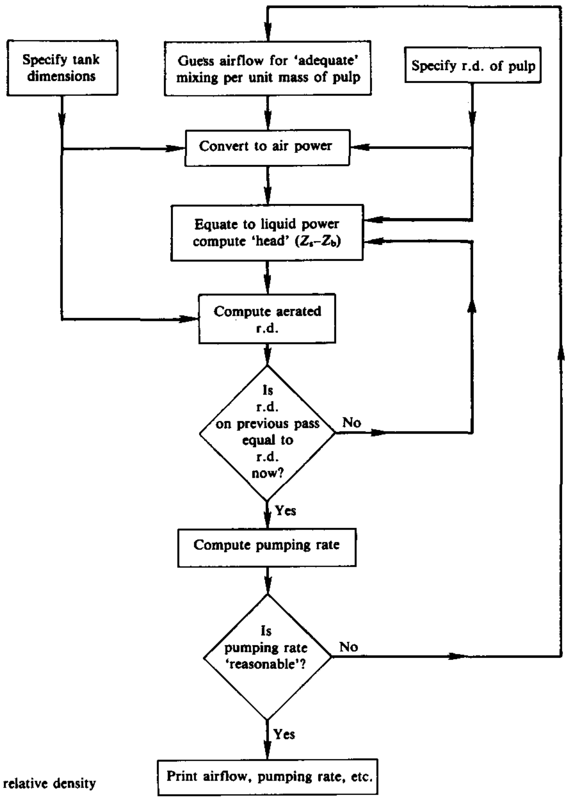
Fig. 3—The circulating velocity of the pulp, calculated from eq. (7), versus the measured volumetric flowrate of the air agitating the pulp in a 1 m test tank

The calculated value for this condition was found to be 56 029 m^3/h, compared with the value of 51 500 m^3/h quoted by Kipke[8]. The calculation procedure adopted is shown in Fig. 4. Kipke's value was derived from the more traditional design method, which is based on a grain-settling velocity of 0,88 cm/s and *downflow* in the draught tube.

TABLE I
COMPUTED VALUES OF PULP VELOCITY

Air rotameter reading no.	Air flowrate (measured)* m^3/h	Head ($Z_s - Z_b$) (measured) m	P_l power from liquid eq. (11) W	P_g power from air eq. (3) W	V_b pulp velocity eq. (7) m/s
2	1,730	0,043	3,8	20,4	0,649
4	2,376	0,063	9,6	28,0	0,786
6	3,060	0,068	11,6	35,7	0,817
8	3,744	0,105	35,6	44,0	1,015
10	4,572	0,110	38,4	53,4	1,039
12	5,328	0,123	50,0	62,3	1,098
14	6,156	0,140	69,6	72,1	1,172
16	7,056	0,150	82,4	82,7	1,213
18	7,956	0,165	104,6	93,5	1,272
20	8,892	0,170	112,0	104,2	1,290

*At normal temperature and pressure



r.d. = relative density

Note:
'Reasonable' pumping rate means that pulp velocity in draught tube is much greater than settling velocity of largest particles.

Fig. 4—Calculation procedures for air-agitated pachucas

The parameters of the impeller were as follows:

Draught-tube diameter	3000 mm
Impeller diameter	2800 mm
Grain-settling velocity	0,88 cm/s
Pumping rate	51500 m^3/h
Stirring speed	37 min^{-1}
Impeller power	57 kW
Tip speed	5,5 m/s.

The boundary conditions were as follows:

Vessel diameter	9250 mm
Filling height	17000 mm
Volume	1140 m^3
Solids density	2700 kg/m^3
Solids concentration	50%
Density of suspension	1460 kg/m^3
Particle size	70 %<75 μm
Temperature of suspension	20°C
Atmospheric pressure	1,01 bar abs.

Conclusions

The superficial circulating velocity of pulp agitated by air can be approximated by measurement of the head to which the aerated plume is elevated in the simple relation

$$V_b = \sqrt{2(Z_s - Z_b)g}, \quad \ldots \ldots \quad (7)$$

where $(Z_s - Z_b)$ *refers to the mean head* $(Z_s - Z_b)/2$.

