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Abrasive and impactive wear of grinding balls in rotary mills

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SYNOPSIS

New distribution functions are derived to describe the size distribution of grinding elements in ball mills. The formulations are based on the assumption that abrasive as well as impactive interactions occur during ball milling—an assumption that is supported by a large body of experimental evidence. It is shown that the functions can be used in the estimation of the magnitudes of abrasive and impactive components in the total rate of ball wear.

The relative magnitudes of the wear components provide a basis for optimization of the chemical and metallurgical properties of the balls in a given milling situation. However, it is contended that these quantities are also useful indicators of the relative intensities of the abrasive and impactive interactions that are operative in the size reduction of mineral particles within ball mills. These quantities can be determined for any industrial ball mill, and their magnitudes provide practical guidelines for mill operation.

The theory is used in an analysis of samples of ball charges from two industrial ball mills, and it is also applied to all the data on ball-size distributions found in the literature. The qualitative correlation between the calculated values of the wear components and the reported operating conditions is good for a variety of industrial ball mills, in conformity with the hypothesis that the relative magnitudes of the wear components are related to the milling conditions.

SAMEVATTING

Daar word nuwe verdelingsfunksies afgelei om die grootteverdeling van maalelemente in balmeule te beskryf. Die formulerings word gegrond op die aanname dat daar sowel skuur- as slagwisselwerking tydens balmaling plaasvind—'n aanname wat deur 'n groot hoeveelheid eksperimentele getuienis gestaaf word. Daar word getoon dat die funksies gebruik kan word om die grootte van die skuur- en slagkomponente in die totale tempo van balslytasie te raam.

Die relatiewe grootte van die slytkomponente verskaf 'n grondslag vir die optimering van die chemiese en metallurgiese eienskappe van die balle in 'n gegewe maalsituasie. Daar word egter aangevoer dat hierdie groothede ook nuttige aanwysers is van die relatiewe intensiteit van die skuur- en slagwisselwerking wat plaasvind tydens die verkleining van mineraalpartikels in balmeule. Hierdie groothede kan vir enige industriële balmeul bepaal word en verskaf praktiese riglyne vir meulbedryf.

Die teorie word gebruik om monsters van balladings wat uit twee industriële balmeule verkry is, te ontleed en word toegepas op al die data oor balgrootteverdelings wat in die literatuur gevind is. Die kwalitatiewe korrelasie tussen die berekende waardes van die slytkomponente en die gerapporteerde bedryfstoestande is goed vir 'n verskeidenheid industriële balmeule in ooreenstemming met die hipotese dat die relatiewe groothede van die slytkomponente met die maaltoestande verband hou.

Introduction

Ball milling has been employed for more than a hundred years in the fine grinding of ores, coal, cement, and other materials. The world's consumption of grinding balls used in this way is about 500 kt a year, and the consumption of balls per ton of material milled varies widely. Ball consumption constitutes a significant proportion of the costs of fine grinding, rising markedly for hard and abrasive ores. In attempts to reduce ball consumption and the high cost of fine grinding, many studies have been directed to the analysis of factors causing ball wear. These studies have almost invariably attributed all the wear to one of the two wear mechanisms that operate in ball milling: abrasive wear and impactive wear.

That abrasion plays a large part in all fine-grinding operations has long been known and acknowledged. Over forty years ago, Prentice[1] reported tests aimed at providing evidence that all the wear of grinding balls is caused by abrasion. Much additional evidence has since been quoted[2] to show the significance of three-body abrasive interactions in fine grinding, which result in the size reduction of minerals and in the wear of grinding media.

The wear of grinding balls by impact[3] in a mill is not nearly as evident nor as widely accepted. Nevertheless, there is ample proof that balls are projected into flight and collide with the *en masse* charge and occasionally with the mill lining. Numerous photographic studies, the recent techniques of instrumented bolts[4] in mills, and the results of tests on balls fitted with accelerometers[5] have all produced irrefutable evidence of impact.

During impactive processes, the rate of ball wear, which is the rate of mass loss by the balls, is proportional to the ball mass, i.e. the cube of the ball diameter[3], whereas the rate of ball wear during abrasive processes is proportional to the surface area[1] of the balls, or the square of the ball diameter.

The general idea of *combined wear*, in which mechanisms of both abrasive and impactive wear are operative, can be attributed to Bond[6]. His work in this area can be summarized[1,7] by the following empirical relationship between the rate of ball wear, $-\mathrm{d}m/\mathrm{d}T$, and the ball diameter, X:

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$$-\frac{dm}{dT} = k'X^q, \qquad (1)$$

where m is the mass of a ball, T the amount of material milled, and k' and q are constants. The value of the exponent q, which by hypothesis should lie in the range $2 \leqslant q \leqslant 3$, provides an indication of the relative intensities of the two components in the rate of ball wear.

In discussions of the formula given in equation (1), Bond[6,8], Hukki[9], and Taggart[10] emphasized the role of mill speed and pointed out that, at low speeds, say in the neighbourhood of 60 per cent of critical speed, cascading will be a prominent feature of the charge motion and q will be very nearly equal to 2; at higher speeds, say 85 per cent of critical speed and higher, cataracting will be a prominent feature and q will be nearly equal to 3.

The idea that the abrasive and impactive components in ball wear are functions not only of the ball material but also of the mill-operating variables is therefore well established in the literature. As ball wear implies size reduction during the residence of balls in a mill, it can be expected that the size distribution of balls in a mill will be related to the relative intensities of the two wear mechanisms. These values can be of practical significance in the selection of the best chemical and mechanical characteristics of balls for a given milling operation; they can also be used to reveal the conditions in the mill in which the balls are operating as the grinding medium. For this reason, the present investigation is based upon data from measurements of ball-size distributions, and includes the derivation of expressions to describe a steady-state ball-size distribution under conditions of abrasive and impactive wear, and the determination of the relative magnitudes of the two components in the rate of ball wear under defined conditions of milling.

Relationships between Wear Mechanisms and Ball-size Distribution

Only ball *wear* is considered here. The production of smaller 'balls' by the fracture of larger ones is beyond the scope of the present work.

Qualitative and quantitative information relating to the relative intensities of the two wear components can be obtained from studies of ball-size distributions by the establishment of a relationship between the number density, $\nu(X)$, of balls of diameter X and the rate of size reduction of these balls, $-dX/dT$ (millimetre per ton of material milled). The number density function, which is often called the 'frequency distribution', is defined by the relation

$$dN = \nu(X)dX, \qquad (2)$$

where dN is the number of balls in the mill whose diameters are in the size interval X to $X + dX$, and X is within the range $X_o \leqslant X \leqslant X_{max}$.

From equation (2) it can be shown that the mass of this aggregate of balls is

$$dM = \frac{1}{6}\rho\pi X^3 \nu(X)dX, \qquad (3)$$

where ρ is the density of the ball material.

