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Pebbles as grinding medium: Interrelationships of some milling parameters

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SYNOPSIS

The use of pebbles as the medium in the grinding of quartz gravel in a laboratory mill provided experimental data that were used to quantify the relationships between the feed rate of ore to the mill, the fineness of grind of the product, and the wear rate and consumption of the grinding medium.

Fineness of grind is expressed as the inverse of the harmonic mean size, a quantity that is calculated from data obtained by sieve analyses. The commonly used criterion for fineness of grind (the percentage of material smaller than 75 μm in the mill product) is shown to correlate approximately linearly with the inverse of the harmonic mean size. An increase in the feed rate was found to cause the harmonic mean size of the product to increase linearly, i.e. the relationship previously found for metallic grinding media is applicable to all grinding media, irrespective of shape or composition.

Simple expressions for the consumption and wear rate of pebbles are derived from the experimental data, and are shown to be functions of the feed rate. The expression for wear rate is a monotonically increasing function, while that for consumption is a hyperbolically decreasing function of the feed rate.

In contrast, a linear relationship was found between the rate of wear of the grinding medium and the rate of production of fines. The expressions that were previously developed in relation to the use of metallic grinding media are shown to be equally applicable to pebble milling.

SAMEVATTING

Die gebruik van rolklippe as maalmedium vir die maal van kwartsgruis in 'n laboratoriummeul het eksperimentele data verskaf wat gebruik is om die verhouding tussen die voertempo van erts na die meul, die maalfynheid van die produk en die slyttempo en verbruik van die maalmedium te kwantifiseer.

Die maalfynheid word uitgedruk as die omgekeerde van die harmoniese gemiddelde grootte, 'n grootheid wat bereken is aan die hand van sifontledingsdata. Daar word getoon dat die kriterium wat algemeen vir maalfynheid gebruik word (die persentasie materiaal kleiner as 75 μm in die meulproduk) min of meer lineêr korreleer met die omgekeerde van die harmoniese gemiddelde grootte. Daar is gevind dat 'n verhoging van die voertempo die harmoniese gemiddelde grootte van die produk lineêr laat toeneem d.w.s. die verhouding wat vroeër vir metaalmaalmedia bepaal is, is op alle maalmedia van toepassing, ongeag die vorm of samestelling daarvan.

Daar word eenvoudige uitdrukkings vir die verbruik en slyttempo van rolklippe van die eksperimentele data afgelei en daar word getoon dat hulle funksies van die voertempo is. Die uitdrukking vir die slyttempo is 'n monotoonstygende funksie, terwyl dié vir die verbruik 'n hiperbolies dalende funksie van die voertempo is.

In teenstelling daarmee is daar 'n lineêre verhouding gevind tussen die slyttempo van die maalmedium en die produksietempo van fynmateriaal. Daar word getoon dat die uitdrukkings wat vroeër in verband met die gebruik van metaalmaalmedia ontwikkel is, eweseer op rolklipmaling van toepassing is.

Introduction

Tube milling or pebble milling of ores has been practised on the gold mines of the Witwatersrand for over 60 years. In the earlier mills (1,98 m in diameter by 6,10 m long), the grinding media consisted of hand-picked pebbles up to 150 mm in diameter. With the introduction of crushing and screening operations, it soon became evident that the larger screened fractions of ore could be used as 'pebbles', and this practice is now in common use. Where the character of the rock in the gold-bearing reef and the mining procedures employed allow pebble milling to be used exclusively, the costs of milling are considerably reduced. In less favourable circumstances where the supply of pebbles may be inadequate for the milling requirements, metallic grinding media are needed to supplement the pebble supply. Run-of-mine (ROM) milling, which is currently used on a large scale in new mines and on extensions to existing plants, is essentially a form of pebble milling, but additions of metallic grinding media are very frequently necessary for efficient operation.

In view of the extensive testwork[1] already carried out at the Council for Mineral Technology (Mintek) with metallic grinding media, it was decided to extend that work to the study of pebbles as grinding media to show whether the findings and relationships established in the earlier work[2] could also be applied to pebble milling.

Test Procedures

Tests were conducted using a laboratory mill (0,6 m by 0,6 m) operating at 70 per cent of the critical speed. The pebbles used in the tests were obtained by the treatment of coarsely crushed screened fractions of gold-bearing ore in a rubber-lined mill (1,0 by 1,4 m) that was fitted with rubber lifter bars and rotated at about 70 per cent of the critical speed. When tumbled in this mill, the pieces of rock were found to be smooth and rounded after 24 hours.

A weighed charge of pebbles (about 99 kg) was placed in the mill at the start of each test. The washed quartz gravel (the sieve analysis of which is shown in Table I) was then fed to the mill at pre-selected rates. Water was added to give a 75 per cent solids content in the mill discharge. At the conclusion of each test, the pebble

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charge was withdrawn from the mill, and the loss in mass was determined. Samples of the pulp discharging from the mill were taken at frequent intervals during the test and, after being composited, were subjected to sieve analysis using the series of Tyler screens 300, 212, 150, 106, 75, 56, and 43 μm. The harmonic mean size (D_m) of the solids discharged from the mill was calculated from the sieve analysis by use of the expression

$$D_m^{-1} = \Sigma\, m_i/\bar{X}_i, \qquad (1)$$

where m_i is the mass fraction of the screen fraction in the ith size interval, and

$\bar{X}_i$, as given by Herdan[3], is

$$\bar{X}_i = [(X_1^2+X_2^2)\,(X_1+X_2)/4]^{1/3}, \qquad (2)$$

X_1 and X_2 being the size limits of the ith size interval.

