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The stability and design of yield pillars located at shallow and moderate depths

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SYNOPSIS

The stability of pillars is analysed on the assumption that pillars shed load in a stable manner if the slope of the strata load-deformation relation is steeper than that of pillars in the post-peak regime. Concepts of local stiffness, mine structural stiffness, and critical stiffness are discussed. Procedures are described for the determination of strata stiffness for various pillar layouts by use of the available boundary-element computer programs. The post-peak stiffness of pillars is assessed from the available data relating to the post-peak load-deformation relationship and width-to-height ratio. Stability analyses are carried out by the contrasting of strata stiffness with post-peak pillar stiffness. It is shown that the strata stiffness becomes unacceptably small for a stable yield-pillar layout when the span exceeds about 5 times the depth. The strata stiffness is found to increase, thereby increasing the stability, as the span-to-depth ratio, number of pillars, and percentage extraction are decreased. It is also found that the stability in a yield pillar becomes difficult to achieve when all the pillars in a layout are in the descending part of their post-peak load-deformation curve.

SAMEVATTING

Die stabiliteit van pilare word ontleed met die aanname dat pilare belasting op 'n stabiele wyse afwerp as die helling van die lae se las-vervormingsverhouding steiler as dié van pilare in die naspitsregime is. Begrippe soos plaaslike styfheid en kritieke styfheid word bespreek. Prosedures vir die bepaling van laagstyfheid vir verskillende pilaaruitlegginge met gebruik van die beskikbare grens-elementrekenaarprogramme word beskryf. Die naspitsstyfheid van pilare word geraam aan die hand van die beskikbare data oor die naspitslas-vervormingsverhouding en die breedte-hoogteverhouding. Stabiliteitsontledings word gedoen deur die laagstyfheid met die naspitspilaarstyfheid te vergelyk. Daar word getoon dat die laagstyfheid onaanvaarbaar klein vir 'n stabiele meegeepilaaruitleg word wanneer die span 5 maal die diepte oorskry. Dit blyk dat die laagstyfheid toeneem, en daardeur die stabiliteit verhoog, as die span-diepteverhouding, die getal pilare en die ekstraksiepersentasie verlaag word. Daar is ook vasgestel dat dit moeilik word om stabiliteit in 'n meegeepilaar te verkry wanneer al die pilare in 'n uitleg in die dalende deel van hul naspitslas-vervormingskromme is.

Introduction

It is well established in rock mechanics that, during laboratory uniaxial compression testing, the stability of the equilibrium between the testing machine load and the specimen resistance in the specimen's post-peak regime depends on the relationship between the stiffness of the testing machine and the post-peak stiffness of the test specimen[1], as shown in Fig. 1. If the stiffness of the testing machine (slope of line B in Fig. 1) is less than the slope of the post-peak load-deformation relation of the specimen, the test is terminated by a violent failure of the specimen. However, violent failure will not occur if the stiffness of the testing machine (slope of line A in Fig. 1) is greater than the minimum slope (most negative) of the post-peak load-deformation relation of the specimen.

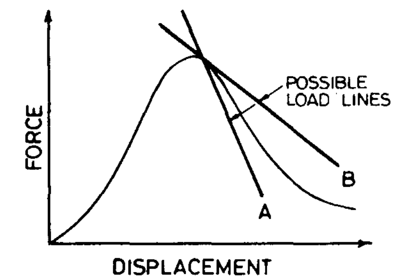

Fig. 1—Stable (A) and unstable (B) loading of a rock specimen under uniaxial compression: lines A and B describe possible stiffness characteristics of the testing machine, load line B being softer than load line A

Using the analogy between uniaxial rock testing and loading of an isolated pillar, Salamon[2] showed that the equilibrium between loading of a pillar and post-peak pillar resistance is 'stable', regardless of the convergence experienced by the pillar, if

$$k + \lambda > 0, \qquad (1)$$

where k is the stiffness of the loading strata and λ is the minimum slope of the post-peak load-deformation relation of the pillar. The strata stiffness is understood to be the force required to cause a unit increment of closure between the hangingwall and the footwall at the position of the pillar. This stiffness depends on the location at which it is computed, the surrounding extraction pattern, and the state of other pillars in the layout.

When pillar layouts are designed to satisfy the above inequality, pillars continue to carry load in their post-peak

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state. Pillars with post-peak load-carrying ability, which are referred to as 'yield pillars' in this paper, can be a practical design tool to permit the mining of tabular deposits. For example, in multireef-mining situations, the occurrence of a tensile zone in the upper reef can be prevented if yield pillars are left in the bottom reef. This is an alternative to the use of conventional 'stiff' support pillars, which may generate excessive stresses during later mining operations at the top reef horizon. In shallow and moderately deep single-reef mines, control of the large tensile zones in the hangingwall can be achieved by the use of yield pillars, which can provide sufficiently stiff support to the immediate hangingwall while permitting acceptable extraction ratios.

The object of this paper is to analyse the stability of yield pillars as local support and to propose a design procedure for such pillars in relatively shallow to moderately deep mines. Assuming inequality (1) to be the basic principle in the designing of yield pillars, the paper is largely concerned with the determination of strata stiffness, and post-peak pillar stiffness, λ. Since inequality (1) is derived by use of the analogy between uniaxial rock testing and the loading of a pillar, it is important to note that the analyses given in this paper assume pillars to be intact when they are first formed at the face.