Ball-size distribution in a given mill is determined conventionally as follows: the entire ball charge is removed from the mill and screened into a number of size intervals; the number and mass in each size interval are then determined. In this way, experimental values are obtained for the distribution functions $n(X)$ and $m(X)$, which are, respectively, the cumulative number fraction and the cumulative mass fraction of balls passing size X. These distribution functions are related theoretically to the number density function, $\nu(X)$, according to the following formulae:

$$n(X) = \frac{\int_{X_o}^{X} \nu(X)dX}{\int_{X_o}^{X_{max}} \nu(X)dX}, \qquad (4)$$

and

$$m(X) = \frac{\int_{X_o}^{X} X^3\nu(X)dX}{\int_{X_o}^{X_{max}} X^3\nu(X)dX}. \qquad (5)$$

The charge of a ball mill has been shown[1,3,9] to tend to a stabilized condition after the mill has been operating under steady-state conditions for a sufficiently long time. *Steady state* implies a constant feed rate of ore to a mill and a steady rate of addition of top-size balls to compensate for the depletion of the ball charge by wear. The time required for the stabilized condition to be achieved is about

$$\frac{-3M}{(dM/dT)(dT/dt)},$$

where M is the total mass of balls within the mill, dM/dT is the rate of ball consumption (kilograms per ton of ore fed to the mill), dT/dt is the milling rate, and the negative sign appears because the mass of balls decreases during the milling operation. It is assumed that, in the stabilized condition, the ball-size distribution in a mill does not vary[1,3]; namely, that during operation the number and mass of balls in any specific size interval remain constant, although these quantities may vary substantially from one size interval to another. The number density, $\nu(X)$, is then related to the rate of ball wear by a consideration of the number of balls in a size interval, as follows.

If N_j is the number of balls in the mill whose diameters are in the size range X_i to X_j, where $X_o \leqslant X_i < X_j \leqslant X_{max}$, then from equation (2)

$$N_j = \int_{X_i}^{X_j} \nu(X)dX. \qquad (6)$$

If N_j is constant, its derivative with respect to the amount of material milled, T, is zero, i.e.

$$\frac{dN_j}{dT} = \frac{d}{dT}\int_{X_i}^{X_j} \nu(X)dX$$

$$= \nu(X_j)\frac{dX_j}{dT} - \nu(X_i)\frac{dX_i}{dT} = 0. \quad\text{.................(7)}$$

Since equation (7) holds good for *any* $X_i < X_j$ within the range X_o to X_{max}, it follows that

$$-\nu(X)\frac{dX}{dT} \text{ is constant. } \quad\text{...............................(8)}$$

From a consideration of the largest size group, it can be shown[11] that the constant in equation (8) is n', the number of top-size balls of diameter X_{max} that are added to the mill in the time required for the mill to grind a unit mass of new feed. Equation (8) can therefore be written in the form

$$\nu(X) = -\frac{n'}{dX/dT} \;. \quad\text{.....................................(9)}$$

Equation (9) shows that, in a stabilized mill, the number density of balls of size X is inversely proportional to the rate at which their diameters are being reduced. This relation, together with equations (2) to (5), can therefore be used to supply information on the relative intensities of the wear mechanisms that are operative in any given stabilized mill.

The exponent q in the empirical formulation of the rate of ball wear given in equation (1) provides important, although only qualitative, information on the relative intensities of the rates of abrasive and impactive wear in ball milling. This is because a value of q between 2 and 3 represents combined wear, in which the mechanisms of both abrasive and impactive wear are operative. The determination of q from measurements of the ball-size distribution requires the derivation of the number density function and the distribution functions appropriate to this formulation, followed by fitting of the derived distribution functions to the measured values of the number and mass fractions of balls passing size X.

From Bond's formulation of the rate of ball wear, which is expressed by

$$-\frac{dm}{dT} = k'X^q, \quad\text{...(1)}$$

the following is obtained:

$$-\frac{dX}{dT} = kX^{q-2}, \quad\text{.......................................(10)}$$

where $k = 2k'/\rho\pi$. Substitution of equation (10) into equation (9) then yields the following expression for the number density function:

$$\nu_C(X,q) = \frac{n'}{kX^{q-2}}, \quad\text{...................................(11)}$$

where the subscript C denotes combined wear.

Table I gives expressions for N_C, the total number of balls in the mill, and M_C, their total mass, and for the functions $n_C(X,q)$ and $m_C(X,q)$, which are number and mass distribution functions respectively. These expressions were obtained simply by the substitution of equation (11) into equations (2) to (5), followed by the required integrations. Table I also shows the forms to which the expressions reduce under hypothetical conditions of purely abrasive wear[1] and purely impactive wear[3], under which the exponent q has the limiting values of 2 and 3 respectively.

As an example of the applicability of the theory, the number distribution function, $n_C(X,q)$ (equation 14 in Table I), was fitted to measured values[7] of the ball-size distribution in the charge of a primary ball mill at Marievale (Table A-1 in the Addendum). The fitting procedure yielded the formula

$$n_C(X,q) = \frac{X^{0,36} - X_o^{0,36}}{X_{max}^{0,36} - X_o^{0,36}}, \quad\text{..........................(16)}$$

where X_o = 1,0 inch and X_{max} = 4,0 inches. For purposes of comparison, the respective number distribution functions $n_A(X)$ and $n_I(X)$ for purely abrasive and purely impactive wear (Table I), with the same values for X_o and X_{max}, are also depicted in Fig. 1. The function $n_C(X,q)$ for combined wear gives the best description of the measured ball-size distribution. Equations (14) and (16) show that q, as determined from the number distribution function, is

$$q(n_C) = 3 - 0,36 = 2,64.$$

A similar analysis, in which the mass distribution function for combined wear $m_C(X,q)$ and the measured mass distribution were used, yielded

$$q(m_C) = 2,28.$$

Despite a discrepancy of about 15 per cent between the two estimates of q, the results suggest fairly strongly that the balls in the mill were subjected to a significant impactive component.

It is clear that exponent q is a number, and that the determination of its value provides only a qualitative indication of the relative intensities of abrasive and impactive wear in a given mill.