The oversize material (larger than 300 μm) presents a small problem in the calculation of the harmonic mean size. In the present investigation, it was assumed that 50 per cent of this material was in the size range 425 to 300 μm, and the remainder in the range 600 to 423 μm. This treatment of the oversize material introduced a small error into the quoted values of the harmonic mean size, which amounts to an over-estimate of less than 5 per cent.

Results

As shown in Table I, six tests were carried out, with a constant charge of about 99 kg of pebbles. The feed rate of ore to the mill ranged from 300 to 1800 g/min (18 to 108 kg/h). Additional data relating to the consumption and wear rate of the pebbles are given in Table II. For pebbles, the wear rate ($\dot{M}$) is given as kilograms per hour of milling time, and the consumption (M') is given as kilograms per ton of ore milled, the consumption obviously being equal to the wear rate divided by the feed rate of ore to the mill.

Fineness of Grind in Relation to Feed Rate

In the gold-mining industry fineness of grind is commonly expressed as the percentage of material smaller than 75 μm in the product. In this paper, this quantity is referred to as *fines*. Although this quantity is very useful as a pragmatic indicator, it does not reflect the size distribution of the product since it merely serves to separate the material under test into two fractions. The harmonic mean size, D_m, calculated by use of equations (1) and (2), is based on the particle-size distribution. The inverse of the calculated harmonic mean size, (D_m^{-1}) can be regarded as an index of the fineness of grind because

TABLE I

DATA RELATING TO THE SIZE DISTRIBUTION OF THE PRODUCT IN THE GRINDING OF QUARTZ GRAVEL WITH QUARTZITE PEBBLES IN AN 0,6 BY 0,6 m MILL

Feed rate kg/h	Product in sieve fraction, %									Rate of production of fines kg/h	Harmonic mean size μm	Inverse of harmonic mean size mm^{-1}
	+300 μm	+211 μm	+150 μm	+106 μm	+75 μm	+53 μm	+45 μm	−45 μm	−75 μm			
16,95	3,52	3,08	6,07	9,67	10,97	10,76	6,35	49,52	66,63	11,2	43,73	22,9
38,02	12,51	7,37	10,35	11,16	9,94	8,45	4,94	35,27	48,66	18,5	56,02	17,85
55,86	26,31	9,02	10,18	9,89	8,60	6,69	2,91	26,39	35,99	20,1	70,59	14,17
67,53	35,75	8,39	9,19	9,12	7,33	5,15	1,87	23,20	30,22	20,4	79,79	12,53
93,24	44,21	7,51	7,95	7,58	6,36	4,65	1,85	19,90	26,40	24,6	86,54	11,56
107,79	51,95	7,68	7,65	6,90	5,50	4,15	2,07	14,10	20,32	21,9	109,90	9,10

	Sieve analysis of the feed								
Mesh, μm	3350	2630	1700	1180	850	600	425	300	<300
Retained, %	15,25	48,84	25,26	6,92	2,34	0,93	0,11	0,09	0,24

TABLE II

DATA RELATING TO THE CONSUMPTION AND WEAR RATE OF PEBBLES USED AS THE GRINDING MEDIUM IN AN 0,6 BY 0,6 m EXPERIMENTAL MILL

Data					Calculated from the data			Calculated from theory	
1	2	3	4	5	6	7	8	9	10
			Pebble mass						
Average feed rate kg/h	Milling time h	Amount milled kg	Initial kg	Final kg	Average consumption kg/t	Average wear rate kg/h	Tonnage parameter τ_{exp} t	Average consumption kg/t	Average wear rate kg/h
16,95	16,46	279	98,14	77,31	74,66	1,27	1,170	76,37	1,295
38,02	8,26	314	98,30	85,33	41,31	1,57	2,218	42,89	1,636
55,86	5,80	324	98,60	88,01	32,69	1,83	2,852	31,40	1,782
67,53	7,13	482	98,64	85,36	27,58	1,86	3,329	26,73	1,805
93,24	6,58	613	98,75	86,36	20,20	1,88	4,576	20,02	1,882
107,79	5,52	595	98,81	88,26	17,73	1,91	5,280	17,89	1,928

it increases with increasing fineness. As the percentage of fines increases with increasing fineness of grind, some correlation between the two would be expected. Fig. 1 shows the values of (D_m^{-1}) plotted against the percentage of material smaller than 75 μm in the mill product. The data for the diagram were derived from a large number of tests carried out on a laboratory mill and on several full-scale mills. The results all follow the same pattern notwithstanding the use of pebbles or of metallic grinding media of widely differing shapes and sizes.

