

# Towards more effective simulation of CIP and CIL processes. 2. A population-balance-based simulation approach

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## SYNOPSIS

Although several models for the carbon-in-pulp (CIP) or carbon-in-leach (CIL) processes have been published, most of these models do not consider in detail the way in which the carbon phase flows on large-scale plants. This paper illustrates the importance of these details if the CIP or CIL process is to be modelled correctly. An appropriate model, based on population-balance techniques, is then derived. Suitable mathematical techniques for the solution of the model equations are also described. Finally, a method for estimating the model parameters for a full-scale plant is described.

## SAMEVATTING

Hoewel daar verskeie modelle vir die koolstof-in-pulp (KIP)- of koolstof-in-loogproses (KIL) gepubliseer is, skenk die meeste van hierdie modelle nie in besonderhede aandag aan die wyse waarop die koolstoffase in grootskaalse aanlegte vloei nie. Hierdie referaat illustreer die belangrikheid van hierdie details vir die korrekte modellering van die KIP- of KIL- proses. Daar word 'n geskikte model wat op populasie-balanstegnieke gebaseer is, afgelei. Verder word geskikte wiskundige tegnieke vir die oplossing van die modelvergelykings ook beskryf en ten slotte word daar 'n metode vir die raming van die modelparameters vir 'n volskaalse aanleg beskryf.

## Introduction

This series of papers introduces a new simulator for carbon-in-pulp (CIP) and carbon-in-leach (CIL) operations. The first in the series<sup>1</sup> considered adsorption, and an evaluation was made of the available quantitative models that predict the rate of adsorption of gold onto activated carbon and the rate of leaching that can occur concurrent with the adsorption. It was shown that the use of the simple film-diffusion adsorption model in its various forms is justified. With regard to leaching, four models were described and two were found to adequately fit the leaching-rate data.

In this paper, attention is focused on detailed modelling of the process configuration, that is the way in which carbon is contacted with the slurry. The nature of the CIP and CIL processes makes this aspect difficult to model satisfactorily. With few exceptions, published work largely ignores the complex way in which the carbon phase flows in an industrial-size plant. The complexity of the problem is due to the manner in which carbon flows within the adsorption circuit. Two major effects can be identified.

### Carbon Transfer

Carbon is transferred in many ways, and operating practices vary widely. The countercurrent transfer of carbon can be carried out continuously or intermittently. If carbon is transferred intermittently, the adsorption process does not operate at steady-state but is fully dynamic, the tenor of the solutions and the carbon loadings in the stages varying in response to the carbon flows.

Such cyclic transfer of carbon is common industrial practice. The length of time taken to complete transfer depends largely on the capacity of the pumps used to make the transfer, the concentration of carbon in the adsorption tanks, and the manner in which the transfer is performed. On industrial-scale plants, this could take several hours but, on small pilot plants, transfer can be carried out almost instantaneously. Under conditions of intermittent transfer, no steady-state is reached and the process should be modelled as a dynamic system. One of the greatest limitations of most existing models is that they assume that carbon transfer is carried out instantaneously.

### Carbon Leakage

Owing to the imperfect nature of the screening units used to retain carbon in each adsorption contactor, leakage of carbon through these screens results in a co-current movement of some of the carbon. Improvements in screening technology mean that this is perhaps not as serious a problem as it was initially. However, there will always be some carbon that flows co-currently—because of breakage, screen overflows, or the passage of platelet-shaped carbon particles through the slots in wedgewire screens. Relatively little countercurrent flow of carbon is required for the efficiency of adsorption to be affected adversely. Whyte *et al.*<sup>2</sup> report the results of a simulation exercise (using the simulation techniques reported here) in which it was found that a concentration of only 0,06 g of carbon per litre in the interstage slurry led to an increase of 21 per cent in the average barren-solution value and a 24 per cent increase in the gold lock-up. A model that accounts for this phenomenon may be valuable in the optimization of plant operation or design. No existing models account for this effect.

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### Modelling of Complex Carbon Flows

The result of the complex movement of carbon is that carbon from different adsorption stages becomes mixed. Owing to the staged nature of the process, carbon from different contactors has different loadings, carbon from the first stage (that into which the pregnant pulp is fed) having the highest loading of gold. This mixing of carbon of different gold loadings causes a distribution of gold loadings on the carbon in each stage. The manner in which the carbon is moved obviously affects the loading distributions (because of carbon transfer and leakage).