Bond's empirical relation, which was shown in equation (1), can be replaced by a more detailed analysis that leads to the determination of the magnitudes of the two components in the rate of ball wear. The rates of abrasive and impactive wear are superposed in the following manner: during abrasion the rate of ball wear is proportional to the surface area of a ball[1], i.e.

$$-\frac{dm}{dT} \propto m^{2/3};$$

during impact the rate of ball wear is proportional to the mass of a ball[3], i.e.

$$-\frac{dm}{dT} \propto m.$$

TABLE I

EXPRESSIONS TO CHARACTERIZE A STABILIZED BALL-SIZE DISTRIBUTION UNDER COMBINED WEAR AND UNDER THE HYPOTHETICAL CONDITIONS OF PURELY ABRASIVE AND PURELY IMPACTIVE WEAR ACCORDING TO THE BOND FORMULATION OF THE RATE OF BALL WEAR

Quantity	General expression for combined (abrasive and impactive) wear*	Purely abrasive wear[7] (limiting form as q→2)	Purely impactive wear[3] (limiting form as q→3)
Formula for wear rate	$-\frac{dm}{dT} \propto X^{q}$ .... (1)	$-\frac{dm}{dT} \propto m^{2/3}$	$-\frac{dm}{dT} \propto m$
Number density function	$\nu_C(X,q) = \frac{n'}{kX^{q-2}}$ .... (11)	$\nu_A(X) = \frac{n'}{k}$	$\nu_I(X) = \frac{n'}{kX}$
Number of balls in the mill	$N_C(q) = \frac{n'(X_{max}^{3-q} - X_o^{3-q})}{k(3-q)}$ ..... (12)	$N_A = \frac{n'}{k}(X_{max} - X_o)$	$N_I = \frac{n'}{k}(\ln X_{max} - \ln X_o)$
Mass of balls in the mill	$M_C(q) = \frac{n'\rho\pi(X_{max}^{6-q} - X_o^{6-q})}{6k(6-q)}$ .... (13)	$M_A = \frac{n'\rho\pi}{24k}(X_{max}^4 - X_o^4)$	$M_I = \frac{n'\rho\pi}{18k}(X_{max}^3 - X_o^3)$
Number distribution function of balls smaller than size X	$n_C(X,q) = \frac{X^{3-q} - X_o^{q-3}}{X_{max}^{3-q} - X_o^{3-q}}$ ...... (14)†	$n_A(X) = \frac{X - X_o}{X_{max} - X_o}$	$n_I(X) = \frac{\ln X - \ln X_o}{\ln X_{max} - \ln X_o}$
Mass distribution function of balls smaller than size X	$m_C(X,q) = \frac{X^{6-q} - X_o^{6-q}}{X_{max}^{6-q} - X_o^{6-q}}$ ...... (15)†	$m_A(X) = \frac{X^4 - X_o^4}{X_{max}^4 - X_o}$	$m_I(X) = \frac{X^3 - X_o^3}{X_{max}^3 - X_o^3}$

*The present investigation

Subscript C denotes combined wear, Subscript A denotes purely abrasive wear, Subscript I denotes purely impactive wear

†Austin and Klimpel[20] recently also derived these functions by different methods

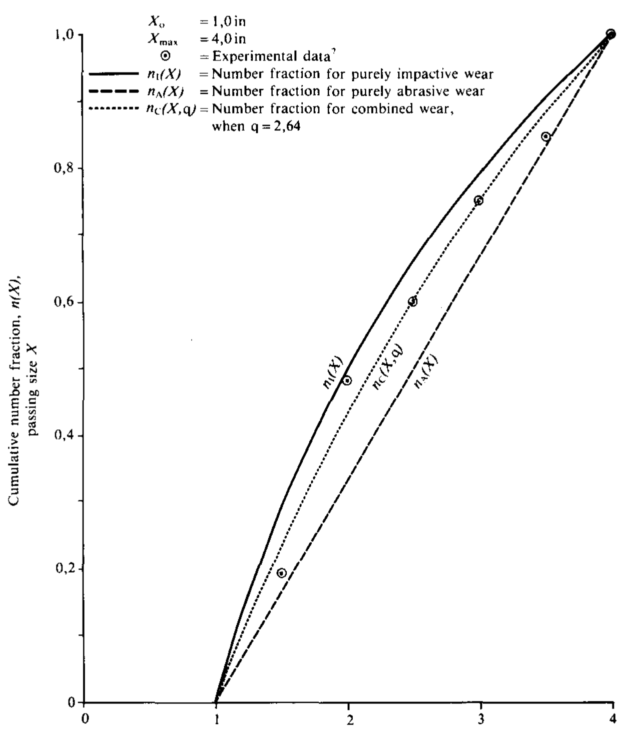

Fig. 1—Comparison of the measured size distributions of balls in the charge of the no. 1 primary ball mill at Marievale with those calculated for three types of wear

Therefore combined wear should, to first order, be given by[2,12]

$$-\frac{dm}{dT} = \alpha m + \beta m^{2/3}, \qquad (17)$$

where α and β are constants.

The two terms on the right-hand side of equation (17) can be expressed in terms of ball size as

$$W_I(X) = \frac{1}{6}\alpha\rho\pi X^3 \qquad (18)$$

and

$$W_A(X) = \beta(\frac{1}{6}\rho\pi X^3)^{2/3}, \qquad (19)$$

where $W_A(X)$ and $W_I(X)$ denote the respective rates of abrasive and impactive wear on balls of size X within the range $X_o \leqslant X \leqslant X_{max}$.

A number density function based upon this formulation of the rate of ball wear can be obtained easily. Manipulation of equation (18) yields

$$-\frac{dX}{dt} = 1/3\ \alpha(X + \lambda), \qquad (20)$$

where

$$\lambda = \frac{\beta}{\alpha(\rho\pi/6)^{1/3}}. \qquad (21)$$

The number density function, obtained by the substitution of equation (20) into equation (9), is

TABLE II

EXPRESSIONS TO CHARACTERIZE A STABILIZED BALL-SIZE DISTRIBUTION UNDER CONDITIONS OF COMBINED WEAR

Quantity	General expression for combined (abrasive and impactive) wear*
Formula for wear rate	$-\dfrac{dm}{dT} = \alpha m + \beta m^{2/3}$ .... (17)
Number density function	$\nu_C(X,\lambda) = \dfrac{3n'}{\alpha(X+\lambda)}$ .... (22)
Number of balls in the mill	$N_C(\lambda) = \dfrac{3n'}{\alpha}\ln\left(\dfrac{X_{max}+\lambda}{X_o+\lambda}\right)$ .... (23)
Mass of balls in the mill	$M_C(\lambda) = \dfrac{n'\rho\pi}{2\alpha}\left[\dfrac{1}{3}(X^3_{max}-X^3_o)-\dfrac{1}{2}\lambda(X^2_{max}-X^2_o)+\lambda^2(X_{max}-X_o)-\lambda^3\ln\left(\dfrac{X_{max}+\lambda}{X_o+\lambda}\right)\right]$ .... (24)
Number distribution function	$n_C(X,\lambda) = \dfrac{\ln(X+\lambda)-\ln(X_o+\lambda)}{\ln(X_{max}+\lambda)-\ln(X_o+\lambda)}$ .... (25)
Mass distribution function	$m_C(X,\lambda) = \dfrac{\frac{1}{3}(X^3-X^3_o)-\frac{1}{2}\lambda(X^2-X^2_o)+\lambda^2(X-X_o)-\lambda^3\ln\left(\frac{X+\lambda}{X_o+\lambda}\right)}{\frac{1}{3}(X^3_{max}-X^3_o)-\frac{1}{2}\lambda(X^2_{max}-X^2_o)+\lambda^2(X_{max}-X_o)-\lambda^3\ln\left(\frac{X_{max}+\lambda}{X_o+\lambda}\right)}$ .... (26)

*Based upon the superposition of abrasive and impactive wear rates

$$\nu_C(X,\lambda) = \frac{3n'}{\alpha(X+\lambda)} \cdot \quad\ldots\ldots\ldots (22)$$

This function is expressed as a function of λ, because the latter has important physical significance within the present context, which will be discussed further.