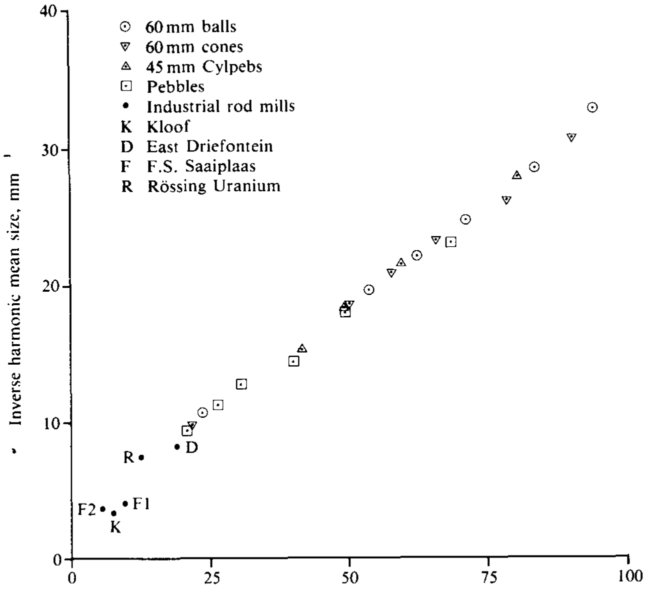

Fig. 1—Inverse of the harmonic mean size of the product as a function of the percentage fines in the product under various milling conditions

As can be seen from Fig. 1, the correlation is substantially linear, and can be expressed as

$$D_m^{-1} \approx A + Bp, \qquad (3)$$

where p is the percentage of material smaller than 75 μm, and A and B are positive constants.

Earlier work[2], in which various metallic grinding media were used, showed that the relationship between the feed rate of ore to the mill and the calculated harmonic mean size of the product is linear. When pebbles were used as grinding media in the present work, the relationship between these quantities was again found to be linear, as shown in Fig. 2. This can be expressed as

$$D_m = d_0 + bF, \qquad (4)$$

where d_0 and b are positive constants. As would be expected, the values of d_0 and b are different for metallic grinding media of different shapes and sizes, and for pebbles, but the linear relationship between the calculated harmonic mean size and the feed rate of the ore appears to obtain for all types of grinding media.

The increase in the calculated harmonic mean size with increasing feed rate provides a quantitative description of the well-known fact that the grind becomes coarser as the feed rate of ore to the mill increases[4]. From Fig. 3, it is clear that the percentage of fines in the product is a markedly decreasing function of the feed rate, but that the proportion of fines in the product is substantially lower with a grinding charge of pebbles than with the same volume of metallic media.

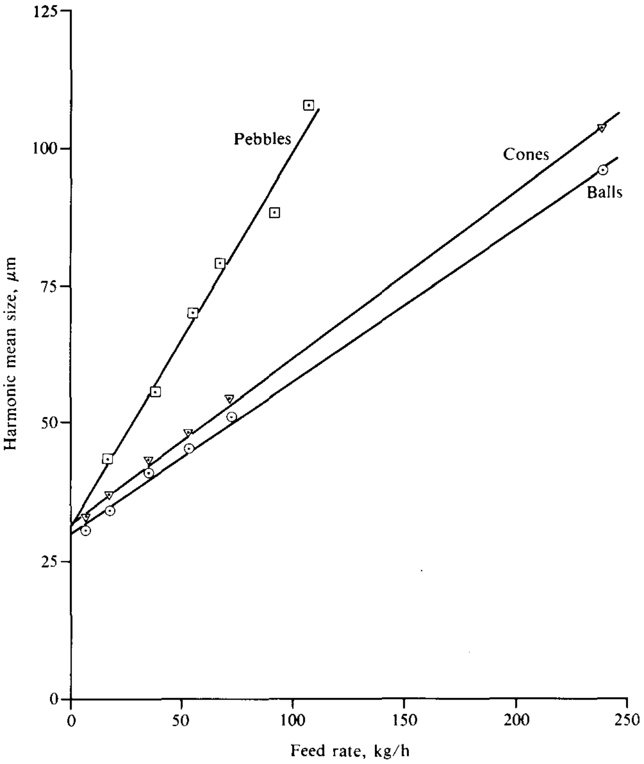

Fig. 2—The dependence of the harmonic mean size on the feed rate of ore to the given mill for various grinding media

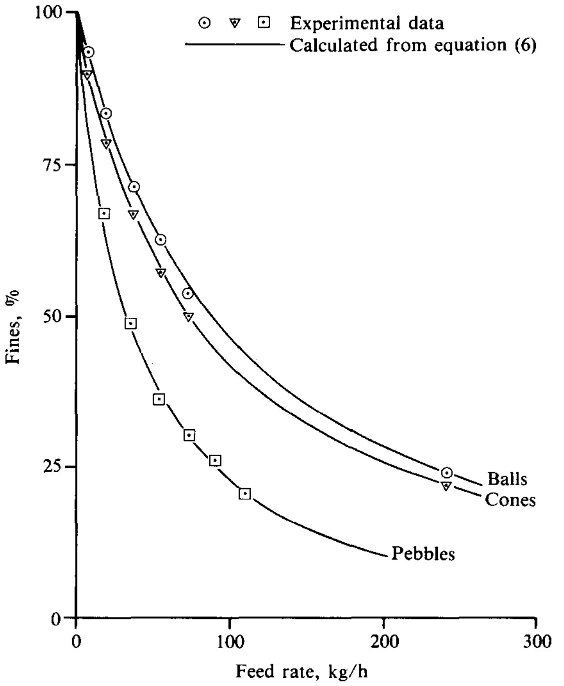

Fig. 3—The percentage fines in the product versus the feed rate of ore

Fig. 4 shows the relationship between the rate of production of fines (π) and the feed rate of ore to the mill. While the rate of production of fines is less than half that obtained with an equal volume of metallic grinding media, the graph relating to pebbles appears to reach a maximum that is more clearly defined than that for metallic grinding media.

