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Economic optimization of the number of boreholes and deflections in deep gold exploration

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SYNOPSIS

Deep gold exploration such as is used for Witwatersrand gold deposits is both expensive and time-consuming. This paper provides quantitative guidelines as to the economic optimum number of boreholes and deflections within each borehole for a certain reef characterized by its variability pattern and depth below surface in order to maximize the level of confidence for a fixed drilling budget or to minimize the total drilling cost for a given level of confidence.

Portions of the Basal, Vaal, and Elsburg Reefs that show a wide range of variability (sample logarithmic variances of 0,63, 1,05, and 2,06 respectively) are examined at depths below surface ranging from 1000 m to 4000 m over a simulated mining property covering 7000 ha (7000 m by 10 000 m).

It is concluded that four intersections per borehole are optimum in all the cases considered, which cover the majority of current Witwatersrand exploration, with the exception of the highly variable Elsburg Reef at depths of 1000 m and 2000 m, where boreholes with one and two intersections respectively should be preferred.

SAMEVATTING

Diep goudeksplorasie soos wat vir die goudafsettings aan die Witwatersrand gebruik word, is duur en tydrowend. Hierdie referaat gee kwantitatiewe riglyne wat betref die ekonomies optimale getal boorgate en defleksies in elke boorgat vir 'n sekere rif wat gekenmerk word deur sy veranderlikheidspatroon en diepte onder die oppervlak, om die vertrouenspeil vir 'n vaste boorbegroting te maksimeer, of die totale boorkoste vir 'n gegewe vertrouenspeil te minimeer.

Gedeeltes van die Basale, Vaal- en Elsburgrif wat 'n groot mate van veranderlikheid toon (logaritmiese variansie van monsters onderskeidelik 0,63, 1,05 en 2,06) word op dieptes onder die oppervlak wat van 1000 m tot 4000 m wissel, ondersoek oor 'n gesimuleerde myneiendom wat 7000 ha (7000 m by 10 000 m) dek.

Die gevolgtrekking is dat vier kruisings per boorgat optimaal is in al die gevalle wat oorweeg is wat die grootste deel van die huidige eksplorasie aan die Witwatersrand dek, met die uitsondering van die uiters veranderlike Elsburgrif op dieptes van 1000 en 2000 m waar daar voorkeur gegee moet word aan boorgate met onderskeidelik een en twee kruisings.

Introduction

Because deep gold exploration as is used for gold deposits on the Witwatersrand is both expensive and time-consuming, this paper attempts to give quantitative guidelines on the economic optimum number of boreholes and deflections within each borehole for a certain reef characterized by its variability pattern and depth below surface. In this way, the level of confidence for a fixed drilling budget can be maximized or the total drilling cost for a given level of confidence can be minimized.

General Assumptions and Application Areas

The main assumptions are that the approximate depth below surface of the reef and its variability pattern as described by the semivariogram and the variance/size of area are known.

These assumptions limit the areas of application of this procedure to extensions of mined-out areas where the target reefs or reefs have been sampled and semivariograms (variance/size of area) have been obtained. These conditions apply to extensions of on-going mines such as new shaft areas or extensions of known goldfields into new lease areas. These new areas are to be explored by surface drilling.

It is felt that the information available in completely new exploration areas is insufficient for the application of this method. However, even in this extreme case, if the reef and its approximate depth are known, sampling results obtained elsewhere for a reef of similar sedimentological conditions could be used as a guideline.

It is assumed that the exploration area is drilled with between five and twenty-five boreholes, and that each borehole yields between one and nine satisfactory intersections. The boreholes within the exploration area are treated as independent. This assumption is reasonable since, in general, surface boreholes are a few kilometres apart while the range of correlation of gold values is known to be of the order of a few hundred metres. Furthermore, it is assumed that intersections within the same borehole fall randomly within a 6 m by 6 m square. The values of these intersections are treated as correlated.

For illustration purposes, calculations are carried out for the Basal, Vaal, and Elsburg Reefs at depths ranging from 1 km to 4 km and covering a surface area of 7000 ha (7 km by 10 km).

The above assumptions are shown pictorially in Fig. 1.

It is further assumed that the drilling details and costs

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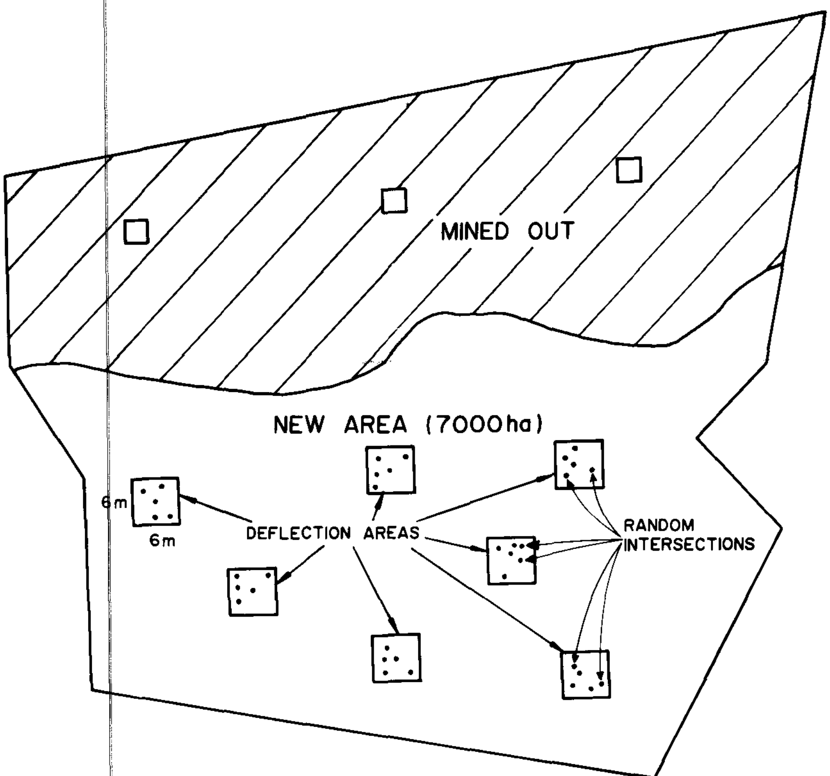


Fig. 1—General assumptions in addition to the assumption that the depth and variability pattern (semivariogram and variance/area relationship) of the reef are known

are known. In the numerical examples presented here, the assumptions and figures shown below are used. However, changes to these do not affect the validity of the method.

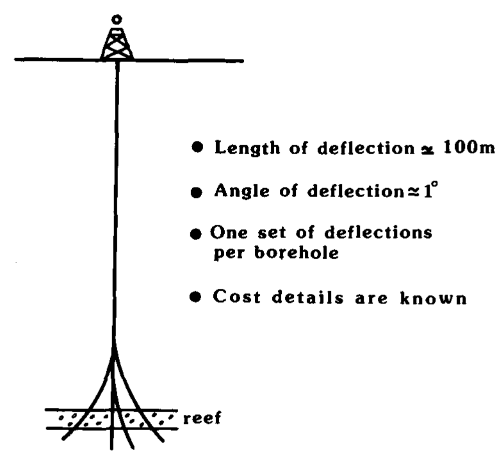
- The deflections are non-directional and between 10 m and 100 m long, differing in length to allow for the installation of the wedges.
- The angle of deflection is approximately 1°.
- Each borehole has one set of deflections, i.e. one target reef. This assumption is not always valid, but more than one target reef per borehole does not affect the method in principle since the different reefs are likely to have different variability patterns and can be treated as independent. The only difficulty arises in apportioning, among the different reefs, the cost of the original borehole before the deflection area is reached.
- The cost details are known including
  - drilling cost in rands per metre at different depths
  - cost of installing wedges at different depths
  - fixed cost of moving the drilling rig once the main borehole and its deflections have been completed.

The total costs used in these examples were obtained from a drilling contractor in April 1985 and are shown in Fig. 2.

It is assumed that, once all the drilling is complete, the most likely mean value of the property with associated confidence limits is calculated with

- the arithmetic mean of all intersections lying within each deflection area (borehole means), and
- Professor D.G. Krige's t'' estimator of the mean value of all boreholes means, which is given by

$$t'' = GM^* \exp\{\sigma^2 (n-1)/2n\} - \beta, \ldots \quad (1)$$



DEPTH OF REEF	NO. OF INTERSECTIONS PER BOREHOLE 1	2	3	4	5	7	9
1000	93	118	131	144	158	188	220
1500	150	176	190	205	220	253	288
2000	220	249	267	286	306	348	393
2500	303	337	359	382	407	457	511
3000	408	446	473	501	530	592	658
3500	545	590	624	660	698	777	862
4000	714	768	812	858	905	1004	1111
4500	907	962	1007	1053	1102	1204	1315

Fig. 2—Cost of a single borehole (R000)

where

GM = geometric mean of borehole means + β (cm.g/t)
σ^2 = logarithmic variance of borehole means, known *a priori* from mined-out areas
n = number of borehole means
β = additive constant (cm.g/t).

The confidence limits for the mean of the mining property, on the assumption of lognormal error distribution, were given by Krige[1] as

$$95\% \text{ upper limit} = \exp\{\ln (t''+\beta) + \sigma_e^2/2 + 1{,}645\ \sigma_e\} - \beta$$

$$95\% \text{ lower limit} = \exp\{\ln (t''+\beta) + \sigma_e^2/2 - 1{,}645\ \sigma_e\} - \beta, \ldots \quad (2)$$

where σ_e^2 is the error variance of estimation of the mean gold value of the property. This is calculated in detail later.