It should be noted that, because of the inhomogeneous nature of the rock mass, conditions in practice can be much more complicated than are assumed in this paper. It is suggested that the results presented in this paper should be used only as guidelines.

Stiffness of the Strata Surrounding Pillar Layouts

Local Stiffness

The stiffness of the strata at an individual pillar location, i.e. local stiffness, is explained in a simple example given by Starfield and Fairhurst[3]. In their example, one of the pillars in a layout is notionally replaced by a hydraulic jack and, as the hydraulic jack is retracted, the closure at its point of application is plotted against the load loss in the jack. The slope of the resulting load–convergence curve is the stiffness of the strata at that pillar location.

By use of the same concept, the local stiffnesses, k_i, for all the pillars in a layout can be obtained by means of available boundary-element computer programs. The procedure is simple: for each pillar in the layout, the elements occupying the pillar area are excited at least twice by uniformly distributed loads, and the resultant displacements are recorded against the applied loads. The slope of the load–displacement curve is then the stiffness of the strata at that pillar location.

Thus, with regard to the stability of a pillar layout, the stability condition given by inequality (1) can be generalized for all the pillars in the layout as[4]

$$k_i + \lambda_i > 0, \quad \text{(2)}$$

where k_i is the local stiffness, λ_i is the pillar post-peak stiffness, $i = 1, 2, \ldots N$, and N is the number of pillars in the layout. It is important to note that, in inequality (2), the numerical values of k_i will depend on the state of other pillars in the layout, which are assumed to be intact and to have a constant value for linear-elastic strata stiffness. However, the strata stiffness at a certain pillar location can change, for example if one of the neighbouring pillars fails. Inequality (2), therefore, does not necessarily imply absolute stability for a pillar layout.

Mine Structural Stiffness

As a more complete approach than local stiffness, Salamon[2] introduced the concept of mine structural stiffness. In his study, it was shown that a set of incremental loads, ΔP, applied to pillars, are related to associated convergences, ΔS, by

$$\Delta P = (K + \Lambda)\Delta S. \quad \text{(3)}$$

In this equation, K is the stiffness matrix describing the stiffness of the strata at the pillar locations and Λ is a diagonal matrix the elements of which are the minimum stiffnesses of the pillars in the layout, that is λ. In the same study, it was also shown that a stable equilibrium between strata loading and pillar response is achieved when $(K + \Lambda)$ is positive-definite, that is

$$(K + \Lambda) > 0. \quad \text{(4)}$$

A procedure for the testing of $(K + \Lambda)$ for positive definiteness has been suggested by Ryder and Napier[5], using a stress analysis program such as MINSIM-D[6]. By use of the analogy between $(K + \Lambda)$ and the MINSIM-D matrix, and the theory of Gauss–Seidel iteration, it was found that the MINSIM-D solution would converge only if $(K + \Lambda)$ is positive-definite. The implementation of this procedure in stability analysis involves the assignment of pre- and post-peak stiffnesses to the elements occupying the pillar locations. The method is general in that it can be used to assess the stability of a pillar layout but a separate run is required for each λ to be tested.

Critical Stiffness

The most conservative approach to the stability analysis of yield-pillar layouts is based on the concept of critical stiffness. Salamon[2] showed that, regardless of the magnitudes of the convergences experienced by the pillars (i.e. all the pillars in the layout can be in a failed state), the layout is perfectly stable if

$$\lambda > \lambda_c, \quad \text{(5)}$$

where $-\lambda_c$ is the critical stiffness (minimum eigenvalue of K) and λ is the minimum (steepest) slope bounding the load–deformation relations of the pillars. The characteristic of critical stiffness is that it represents the worst case when all the pillars in the system are in their post-peak state.

The practical aspect in the use of critical stiffness for stability analyses is that, once λ_c has been established, it can immediately be contrasted with several values of λ. The determination of λ_c can be simplified by the use of available boundary-element stress-analysis computer programs, as discussed in the next section of this paper.

Determination of Critical Stiffness

As already mentioned, critical stiffness is the minimum eigenvalue of the stiffness matrix, K. It can be obtained directly by taking the reciprocal of the maximum eigenvalue of the influence coefficient matrix, C. The elements c_{ij} of the influence coefficient matrix are defined by the equation

$$s_i = \sum_{j=1}^{N} c_{ij} P_j, \qquad (6)$$

where $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, N$ and, for a given pillar layout, s_i is the convergence at the i^{th} pillar location induced by the pillar forces $P_1, P_2, \ldots, P_N$, and N is the number of pillars. Equation (6) can be written in matrix form:

$$S = CP, \qquad (7)$$

where S and P are $N \times 1$ column matrices of displacements and loads respectively.

The influence matrix, C, can be obtained by numerical techniques using boundary-element computer programs: the elements occupying the j^{th} pillar area are excited by a force field P_j on the assumption that there are no other pillars present in the layout. The convergences induced at all the pillar locations are first divided by P_j and then recorded as the j^{th} column of the influence coefficient matrix. The other elements of the influence coefficient matrix are determined by a repetition of the same procedure at other pillar locations in the layout. The critical stiffness can then be determined from the reciprocal of the maximum eigenvalue of matrix C.