The first paper in this series showed that all rate models for adsorption, except those so simple that they are of little value, contain a loading term. Thus, the rate at which gold is loaded onto a carbon particle depends on the amount of gold already on that particle. If two carbon particles that are otherwise identical in identical solution environments have different loadings, they will adsorb gold at different rates. If there is a distribution of loadings in an adsorption vessel, which will nearly always be the case, there will also be a distribution of adsorption rates. The existing simulation models for CIP and CIL are therefore inadequate in two important respects.

- (1) They do not account for the movement of carbon due to leakage and transfer in detail, and cannot therefore describe the loading distributions that develop. These loading distributions lead to a distribution of adsorption rates in each adsorption vessel.
- (2) Many of the models assume that these distributions of loadings can be ignored. Instead of calculating the weighted sum of the distributed rates, they simply calculate the rate based on the average loading of gold on the carbon. It will now be shown that, for certain rate expressions, there is no difference between these methods. However, for other commonly used rate expressions, there can be a significant difference.

#### Errors Associated with Distributed Rates

Methods for the calculation of loading distributions have been described in previous work<sup>3</sup>. Figs. 1 and 2

show the loading distributions calculated for a particular set of conditions at different times. It can be seen that these distributions are complex. To show the possible effect of loading distributions in the modelling of adsorption, let us define the loading distribution function for any tank at any time as

$$\Psi(y)dy,$$

the mass fraction of carbon in the tank having a loading between  $y$  and  $y + dy$ , where the loading  $y$  is the mass of gold per mass of carbon. It should be noted that

$$\int_0^{\infty} \Psi(y)dy = 1.$$

$R(y, C)$  is the rate of adsorption expressed as the mass of gold adsorbed per mass of carbon per unit time for a particle of carbon having a gold loading  $y$  in a solution of bulk tenor  $C$  (mass of gold per mass of solution).

The weighted rate of adsorption,  $R_w$ , is calculated as the sum of all the rates at each possible loading multiplied by the fraction of carbon having that particular loading. This is the overall rate at which gold is adsorbed from the solution phase. That is,

$$R_w = \int_0^{\infty} R(y, C)\Psi(y)dy. \dots\dots\dots (1)$$

The rate evaluated at the average loading  $R_a$  is written as  $R(\bar{y}, C)$ , where the average loading  $\bar{y}$  is defined in the normal way:

$$\bar{y} = \int_0^{\infty} y\Psi(y)dy. \dots\dots\dots (2)$$

If the weighted rate is to be equivalent to the rate at the average loading, then  $R_w = R_a$ . From the definition of the average loading in equation (2), it can be seen that this condition will be true if all the loading terms in the rate expression  $R(y, C)$  are linear. Thus, if all loading terms  $y$  in the rate expression are linear, then the rate at the average loading  $R_a$  is equivalent to the distributed rate  $R_w$ .

If the loading terms in the rate expression are not linear, then  $R_a$  is not equal to  $R_w$ . From the curves