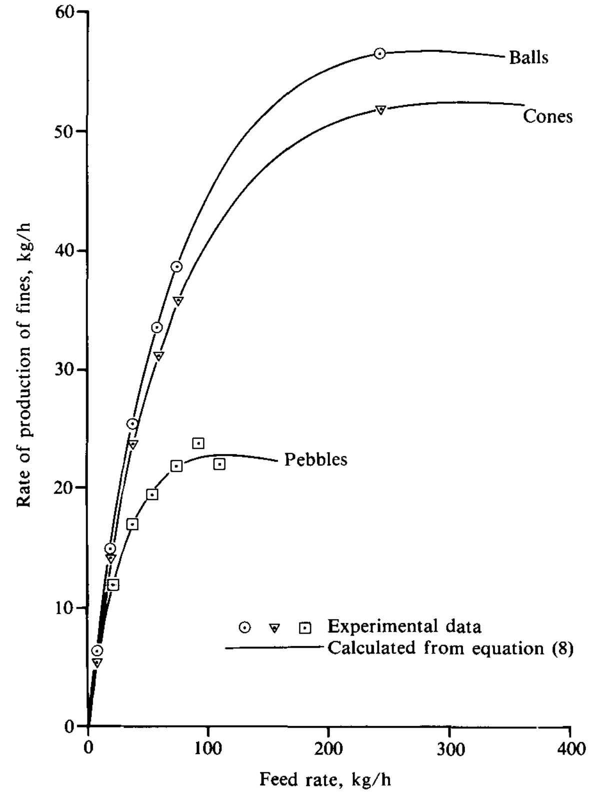

Fig. 4—The rate of production of fine material (π) versus the feed rate of ore

Consumption and Wear Rate of Media in Relation to Feed Rate and Fineness of Grind

The data relating to the average wear rate and consumption of pebbles, which are shown in Table II in columns 6 and 7 respectively, show that the wear rate was between 1 and 2 kg/h, and that it increased with increasing feed rate. The consumption was between 10 and 70 kg per ton of ore milled, and decreased with increasing feed rate. Similar behaviour was observed previously[2] with metallic grinding media, but the values were at least an order of magnitude smaller.

As is evident from Fig. 5, the relationship between the wear rate of grinding media (pebbles and steel balls) and the rate of production of fine material is approximately linear.

Discussion

Two relationships can be considered for the calculated harmonic mean size, D_m. As is evident from Fig. 2, this quantity increases linearly with the feed rate over the full range of feed rates that could be accepted by the experimental mill. In Fig. 1, the inverse of this quantity, D_m^{-1}, is shown to be approximately linearly correlated with the percentage of material smaller than 75 μm, $p(F)$, in the mill products. The elimination of the harmonic mean size from equations (3) and (4) yields the following expression for $p(F)$:

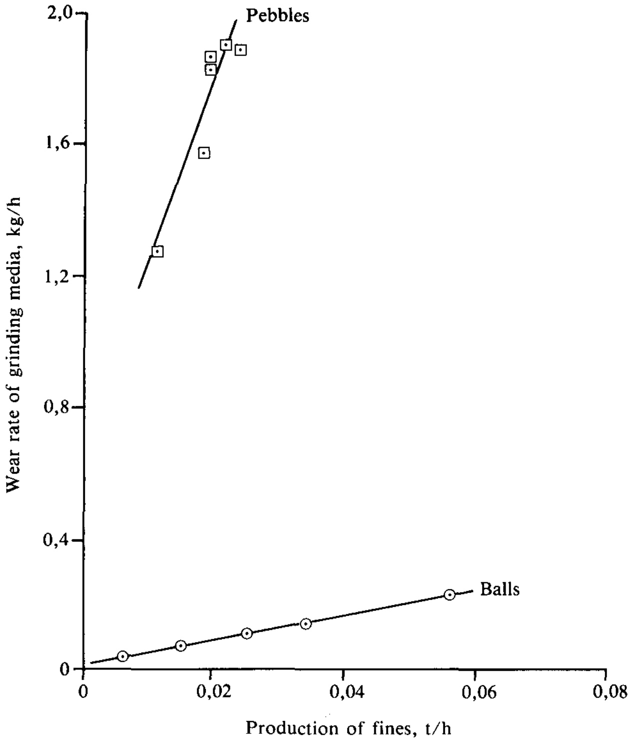

Fig. 5—The relationship between the production rate of fines and the wear rate of grinding media

$$p(F) = \frac{(1 - Ad_0) - AbF}{B(d_0 + bF)} \text{ . } \qquad (5)$$

This is essentially a three-parameter description of the form

$$p(F) = \frac{C_1 - C_2F}{1 + C_3F}, \qquad (6)$$

where

$C_1 = (1 - Ad_0)/(Bd_0)$,
$C_2 = (Ab)/(Bd_0)$, and
$C_3 = b/(Bd_0)$.