Expressions equivalent to those listed in Table I, derived by the substitution of equation (22) into equations (2) to (5), are shown in Table II. In addition to these equations—(23) to (26)—expressions can be derived for the cumulative number and cumulative mass of balls smaller or larger than a given size, the number and mass of balls in a size interval, the mean ball diameter, the surface area of the ball charge, and so on.

The parameter λ in the expressions above is proportional to β/α so that, if β tends to zero, then λ also tends to zero. The expressions in Table II then reduce to the corresponding expressions that are applicable under hypothetical conditions of purely impactive wear (Table I). On the other hand, if β is finite and α tends to zero, then λ tends to infinity. The expressions in Table II then reduce to the corresponding expressions that are valid under hypothetical conditions of purely abrasive wear (Table I).

The physical meaning of λ becomes apparent from equations (18), (19), and (21) when the rate of abrasive wear is equated to that of impactive wear. This shows that λ represents a ball size for which, under the given milling conditions, the ratio of abrasive to impactive wear is unity. If $\lambda/X_{max} > 1$, then all the balls in the mill experience primarily abrasive wear, i.e. the action of the mill is primarily abrasive, but there is a finite impactive component as well. If $\lambda/X_{max} < 1$, then all the balls in the mill with diameters larger than λ experience primarily impactive wear, and all those with diameters less than λ undergo primarily abrasive wear. The action of the mill is then clearly such that balls are subject to both wear mechanisms, but the impactive component is now enhanced relative to the previous case, in which λ/X_{max} was greater than 1.

Equations (18) and (19) give the respective rates of abrasive and impactive wear on balls of size X. Summation over all the balls in the mill gives the values for W_A(charge) and W_I(charge), the respective rates of abrasive and impactive wear on *the whole ball charge*, as shown in equations (27) and (28).

$$W_A(\text{charge}) = \int_{X_o}^{X_{max}} W_A(X)\nu(X,\lambda)dX$$

$$= \tfrac{1}{2}\, n'\rho\pi\lambda\left[\tfrac{1}{2}(X^2_{max} - X^2_o) - \lambda(X_{max} - X_o) + \lambda^2\ln\frac{X_{max}+\lambda}{X_o+\lambda}\right] \cdot \quad\ldots\ldots\ldots (27)$$

Similarly,

$$W_I(\text{charge}) = \tfrac{1}{2}\, n'\rho\pi\left[1/3(X^3_{max} - X^3_o) - \tfrac{1}{2}\lambda(X^2_{max} - X^2_o) + \lambda^2(X_{max} - X_o) - \lambda^3\ln\frac{X_{max}+\lambda}{X_o+\lambda}\right] \cdot \quad\ldots\ldots (28)$$

Hence, the ratio of the rates of abrasive to impactive wear on the ball charge is

$$R_{AI}(\text{charge}) = \frac{\tfrac{1}{2}\lambda(X^2_{max} - X^2_o) - \lambda^2(X_{max} - X_o) + \lambda^3 \ln \dfrac{X_{max} + \lambda}{X_o + \lambda}}{1/3(X^3_{max} - X^3_o) - \tfrac{1}{2}\lambda(X^2_{max} - X^2_o) + \lambda^2(X_{max} - X_o) - \lambda^3 \ln \dfrac{X_{max} + \lambda}{X_o + \lambda}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Equation (29) shows that the ratio R_{AI}(charge) is independent of α, β, and the rate of ball consumption. It is a function only of λ, X_o, and X_{max}. The quantity λ can therefore be used to express quantitatively the relative magnitudes of the rates of abrasive and impactive wear that are operative on the whole ball charge. It is contended that not only are these quantities related to ball wear, but they are also important indicators of the mechanisms of size reduction of mineral particles in a given milling situation. Hence, their calculated magnitudes should correlate with the milling conditions, e.g. mill diameter, mill speed, liner configuration, pulp density, ball size.

Furthermore, λ can also be related to the exponent q in the Bond formulation of ball wear. The relationship is found by the use of equations (12) and (13) in Table I in conjunction with equations (23) and (24) in Table II, to yield equation (30):

$$\frac{1/3(X^3_{max} - X^3_o) - \tfrac{1}{2}\lambda(X^2_{max} - X^2_o) + \lambda(X_{max} - X_o) - \lambda^3 \ln \dfrac{X_{max} + \lambda}{X_o + \lambda}}{\ln(X_{max} + \lambda) - \ln(X_o + \lambda)} = \frac{(3 - q)(X^{6-q}_{max} - X^{6-q}_o)}{(6 - q)(X^{3-q}_{max} - X^{3-q}_o)} \cdot (30)$$

This relationship shows that, if the value of λ is known, q can be determined, and *vice versa.*

The new distribution functions $n_C(X,q)$, $m_C(X,q)$, $n_C(X,\lambda)$, and $m_C(X,\lambda)$ shown in Tables I and II therefore provide various methods for the determination of λ, and hence of the rates of abrasive and impactive wear during ball milling.

In mathematical integrations over a range of ball sizes, the quantities k, α, and β in the two formulations of ball wear were regarded as constants, this assumption being common to the present work and all previously reported investigations[1,3,20]. These quantities are essentially shape factors, implying that the derived expressions are applicable strictly only to size distributions in which the grinding elements remain spherical. Examination of the ball charge in any industrial mill will show that a substantial proportion of the smaller grinding elements are not spherical, and some degree of error therefore arises when the derived distribution functions are applied to ball charges.

The complete evaluation of these errors is difficult, but the constant-shape approximation is good for balls larger than one-third of the size of top-size balls because these balls are fairly well rounded. The smaller, non-spherical elements usually constitute less than one-third of the total number and a much smaller proportion of the total mass. The non-sphericity factor can be taken into account if the total mass is calculated according to equations (13) or (24) and the results are compared with the measured total mass of the charge. The value of the calculated mass will always be smaller than the value of the measured mass unless all the grinding elements are truly spherical. The degree of uncertainty to be associated with theoretical assessments of the abrasive and impactive components in ball wear can be estimated by the use of this difference between the measured and the calculated ball masses.