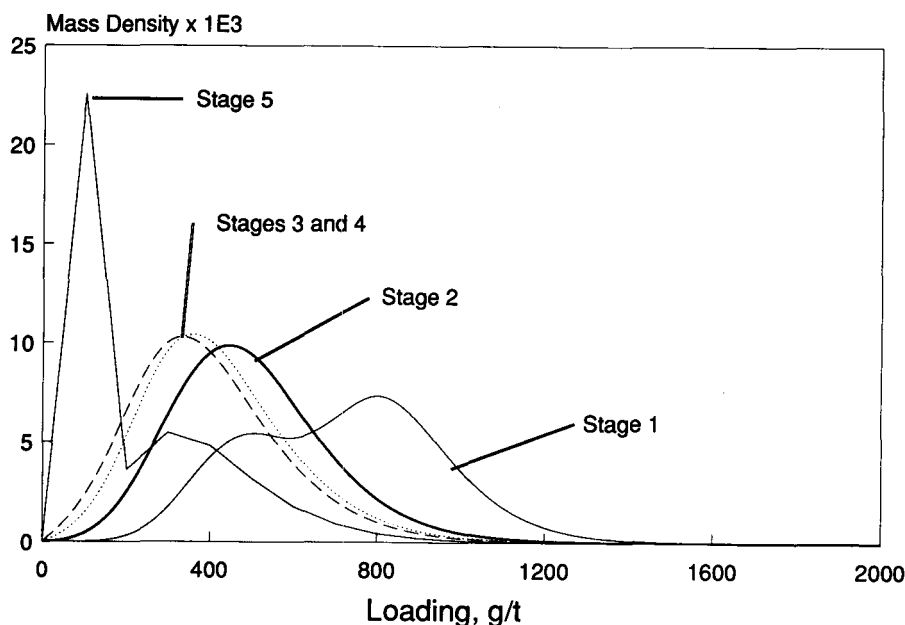


Fig. 1—Loading distributions after one cycle

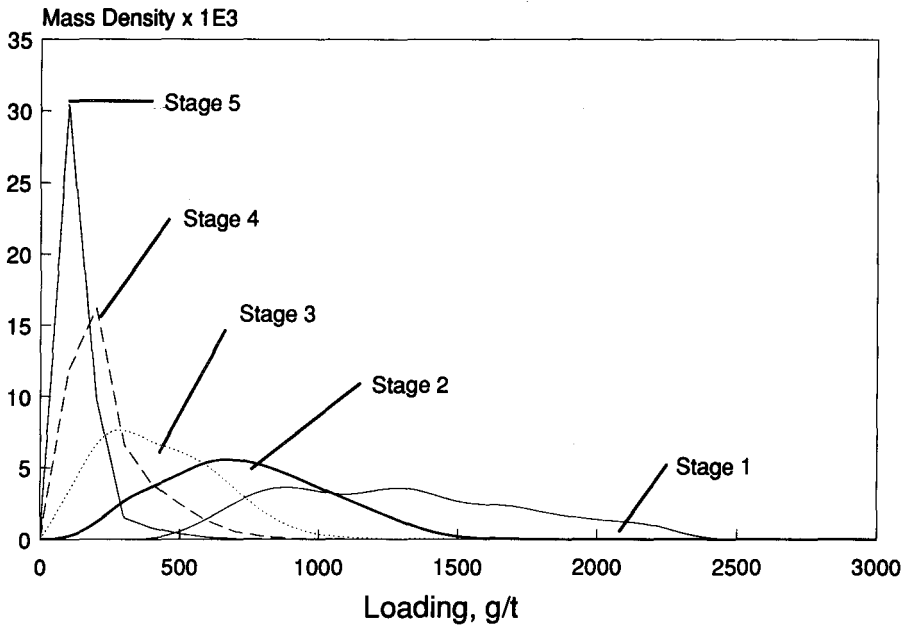


Fig. 2—Loading distributions after five cycles

shown in Figs. 1 and 2, it can be shown that in this case the differences between  $R_a$  and  $R_w$  are significant. The form chosen for a rate expression that is not linear in the loading term  $y$  has been widely used and is as follows:

$$R(y, C) = k \left( C - \left( \frac{y}{A} \right)^{1/b} \right) \dots \dots \dots (3)$$

The parameters  $A$  and  $b$  were taken to be 7500 and 0,76, while  $k$  was assumed<sup>4</sup> to be approximately 150. Values of  $R_a$  and  $R_w$  for the various curves were found to differ by more than 50 per cent in certain cases. The relative errors between  $R_a$  and  $R_w$  for the five distribution curves in Fig. 1 are shown in Table I at different values for the tenor of the bulk solution<sup>5</sup>.

TABLE I  
PERCENTAGE RELATIVE ERROR BETWEEN  $R_a$  AND  $R_w$

Solution tenor, $C$ g/t	Loading distribution curve (see Fig. 1)				
	1 $\bar{y} =$ 3095 g/t	2 $\bar{y} =$ 2156 g/t	3 $\bar{y} =$ 1827 g/t	4 $\bar{y} =$ 1711 g/t	5 $\bar{y} =$ 1056 g/t
5	5,4	3,3	2,7	2,5	1,3
2	13,6	8,4	6,7	6,2	3,2
1	27,9	17,1	13,6	12,5	6,5
0,5	58,8	35,2	27,9	25,5	13,1

It can be seen that the errors are highest at low solution tenors and/or high loadings of gold on the carbon. In general, it can be concluded that serious errors can result if the distribution of loadings on the carbon is ignored during the modelling of adsorption. This implies that, for accurate simulation, it is essential to model the way in which the carbon-loading distributions are affected by the manner in which the carbon is transferred and by the leakage of carbon. Existing simulation models do not account for these effects.

The remainder of this paper deals with the development of a simulation framework that allows for the gold-

loading distribution on the carbon phase, as well as factors such as carbon transfer and leakage that affect these distributions. In addition to the distributions of loading on the carbon, the nature of the carbon is such that a distribution of carbon particle sizes exists. The rate of adsorption is strongly influenced by the particle size of the carbon. The model must also allow for this second distributed property, viz. carbon particle size.