In Table I, the influence coefficient matrix obtained by use of MINAP[7] and the procedure described above is compared with that obtained analytically by Salamon[2] for a layout at 'infinite depth' with the geometry shown in Fig. 2. The differences between the values for the influence coefficients obtained by the two methods are less than 6 per cent; these differences are at least partially due to differences in the assumed loading conditions of the pillars. Salamon's analytic method assumes a constant convergence excitation at the pillar locations, whereas in MINAP the excitation is a uniform force field along the pillar–strata contact.

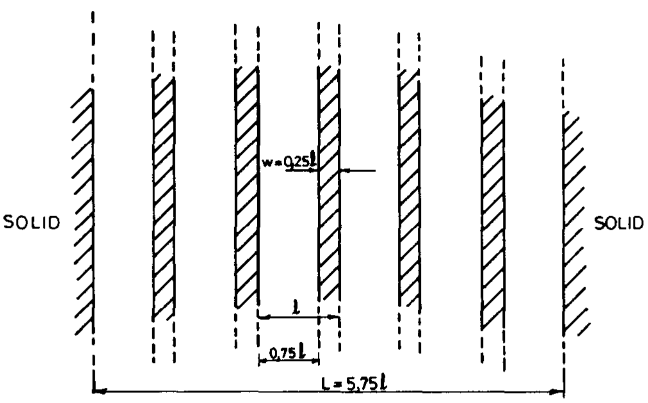

Fig. 2—Pillar layout for the influence coefficient matrices given in Table I (w = pillar width, ℓ = pillar centre-to-centre spacing, L = span)

TABLE I
INFLUENCE COEFFICIENT MATRICES*

Analytic method[2]					MINAP				
1,227	0,450	0,287	0,193	0,115	1,230	0,452	0,291	0,198	0,122
0,452	1,398	0,547	0,336	0,193	0,452	1,402	0,557	0,356	0,198
0,288	0,548	1,439	0,547	0,288	0,291	0,557	1,491	0,552	0,291
0,193	0,336	0,547	1,398	0,452	0,198	0,356	0,552	1,402	0,452
0,115	0,193	0,287	0,450	1,227	0,122	0,198	0,291	0,452	1,230

* Normalized with respect to $\Omega = 4(1 - \nu^2)/E_S$ for the layout given in Fig. 2, obtained by the use of two different methods (E_S = elastic modulus of strata in GPa, ν = Poisson's ratio)

Critical Stiffnesses for Two-dimensional Pillar Layouts

The procedure described above for the determination of critical stiffness can be executed by use of two- or three-dimensional computer programs such as MINAP or MINSIM-D, which can model most practical mining layouts. In Fig. 3, critical stiffnesses normalized with respect to the strata elastic modulus, E_s, and Poisson's ratio, ν, in the form $\Omega = 4(1 - \nu^2)/E_s$ are plotted against various ratios of mining span (L) to mining depth (H) for various two-dimensional pillar layouts. The w/ℓ parameter in these plots represents the ratio of pillar width to pillar centres. It can be shown by means of numerical modelling that, in Fig. 3, the normalization of L with respect to H allows λ_c to be determined for any combination of absolute values of L and H. For example, λ_c values obtained for the cases of L = 200 m and H = 100 m, and of L = 2000 m and H = 1000 m, will be the same since $L/H = 2$ in both cases, as long as the number of pillars is kept constant and the pillar sizes are increased in proportion to the mining span.

To estimate the point of intersection of the curves in Fig. 3 with the vertical axis, i.e. $L/H = 0$ (infinite depth), consider the influence coefficient matrices given in Table I. Note that these matrices are obtained for a pillar layout located at infinite depth, i.e. $L/H = 0$, with the geometry given in Fig. 2, which is the same as the five-pillar case of the diagram labelled $w/\ell = 0,25$ in Fig. 3. Eigenvalue analysis of the matrices given in Table I gives values for $-\lambda_c\Omega$ of 0,38 and 0,36 for the analytic and the numerical cases respectively. When these values of $-\lambda_c\Omega$ are placed on the vertical axis of the graph labelled $w/\ell = 0,25$, it can be seen that they are located close to the point where the curve given for the 5-pillar case would intersect the $L/H = 0$ axis if it were extended towards this axis.

From the argument above, it can be concluded that the curves in Fig. 3 can be extended to intersect the $L/H = 0$ axis to give the values of $-\lambda_c\Omega$ for pillar layouts at infinite depth. To further illustrate the validity of this assumption, the variation of $-\lambda_c\Omega$ with the number of pillars in the layout, as obtained by Salamon[2], who used an infinite depth analytic solution, is given in Fig. 4. The layouts used for the 3- and 5-pillar cases in Fig. 4 are the same as the 3- and 5-pillar cases in the diagram labelled $w/\ell = 0,25$ in Fig. 3. Since 'infinite depth' in Salamon's solution implies $L/H = 0$, the values of $-\lambda_c\Omega$ for the 3- and 5-pillar cases in Fig. 4 should fall on the vertical axis of the diagram labelled $w/\ell = 0,25$ in Fig. 3, at the points marked *. As can be seen, the values of $-\lambda_c\Omega$ are within close proximity of the points if the 3- and 5-pillar curves are extended to intersect the vertical axis.