The rate of production of material smaller than 75 μm is given by

$$\pi = pF. \qquad (7)$$

Equation (7) is completely valid for ball milling. However, when pebbles are used as the grinding medium, a proportion of the material smaller than 75 μm in the discharge from the mill is derived from the pebbles, the precise contribution due to pebble wear being unknown. Even if it is assumed that all the product resulting from the wear of the pebbles is smaller than 75 μm, the data in Tables I and II indicate clearly that the maximum contribution from the wear of the pebbles is less than 7,5 per cent at the lowest feed rate, and decreases to less than 2 per cent at the highest feed rate. For these reasons, this contribution to the fines has been ignored, and all subsequent calculations are based on equation (7). Equations

(6) and (7) show that π can be expressed as a function of the feed rate as follows:

$$\pi(F) = \frac{C_1F - C_2F^2}{1 + C_3F}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

Expressions (6) and (8) are valid only for feed rates less than C_1/C_2 and, when these expressions are fitted to the experimental data given in Table I, the values for the fineness parameters C_1, C_2, and C_3 can be calculated (Table III).

TABLE III
CALCULATED VALUES OF THE FINENESS PARAMETERS FOR DIFFERENT GRINDING MEDIA

Grinding medium	Parameter		
	C_1	C_2 kg^2/h	C_3 kg^3/h
Steel balls	1,002	0,745	10,404
Cones	0,968	0,556	11,956
Pebbles	1,025	1,710	27,680

While the calculated values for C_1 are virtually independent of the type or shape of the grinding medium used, C_2 and C_3 show significant variations associated with changes in the type and shape of the grinding media. At very low feed rates, nearly all the product from the mill will be fine and, as shown in expression (5), the value of p tends to C_1 as F tends to zero, i.e. C_1 will approach unity as indicated in Table III.

Using the values for C_1, C_2, and C_3 as given in Table III, graphs were calculated from equations (6) and (8) for the functions $p(F)$ and $\pi(F)$ respectively. These graphs, shown in Fig. 3 and 4 respectively, are in good agreement with the experimental data obtained in the tests in which the three types of grinding media were used.

The effect of changes in the feed rate of the ore on the percentage and rate of production of fines can be estimated from the differential form of equation (7), i.e.

$$\Delta\pi/\pi \approx \Delta F/F + \Delta p/p. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

Since $\Delta p/p$ and $\Delta F/F$ are of opposite sign and p is a decreasing function of increasing F, it follows that the relative change in the rate of production of fines will always be *smaller* than the relative change in feed rate. Indeed, it may be negligibly small, and even negative. The latter situation can arise in mills operating close to their maximum possible throughput.

The interrelationships between the feed rate, the fines fraction in the product, and the rate of production of fines can be shown by ternary curves in π-F-p space. Curves for steel balls and pebbles, which are shown in Fig. 6, indicate the changes in p and π that can be expected for given changes in F (the only independent variable). Hence, if the feed rate is increased by 20 per cent, say from 100 to 120 kg/h, then, with balls as the grinding medium, the percentage of material smaller than 75 μm in the product will decrease by about 10 per cent, and the rate of production of fines will increase by about 7 per cent, i.e. $\Delta\pi/\pi$ will be less than $\Delta F/F$.

On the other hand, with pebbles as the grinding medium, a similar change in the feed rate will result in a decrease in the fines of 16,3 per cent, and the production rate of fines will increase by only 0,4 per cent. This almost negligible increase in the production rate of fines is due to the fact that, when a given mill is operated with pebbles as the grinding medium, a feed rate of 100 kg/h is close to the maximum rate at which material can be accepted by the mill. It can be deduced that, when the relative changes in feed rate and in percentage of fines in the product are nearly equal, the change in the production rate of fines will be extremely small. This finding is of practical value when mills are operated near their maximum capacity.

Consumption and Wear Rate of Pebbles

Earlier work[5,6] on the wear rate of metallic grinding media showed that the decrease in mass of the grinding elements can be modelled fairly accurately by an exponential function of the form

$$M(\Delta T) = M_0 \exp(-\Delta T/\tau), \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

where M_0 is the original mass of the grinding medium,
$M(\Delta T)$ is the mass of the grinding medium after ΔT tons of material have been milled in a time Δt hours, and
τ is the tonnage parameter.

The above expression is essentially time-dependent, since ΔT is the feed rate of ore to the mill multiplied by the milling time. The application of equation (10) to the wear of pebbles in milling is discussed below.

The experimental data given in columns 1 to 5 of Table II were used in the calculation of the values of the average wear rate ($\dot{M}_{\text{AV}}$), the average consumption (M'_{AV}), and the experimental values of the tonnage parameter τ_{exp}. It is clear that these values, which are shown in Table II in columns 6, 7, and 8 respectively, vary systematically with the feed rate of ore to the mill.