The complexity outlined above is not unusual in processes containing particulate phases. Van Deventer<sup>5</sup> developed a method for simulating the loading distributions that result when carbon transfer takes place instantaneously, as may be the case on a pilot plant. A more general technique, known as the population-balance method, has been developed for the modelling of complex particulate systems.

The application of the population-balance approach to the CIP system for a simplified system (where instantaneous transfer is assumed) is discussed by Stange and King<sup>3</sup>. The application of this technique to a general CIP or CIL system in which realistic carbon transfer and interstage carbon leakage may take place is described in the present paper. The resulting model forms part of a mineral-processing simulation package that runs on IBM personal computers and compatibles<sup>6</sup>. The problem of applying the model to actual operations requires that the appropriate parameters be measured. The estimation of the parameters for full-scale operations will be discussed in some depth. In the third paper in this series, the capabilities of this simulator will be explored in more detail.

#### The Method of Population Balance

The population-balance method is an accounting technique for particulate systems that allows the monitoring, in time (for a dynamic system) and space (if the particle properties are spatially distributed), of the changes in the distributions of various particulate properties of interest.

The technique was first used to model the kinetics of crystallization, and has since been used for other particulate processes such as grinding, flotation, polymeriza-

tion kinetics, gas-solid reactions, and leaching. Herbst<sup>7</sup> provides an introduction to population-balance modelling as applied to various metallurgical systems, while Sepulveda<sup>8</sup> presents an in-depth analysis of the use of the population balance as applied to the modelling of leaching. Interested readers are referred to these publications and that by Stange and King<sup>3</sup> for detailed explanations of this technique.

The population-balance method is based on the concept of a multidimensional particle-phase space. The particles are all contained in this space. The coordinates of the space quantify the properties of the particles. The minimum number of independent properties to allow a complete description of the problem are chosen.

Before a particulate system such as the CIP adsorption process can be described, the following quantities need to be defined:

- (a) a vector  $v$  that quantifies the rate of change of the particulate properties due to continuous changes:

$$v = [v_1, v_2, \dots, v_J],$$

where

$$v_1 = \frac{dI_1(t)}{dt} \quad v_2 = \frac{dI_2(t)}{dt} \quad \dots \quad v_J = \frac{dI_J(t)}{dt};$$

it is assumed there are a total of  $J$  properties of interest, and  $I_n(t)$  represents the value of the  $n$ th property at time  $t$ ;

- (b)  $B dP$  as the number of particles created (birth events) per unit volume per unit time owing to discrete events;  
 (c)  $D dP$  as the number of particles destroyed (death events) per unit volume per unit time owing to discrete events;  $dP$  is the differential phase space defined as  $dP = dI_1 dI_2 \dots dI_J$ ;  
 (d)  $\Psi dP$  as the number of particles per unit volume having property values in the range  $I_1$  to  $I_1 + dI_1$ ,  $I_2$  to  $I_2 + dI_2$ , ...,  $I_J$  to  $I_J + dI_J$ .

### The Number Balance

The equation for the macroscopic population balance for a particulate process in a reactor can be derived from a number balance. The resulting equation can be written as follows<sup>8</sup>:

$$\frac{1}{V} \frac{\partial(V\Psi)}{\partial t} + \nabla \cdot (v\Psi) - (B - D) = \frac{1}{V} \left( \sum_p Q_{in}^p \Psi_{in}^p - \sum_q Q_{out}^q \Psi_{out}^q \right) \dots \dots \dots (4)$$

The volume of the reactor is denoted by  $V$ , while  $Q$  is the volume flowrate of the suspension entering or leaving the reactor (denoted by subscripts 'in' and 'out'). The system may have multiple feed and product streams, and these different input and output streams are denoted by the subscripts and superscripts 'p' and 'q'.

Equation (4) is a number-conservation equation. The first term on the left-hand side arises from the accumulation that results from the dynamic nature of the system. The second term is due to the changes caused by continuous processes, and the third represents the changes due to discontinuous events. The terms on the right-hand side represent the changes in the property distributions

due to particles entering or leaving the reactor in all possible feed and product streams.

### The Population Balance and Adsorption Circuits

Fig. 3 illustrates the flows of pulp and carbon that may take place on a CIP plant. It should be noted that the configuration illustrated in Fig. 3 is a general representation of a number of different types of staged mass-transfer processes such as the following:

- the conventional batch countercurrent CIP, CIL, or resin-in-pulp (RIP) circuit including the carousel\* configuration;
- continuous countercurrent CIP, CIL, or RIP circuits in which the adsorbent is moved continuously;
- circuits in which the particulate and aqueous phases flow co-currently and where there is mass transfer between the phases;
- circuits in which the particulate phase flows in both countercurrent and co-current directions.