The power required for pulp to be circulated at this velocity is the product of the values for the kinetic head and the mass flowrate. This latter value is the product of the horizontal discharge area of the exposed tip of the plume, the superficial velocity, and the density of the aerated pulp:

$$P_l = \frac{Z_b^2}{2} \cdot V_b \cdot (XSA) \cdot \varrho_l^* \cdot \quad \ldots \ldots \quad (9)$$

The combination of equations (7) and (9) for the particular conditions gave the following:

$$P_l = 9922{,}8\,(Z_s - Z_b)^{5/2}, (\varrho_l^* = 1285\ \mathrm{kg/m^3}). \quad \ldots \quad (12)$$

Addendum: Derivation of the Equations

Power of the Gas Stream

Bernoulli's equation for the gas between location o (at the air inlet) and location s (at the liquid surface) is

$$\frac{V_s^2 - V_o^2}{2} + (Z_s - Z_o)g + \int_o^s \frac{dp}{\varrho_g} + W_g + H = 0. \quad (1)$$

This equation can be simplified if the following assumptions are made.

(i) The velocity of the gas at the surface can be neglected, i.e. $V_s = 0$,

(ii) The friction term can be neglected, i.e. $H = 0$.

(iii) The gas density can be approximated by the ideal gas law, i.e. $p\varrho^{-1}$ = constant, so that

$$\int_o^s \frac{dp}{\varrho_g} \approx \frac{p_o}{\varrho_{go}} \ln \frac{p_s}{p_o}. \quad \ldots \ldots \quad (2)$$

By substitution of these assumptions into equation (2) and rearrangement, the following relation is obtained:

$$W_g = \frac{V_o^2}{2} + \frac{p_o}{\varrho_{go}} \ln \frac{p_o}{p_s} + (Z_o - Z_s)g,$$

where W_g is the work done by the gas on the vessel's contents per unit mass of gas.

Thus,

$$P_g \text{ (power from air)} = \text{mass flowrate} \times W_g. \quad \ldots \ldots \quad (3)$$

Power of the Liquid Stream

The only work done by the gas on the liquid is to circulate it, as follows.

(i) The air lifts the liquid, and the 'head' $(Z_s - Z_b)$ is created by energy transfer.

(ii) The liquid flows from s to b (Fig. 1) and, by continuity, circulation occurs.

The Bernoulli equation for the liquid stream is:

$$\frac{V_s^2 - V_b^2}{2} + (Z_s - Z_b)g + \int_b^s \frac{dp}{\varrho_{lb}} + W_l + H = 0.$$

This equation can be simplified by the making of the following assumptions.

(i) The liquid is incompressible, thus

$$\int_b^s \frac{dp}{\varrho} \approx \frac{1}{\varrho_l}(p_s - p_b). \quad \ldots \ldots \quad (4)$$

(ii) The friction term can be neglected, i.e. $H = 0$.

(iii) As shown in Fig. 5, the horizontal liquid velocity at s is zero ($V_s = 0$), and that at the draught tube is V_b.

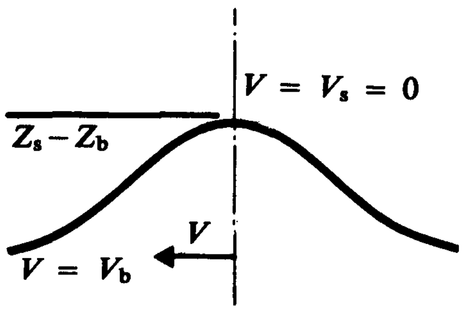


Fig. 5—The horizontal liquid velocity at s

(iv) W_l The work done by the pulp on its surroundings is nil, i.e. $W_l = 0$.

The simplified equation is then

$$-\frac{V_b^2}{2} + (Z_s - Z_b)g + \frac{1}{\varrho_l}(p_s - p_b) = 0. \quad \ldots \quad (5)$$

However, because $p_s = p_b$ = atmospheric pressure,

$$\frac{V_b^2}{2} = (Z_s - Z_b)g. \quad \ldots\ldots\ldots\ldots \quad (6)$$

Equation (6) states that the kinetic energy of the pulp at b derives solely from the difference in levels between s and b. This difference is created by the air flow.