To account for this dependence on the feed rate, the following model was proposed[2]:

$$\begin{array}{c}\text{Wear rate}\\ \text{of grinding}\\ \text{medium} =\end{array} \left[\begin{array}{c}\text{Rate at which}\\ \text{work is done by}\\ \text{the charge on the}\\ \text{material passing}\\ \text{through the mill}\end{array}\right] + \left[\begin{array}{c}\text{Wear rate at}\\ \text{zero feed rate}\\ \text{when the}\\ \text{hold-up is}\\ \text{being milled}\end{array}\right].$$

This can be expressed as

$$\dot{M}(F) = \frac{MF}{\tau_{\text{F}}} + \frac{M}{\theta}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

where θ is a constant (with the dimensions of time), and τ_{F} is a parameter that depends on the feed rate. To first order, τ_{F} is of the form

$$\tau_{\text{F}} = \tau_0 + gF, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

where τ_0 and g are constants. Hence

$$\dot{M}(F) = M\left(\frac{F}{\tau_0 + gF} + \frac{1}{\theta}\right). \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

As the consumption is the wear rate divided by the feed rate, it follows that the tonnage parameter, $\tau(F)$, express-

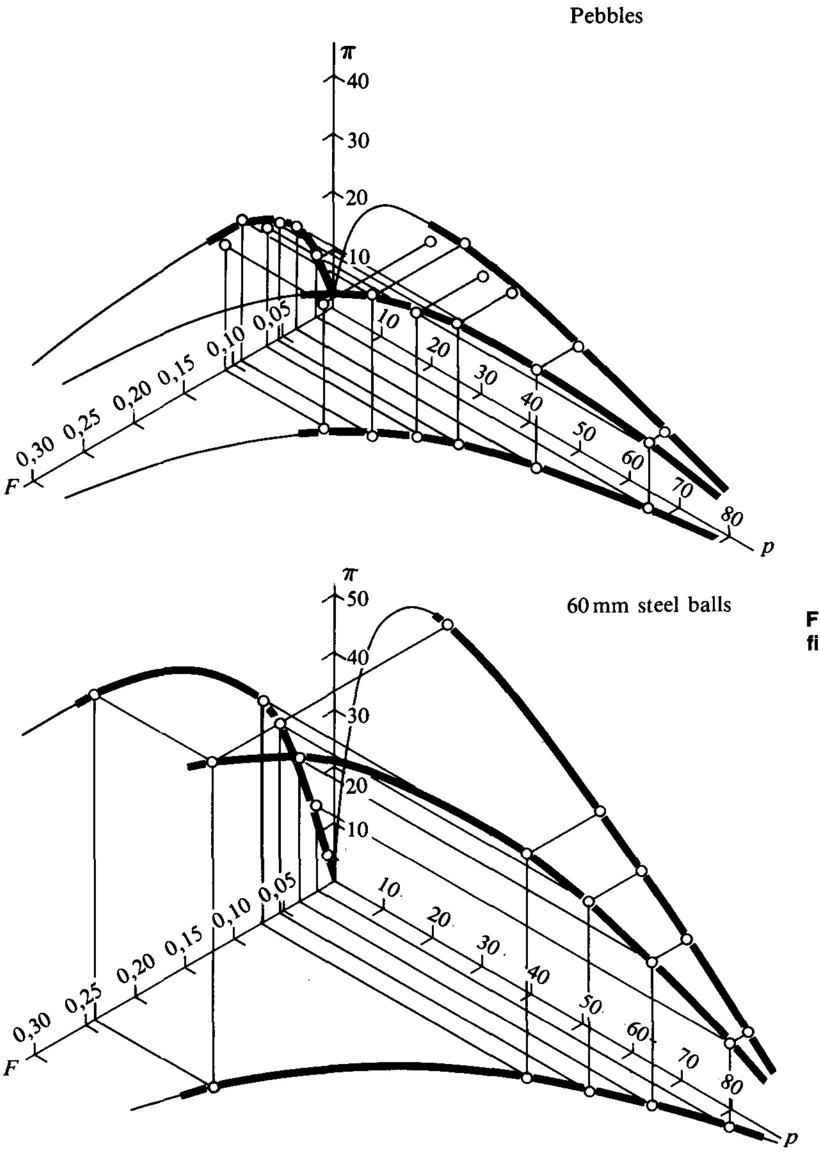

Fig. 6—Ternary curves of the production rate of fines, the feed rate, and the fines fraction in the product

ed as a function of the feed rate, is given by

$$\frac{1}{\tau(F)} = \frac{1}{\tau_0 + gF} + \frac{1}{\theta F}. \quad \text{.............................} \quad (14)$$

The fitting of expression (14) to the values of τ_{exp} in Table II permits the wear constants θ, τ_0, and g to be calculated. Table IV compares these wear constants for pebbles with those for a metallic grinding medium[2], and shows that the values of the wear constants for pebbles are between one and two orders of magnitude smaller than those for a metallic grinding medium. This is in conformity with the much higher wear rate and consumption that are characteristic of pebbles. The graphs of equation (14) shown in Fig. 7 are in good agreement with the values of τ_{exp} for pebbles and a metallic grinding medium.

Equation (14) permits the initial consumption and the initial wear rate to be calculated as functions of the feed rate as follows:

Initial consumption

$$M_0'(F) = M_0\left(\frac{1}{\theta F} + \frac{1}{\tau_0 + gF}\right). \quad \text{..................} \quad (15)$$

Initial wear rate

$$\dot{M}(F) = M_0\left(\frac{1}{\theta} + \frac{F}{\tau_0 + gF}\right). \quad \text{....................} \quad (16)$$

Figs. 8 and 9 show the graphs of the above expressions for pebbles and for steel balls respectively, and include the data points for the experimentally determined values of the initial consumption (M_0/τ_{exp}) and the initial wear rate (M_0F/τ_{exp}). The curves are in good agreement with the data points, but the two relationships differ markedly. The initial wear rate is a monotonically increasing (although bounded) function of the feed rate, having the values M_0/θ at zero feed rate and $M_0(1/\theta + 1/g)$ at infinite feed rate. In contrast, the initial consumption is an essentially hyperbolically decreasing function of the feed rate because the consumption is the wear rate divided by the feed rate.