In order to apply the population balance to the adsorption circuit shown in Fig. 3, particle properties must be chosen to define the problem completely. For this application, the distributed properties of gold loading on the carbon and the particle size are chosen, and it is assumed that there is no carbon breakage. This implies that no birth or death events take place. Although carbon degradation does occur in practice, no model for carbon abrasion and breakage kinetics has been published, so that this aspect cannot be considered at this point. Once available, the kinetic model for carbon degradation could be used within the population-balance framework to develop a more general adsorption model.

The application of equation (4) to the adsorption situation for the assumptions just stated results in the following equation for contactor  $x$ :

$$\Psi_x \frac{\partial V_x}{\partial t} + V_x \frac{\partial \Psi_x}{\partial t} = Q_x^{in} \Psi_x^{in} - Q_x^{out} \Psi_x^{out} - V_x \left( \Psi_x \frac{\partial R}{\partial y} + R \frac{\partial \Psi_x}{\partial y} \right) \dots \dots \dots (5)$$

As it has been assumed that no degradation of the particles takes place, the particle-size distribution is fixed. This means that, as the size distribution is not changed by equation (5), the particle size is a parameter rather than a distributed variable. Thus, the size distribution is discretized into a number of size classes with equation (5) applying for each size class in each contactor. The distribution function describing the variation of the coordinates for the carbon phase with time can now be defined more appropriately. Thus,  $P_x(y, \delta, t) dy$  is the mass of carbon per unit of active tank volume having a gold loading between  $y$  and  $y + dy$  and representative carbon particle size  $\delta$  at time  $t$ .

$R(y, \delta, C)$  is the rate of adsorption (mass of gold per mass of carbon per unit time) experienced by a carbon particle of representative size  $\delta$  with a gold loading  $y$  in a solution having a bulk tenor of  $C$ .

\* A carousel adsorption circuit is engineered so that the feeding point for pulp can be moved to each contactor in the train. This results in a countercurrent movement of carbon to pulp without movement of the carbon. Each batch of carbon remains in a particular contactor without any carbon mixing taking place.

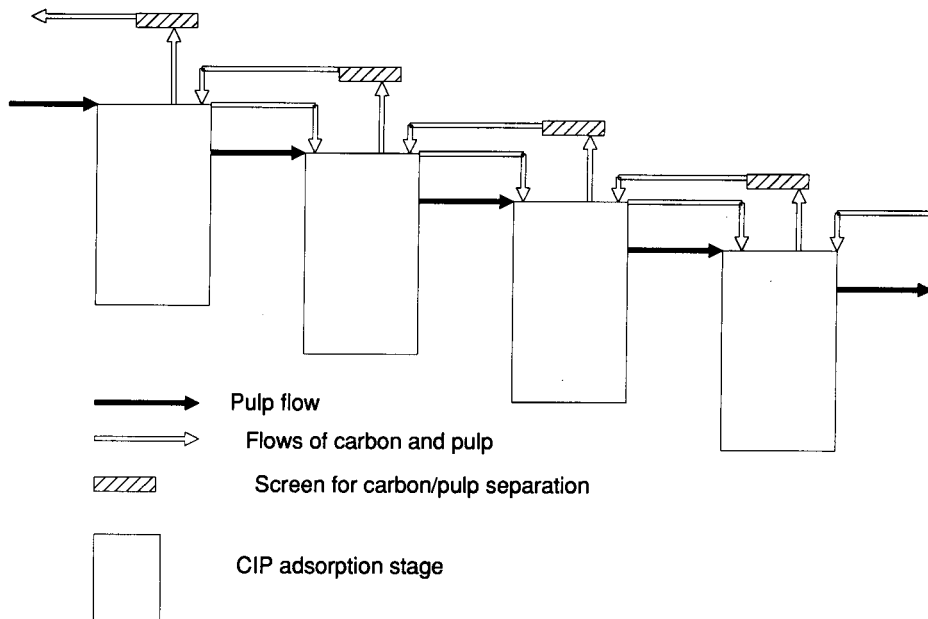


Fig. 3—The CIP adsorption process

For the development of the model, a functional form needs to be chosen for the rate expression  $R$ . Although it is possible to implement the model for an arbitrary rate expression<sup>3</sup>, this leads to a large system of differential equations that must be solved simultaneously, resulting in significant calculation times. The use of a functional form for the rate expression leads to less generality, and also allows considerable simplification of equation (5), resulting in a model that is practical for use on a micro-computer.