The Nature of Ball Wear in Some Industrial Mills

The determination of the ball-size distribution in an industrial mill is a lengthy process involving disruption of the milling process and possibly some loss of production. Although the alternative procedure, i.e. sampling of the ball charge, introduces an element of uncertainty, it was carried out in two industrial mills—a 9 ft by 10 ft ball mill at the Libanon Gold Mine and an 8 ft by 8 ft ball mill at the Marikana Mine of Western Platinum Limited—and the data obtained were analysed. In addition, the literature was searched for all the relevant data that could be used in testing the validity of the theory of combined wear.

Data reported by Prentice[1] for the whole ball charge in a 6½ ft by 12 ft mill at Blyvooruitsig Gold Mine included the numbers and masses of balls in every ½-inch size interval between 1 and 3 inches, the total number and total mass of the balls, and the rate of ball consumption, which was equivalent to 300 top-size balls per day. The mill had been operating under steady-state conditions for 8 months, and it seems a safe assumption that the charge was stabilized, since about 9,5 times the total ball load had been consumed in that period. The throughput of the mill was not reported, but it has been suggested[13] that it was about 450 t/d, giving a rate of ball consumption of 0,667 balls or 1,17 kg of balls per ton milled. The original data[1] are reported in Table A-2 of the Addendum. Such comprehensive data provide a basis for illustrating the application of the various formulae in the calculation of the rates of abrasive and impactive wear under given milling conditions.

The cumulative forms of Prentice's data are shown by the points in Figs. 2(a) to 2(d). The distribution functions $n_C(X,\lambda)$ and $m_C(X,\lambda)$, shown in equations (25) and (26), were adapted to the data. The fitting procedures, with X_o equal to 0,6 inch, gave the following values:

$\lambda(n_C)$ = 8,1 inches = 206 mm
$\lambda(m_C)$ = 8,9 inches = 226 mm.

In (a) and (b) of Fig. 2, the cumulative forms of the Blyvooruitsig values[1] are compared with graphs of the

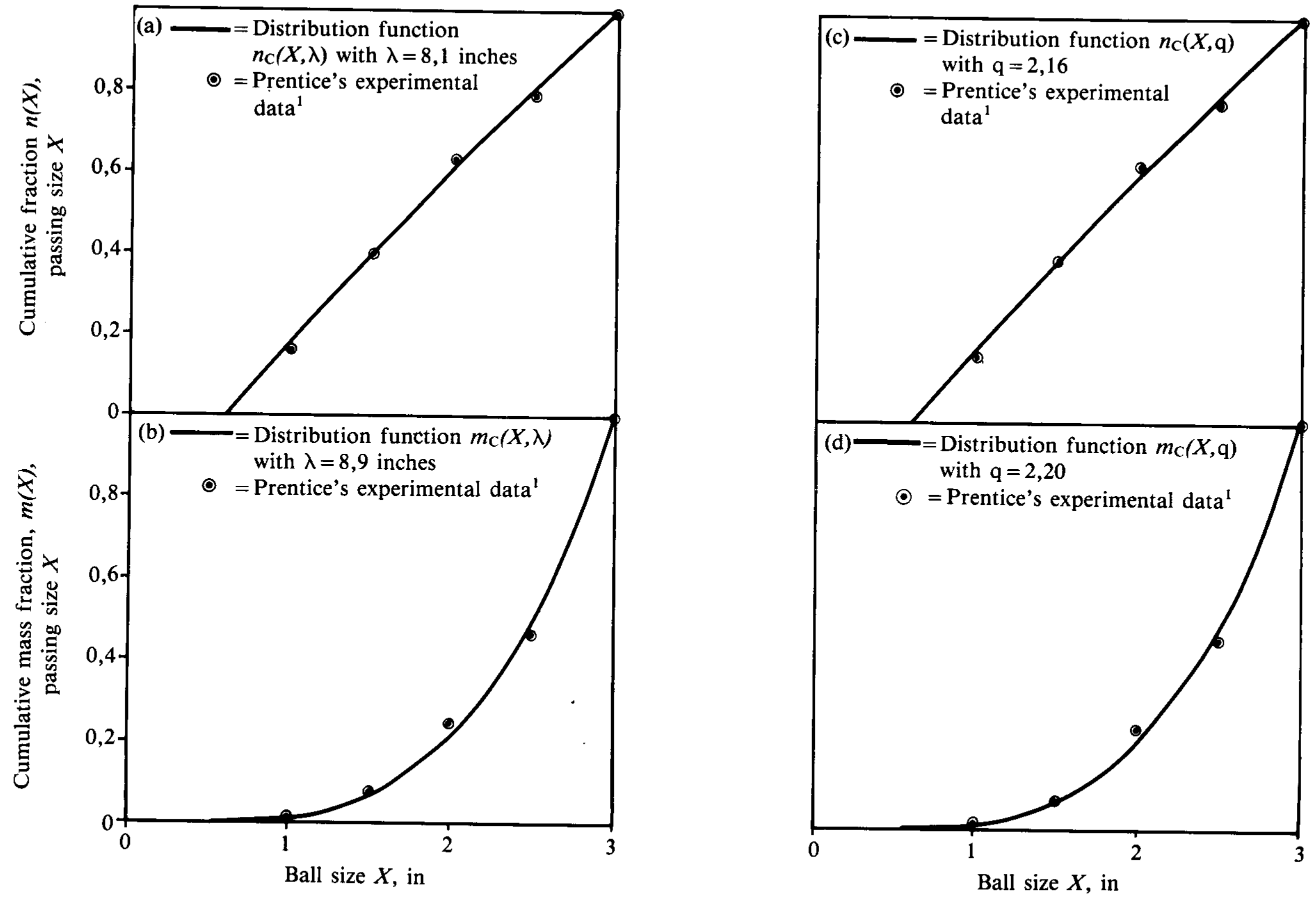

Fig. 2—Comparison of measured and calculated size distribution of balls in the charge of the ball mill at Blyvooruitsig Gold Mine

functions $n_C(X,\lambda)$ and $m_C(X,\lambda)$ plotted from the above values of the parameters. It can be seen that the curves are good descriptions of the measured values. The discrepancy between the two values of λ is less than 10 per cent, and the mean value (216 mm) is nearly three times larger than that of the top-size balls fed to the mill. This indicates that, although abrasive processes were dominant in the mill, there was a finite impactive component as well.

The distribution functions $n_C(X,q)$ and $m_C(X,q)$ as shown in equations (14) and (15) respectively (Table I) were also fitted to Prentice's data. The fitting procedures yielded the formulae

$$n_C(X,q) = \frac{X^{0,84} - 0,6^{0,84}}{3^{0,84} - 0,6^{0,84}}$$

and

$$m_C(X,q) = \frac{X^{3,80} - 0,6^{3,80}}{3^{3,80} - 0,6^{3,80}}\,.$$

In (c) and (d) of Fig. 2, the cumulative forms of the Blyvooruitsig values[1] are compared with graphs of these formulae for $n_C(X,q)$ and $m_C(X, q)$. It can be seen that the graphs are also good descriptions of the measured values, and the above functions suggest that

$$q(n_C) = 3 - 0,84 = 2,16 \text{ and}$$
$$q(m_C) = 6 - 3,80 = 2,20.$$

The two values obtained for q are in good agreement and, being close to 2, indicate again that abrasive processes were predominant in the mill. The mean value of q was used in equation (30) for the calculation of

$$\lambda(q) = 193 \text{ mm } (7,6 \text{ inches}).$$

This value of λ is in good agreement with those determined previously, which confirms the internal consistency of the theory.