Although the t'' method is used in the numerical examples given here, the more conservative Sichel t estimator[2] with associated confidence limits could be used. However, it is advisable to use an *a priori* knowledge of the logarithmic variance σ^2 since the estimate of this parameter obtained from borehole results can be seriously affected by outliers, resulting in unrealistically wide confidence limits.

The present study gives a relative assessment; therefore, if either the t'' estimator or the Sichel t estimator of the mean with associated confidence limits is used consistently throughout, the same conclusions should be reached.

Objective

The criteria used to measure the performance of a drilling pattern are

- the error variance of estimation of the mean of the property σ_e^2; or, equivalently, the confidence limits expressed as a percentage of the mean value (confidence limit/mean value)*100; and
- the total drilling cost.

These two criteria are calculated for all possible combinations of boreholes and numbers of intersections within boreholes. Then, one of the following two routes is followed:

- choose the least cost option among all the drilling patterns that yield a common acceptable level of error variance of estimation of the mean gold value of the property; or
- choose the drilling pattern that gives the least error variance of estimation of the mean gold value of the property for a given cost (budget).

Analysis of Reefs

In order to produce realistic numerical calculations from which useful conclusions could be obtained, it was decided to use reefs that display a wide range of logarithmic variances of individual sampling sections. The main statistical characteristics of these reefs are shown in Table I. Those for the Basal and Elsburg Reefs were obtained from mined-out portions of Loraine Gold Mines, while those for the Vaal Reef were obtained from a section of Hartebeestfontein Gold Mine. It should be noted that the values shown in Table I do not reflect current ore-reserve figures for the above mines.

TABLE I
CHARACTERISTICS OF THE REEFS

Reef	Number of Samples	Mean cm.g/t	Additive Constant β(cm.g/t)	Logarithmic Variance σ^2	Coefficient of Variation	80% of Values Between	
Basal	9861	697	100	0.632	1.07	110	1500
Vaal	3991	900	50	1.048	1.44	100	2000
Elsburg	1291	1444	0	2.059	2.62	80	3200

By use of the principles of stratified random sampling, the total logarithmic error variance of estimation of the mean gold value of the mining property drilled by *a single borehole* and its deflections (σ_{el}^2) can be decomposed into two parts:

- the logarithmic error variance of estimation of the mean gold value of the deflection area itself estimated by the arithmetic mean of the deflections (σ_{ed}^2); and
- the logarithmic extension variance of the true value of the 6 m by 6 m deflection area to the mining property ($\sigma^2\ d/p$).

Mathematically,

$$\sigma_{el}^2 = \sigma_{ed}^2 + \sigma_{d/p}^2. \quad (3)$$

Finally, since the mining property is drilled by n boreholes with their respective deflections and they are assumed independent, the total logarithmic error variance of estimation of the mining property is given by

$$\sigma_e^2 = n(\sigma_{ed}^2 + \sigma_{d/p}^2)/n. \quad (4)$$

This, in principle, is similar to the approach adopted by Krige[3] and Rendu[4].

Estimation of σ_{ed}^2

Since the borehole intersections that fall within a deflection area are randomly spaced, but are too close to each other to be considered independent (Fig. 1), the logarithmic error variance of estimation of the mean gold value of the 6 m by 6 m deflection area is calculated using the semivariogram applicable to the reef being analysed and the general equation[4]:

$$\sigma_{ed}^2 = -\bar{\gamma}\ dd - \bar{\gamma}\ bb + 2\,\bar{\gamma}\ bd, \quad (5)$$

where $\bar{\gamma}\ dd$ = average semivariogram within the 6 m by 6 m deflection area
$\bar{\gamma}\ bb$ = average semivariogram between all possible pairs of borehole intersections within the deflection area
$\bar{\gamma}\ bd$ = average semivariogram between all the borehole intersections within the deflection area and the deflection area itself.

The three terms mentioned above can be computed using the appropriate semivariogram model and dividing the deflection area into a finite number of points (discretization). The observed semivariogram values and the mathematical models fitted to them for each of the reefs considered can be found in Figs. 3 to 5.

Examples of the numerical calculations involved in the application of the above equations can be found in Rendu[4].

Estimation of $\sigma_{d/p}^2$

The logarithmic extension variance of the true value of the 6 m by 6 m deflection area to the mining property can be estimated directly from the relationship between variance and area obtained for each reef[5,6].

This procedure is illustrated for the three reefs under consideration in Figs. 6 to 8. In these cases, the following points should be noted.

- The abscissa of the graphs is a logarithmic scale representing the linear equivalent of the areas under

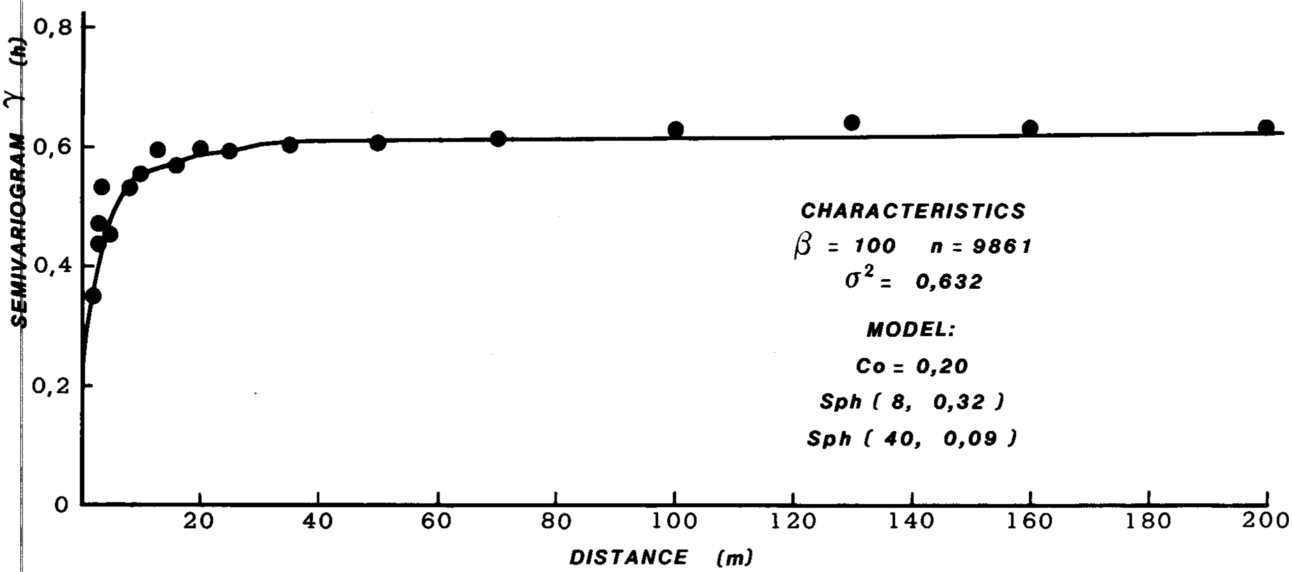


Fig. 3—Semivariogram of the Basal Reef

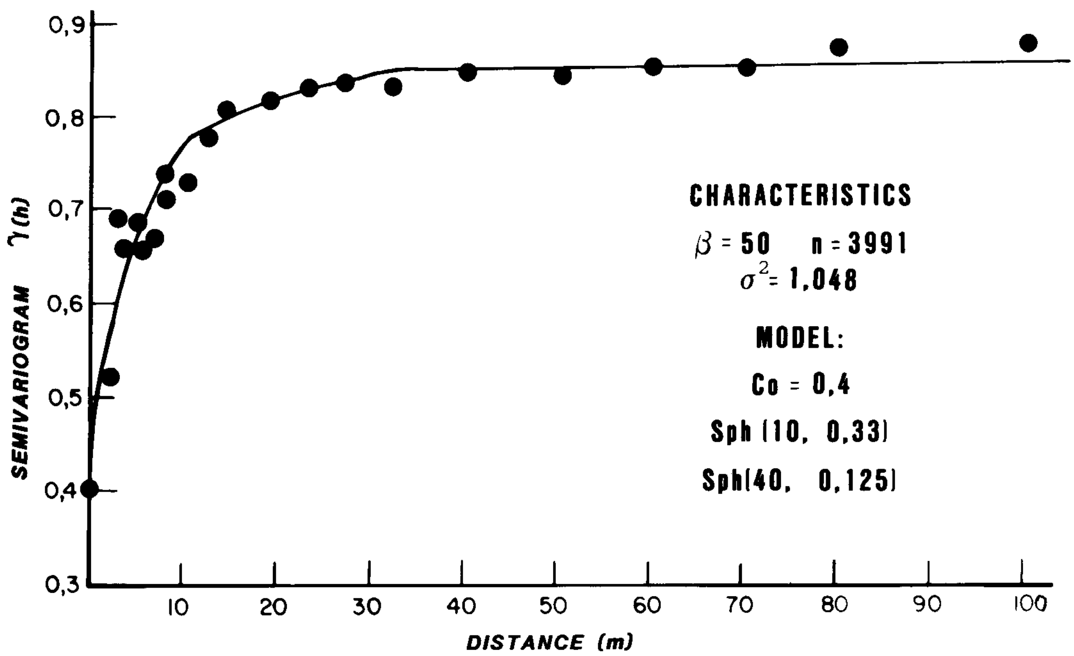


Fig. 4—Semivariogram of the Vaal Reef

consideration (the linear equivalent of a rectangle of sides a and b is simply $a + b$). This was done because the range of the areas involved is very large.