Hulbert and Katz<sup>9</sup> describe a method that allows the partial differential equations (PDEs) arising from the application of the population balance to be transformed into a system of ordinary differential equations (ODEs). The system of ODEs describes how the leading moments of the particle distribution change with time, whereas the original PDEs (equation (5)) describe how the entire distribution changes with time. In a practical situation, a knowledge of the first moment of the loading distribution (the average gold loading on the carbon in each contactor) should suffice. Application of the method of moments thus implies that the detailed information regarding the distribution of gold loadings is lost. The consequent reduction in computation effort makes this simplification worth while.

In the application of the method of moments, a rate expression that is linear in gold-loading term(s) must be chosen. Two such expressions have been reported in the literature<sup>1</sup>.

One is due to Dixon, Cho, and Pitt<sup>10</sup> and is written

$$R(y, \delta, C) = k_1(\delta)C(y^* - y) - k_2(\delta)y. \dots\dots\dots (6)$$

The other, due to Nicol, Fleming, and Cromberge<sup>11</sup>, can be written

$$R(y, \delta, C) = k(\delta) \left( C - \frac{y}{A} \right). \dots\dots\dots (7)$$

In general, the terms  $k_1$ ,  $k_2$ , and  $k$  are likely to depend on particle size. The relationship between these terms and the mass-transfer coefficient and equilibrium parameters was shown in the previous paper<sup>1</sup>.

Although the modelling methods that will now be described can be used with any rate expression containing only linear forms of the gold loading on carbon,  $y$ , the development of the method will be illustrated by use of the rate expression written as equation (6).

When rate expression (6) is substituted into (5), the method of moments is applied, and some manipulation is performed<sup>12</sup>, the following ODE is obtained for contactor  $x$ :

$$\frac{d\bar{y}(\delta, t)}{dt} = \frac{Q\epsilon_x^{in}(t, \delta)}{V\epsilon_x(t, \delta)} \{\bar{y}(\delta)^{in} - \bar{y}(\delta)\} + k_1(\delta)C_x(t)\{y^* - \bar{y}(\delta)\} - k_2(\delta)\bar{y}(\delta), \dots\dots\dots (8)$$

where

- $\epsilon_x(t, \delta)$  is defined as the mass of carbon having representative size  $\delta$  per unit volume of pulp in contactor  $x$ ,
- $\epsilon_x^{in}(t, \delta)$  is defined as the mass of carbon having representative size  $\delta$  per unit volume of pulp entering contactor  $x$ ,
- $\bar{y}(\delta, t)$  is the average loading of gold on carbon of size  $\delta$  in contactor  $x$  at time  $t$ ,
- $\bar{y}^{in}(\delta, t)$  is the average loading of gold on carbon of size  $\delta$  in the pulp stream(s) entering contactor  $x$  at time  $t$ ,
- $C_x(t)$  is the tenor of the bulk gold solution in contactor  $x$  at time  $t$ ,
- $Q$  is the volume flowrate of pulp through the contactors, which is assumed to be constant,
- $V_x$  is the total active volume for contactor  $x$ , which is assumed to be constant.

Equation (8) is similar to that obtained by Carrier, Houdouin, and Courchesne<sup>13</sup>. They obtained their result by performing a differential mass balance on the bulk carbon phase, ignoring the loading distributions present on the carbon, and writing the balance in terms of average loadings. However, as they used the Dixon rate expression, which is linear in the loading term, their results are correct since  $R_a = R_w$  for this case. If their approach were used with a rate expression that is not linear in the

loading term, such as equation (3), the resulting equation would not be correct. The errors using this approach may be significant as demonstrated earlier in the paper.

Equation (8) is a system of ODEs describing the average carbon loading in each size class in each contactor as a function of time. This system of equations must be solved simultaneously with any other equations needed for a complete problem definition, such as the solution mass balance and the bulk mass balance of the carbon. In order to complete the mathematical description of the process, equations describing the amount of carbon in each stage and the solution tenor as functions of time need to be derived. The bulk mass balance of carbon in each stage is derived from the mass conservation equation:

$$V_x \frac{d\epsilon_x(t, \delta)}{dt} = Q(\epsilon_x^{\text{in}}(t, \delta) - \epsilon_x^{\text{out}}(t, \delta)) \dots \dots \dots (9)$$