It appears from the diagrams given in Fig. 3 that, in the region $0 < L/H < ¼$, the strata stiffness will not increase significantly if the mining span is reduced further except, possibly, for the 3-pillar, $w/\ell = 0,50$ case. In the region $¼ < L/H < 5$, however, the strata stiffness

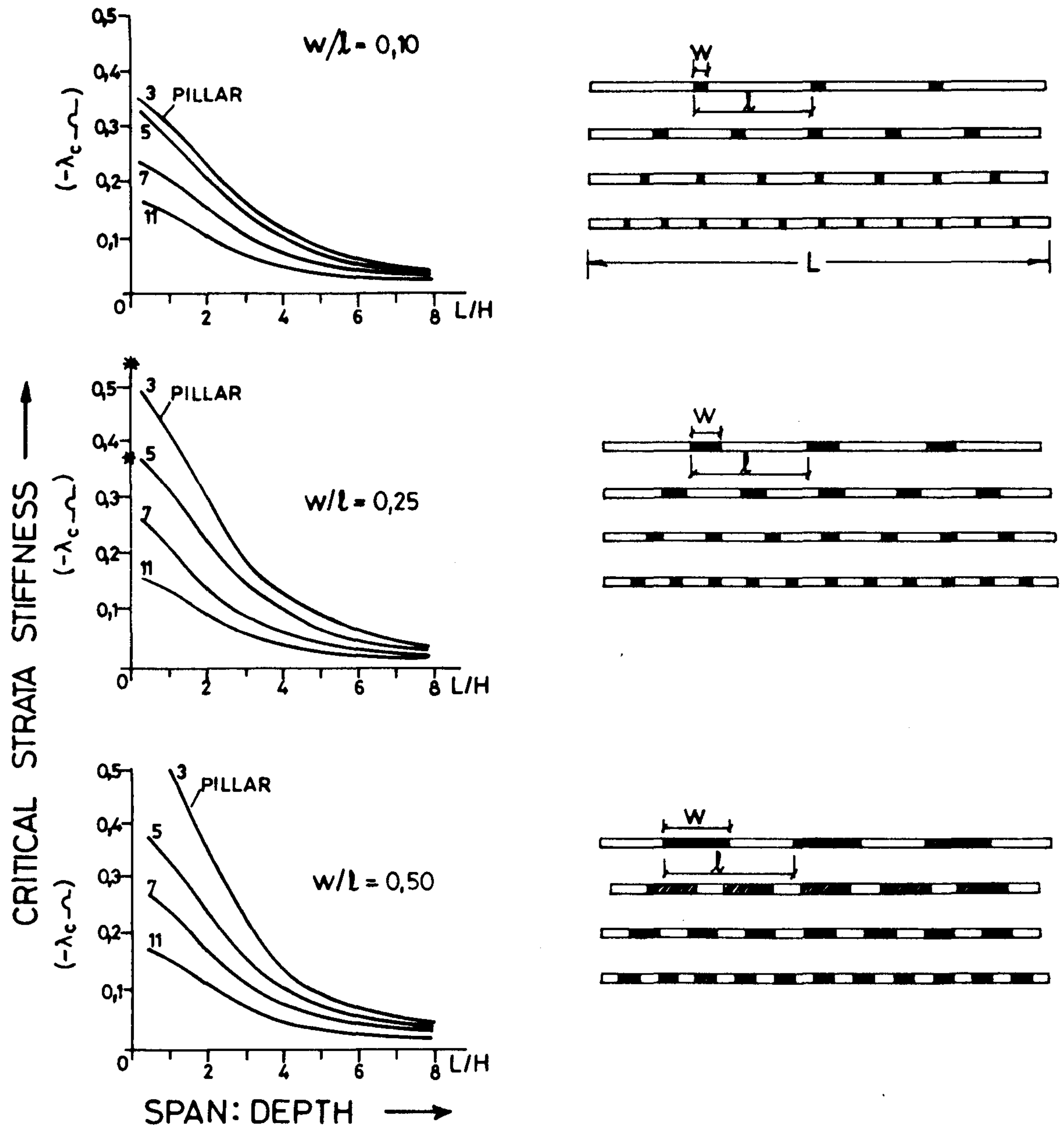

Fig. 3—The variation of critical stiffness with span-to-depth ratio, determined by use of the MINAP computer program

decreases rapidly with increasing L/H ratio. When $L/H > \approx 5$, the stiffness of the strata approaches the value of zero asymptotically with increasing mining span. According to inequality (1), when the stiffness of the strata is zero, a stable equilibrium between strata loading and pillar resistance is not possible if the load–deformation curve of the pillars has a negative slope of any magnitude, i.e. if the pillars 'fail'. Therefore, in the region $L/H > 5$, pillars have to be designed not to fail under the tributary-area load.

The graphs in Fig. 3 also show that, if 1-(w/ℓ) represents the percentage extraction, the extraction ratio does not have a significant effect on the stiffness of the strata when the number of pillars in a layout is greater than 5. For the case of 3 pillars, however, the strata stiffness increases noticeably with decreased extraction ratio.

Finally, the curves in Fig. 3 imply, contrary to intuition, that the strata stiffness *decreases* with an increasing number of pillars in the layout.