- A certain amount of extrapolation of the linear trend of the graphs was necessary since no mined-out areas of 7 km by 10 km were available.
- According to the additivity of variances theorem known as Krige's relationship[5], the logarithmic extension variance $\sigma^2_{d/p}$ is obtained by subtracting the logarithmic variance of samples within a deflection area, from the logarithmic variance of samples within the mining property. For example, in the case of the Vaal Reef shown in Fig. 7, the calculation would be

$$\sigma^2_{d/p} = 1{,}145 - 0{,}685 = 0{,}460.$$

Estimation of σ^2_e

The final step in the calculation of σ^2_e is the application of equation (4), in which the two error terms obtained earlier are added to form the total logarithmic error variance of estimating the mean of the mining property if only one borehole with deflections is drilled and the value of the property is accepted as the arithmetic mean value of these intersections. Where a number (n) of independent borehole clusters are drilled, the quantity obtained above is then divided by the number of borehole clusters.

It is worth noting that, as shown by equation (4), excessive drilling of deflections within a borehole deflection area gives a better estimate of the deflection area but does not reduce the second term of the equation, i.e. the

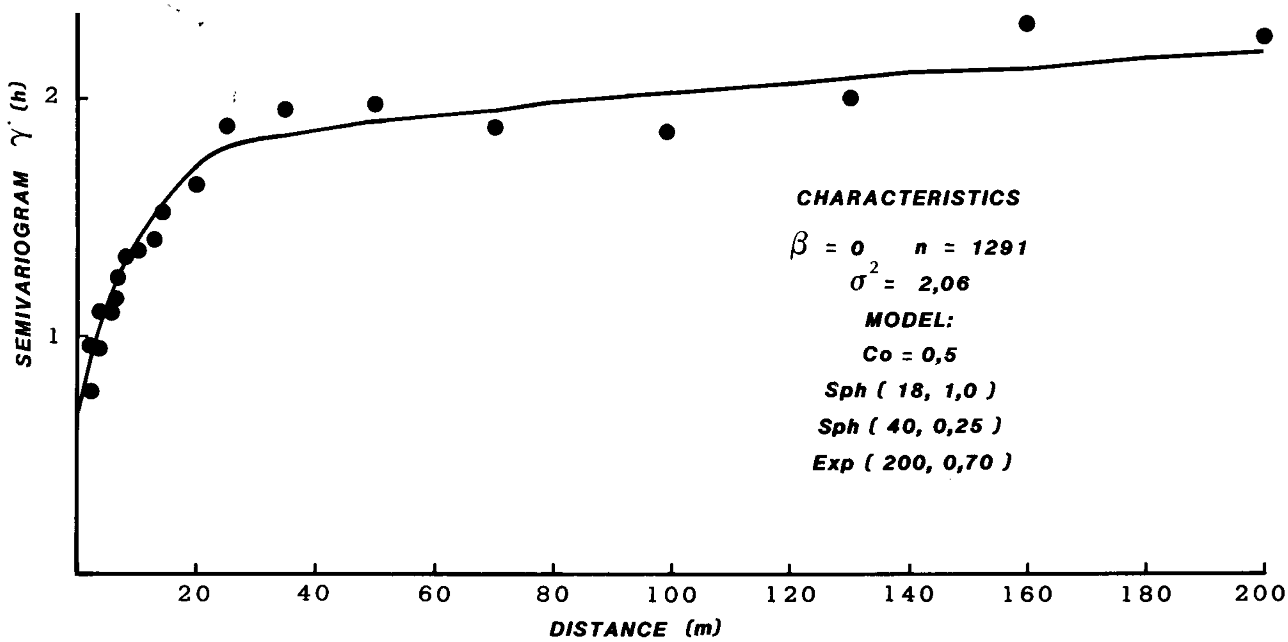


Fig. 5—Semivariogram of the Elsburg Reef

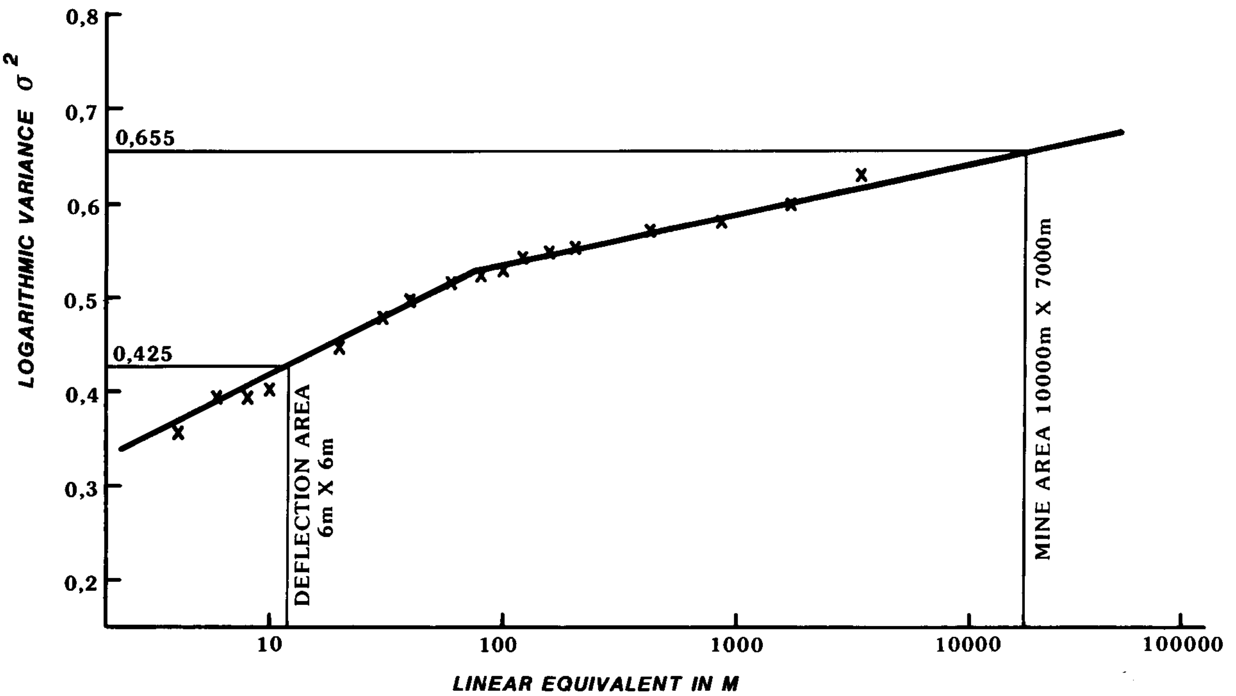


Fig. 6—Variance/area for the Basal Reef

logarithmic extension error variance of the value of the deflection area to the mining property. This term is often the larger of the two, and can be reduced only by increasing the size of the deflection area, i.e. by drilling longer deflections.

The logarithmic error variance of estimating (σ_e^2) the mean gold value of the mining property, calculated as the t'' estimator of the n borehole means, can be expressed as confidence limits using the set of equations shown as (2). This is valid since the error distribution of the t'' estimator is lognormal.

The above confidence limits were calculated for each borehole configuration (from 5 to 25 boreholes with 1 to 9 intersections per borehole) for each of the three reefs, and were expressed as percentages of the known reef means (Table I). These are shown in Figs. 9 to 11 for the Basal, Vaal, and Elsburg Reefs respectively. The confidence limits for borehole configurations involving 7 and 9 intersections are not shown since they are very similar to those with 5 intersections. Since the lower limits are the critical ones from the economic point of view, these have been drawn to a larger scale in Figs. 12 to 14 for the three reefs respectively.