The mean of all the values of λ obtained so far gives

$$\bar{\lambda}_{\text{Blyvooruitsig}} = 208 \text{ mm}.$$

Quantitative values of α and β in equation (17), the expression for combined wear, can be determined for the mill because the total number, total mass, and total rate of consumption of the balls are known. When the mean value of λ is substituted into equations (21), (24), and (25), it is found (by use of the data in Table A-2 and the assumption $\rho = 7,6 \times 10^3$ kg m^{-3}), that

$$\alpha \approx 1,97 \times 10^{-5}\ t^{-1},$$
$$\beta \approx 6,5 \times 10^{-5}\ kg^{1/3}\ t^{-1}.$$

These values of α and β permit the calculation of the rate of abrasive wear, $W_A(m)$, and the rate of impactive wear, $W_I(m)$, to which any ball of mass m in the mill was subject. For example, for balls of average mass $\bar{m}$ (0,558 kg),

$$W_A(\bar{m}) = \beta\bar{m}^{2/3} \approx 4,4 \times 10^{-5} \text{ kg t}^{-1},$$

and

$$W_I(\bar{m}) = \alpha\bar{m} \approx 1,1 \times 10^{-5} \text{ kg t}^{-1}.$$

These values, for balls of average mass, indicate that abrasive interactions were predominant in the given mill. That this was characteristic of the whole charge in the mill can be confirmed by calculation of the rates of abrasive wear and impactive wear on the whole ball charge, viz W_A(charge) and W_I(charge). Quick estimates, which can be confirmed by detailed calculations based on equations (27) and (28), indicate that, since $N\bar{m}$ is the total mass of the charge (13 309 kg), the charge experienced an impactive wear rate of

$$W_I(\text{charge} = \alpha N\bar{m} \approx 0,26 \text{ kg t}^{-1}.$$

Hence the abrasive wear rate of the whole charge was

$$\begin{aligned} W_A(\text{charge}) &= \text{rate of ball consumption minus } W_I(\text{charge}) \\ &= 1,17 - 0,26 \\ &\approx 0,91 \text{ kg t}^{-1}. \end{aligned}$$

The ratio R_{AI}(charge) of the rates of abrasive to impactive wear on the ball charge was therefore

$$R_{AI}(\text{charge}) \approx 3,5.$$

Substitution of $\bar{\lambda}$ into equation (29) yields a value of 3,6 for R_{AI}(charge), which is in good agreement with the above.

The value of R_{AI}(charge) clearly indicates that abrasion was probably the predominant comminution mechanism in that mill and explains why Prentice[1] was able to claim that the Blyvooruitsig data supported his theory of ball wear. However, the results of the present work show that about 23 per cent of the total ball consumption in the given mill was due to an impactive component.

The influence of the proportion of non-spherical grinding elements on the above results should be considered. The measured mass of the charge was 13 309 kg, and the mass, as calculated from equation (24), was 12 334 kg, i.e. the calculated mass was smaller than the measured mass, as expected. The discrepancy of only 8 per cent is due to the non-sphericity of the smaller, worn grinding elements in the charge. This suggests that the error in the results for the abrasive component is about $8^{2/3}$ per cent, i.e. 4 per cent, and that the impactive component is too high by about 8 per cent. If allowances are made for uncertainties in λ and X_o, it can be concluded that the estimate of the ratio of the rates of abrasive to impactive wear in the mill at Blyvooruitsig is, as far as is known at present, reliable to about 20 per cent.

In the present work, ball samples were drawn from the charges of the mills at the Libanon Gold Mine and the Marikana Mine of Western Platinum Ltd. The data describing the size distribution in the two ball samples are given in Tables A–III and A–IV of the Addendum. Fig. 3 compares the cumulative forms of the data (i.e. the number fractions of balls smaller than size X) relating to these mills with curves representing the extremes of purely abrasive wear and purely impactive wear, $n_A(X)$ and $n_I(X)$ respectively. The data points are clearly intermediate between these two extremes, and the best descriptions of the data are provided in each instance by the graphs for the theory of combined wear, i.e. the function $n_C(X,\lambda)$ as given by equation (25) in Table II, with the values

$$\lambda_{\text{Libanon}} = 149 \text{ mm}$$
$$\lambda_{\text{Marikana}} = 10,2 \text{ mm}.$$

These values of λ and the relevant data were substituted into equation (29), and the ratio of the rates of abrasive to impactive wear on the charge R_{AI}(charge) was calculated for both ball samples. The results (Table III) show that the impactive component accounted for 74 and 34 per cent of the ball consumption in the Marikana and Libanon mills respectively. Table III also shows that the differences in the milling conditions at the two mines are quite consistent with these results: the mill at Marikana was equipped with lifter bars, which provided a strong impactive component, whereas the mill at Libanon had a grid liner, which was effectively smooth and which exhibited marked circumferential grooving due to slip and therefore marked abrasion of the grinding charge. These results are also consistent with those of Blyvooruitsig, where the mill speed and diameter, volume of the charge and top-size balls, and the impactive component (23 per cent) were all less than at Libanon. The relationship between the magnitudes of the wear components and the operating conditions of ball mills is an important corollary to the present work. The salient milling conditions include the diameter and speed of the mill, the size of the balls fed, the nature of the mill lining, and the pulp density. It is contended that the relative magnitudes of the wear components provide an indication of the grinding mechanisms that are operative in ball mills, and that the relative magnitudes can be changed by variation of one or more of the salient milling conditions.