When the carbon is assumed to be discretized into  $N$  size classes, the differential mass balance for the gold in solution can be written as

$$\frac{dC_x(t)}{dt} = \frac{Q_s^{\text{in}} C_s^{\text{in}}}{V_s} - \frac{Q_s^{\text{out}} C_x}{V_s} + \frac{F_s}{V_s} (S_{x-1} - S_x) - \frac{V_x}{V_s} \sum_{i=1}^N \epsilon_x(t, \delta_i) R(\bar{y}(\delta_i), \delta_i, C_x), \dots \dots \dots (10)$$

where

- $Q_s^{\text{in}}$  is the total flowrate of solution into contactor  $x$
- $Q_s^{\text{out}}$  is the total flowrate of solution out of contactor  $x$
- $C_s^{\text{in}}$  is the weighted average gold tenor of the solution in the streams entering contactor  $x$
- $F_s$  is the flowrate of ore through the series of contactors (assumed to be constant)
- $S_x$  is the gold grade in the ore stream in the exit of stream  $x$

$$R(y, \delta, C) = k_1(\delta)C(y^* - y) - k_2(\delta)y \dots \dots \dots (11)$$

The term on the left-hand side of equation (10) represents the accumulation of gold in the solution in contactor  $x$  due to the dynamic nature of the process. The first term on the right-hand side represents the gold flowing in the contactor feed stream(s) in solution, while the second term represents the gold in solution flowing out in the product stream(s). The third term on the right-hand side represents the amount of gold that has been transferred into the solution phase as a result of the leaching of the ore. The last term represents the amount of gold that has been adsorbed onto the carbon in the contactor.

The term in equation (10) that accounts for the gold leached into solution can be evaluated from experimental data if these are available. However, it is useful in many cases to be able to predict leaching behaviour by use of a model.

### Continuous Leaching Models

The first paper in this series<sup>1</sup> described various rate expressions that were fitted to a number of sets of batch gold-leaching data. It was found that the Loveday expression fitted the data best, and the Mintek expression was also found to give reasonable fits. The expressions presented in the previous paper applied only to the batch

configuration; equations based on these expressions that are applicable to the continuous situation must be derived for use in equation (10).

The simple form of the Mintek expression and the fact that time does not appear explicitly in the expressions allow the following model to be derived for continuous leaching by the performance of a simple mass balance for gold across CSTR  $x$ :

$$S_x = \frac{(S_m - \alpha) + \sqrt{\alpha^2 + 4\alpha(S_{x-1} - S_m)}}{2}, \dots \dots \dots (12)$$

in which  $\alpha = \frac{1}{\tau k_p}$ ,

where

- $\tau$  is the nominal residence time of the ore in a single vessel
- $k_p$  is the leaching rate constant
- $S_x$  is the gold grade in the ore leaving contactor  $x$
- $S_m$  is the minimum achievable residue by cyanidation, i.e. the amount of gold left in the residue after infinite leaching.

Because time appears explicitly in the Loveday expression, the calculation of the gold grade of the bulk ore in the exit stream of the last CSTR in the series requires the batch expression to be weighted by the CSTR residence-time distribution and to be integrated over all possible times that a single particle may spend in a series of CSTRs. If this is done, it can be shown<sup>12</sup> that

$$S_x = (S_o - S_m)A^x + S_m, \dots \dots \dots (13)$$

where

$$A = (1 - \beta) - \frac{nk_{\text{max}}}{n + 1} \left( \tau(1 - \beta) - \frac{\beta}{k_{\text{max}}} \right) +$$

$$\frac{1}{n + 1} \left( \frac{1}{k_{\text{max}}} \right)^n \beta I$$

$$\beta = e^{-1/\tau k_{\text{max}}}$$

$$I = \int_0^1 \frac{dz}{(-\tau \log(z) + 1/k_{\text{max}})^n}$$

Thus, the CIP or CIL process can be simulated by the solving of ordinary differential equations (8), (9), and (10). Given the appropriate parameters, equation (12) or (13) can be used in the calculation of the leaching terms required in equation (10). Standard Runge-Kutta methods, preferably implemented with automatic step-length control, can be used in the integration of these equations<sup>12</sup>.

### Use of the Simulator

It is most important that the parameters for models can be measured from full-scale plants. This allows simulation to be used to optimize the operation of existing plants. For the purposes of design, parameters for full-scale plants cannot be measured, but can be established empirically if kinetic parameters for the same ore and carbon have been measured in the laboratory and on a corresponding full-scale plant for a number of plants. Alternatively, appropriate mass-transfer correlations can

be used. Thus, estimates of plant-adsorption parameters are very useful when simulation is used in the design of a new plant and in the improvement of existing plant operation.