From equation (6), therefore,

$$V_b = \left[2(Z_s - Z_b)g\right]^{\frac{1}{2}}. \quad \ldots\ldots\ldots\ldots \quad (7)$$

Density of the Aerated Column

At plane A (Fig. 6) under static conditions, the pressures outside and inside the tube are equal, i.e.

$$\varrho_l^* g Z_s = \varrho_l g Z_b, \text{ or } \varrho_l^* = \varrho \frac{Z_b}{Z_s}, \quad \ldots\ldots\ldots \quad (8)$$

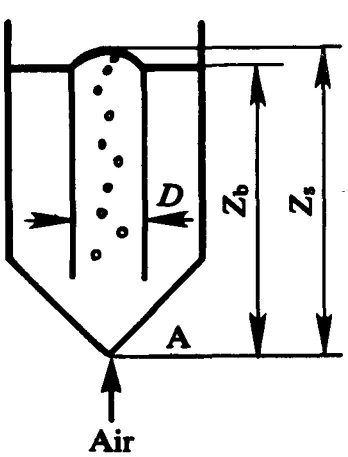


Fig. 6—Pressures at plane A (outside and inside the draught tube) under static conditions are equal

The Overflow Area at the Top of the Aerated Plume

The overflow area is the exposed tip of the plume, which is equivalent to the flow area of a submerged, circular weir, i.e.

$$(XSA) = \pi D(Z_s - Z_b).$$

From equation (6), the power of the liquid can be expressed as follows:

$$P_l = \frac{V_b^2}{2} \cdot V_b \cdot (XSA) \cdot \varrho_l^* \quad \ldots\ldots\ldots\ldots \quad (9)$$

$$= \frac{V_b^3}{2} \cdot (XSA) \cdot \varrho_l^*$$

$$= \frac{V_b^3}{2} \cdot \pi D(Z_s - Z_b) \cdot \varrho_l^*. \quad \ldots\ldots\ldots\ldots \quad (10)$$

Since the mean head $(Z_s - Z_b)/2$ is applicable in equation (7), the substitution for V_b in (10) gives the following:

$$P_l = \tfrac{1}{2}\left[2(Z_s - Z_b)\, g/2\right]^{3/2} \cdot \pi D(Z_s - Z_b) \cdot \varrho_l^*$$

$$= \tfrac{1}{2}(Z_s - Z_b)^{5/2} \cdot g^{3/2} \cdot \pi D \cdot \varrho_l^*. \quad \ldots\ldots\ldots \quad (11)$$

Substitution of the values for the 1 m tank gave

$$P_l = 9922{,}8\ (Z_s - Z_b)^{5/2},\ (\varrho_l^* = 1285 \text{ kg/m}^3) \quad \ldots \quad (12)$$

$$= P_g$$

i.e. all the liquid power is derived from the gas.

Nomenclature

D	Diameter of draught tube
g	Acceleration due to gravity, 9,81 m/s^2
H	Friction loss, J/kg
p	Pressure, N/m^2
P	Power, W
ϱ	Density, kg/m^3
V	Velocity, m/s
W	Work, J/kg
XSA	Cross-sectional area of exposed plume, m^2
Z	Displacement, m

Subscripts

b	At surface adjacent to wall
g	Gas
l	Liquid
s	At surface on centre line
o	At air inlet

Superscripts

*	Aerated

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The 22nd annual short course in modern X-ray powder diffraction will be offered at the State University of New York at Albany from 17th to 28th June, 1985. The course will be instructional, and will develop the basic theory and techniques starting from elementary principles. No previous knowledge or experience is required.

The first week will cover basic principles, techniques, and practical applications; and the second week will continue with further fundamentals and practical applications. Emphasis in the first week will be on camera and film techniques, X-ray instrumentation especially the diffractometer, identification of powder patterns, multi-phase identifications using several indices, and fundamentals of quantitative analyses. The second week will cover more advanced principles and techniques with emphasis on diffractometer alignment, complex quantitative analysis, complex powder identifications, computer automation of diffractometers, and computer search–match methods. Equal time will be devoted to lectures and laboratory/problem-solving sessions. A suitable amount of time will be set aside for individual sessions.

Registration may be made for one week, either week, at a registration fee of $850.00 or for the entire two-week session at a registration fee of $1,600.00, payable in U.S. dollars drawn on a U.S. bank. For further information and to register, please communicate with

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Registration may be made for one week, either week, at a registration fee of $850.00 or for the entire two-week session at a registration fee of $1,600.00, payable in U.S. dollars drawn on a U.S. bank. For further information and to register, please communicate with

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