This matter was investigated further by the application of the theory of combined wear to all the ball-size distributions found in the literature, and the values of q, λ, and the relative magnitudes of the components in the rate of ball wear were determined. The results of this analysis are shown in Fig. 4, which was obtained by use of the fact that λ is a function of q and of the ball-size reduction ratio, X_{max}/X_o, as shown in equation (30). The points in Fig. 4 show values of λ/X_{max} for the various mills plotted as a function of the corresponding values of q. The continuous curves in Fig. 4 show the theoretical relationship—equation (30)—between λ/X_{max} and q for various specific values of X_{max}/X_o. In spite of the limited information and sometimes inadequate data (e.g. the size range of the balls in four ball-tube mills at Lake Shore[14] was divided into only three intervals), the measured values exhibit a trend that provides strong confirmation of the validity of the theory of combined wear proposed here. At first sight, the measured values corresponding to ball

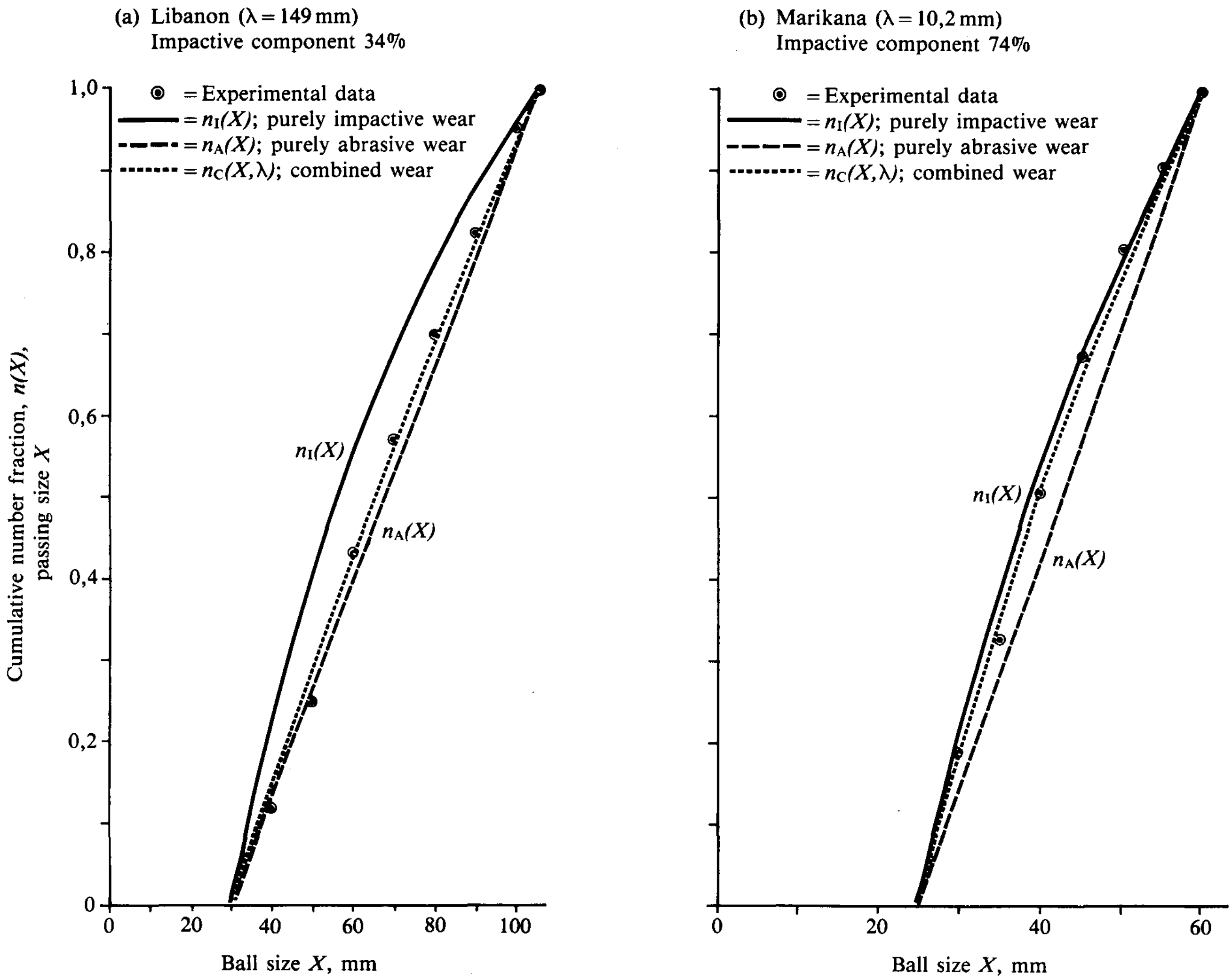

Fig. 3—Comparison of the measured and calculated size distributions in samples of ball charges at Libanon and Marikana Mines

charges at Sub Nigel[1] and Hollinger[15] appear to deviate from the general trend, but this anomaly is resolved when one notes that the ball scrap at these mills was recirculated until it was completely worn away. These values are clearly associated with curves appropriate to an infinite or very large reduction ratio.

The majority of the points shown in Fig. 4 were calculated from data reported by various authors, most of whom interpreted their results in terms of either purely abrasive wear or purely impactive wear. The only exception was a guarded reference by Crocker[14] to the possibility of combined wear in the no. 7 ball mill at Lake Shore. Fig. 4 shows that none of the data corresponds to values of q equal to 2 or 3; on the contrary, the points are fairly uniformly distributed between these two values. Therefore, the present analysis and the physical evidence discussed briefly in the introduction suggest strongly that both wear mechanisms are present in all industrial mills. In particular, although it was claimed[1,7] that the ball wear at Marievale[7], Hollinger[15], Blyvooruitsig[1], and Sub Nigel[1] was due to purely abrasive interactions, the present re-analysis of the data confirmed that abrasion was indeed predominant at Hollinger, Blyvooruitsig, and Sub Nigel, but that impactive processes were predominant at Marievale. Information from the archives of General Mining Union Corporation Limited[17] has shown that, at Marievale, the mill speed (more than 80 per cent of critical speed), the volume of the ball charge (nearly 50 per cent of the internal mill volume), and the top-size balls (4 inches) were consistent with the existence of an impactive component when the ball-size distribution in that mill was measured.

Table III gives values of q, λ, and the relative magnitudes of the wear components for some of the mills. Data for the four ball-tube mills at Lake Shore[14] were ignored for the reasons given above. As indicated in Table III, the gradual transition from predominantly impactive to strongly abrasive conditions is consistent with accompanying changes in the conditions obtaining in the various mills, and provides strong support for the contention that the relative magnitudes of the wear components are related to the milling conditions.

Summary and Conclusions

With the exception of Bond[6,8], previous investigators[1,3,7,18,19] ascribed the wear of balls in milling to only one wear mechanism. The most recent paper[20] on the subject, which apparently found 'the situation clear-

TABLE III MILLING CONDITIONS AND THE MAGNITUDE OF THE IMPACTIVE COMPONENT IN SOME INDUSTRIAL BALL MILLS

	Mill				Grinding balls		Calculated parameters				
Mine	Diameter ft	Length ft	Speed % of critical	Volume of charge %	Top size mm	Reject size mm	q	λ	λ/X_{max}	Ratio of abrasive to impactive wear	Impactive wear % of total wear
Miami Copper[3] (Hardinge conical mill)	8	2	82	20	50,8	0	2,97	0,0047	Small	0,0033	99,7
Marikana Western Platinum Limited	8	8	80	45	60	25	2,78	10,17	0,203	0,26	74
Marievale[7]	8	8	83	47	120	25	2,46	67,5	0,66	0,91	52
Lakeshore[14] no. 7 ball mill	7	6	85	45	108	25	2,31	138	1,28	1,72	36
Libanon	9	10	79	48	108	30	2,30	149	1,42	1,88	34,7
Hollinger[15]	6½	12	80	50	76	0	2,17	127	1,67	2,18	30,5
Blyvoortuitsig[1]	6½	12	70	25	76	15	2,17	207	2,73	3,65	21,5
Sub Nigel[1]	6½	9	68	33	100	0	2,13	483	4,75	6,4	13,5