Post-Peak Stiffness of Pillars

It appears from the literature that the post-peak stiffness of pillars has been investigated mainly by means of pillar-model tests carried out in the laboratory[8–11] and by *in situ* tests conducted on coal pillars[12,13]. These investigations indicate that the slope of the post-peak load-deformation relationship of pillars becomes flatter with increasing width-to-height ratio, w/h. It has been also shown that samples consisting of jointed or fractured rock-like material have flatter post-peak moduli than samples made up of initially competent specimens[11].

For a quantitative assessment of the post-peak stiffness of pillars, the available data on post-peak stiffness $(-\lambda)$ normalized with respect to their initial elastic

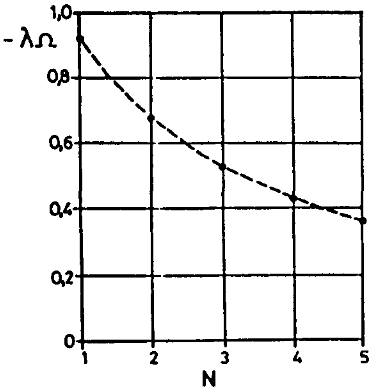

Fig. 4—Variation, after Salamon[2], of the critical stiffness parameter, $-\lambda_c\Omega$, with the number of pillars, N, in a panel ($w/\ell = 0{,}25$ at infinite depth)

modulus (E) are plotted against w/h (Fig. 5). The data consist of the results obtained from two *in situ* tests on coal pillars, as well as laboratory tests on model pillars. In his *in situ* tests, Wagner[12] reproduced the compression of a pillar by forcing its upper and lower halves apart by means of a series of hydraulic jacks located at the horizontal plane of symmetry of the pillars. In Van Heerden's *in situ* tests[13], the simulation of the end-constraint between the pillar and the country rock was achieved by the placing of a concrete block at the top of the pillar. The rest of the data presented in Fig. 5 were obtained from laboratory tests. The data reported for marble[8], sandstone[9], and norite[9] were obtained from tests on model pillars, which were prepared by the cutting of grooves at the centre of cylindrical specimens. The data reported by Wagner and Madden[10] and Hudson *et al.*[14] were obtained from tests on cylindrical sandstone samples with a fixed diameter (24 mm and 50,8 mm) and variable height in direct contact with the machine platens.

The large scatter of the data given in Fig. 5 can be attributed, at least in part, to the differences in the experimental techniques and the rock types used by the researchers. When regression lines are plotted for each set of data, however, it can be seen that the value of $-\lambda/E$ decreases with increasing w/h. The rate of decrease is similar in most of the tests, with the possible exception of the results given by Wagner[12].

Although Fig. 5 does not include data in the range $0 > w/h > 0{,}67$, the value of $-\lambda/E$ is expected to increase rapidly, becoming asymptotic to the vertical axis as w/h approaches zero. Moreover, according to the trends of the regression lines, the transition from negative λ/E to positive λ/E appears to take place in the range $4 < w/h < 7$. For example, according to the data of Wagner and Madden, the post-peak slope of pillars becomes zero when $w/h = 7$.

Considering the group of four regression lines that intersect the $-\lambda/E$ axis around 0,8, and assuming λ/E becomes zero at approximately $w/h = 5$, one can assume the governing relationship in the sequel to be

$$-\lambda/E = (0{,}8 - 0{,}16\,w/h), \qquad (8)$$

within the domain of $0{,}7 < w/h < 5$, which defines the dashed line given in Fig. 5. (It should be noted that λ is given in terms of force per unit pillar length per displacement.)

Design Procedure for Yield-pillar Layouts

The basic requirement in the designing of stable yield pillars is that inequality (1) should be satisfied for all the pillars in a layout at any stage of mining. One method of satisfying inequality (1) is to ensure that the pillars do not experience post-peak strain-softening behaviour in compression. According to Fig. 5, the post-peak stiffness of pillars approaches zero when $w/h > 5$. An unstable equilibrium would therefore not be expected in a layout where the width-to-height ratio of the pillars is greater than 5.

The optimum design of yield pillars, in terms of support and extraction ratio, is achieved when the pillars are designed to reach their strength at the completion of the mining-out of the panels. Such a limiting design may require pillars with width-to-height ratios of less than 5, in which case the pillars would probably have a negative post-peak load–deformation relationship. The stability of the layout then has to be achieved by the selection of an appropriate layout in which the parameters λ and k satisfy the stable equilibrium condition given by inequality (1).

With regard to parameter λ, the empirical equation (8) can be used initially in the selection of approximate pillar sizes. Care should be taken in the use of this equation since the data from which it is derived are highly scattered. The stiffness of the strata surrounding pillar layouts can be estimated by one of the three methods described earlier. Local stiffness is simple to determine and can be used initially in the testing of various layouts, i.e. span, pillar sizes, and spacing, or in an evaluation of the stability of some individual pillars. Analysis based on the concept of critical stiffness aims at the 'perfect stability' condition described by inequality (5).

The implications of the concepts discussed are further illustrated by the example given below.

Conditions:
Mining depth, $H = 400\,\text{m}$
Stoping height (pillar height), $h = 1\,\text{m}$
Elastic modulus of the surrounding strata, $E_s = 70\,\text{GPa}$
Poisson's ratio of the surrounding strata, $\nu = 0{,}2$
Elastic modulus of the reef, $E = 25\,\text{GPa}$
Percentage extraction $= 0{,}75$.