Theoretical values of the average consumption and the average wear rate were calculated from the following formulae:

Average consumption

$$M_{av}'(F) = M_0\{1 - \exp(-\Delta T/\tau)\}/\Delta T. \quad \text{.........} \quad (17)$$

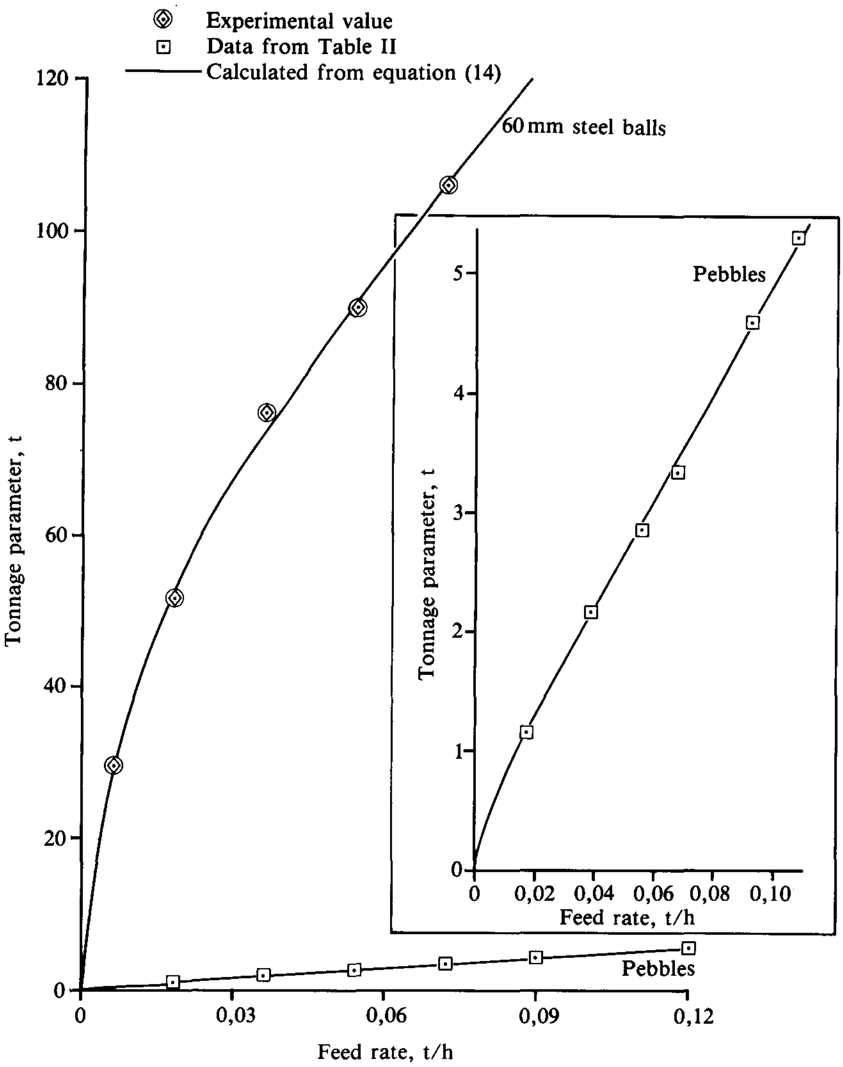

Fig. 7—The tonnage parameter as a function of the feed rate in milling with steel balls and with pebbles

TABLE IV
VALUES OF THE WEAR CONSTANTS FOR VARIOUS GRINDING MEDIA UNDER THE GIVEN MILLING CONDITIONS

Grinding medium	Wear constant		
	θ h	τ_0 t	g h
Pebbles	139	1,125	63,9
Balls	8400	65,64	881
Cones	8077	55,1	1199

Average wear rate

$$\dot{M}_{av}(F) = M_0\{1 - \exp(-\Delta T/\tau)\}/\Delta t. \quad \text{..........} \quad (18)$$

The calculated values given in columns 9 and 10 of Table II are in good agreement with the experimentally derived values, the maximum discrepancy of less than 5 per cent probably being due to experimental scatter.

The above considerations have shown that the same expressions, with appropriate values of the constants, can be used to model the wear rate and consumption of any type of grinding medium in milling, irrespective of its shape or composition.