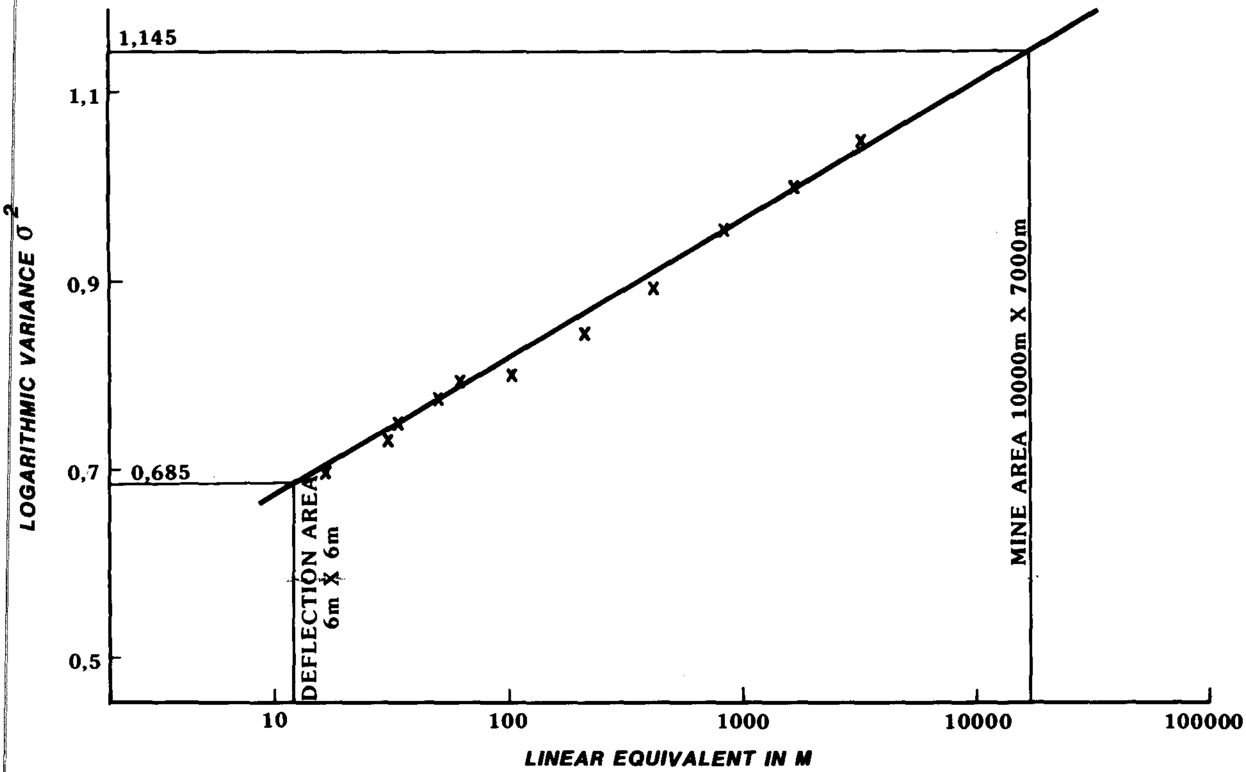


Fig. 7—Variance/area for the Vaal Reef

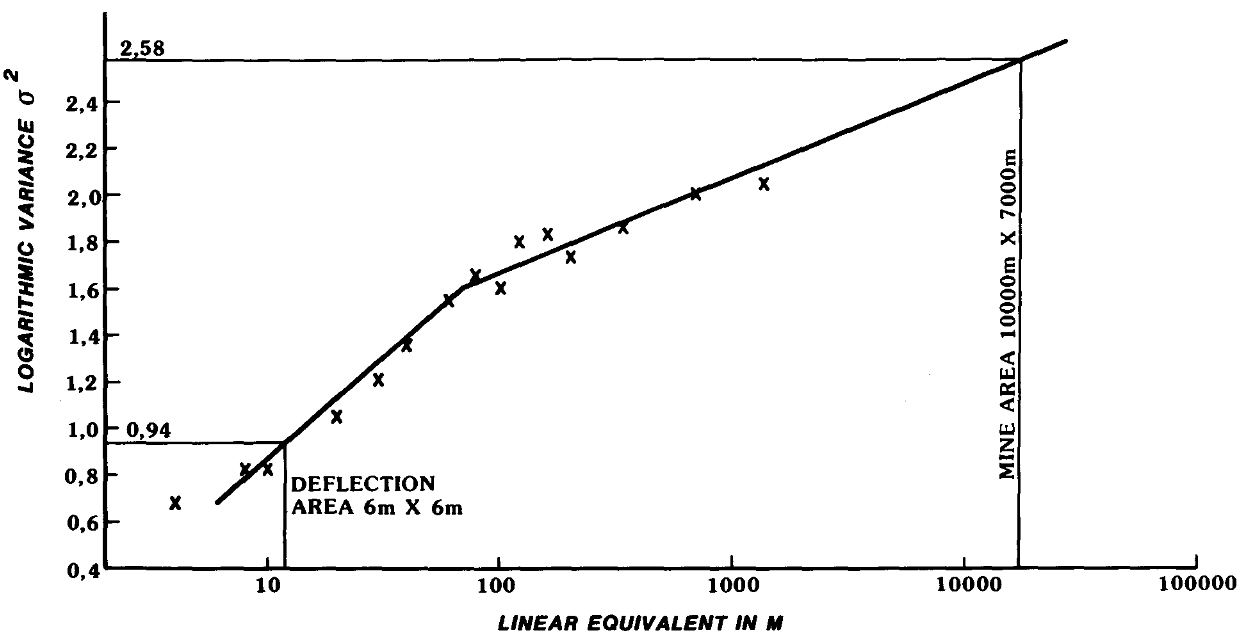


Fig. 8—Variance/area for the Elsburg Reef

A few points concerning the above confidence limits are worthy of mention.

- Reefs with higher inherent variability produce wider final confidence limits for the mean gold value of the mining property. Thus, the confidence limits for the Elsburg Reef are wider than those for the Vaal and Basal Reefs.
- As the confidence limit interval has the typical 'trumpet' shape, a marked improvement in the width of the confidence interval is achieved by going from 5 to 10 boreholes. However, the confidence limit curves are almost 'flat' beyond 20 boreholes. This concept can be used to obtain an approximate borehole density over the mining property.
- Owing to the assumption of lognormal error distribution used in the computation of the confidence limits —equations (2)—the resulting confidence limits are asymmetric. Thus, the upper limit is much more sen-

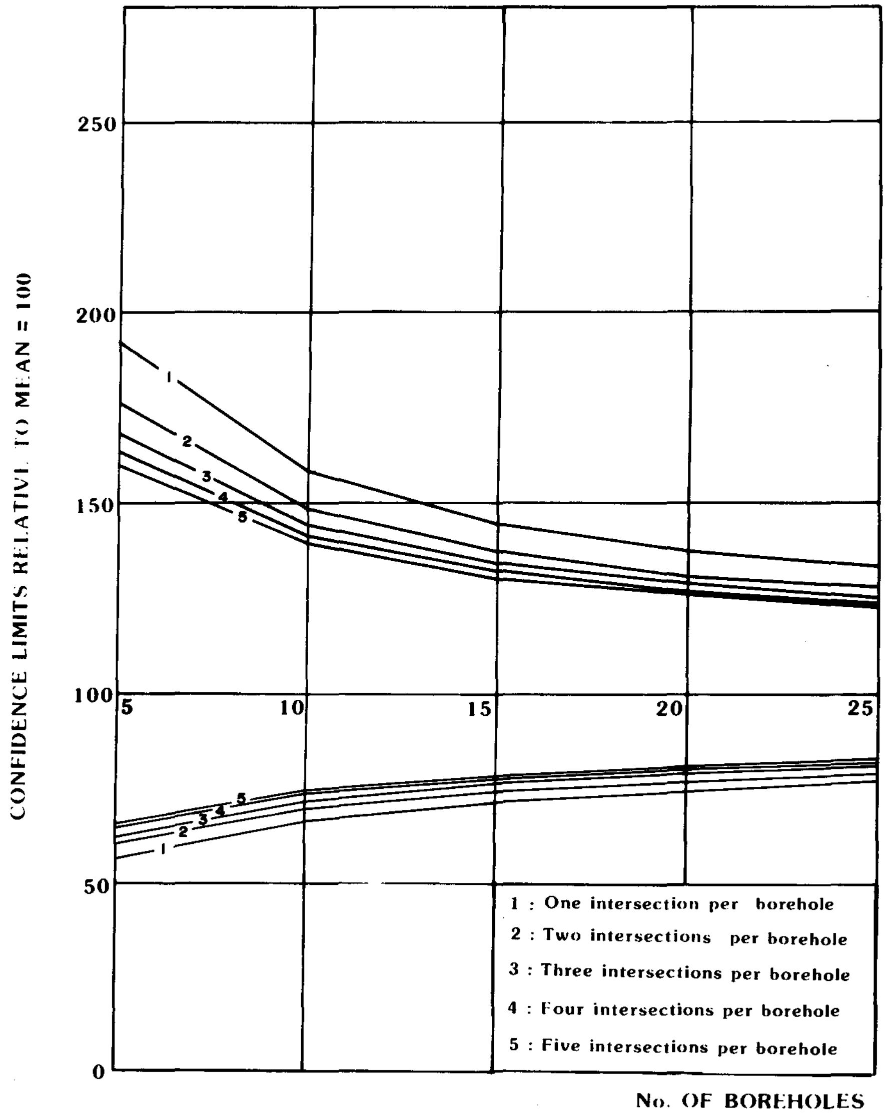


Fig. 9—90 per cent central confidence limits for the Basal Reef

sitive to the number of boreholes than is the lower limit.

- Different borehole configurations produce similar confidence intervals; for example, for Vaal Reef as shown in Fig. 13, 20 boreholes with a single intersection, 15 boreholes with 2 intersections, 13 boreholes with 4 intersections, and 12 boreholes with 5 intersections produce lower confidence limits of approximately 70 per cent of the mean. Thus, by choosing the most economical of these configurations there is scope for optimization.
- The shape of the confidence limit interval is independent of the actual gold value of the reef intersections. It depends only on the variability pattern of the reef (semivariogram), the number of boreholes within the property, the size of the deflection area (i.e. the length of the deflections), the number of intersections per borehole, and, to a much lesser degree, the location of the intersections within the deflection area. Thus, the number of acceptable intersections drilled per borehole should not be influenced by the gold values obtained, but only by geological factors such as faulting and drilling; factors such as core grinding should be taken into account to decide on the validity of an intersection. If gold values are taken into account when deciding on the number of intersections per borehole, serious valuation biases may be introduced, as indicated by Krige[7].
- From the valuation point of view, drilling should be a sequential process in which the number of acceptable intersections per borehole is defined beforehand. Initially, a number of boreholes (say 10) are drilled to cover the mining property. Results should then be reviewed, and at that stage the relative position of the 'pay limit' or cut-off grade to the lower confidence limit should be examined. For example, in the hypothetical case shown in Fig. 13, after 10 boreholes with 4 intersections each have been completed, the lower confidence limit is lower than the pay limit. Therefore, the risk of going ahead with a non-viable venture may be unacceptably high. If a further 5 or 10 boreholes (with 4 intersections each) are drilled, the lower confidence limit increases and becomes higher than the pay limit, thus reducing the risk of an uneconomic