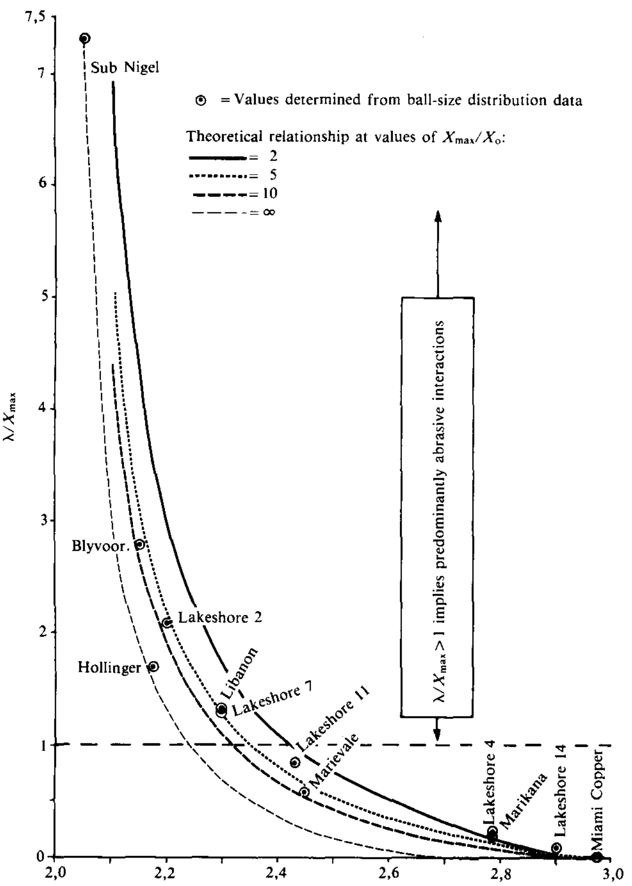

Fig. 4—Values of λ/X_{max} as a function of q for the ball-size distributions in a number of mills

ly confused', also subscribed to this view. The present work has evidence that abrasive as well as provided impactive interactions occur among the grinding elements in ball mills. A theory of combined wear was formulated by the superposition of the wear rates of balls due to the two types of interaction. New distribution functions based upon this theory of combined wear were derived, and the values determined from these functions were compared with measurements of the ball-size distributions in a variety of industrial ball mills. The distribution of ball sizes was shown to depend upon the relative magitudes of the two wear mechanisms operating in ball mills, and the relative magnitude of the two components in the rate of ball wear were determined quantitatively for a large number of industrial ball mills.

The relative intensities of the two wear mechanisms are determined by the mechanical properties of the balls and by such factors as the mill diameter and speed, the top-size balls added, the design of the mill lining, and the density of the mineral pulp. Hence, the magnitudes of the components can be used as a basis for the selection of the optimum chemical and metallurgical properties of balls. They also provide indications of the mechanisms of size reduction that are operating in any industrial ball mill. This was confirmed by a comparison of measured ball-size distributions with the milling conditions in a number of industrial mills.

It was shown that the new formulae can be used to determine the relative magnitudes of the wear components from the size distribution in a representative sample of balls from a mill. This obviates the laborious sizing of the entire grinding charge of a ball mill. Rapid and

TABLE III (cont.)

Remarks on milling conditions in relation to the impactive and abrasive components
High-speed cataracting conditions, low solids content in pulp[16], and high ball-to-ball contact; hence impactive component almost totally predominant
High speed with lifter bars in linings; hence cataracting conditions and strong impactive component
High speed and large grinding balls enhance impactive component, whereas smooth lining limits cataracting; hence impactive and abrasive components almost equal
Conditions similar to those at Marievale, but smaller mill diameter increases abrasive component
Larger mill diameter, but slower speed tends to increase abrasive component; circumferential grooving on the lining is evident
Smaller mill diameter and smaller ball size increase abrasive component
Conditions similar to those at Hollinger but much smaller ball charge increases abrasive component still further
Mill of slowest speed gives highest abrasive component

efficient methods for the taking of representative samples would be of practical value.

The formulae and techniques of analysis developed in the present work could also be useful in other applications where particles are consumed at rates proportional to their dimensions, e.g. in chemical reactors, where the dissolution of particles is a function, not only of their exposed surfaces, but of the diffusion of the reactants into the particles, i.e. a volume effect.

Addendum

Ball-size distribution data that were analysed by the theory of combined wear are given in Tables A–I to A–IV.

TABLE A–I
BALL-SIZE DISTRIBUTION AT MARIEVALE (AFTER WHITE[7])

Ball size inch	Mass	Number of balls	Prentice theory[1]	Davis theory[3]
<4 >3½	45,3	2 282	2 185	1 685
<3½ >3	18,7	1 423	2 185	1 947
<3 >2½	17,6	2 240	2 185	2 304
<2½ >2	7,7	1 792	2 185	2 820
<2 >1½	8,6	4 268	2 185	3 636
<1½ >1	2,1	2 935	2 185	5 124
Total	100,0	14 940	13 110	17 516

TABLE A–II
BALL-SIZE DISTRIBUTION AT THE BLYVOORUITSIG GOLD MINE, 18TH OCTOBER, 1942 (AFTER PRENTICE[1])

Grading of charge inch	Mass %	Total mass lb	Average mass per ball lb	Number of balls
<3 >2½	53,34	15 650	3 114	5 026
<2½ >2	22 25	6 528	1,774	3 680
<2 >1½	17,05	5 002	0,890	5 623
<1½ >1	5,90	1 732	0,333	5 198
<1	1,46	428	0,099	4 316
Total	100,00	29 340	1,231*	23 843

*Average

TABLE A–III
SIZE DISTRIBUTION IN A SAMPLE OF BALLS FROM THE CHARGE OF NO. 2 BALL MILL AT LIBANON GOLD MINE

Size range mm	Number of balls
20 to 30	31
30 to 40	132
40 to 50	185
50 to 60	249
60 to 70	190
70 to 80	183
80 to 90	170
90 to 100	178
100 to 105	64
Total	1382

TABLE A–IV
SIZE DISTRIBUTION IN A 344 kg SAMPLE OF BALLS FROM THE CHARGE IN THE NO. 3A-STREAM BALL MILL AT THE MARIKANA MINE OF WESTERN PLATINUM LIMITED

Ball size mm	Number of balls
<30	204
>30 <35	156
>35 <40	189
>40 <45	179
>45 <50	141
>50 <55	101
>55 <60	100
Total	1070

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