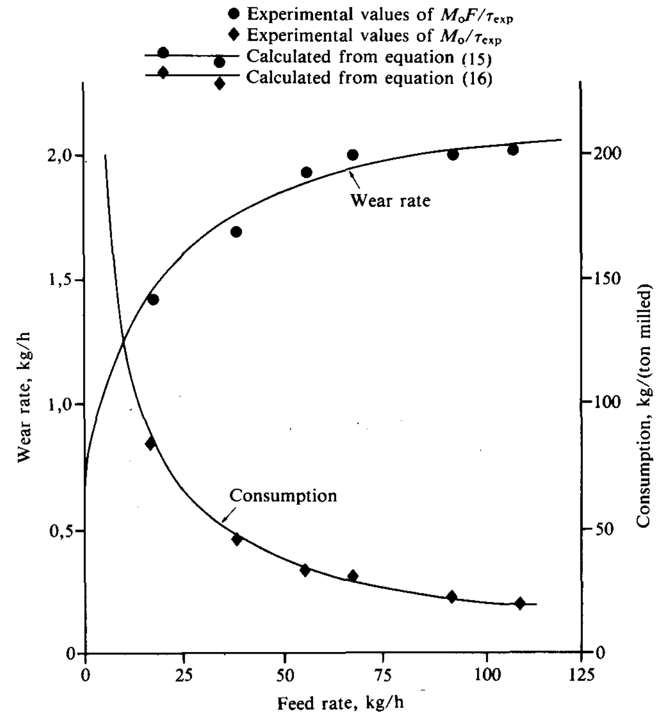

Fig. 8—The initial wear rate and consumption of pebbles as functions of the feed rate

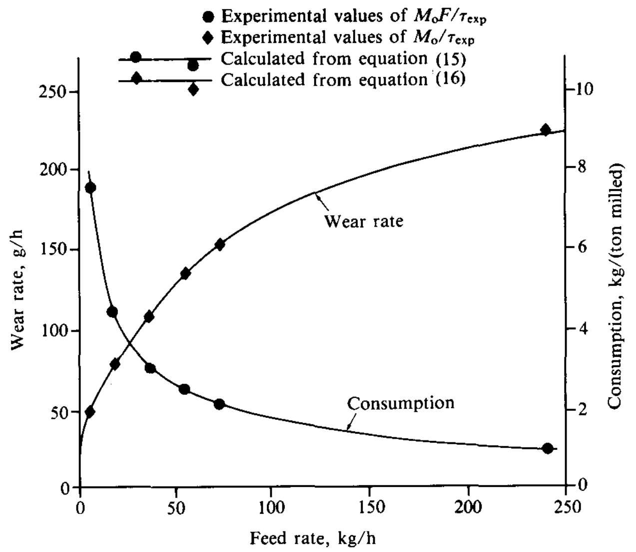

Fig. 9—The consumption and wear rate of 60 mm steel balls as functions of the feed rate

Wear Rate and Fineness of Grind

The wear rate as a function of the rate of production of fines is shown for pebbles and for metallic grinding media[2] in Fig. 5, which indicates clearly that this relationship is approximately linear for both types of grinding medium. An increase in the rate of production of fines therefore involves a proportionate increase in the wear rate of grinding media. For pebbles, the rate of increase in the wear rate is about 53 kg per ton of fines produced, while it is only 3,5 kg per ton of fines for metallic grinding media. This again underlines the very high wear rate of pebbles compared with that of metallic grinding media.

The approximately linear relationship between the wear rate of grinding media and the production rate of fines may be understood through the use of a first-order approximation to equation (8), which expresses the production rate of fines as a function of the feed rate of ore to the mill. The approximation is

$$\pi(F) \simeq \frac{C_1 F}{1 + C_3 F}. \quad \text{.......................................} \quad (19)$$

This expression can be compared with equation (16), which expresses the dependence of the wear rate of grinding media on the feed rate of ore to the mill. Equation (16) can be written in the form

$$\dot{M}(F) = k_1 + \frac{k_2 F}{1 + k_3 F}, \quad \text{...............................} \quad (20)$$

where k_1, k_2, and k_3 are positive constants.

Apart from an additive constant, equation (20) is of the same form as the first-order approximation to $\pi(F)$ given above. Moreover, the curves of $\pi(F)$ and $\dot{M}(F)$ in Figs. 4 and 8 are roughly similar over the same range of feed rates, thus indicating that the wear rate of grinding media is an approximately linear function of the rate of production of fines, as shown in Fig. 5.

Conclusions

(1) The inverse of the harmonic mean size is approximately linearly related to the percentage of material smaller than 75 μm in milled products obtained under a wide variety of milling conditions.

(2) The harmonic mean size of the product increases linearly with increasing feed rate up to the maximum possible rate at which material can be accepted by the mill.

(3) The above relationships can be combined to yield simple three-parameter descriptions of the percentage of fines and the rate of production of fines as functions of the feed rate. The expressions are in good agreement with the experimental results.

(4) It has been quantitatively demonstrated that the relative increase in the rate of production of fine material will always be smaller than the relative increase in the feed rate. Indeed, the former can be negligibly small or even negative. The latter situation can arise when the relative decrease in the percentage of fines is approximately equal to the percentage increase in the feed rate.

(5) The wear rate and consumption of all grinding media can be accurately modelled by an exponential function in which the tonnage parameter is a simple function of the feed rate.

(6) The linear relationship between the rate of wear of grinding media and the rate of production of fine material indicates that increases in the latter can be achieved only at the expense of proportionate increases in the wear rate of grinding media.

(7) Although the wear rate and consumption of pebbles are more than an order of magnitude larger than those of metallic grinding media, the above relationships are applicable both to metallic grinding media of widely different shapes and to pebbles.

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