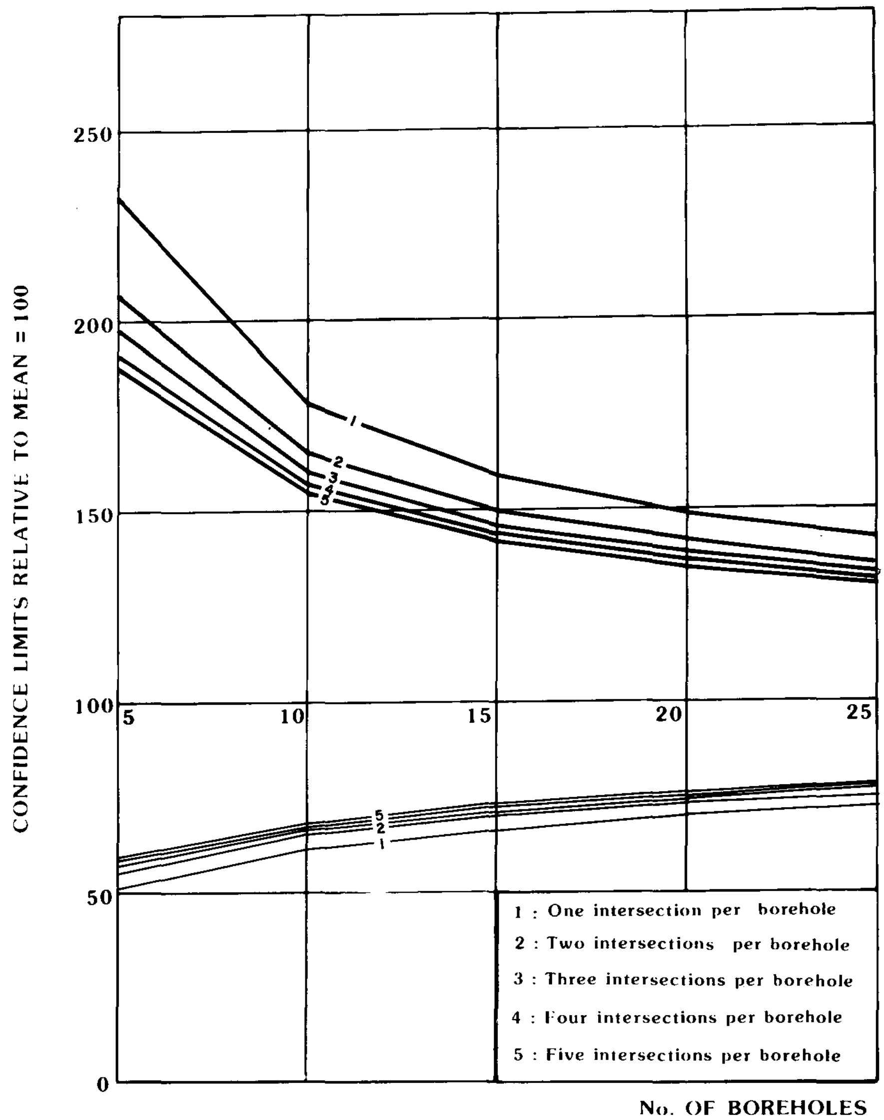


Fig. 10—90 per cent central confidence limits for the Vaal Reef

TABLE II
RESULTS FOR THE VAAL REEF AT A DEPTH OF 1000 m

No OF BOREHOLES	No OF INTERSECTIONS PER BOREHOLE						
	1	2	3	4	5	7	9
5	0,191 233 51 0,47	0,146 207 55 0,59	0,129 198 57 0,66	0,118 191 58 0,72	0,112 188 59 0,79	0,107 185 59 0,94	0,104 183 60 1 '0
10	0,095 178 61 0,93	0,073 165 65 1,18	0,065 160 66 1,31	0,059 157 67 1,44	0,056 155 68 1,58	0,054 153 69 1,88	0,052 152 69 2,20
11	0,087 173 62 1,02	0,066 161 66 1,30	0,059 156 67 1,44	0,054 153 69 1,58	0,051 151 69 1,74	0,049 150 70 2,07	0,047 149 70 2,42
12	0,079 169 64 1,12	0,061 158 67 1,42	0,054 153 68 1,57	0,049 150 70 1,73	0,047 149 70 1,90	0,045 147 71 2,26	0,043 146 71 2,64
13	0,073 165 65 1,21	0,056 155 68 1,53	0,050 151 69 1,70	0,045 148 70 1,87	0,043 146 71 2,05	0,041 145 72 2,44	0,040 144 72 2,86
14	0,068 162 66 1,30	0,052 152 69 1,65	0,046 148 70 1,83	0,042 146 71 2,02	0,040 144 72 2,21	0,038 143 72 2,63	0,037 142 73 3,08
15	0,064 159 66 1,40	0,049 150 70 1,77	0,043 146 71 1,97	0,039 144 72 2,16	0,037 142 73 2,37	0,036 141 73 2,82	0,035 140 74 3,30
20	0,048 149 70 1,86	0,037 142 73 2,36	0,032 139 74 2,62	0,030 137 75 2,88	0,028 135 76 3,16	0,027 134 76 3,76	0,026 134 76 4,40
25	0,038 143 72 2,33	0,029 136 75 2,95	0,026 134 77 3,28	0,024 132 77 3,60	0,022 131 78 3,95	0,021 130 78 4,70	0,021 130 79 5,50

LOG VARIANCE
95% LIMITS
COST RM

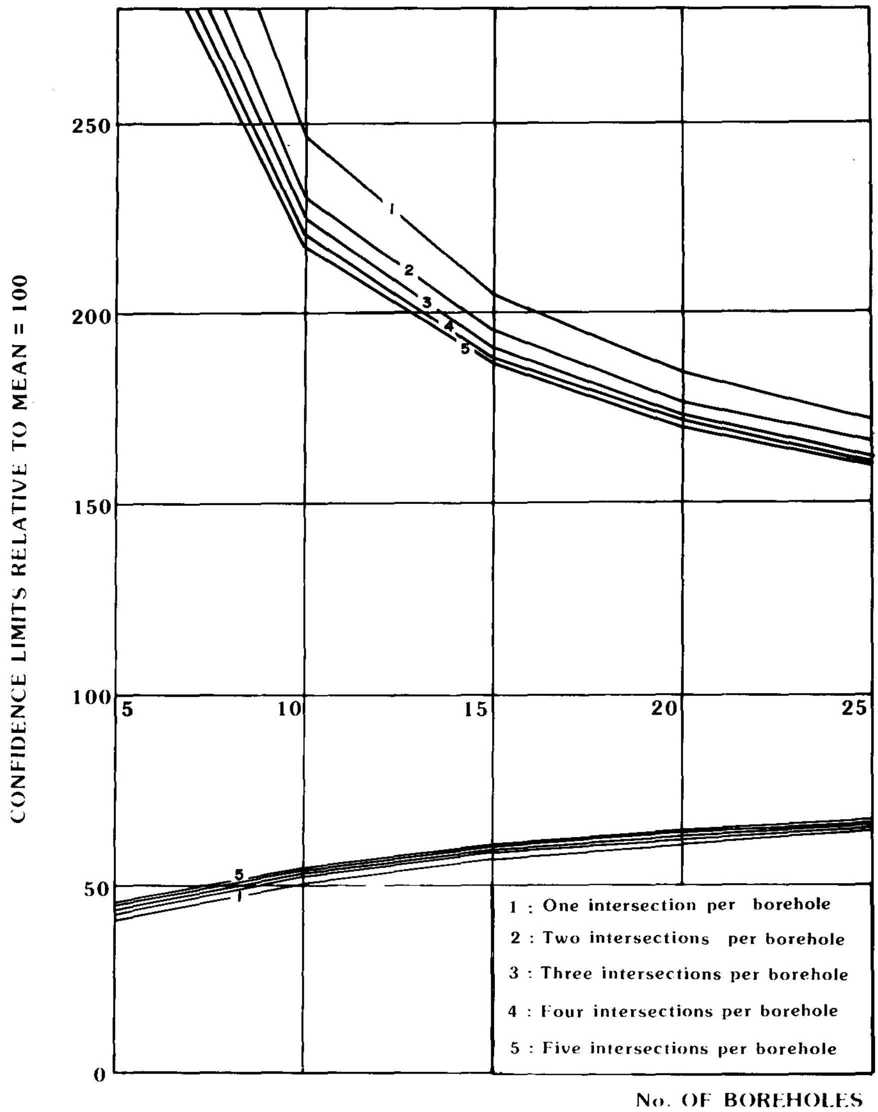


Fig. 11—90 per cent central confidence limits for the Elsburg Reef

TABLE III
RESULTS FOR THE VAAL REEF AT A DEPTH OF 2000 m

No OF BOREHOLES	No OF INTERSECTIONS PER BOREHOLE						
	1	2	3	4	5	7	9
5	0,191 233 51 1,10	0,146 207 55 1,25	0,129 198 57 1,34	0,118 191 58 1,43	0,112 188 59 1,53	0,107 185 59 1,74	0,104 183 60 1,97
10	0,095 178 61 2,20	0,073 165 65 2,49	0,065 160 66 2,67	0,059 157 67 2,86	0,056 155 68 3,06	0,054 153 69 3,48	0,052 152 69 3,93
11	0,087 173 62 2,42	0,066 161 66 2,74	0,059 156 67 2,94	0,054 153 69 3,15	0,051 151 69 3,37	0,049 150 70 3,83	0,047 149 70 4,32
12	0,079 169 64 2,64	0,061 158 67 2,99	0,054 153 68 3,20	0,049 150 70 3,43	0,047 149 70 3,67	0,045 147 71 4,18	0,043 146 71 4,72
13	0,073 165 65 2,86	0,056 155 68 3,24	0,050 151 69 3,47	0,045 148 70 3,72	0,043 146 71 3,98	0,041 145 72 4,52	0,040 144 72 5,11
14	0,068 162 66 3,08	0,052 152 69 3,49	0,046 148 70 3,74	0,042 146 71 4,00	0,040 144 72 4,28	0,038 143 72 4,87	0,037 142 73 5,50
15	0,064 159 66 3,30	0,049 150 70 3,74	0,043 146 71 4,01	0,039 144 72 4,29	0,037 142 73 4,59	0,036 141 73 5,22	0,035 140 74 5,90
20	0,048 149 70 4,40	0,037 142 73 4,98	0,032 139 74 5,34	0,030 137 75 5,72	0,028 135 76 6,12	0,027 134 76 6,96	0,026 134 76 7,86
25	0,038 143 72 5,50	0,029 136 75 6,23	0,026 134 77 6,68	0,024 132 77 7,15	0,022 131 78 7,65	0,021 130 78 8,70	0,021 130 79 9,83