Based on the earlier discussion, the L/H ratio is selected to be around 0,25. If, for example, 7 pillars are to be employed in a panel to provide 75 per cent extraction, the required pillar width, w, and the distance between the centres of the pillars, ℓ, are $w = 3{,}5\,\text{m}$ and $\ell = 14\,\text{m}$, which will result in $L = 108{,}5\,\text{m}$ and $L/H = 0{,}27$.

Local Stiffness: The local stiffness, k_i, at the centre pillar position ($i = 4$) is determined as $k_i = 11{,}00$ GN/m.

Critical Stiffness: From the 7-pillar curve in the diagram for $w/\ell = 0{,}25$ in Fig. 3, the value of $-\lambda_c\Omega$ when $L/H = 0{,}27$ can be found as 0,26. Since $\Omega = 4(1-\nu^2)/E_s$, $\lambda_c = -4{,}74$ GN/m.

Post-peak Pillar Stiffness: If $-\lambda = E(0{,}8 - 0{,}16\,w/h)$, $w/h = 3{,}5$, and $E = 25$, it is found that $\lambda = -6{,}0$

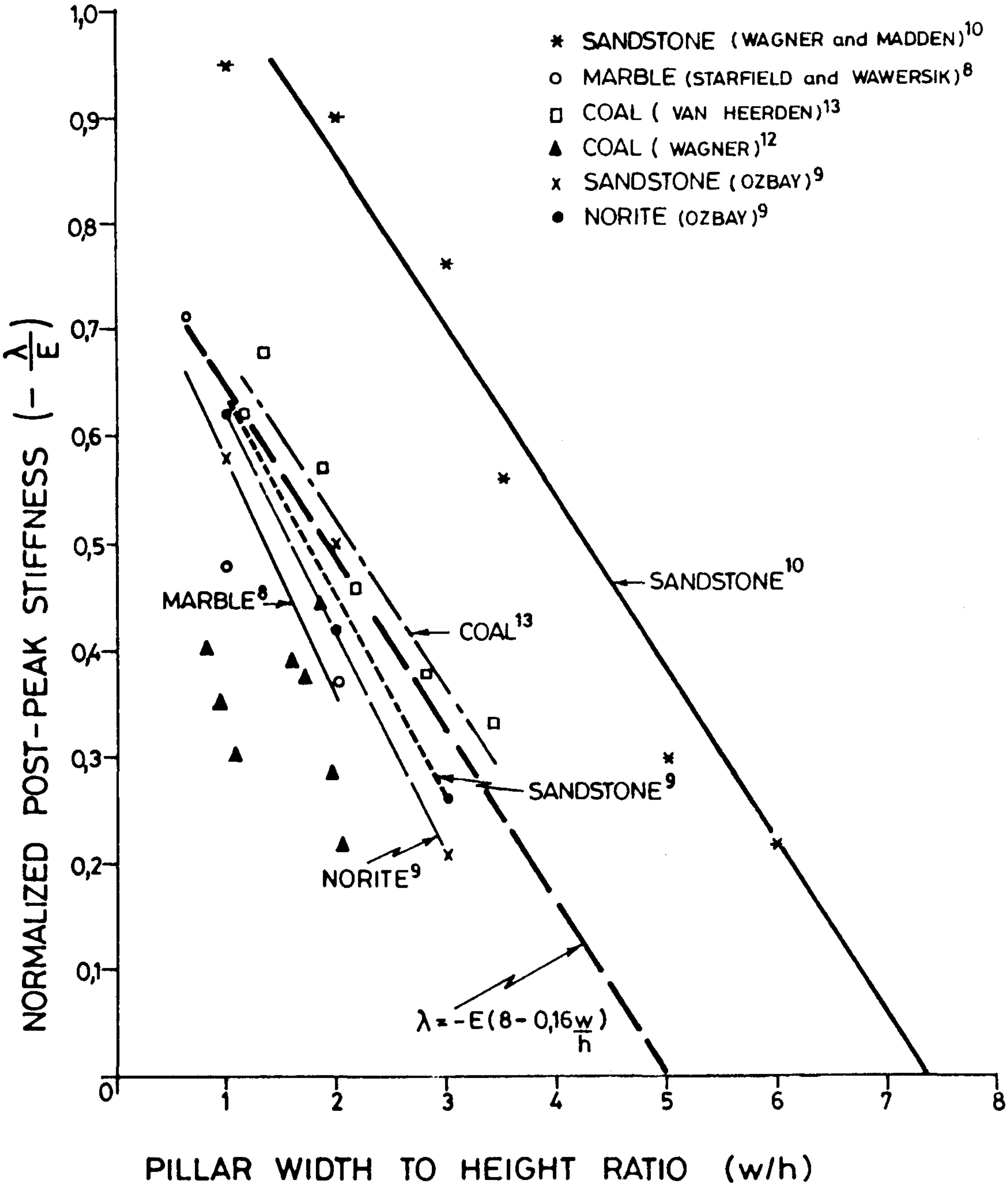

Fig. 5—Post-peak stiffness, $-\lambda$, normalized with respect to the elastic modulus, E, as a function of width-to-height ratio