LOG VARIANCE
95% LIMITS
COST RM

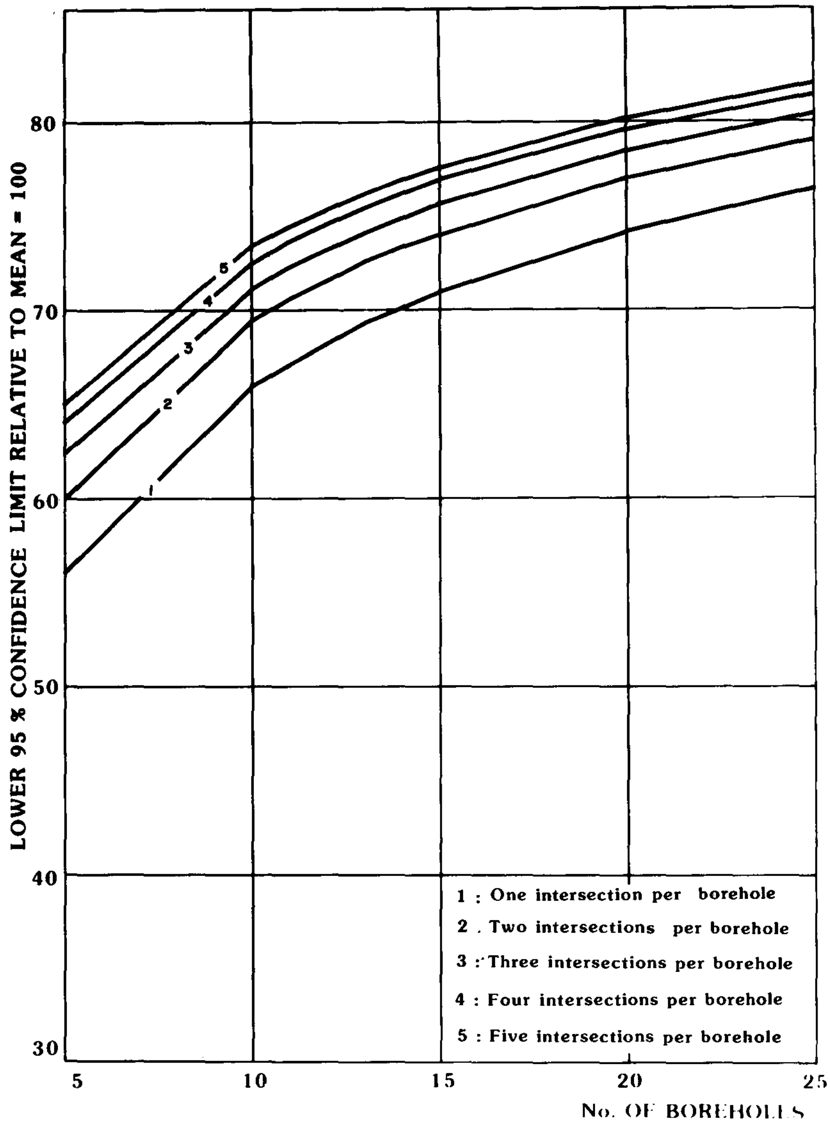


Fig. 12—Lower 95 per cent confidence limit for the Basal Reef (mean = 100)

TABLE IV

RESULTS FOR THE VAAL REEF AT A DEPTH OF 3000 m

No OF BOREHOLES	No OF INTERSECTIONS PER BOREHOLE						
	1	2	3	4	5	7	9
5	0,191 233 51 2,04	0,146 207 55 2,23	0,129 198 57 2,37	0,118 191 58 2,51	0,112 188 59 2,65	0,107 185 59 2,96	0,104 183 60 3,29
10	0,095 178 61 4,08	0,073 165 65 4,46	0,065 160 66 4,73	0,059 157 67 5,01	0,056 155 68 5,30	0,054 153 69 5,92	0,052 152 69 6,58
11	0,087 173 62 4,49	0,066 161 66 4,91	0,059 156 67 5,20	0,054 153 69 5,61	0,051 151 69 5,83	0,049 150 70 6,51	0,047 149 70 7,24
12	0,079 169 64 4,90	0,061 158 67 5,35	0,054 153 68 5,68	0,049 150 70 6,01	0,047 149 70 6.36	0,045 147 71 7,10	0,043 146 71 7,90
13	0,073 165 65 5,30	0,056 155 68 5,80	0,050 151 69 6,15	0,045 148 70 6,51	0,043 146 71 6,89	0,041 145 72 7,70	0,040 144 72 8,55
14	0,068 162 66 5,71	0,052 152 69 6,24	0,046 148 70 6,62	0,042 146 71 7,01	0,040 144 72 7,42	0,038 143 72 8,29	0,037 142 73 9,21
15	0,064 159 66 6,12	0,049 150 70 6,69	0,043 146 71 7,10	0,039 144 72 7,52	0,037 142 73 7,95	0,036 141 73 8,88	0,035 140 74 9,87
20	0,048 149 70 8,16	0,037 142 73 8,92	0,032 139 74 9,46	0,030 137 75 10,02	0,028 135 76 10,60	0,027 134 76 11,84	0,026 134 76 13,16
25	0,038 143 72 10,20	0,029 136 75 11,15	0,026 134 77 11,83	0,024 132 77 12,53	0,022 131 78 13,26	0,021 130 78 14,80	0,021 130 79 16,45

LOG VARIANCE
95% LIMITS
COST RM

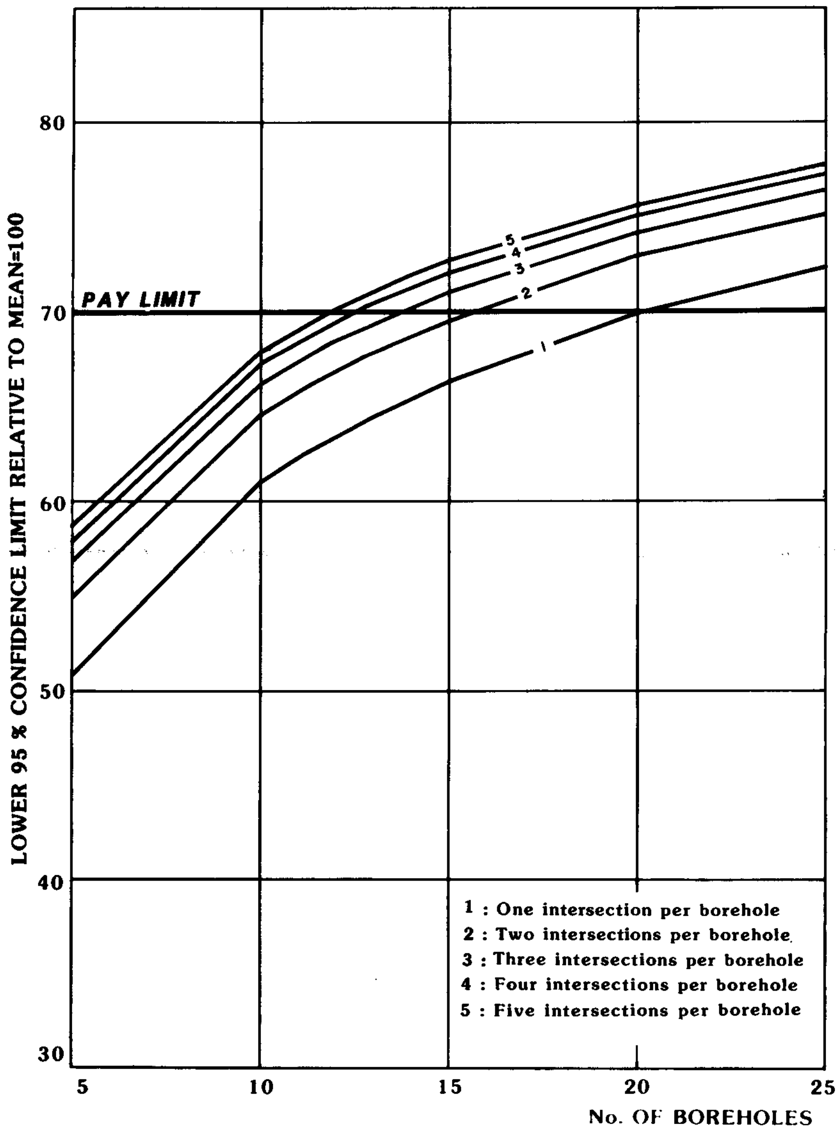


Fig. 13—Lower 95 per cent confidence limit for the Vaal Reef (mean = 100)

TABLE V

RESULTS FOR THE VAAL REEF AT A DEPTH OF 4000 m

No OF BOREHOLES	No OF INTERSECTIONS PER BOREHOLE						
	1	2	3	4	5	7	9
5	0,191 233 51 3,57	0,146 207 55 3,84	0,129 198 57 4,06	0,118 191 58 4,29	0,112 188 59 4,53	0,107 185 59 5,02	0,104 183 60 5,56
10	0,095 178 61 7,14	0,073 165 65 7,68	0,065 160 66 8,12	0,059 157 67 8,58	0,056 155 68 9,05	0,054 153 69 10,04	0,052 152 69 11,11
11	0,087 173 62 7,85	0,066 161 66 8,45	0,059 156 67 8,93	0,054 153 69 9,44	0,051 151 69 9,96	0,049 150 70 11,04	0,047 149 70 12,22
12	0,079 169 64 8,57	0,061 158 67 9,22	0,054 153 68 9,74	0,049 150 70 10,30	0,047 149 70 10,86	0,045 147 71 12,05	0,043 146 71 13,33
13	0,073 165 65 9,28	0,056 155 68 9,98	0,050 151 69 10,56	0,045 148 70 11,15	0,043 146 71 11,77	0,041 145 72 13,05	0,040 144 72 14,44
14	0,068 162 66 10,00	0,052 152 69 10,75	0,046 148 70 11,37	0,042 146 71 12,01	0,040 144 72 12,67	0,038 143 72 14,06	0,037 142 73 15,55
15	0,064 159 66 10,71	0,049 150 70 11,52	0,043 146 71 12,18	0,039 144 72 12,87	0,037 142 73 13,58	0,036 141 73 15,06	0,035 140 74 16,67
20	0,048 149 70 14,28	0,037 142 73 15,36	0,032 139 74 16,24	0,030 137 75 17,16	0,028 135 76 18,10	0,027 134 76 20,08	0,026 134 76 22,22
25	0,038 143 72 17,85	0,029 136 75 19,20	0,026 134 77 20,30	0,024 132 77 21,45	0,022 131 78 22,63	0,021 130 78 25,10	0,021 130 79 27,78

LOG VARIANCE
95% LIMITS
COST RM

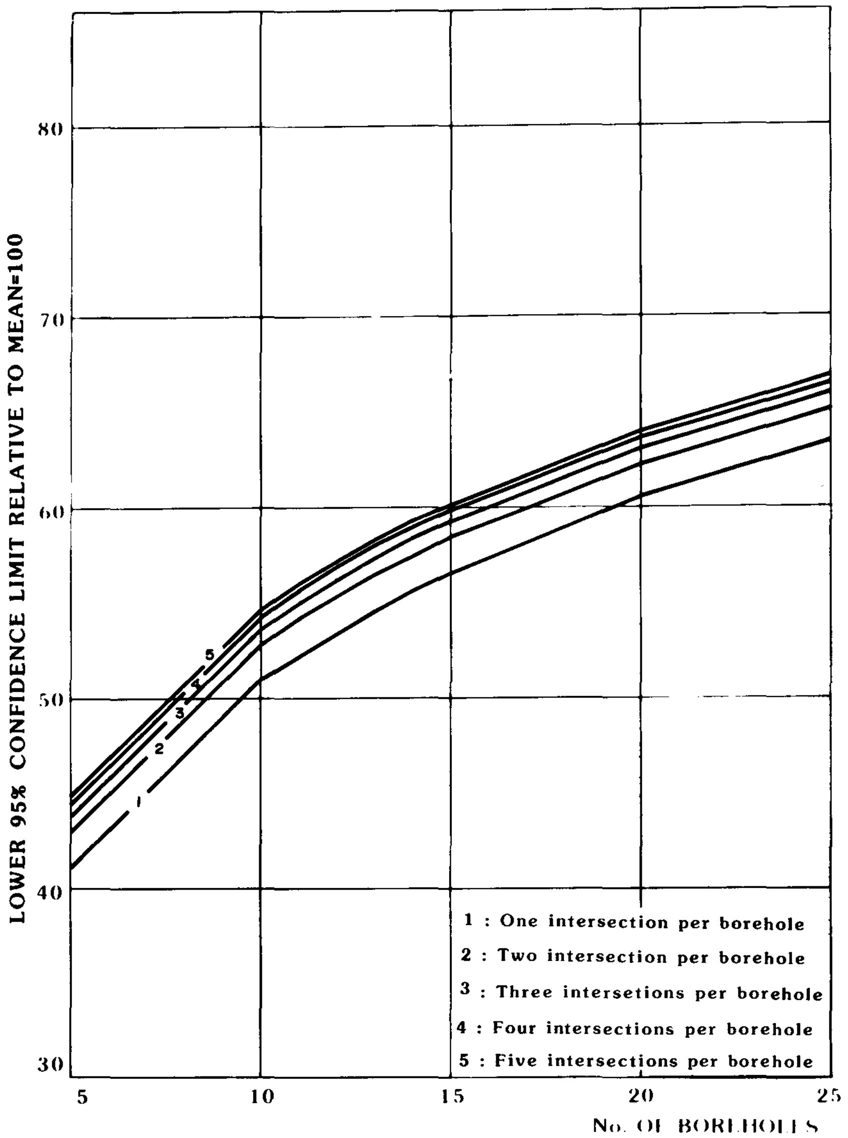


Fig. 14—Lower 95 per cent confidence limit for the Elsburg Reef (mean = 100)

TABLE VI

OPTIMUM NUMBER OF DEFLECTIONS FOR THE DIFFERENT REEFS AT VARIOUS DEPTHS

DEPTH	REEF AND SAMPLE LOGARITHMIC VARIANCE		
	BASAL 0,632	VAAL 1,048	ELSBURG 2,059
1000	3 - 5	2 - 4	1
2000	4 - 5	3 - 5	2 - 3
3000	4 - 5	4 - 5	2 - 4
4000	4 - 5	4 - 5	2 - 4

venture. This, of course, is somewhat theoretical since the estimate of the mean value of the property after the continuation of the drilling campaign can well be different from the original estimate.

- From the valuation point of view, judging by the shape of the confidence interval, there is little gain in drilling more than, say, 25 boreholes. However, it is fully recognized that valuation is not the only objective of a drilling campaign, and that further boreholes might be necessary for the identification of geological features.

Economic Optimization

As mentioned earlier, since different borehole configurations yield similar confidence limits, there is scope for economic optimization by choosing the lowest cost option among these. Alternatively, the borehole configuration that yields the lowest logarithmic error variance of estimation among those that have a similar cost (budget) can be identified.

Since the determination of the optimum borehole configuration involves the cost of the different options, and this in turn depends on the depth of the reef, Tables II to V were prepared to show the results for the Vaal Reef at depths ranging from 1000 m to 4000 m.

In these tables, each block represents a drilling configuration, and the values shown in it are the logarithmic error variance of estimating the mean of the property (t'' estimator), the upper and lower 90 per cent central confidence limits expressed as percentages of the mean value, and the drilling cost of the configuration.

In order to find the optimum number of deflections, the drilling cost was plotted against the corresponding logarithmic error variance of estimation of the mean of the mining property. Different curves were prepared for boreholes with different numbers of intersections, and were drawn as a 'family' of curves for each reef at each of the depths considered. Figs. 15 to 17 are examples of the family of curves obtained for the three different reefs at a depth of 3000 m. It can be seen that the curve corresponding to four intersections is the lowest and therefore the optimum. This optimum curve is not unique in the above three examples.

Similar analyses were conducted for each reef at the four depths considered, and the optimum curves were identified. A summary of the findings is shown in Table VI. It can be seen that four intersections are included in the optimum set in all cases, except for the highly variable Elsburg Reef at depths of 1000 m and 2000 m.

As an example, consider that the Vaal Reef is being drilled at a depth of 3000 m. Table VI shows that the optimum is four or five intersections per borehole.

If a budget of seven million rands is available, Table IV shows that 14 boreholes with four intersections each and 13 boreholes with five intersections each would yield similar results. If, on the other hand, an error variance of 0,04 yielding confidence limits of 72 per cent and 144 per cent is considered acceptable, Table IV shows that 15 boreholes with four intersections each and 14 boreholes with five intersections each would both be acceptable solutions. Table IV also shows that, if this same level of error variance were achieved using 13 boreholes with nine intersections each, the cost involved would be 8,55 million rands instead of the 7,52 million rands that can be achieved by using 15 boreholes with four intersections each.