TABLE II
STABILITY ANALYSIS OF A YIELD-PILLAR PANEL*

w/ℓ	No. of pillars	w m	L m	L/H	w/h	k_i GN/m	$-\lambda_c$ GN/m	$-\lambda$ GN/m	$k_i+\lambda>0$	$\lambda>\lambda_c$
	3	3,0	117,0	0,29	3,0	11,11	6,38	8,00	Yes	No
0,10	5	2,0	110,0	0,28	2,0	11,08	5,83	12,00	No	No
	7	1,4	111,0	0,28	1,4	11,00	4,19	15,00	No	No
	11	0,9	107,0	0,27	0,9	10,99	2,73	16,50	No	No
	3	7,5	112,5	0,27	7,5	14,03	6,93	−10,00	Yes	Yes
0,25	5	5,0	115,0	0,28	5,0	13,94	6,56	0,00	Yes	Yes
	7	3,5	108,5	0,27	3,5	13,86	4,74	6,00	Yes	No
	11	2,3	105,5	0,26	2,3	13,84	2,92	10,75	Yes	No

* Located at a depth of 400 m and with a span of about 110 m

GN/m.

Test for Stability: By use of the criterion of local stiffness stability, that is $k + \lambda > 0$, it can be seen that the values $k_i = 11{,}00$ and $\lambda = -6{,}00$ satisfy the condition for stability. However, the values $\lambda_c = -4{,}74$ and $\lambda = -6{,}0$ fail to satisfy the condition for 'perfect stability'.

Table II gives the values of k_i, λ_c, and λ for various layouts and 75 and 90 per cent extraction, as obtained by the procedure given in the example.

Table II shows that, when local stiffness, k_i, is used to represent the strata stiffness, five of the eight cases considered appear to be stable (see $k_i + \lambda$). However, when the criterion of critical stiffness ($\lambda > \lambda_c$) is used, all but two of the layouts (those with $\lambda > 0$) are unstable.

It is worth noting that it becomes extremely difficult to increase the critical stiffness $-\lambda_c$ beyond 7,0 GN/m, on the assumption of $E_s = 70$ GPa, if $w/\ell < 0{,}25$ is to be maintained. Also, in Table II, the elastic modulus of the pillar material is assumed to be approximately three times smaller than the elastic modulus of the surrounding strata. If the elastic modulus of the reef is taken to be equal to that of the surrounding strata, for example $E = E_s = 70$ GPa, the λ values in Table II would increase by a factor of 2,8. If the $-\lambda$ values are multiplied by 2,8, it can be seen from Table II that neither local nor perfect stability would be achieved except when $w/h > 5$ (i.e. λ is positive).

As stated earlier, the analyses given in this paper are based on the assumption that the pillars are in the ascending part of their load–deformation curve when they are first formed. In moderately deep mines, however, pillars can be designed in such a way that they are fractured, prior to cutting, by high face-abutment stresses and thus, when formed, will already have yielded and reached their 'residual' strength. Further compression of these pillars can be associated with strain hardening, i.e. λ becomes positive. This aspect of yield-pillar design will be discussed in another paper, which is being prepared.

Conclusions

It was shown that the use of the available data on the load–deformation behaviour of pillars and of boundary-element computer programs can give rise to a first approximation towards the design of yield pillars at moderate to shallow depths. The stability of yield-pillar layouts was found to increase with increasing strata stiffness. Based on the concept of critical stiffness, calculations showed that, for extraction ratios of more than 75, strata stiffness comes close to its maximum value when the depth is four times the span, and to its minimum value when the depth is five times smaller then the span.

It was found that pillar layouts tend to become stable as the width-to-height ratio of the pillars increases beyond about 5:1. When the width-to-height ratio is less than 5, the analyses indicate that the stability of yield-pillar layouts becomes critical.

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GUIDE TO THE PREPARATION OF PAPERS FOR PUBLICATION IN THE JOURNAL OF THE SOUTH AFRICAN INSTITUTE OF MINING AND METALLURGY

The following notes have been compiled to assist authors in the preparation of papers for presentation to the Institute and for publication in the *Journal*. All papers must meet the standards set by the Council of the Institute, and for this purpose all papers are referred to at least two referees appointed by the Council.

Although the worldwide readership of the *Journal* results in a preference for papers in English, the Council treats papers in Afrikaans on an equal basis, but, to meet the needs of the majority of readers, an English summary of some 500 to 750 words should be provided.

STANDARDS FOR ACCEPTANCE

To merit consideration, papers should conform to the high standards that have been established for publication over many years. Papers on research should contain matter that is new, interpretations that are novel or of new significance, and conclusions that cast a fresh light on old ideas. Descriptive papers should not be a repetition of well-known practices or ideas, but should incorporate developments that would be of real interest to technical men and of benefit to the mining and metallurgical industry.

In some cases, a well-prepared review paper can be of value and will be considered for publication. All papers, particularly research papers, no matter how technical the subject, should be written with the average reader of the *Journal* in mind, to ensure wide interest.

The amount of textbook material included in a contribution should be the minimum essential to the argument. The length of a paper is not the criterion of its worth, and it should be as brief and concise as possible consistent with the lucid presentation of the subject. Only in very exceptional circumstances should a paper exceed 15 pages of the *Journal* (15 000 words if there are no tables or diagrams). Six to ten pages is more normal.

NOTE: Papers in the *Journal* are printed in 10 point type, which is larger than the 8 point type used on this page. For special publications, Council may decide on page sizes smaller than A4 used for this *Journal*.

The text should be typewritten, double-spaced, on one side only on A4 size paper, leaving a left-hand margin of 4 cm, and should be submitted in triplicate to facilitate the work of the referees and editors.

LAYOUT AND STYLE

Orthodox sequence

Title and author's name, with author's degrees, titles, position.

Synopsis, including a brief statement of conclusions.

An Afrikaans translation of the synopsis.

Introduction.

Development of the main substance.

Conclusions, in more detail.

Acknowledgements.

References.

Title: This should be as brief as possible, yet give a good idea of the subject and character of the paper.

Style: Writing should conform to certain prescribed standards. The Institute is guided in its requirements by:

Collins, F. H. *Authors & Printers' Dictionary*—Oxford University Press.

Hart, H. *Rules for Compositors and Readers*—Humphrey Milford (familiarly known as the *Oxford Rules*).

Fowler, H. W. & F. G. *The King's English*—Oxford University Press.

General: A few well-selected diagrams and illustrations are often more pertinent than an amorphous mass of text. Overstatement and dogmatism are jarring and have no place in technical writing. Be objective, and do not include irrelevant or extraneous matter. Avoid unnecessary use of capitals and hyphens; punctuation should be used sparingly and be governed by the needs of sense and diction. Sentences should be short, uninvolved, and unambiguous. Paragraphs should also be short and serve to separate basic ideas into compact groups. Quotation marks should be of the 'single' type for quotations and "double" for quoted matter within quotations.

Interpretations in the text should be marked off by parentheses ( ), whereas brackets [ ] are employed to enclose explanatory matter in the text.

Words to be printed in italics should be underlined *singly*. For small capitals they are to be underlined DOUBLY and for large capitals TREBLY.

If there is any problem in producing formulae accurately by typewriter, they should be handwritten in ink.

Abbreviations and symbols are laid down in *British Standard* 1991. Abbreviations are the same for the singular and plural, e.g., cm for centimetre and centimetres, kg for kilogram and kilograms. Percentages are written in the text as per cent; the symbol % is restricted to tables. A full stop after an abbreviation is used only if there is likely to be confusion of meaning.

Metric System: The Système International d'Unités (SI) is to be used for expressing quantities. This is a coherent system of metric units derived from six basic units (metre, kilogram, second, ampere, kelvin, and candela), from which are derived all other units, e.g., the unit of force is the newton (N) for kilogram metre per square second (kg m/s^2). Always use the standard metric abbreviations.

The comma must be used as a decimal indicator and must not be used for separating groups of digits. For ease of reading, digits should be grouped in threes counting from the decimal indicator towards the left and right. However, where there are only four digits to the left or right of the decimal indicator, there should be no grouping.

Illustrations: Drawings and diagrams are to be in black India ink and should be about 18 cm wide. When submitting graphical representations, avoid a fine grid if possible. Curves should be in heavy line to stand out. Lettering too should be bold, as a reduction in size is often involved in the printing process.

Numbering of tables should be in Roman numerals: I, II, etc., and figures in Arabic numerals: Fig. 1, Fig. 2, etc. (Always use the abbreviation for figure.) Photographs should be black and white glossy prints.

As a guide to the printer, the author should indicate by means of notes in the typescript where tables and figures, etc. are to appear in the text.

Paragraphs: A decimal system of numbering paragraphs may be used when the paper is long and complicated and there is a need for frequent reference to other parts of the paper.

Proof correction: Galley proofs are sent to authors for the correction of printers' errors and not for the purpose of making alterations and additions, which may be expensive. Should an author make alterations that are considered excessive, he may be required to pay for them. Standard symbols as laid down in *British Standard* 1219C should be used.

SYNOPSIS

It is most important that the synopsis should provide a clear outline of the contents of the paper, the results obtained, and the author's conclusions. It should be written concisely and in normal, rather than abbreviated, English, and should not exceed 250 words, except when an English summary of an Afrikaans paper is involved. While the emphasis is on brevity, this should not be laboured to the extent of leaving out important matter or impairing intelligibility. Summaries simplify the task of abstractors and therefore should present a balanced and complete picture. It is preferable to use standard rather than proprietary terms.

FOOTNOTES AND REFERENCES

Footnotes should be used only when they are indispensable. In the typescript they should appear immediately below the line to which they refer and not at the foot of the page.

References should be indicated by superscripts, thus . . .[1] . . .[2]. Do not use the word *Bibliography*. When authors cite publications of other societies or technical and trade journals, titles should be abbreviated in accordance with the standards adopted in this *Journal*.

GENERAL

The Council will consider the publication of technical notes taking up to three pages (maximum 3000 words).

Written contributions are invited to the discussion of all papers published in the *Journal*. The editor, however, is empowered by the Council to edit all contributions. Once a paper or a note has been submitted to the Institute, that document becomes the property of the Institute, which then holds the copyright when it is published. The Institute as a body is, however, not responsible for the statements made or opinions expressed in any of its publications. Reproduction from the *Journal* is permitted provided there is full acknowledgement of the source. These points should be borne in mind by authors who submit their work to other organizations as well as to the Institute.