Conclusions

As shown in Table VI, it can be concluded that four intersections per borehole are optimum in all the cases considered, which cover the majority of current Witwatersrand exploration, i.e. reefs with sample logarithmic variances ranging from 0,632 to 2,059 and lying at depths between 1000 m and 4000 m, with the exception of the highly variable Elsburg Reef at depths of 1000 m and 2000 m, where boreholes with one and two intersections

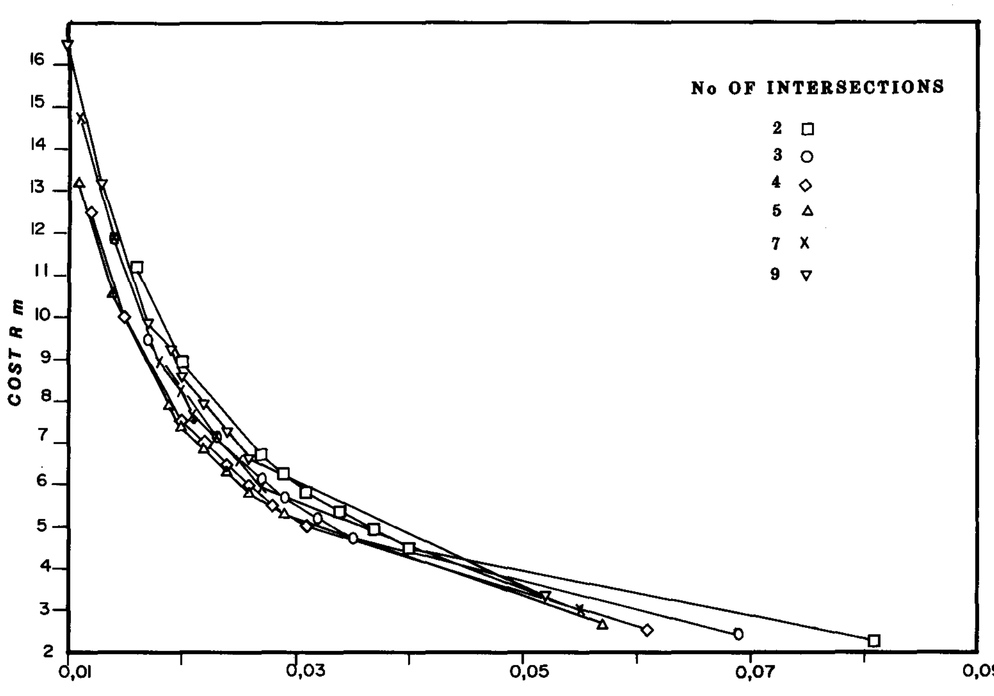


Fig. 15—Analysis of borehole deflections for the Basal Reef at a depth of 3000 m

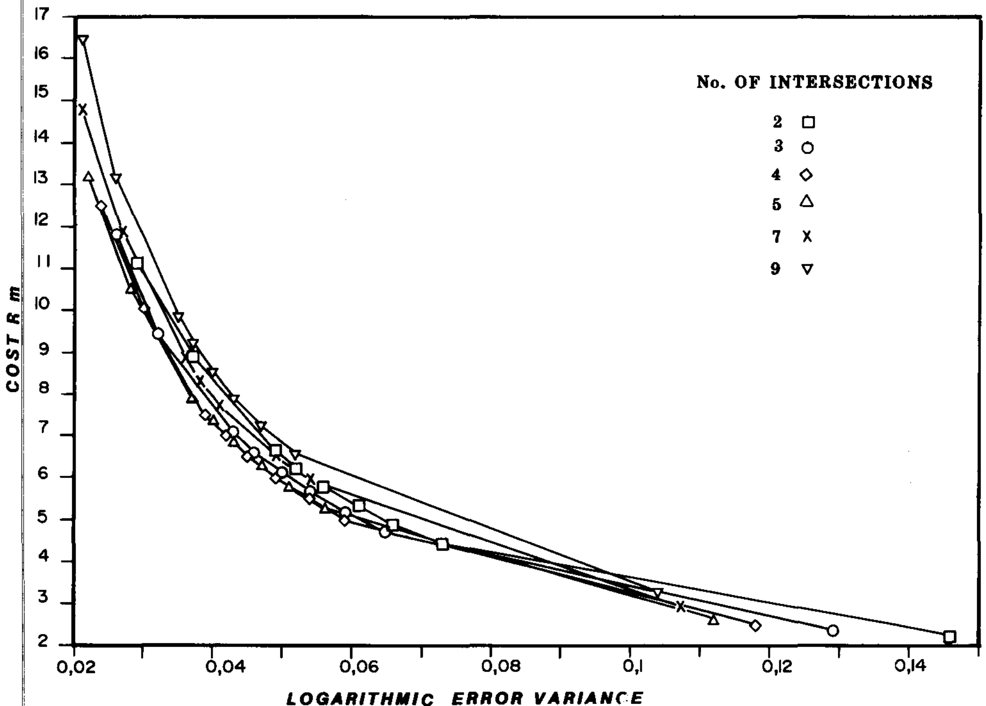


Fig. 16—Analysis of borehole deflections for the Vaal Reef at a depth of 3000 m

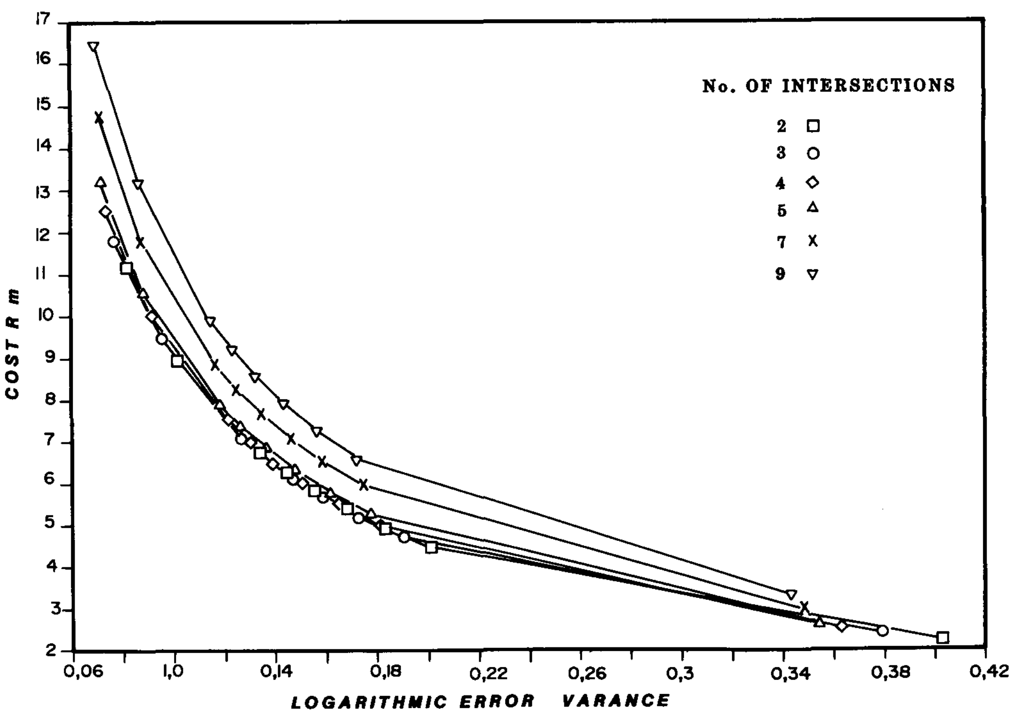


Fig. 17—Analysis of borehole deflections for the Elsburg Reef at a depth of 3000 m

respectively should be preferred.

Acknowledgements

The author expresses his sincere appreciation to the Geology Department of Anglovaal Limited, particularly Mr D.L. Kyle, Mr J. Viljoen, and Mr D. Cloete for their help and guidance; to Professor D.G. Krige for his helpful comments; to Mrs G. Knox and Mr C. Hunt for some of the computer work involved; to Mr D. Schoeman, Mr F. Lyons, and Mrs A.C. Klokow for draughting the figures; and to the Management of Anglovaal Limited for the opportunity they afforded him of carrying out this research work and for permission to publish the results.

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New FSPE President*

Professor A.N. Brown was inducted as President of The Federation of Societies of Professional Engineers (FSPE) at a cocktail party held at Kelvin House, Johannesburg, on 10th August, 1987, by the Immediate Past President, Mr R.G. St Leger.

Professor Brown is Head of the Department of Mining Engineering at the University of Pretoria, a position he has held since 1981. He was born in Bloemfontein in 1933, and educated at Forest High School in Johannesburg. His mining career began at Crown Mines in 1951 as a learner official. He received a bursary from the Chamber of Mines and graduated in 1955 with the B.Sc. degree in mining engineering. In 1957, he was awarded a research scholarship by the Chamber and undertook research in rock mechanics and strata movements associated with the removal of the Turf Shaft Pillar at Robinson Deep Gold Mine. He was awarded the M.Sc. Engineering degree in 1960.

In 1961, he was transferred to the Rand Mines head office, where he came under the influence of leading consulting engineers. In 1965, he returned to the University of the Witwatersrand to take up the appointment as a research fellow. He was appointed Manager of the newly formed Raise and Tunnel Boring Company in 1969, and was also involved with the sinking of the shafts at Cleveland Potash in England. From 1971 to 1976, Professor Brown was in private practice, after which he took up an appointment with Gold Fields of South Africa, being promoted to the position of Consulting Engineer after a year. His brief was mainly concerned with mechanization, technical services, and research.

Professor Brown serves on many committees and councils concerned with The South African Council for Professional Engineers (SACPE), The South African Institute of Mining and Metallurgy (SAIMM), FSPE, and The Associated Scientific and Technical Societies of South Africa (AS&TS). He was President of SAIMM in 1982, and has been Chairman of the FSPE Education and Careers Guidance Committee since July 1985.

He is married and he and his wife, Joyce, have one son and two daughters.

Mr St Leger referred to Professor Brown's wide experience in the mining industry. This, coupled with his humanity and understanding, makes him an extremely appropriate choice as President at a time when the structure of the profession is being rearranged.

Professor A.N. Brown, President of FSPE

In reply, Professor Brown appealed to the profession to continue supporting FSPE financially and morally, until the new structures are working smoothly. The functions at present being undertaken by FSPE, such as publicity and career guidance, need to carry on without interruption. He drew attention to the benefit that engineers derive from membership of their learned societies, and pointed out that subscription rates, which are tax deductible, are low in comparison with those of other professions.

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