One way of estimating the parameters is to isolate a vessel in the leaching or adsorption train, and to use this vessel for batch tests. This is normally easier to do for leaching than for adsorption, and is recommended for the estimation of kinetic leaching parameters. It is more difficult to use this technique for adsorption. Ideally, the batch adsorption test should be carried out for a relatively lengthy period (about 24 hours) at a low carbon concentration (2 to 5 g/l). As the setting up of such conditions may disrupt plant operation to an unacceptable degree, a method that allows simple estimation of kinetic adsorption parameters from plant data without any disturbance of plant operation has been developed.

The manner in which conventional adsorption plants are operated implies that the solution accumulation term  $dC/dt$  in equation (10) is negligible when compared with the other terms in equation (10). When this term is set at zero and equation (10) is manipulated and rewritten,

$$C_x = \frac{Q_s^{\text{in}} C^{\text{in}} + F(S^{\text{in}} - S_x) + V_x \sum_{i=1}^N \epsilon_x(\delta_i) k_2(\delta_i) \bar{y}(\delta_i)}{Q_s^{\text{out}} + V_x \sum_{i=1}^N \epsilon_x(\delta_i) k_1(\delta_i) \{y^* - \bar{y}(\delta_i)\}} \quad (14)$$

It should be noted that, for equation (14), it was assumed that no carbon was being transferred into or out of contactor  $x$ , i.e.  $Q_s^{\text{in}} = Q_s^{\text{out}} = Q_s^{\text{sol}}$ . It is assumed that simultaneous samples are taken from the feed and exit slurry of contactor  $x$  under conditions of known solution flowrate,  $Q_s^{\text{sol}}$ , ore flowrate,  $F$ , and volume,  $V_x$ . It is assumed that no carbon is being transferred into or out of contactor  $x$ , making it possible to measure  $\epsilon_x(\delta_i)$ , the concentration of carbon in the vessel in each size class. If the samples are processed so that values for  $C^{\text{in}}$ ,  $C_x$ ,  $S^{\text{in}}$ ,  $S_x$ , and  $\bar{y}(\delta_i)$  are measured, it will be seen that all the quantities in equation (14) (except the unknown adsorption parameters) are measured. With this in mind, a sampling scheme can be constructed as follows.

- Select a contactor from which and to which no carbon is being transferred, and take the samples and measurements described above.
- If a number of samples are taken from each tank for all vessels in the circuit, a range of measurements for adsorption rates under various solution tenors and carbon loadings will be obtained. In order to ensure that the values are collected under a wide range of rate conditions, the interval between the taking of samples from the same vessel should be lengthy. For example, if parameters are to be estimated for a six-stage circuit, six samples are taken per day, one from each vessel. This is continued for a number of days until the samples are sufficient for the number of parameters to be estimated with some confidence. After six days of such sampling, 36 samples would be available for the estimation. The advantage of a large number of samples is that this results in more confidence in the estimates, as well as making it possible for the adsorption parameters to be estimated for each contactor.

- The parameters can then be estimated from equation (14) and suitable nonlinear regression techniques.
- Not all the adsorption parameters are estimated from kinetic data. Equilibrium data should be used for the estimation of the equilibrium parameters. For the Dixon expression—equation (6)—at equilibrium ( $R=0$ ), the well known Langmuir isotherm is obtained:

$$C_e = \frac{k_2(\delta_i) y_e}{k_1(\delta_i) (y^* - y_e)}, \dots\dots\dots (15)$$

in which the subscript  $e$  denotes equilibrium conditions. This is illustrated in more detail in the next paper in the series.

From equation (15) it can be seen that the parameter  $y^*$  and the ratio  $k_2/k_1$  can be estimated from equilibrium data. These parameters may vary depending on the position of the carbon in the circuit, i.e. older carbon in the stages near the pulp feed would probably be more fouled than carbon that has just been added to the system. This fouling effect would result in a decrease in the capacity constants measured in equilibrium tests. It may be necessary to perform an isotherm for each tank in the plant. Obviously, the conditions under which the isotherm is established should approximate plant conditions as closely as possible. Thus, only one parameter needs to be estimated from the kinetic data sampled on the plant.

Some interesting observations can be made about this sampling scheme and the rate expression to be used in the simulation framework. Firstly, the technique becomes far more complicated when used for the estimation of parameters in rate expressions that are not linear in the loading term. For this case, the rate expression evaluated at the average loading cannot be used, but the weighted rate must be used as described earlier in the paper. This entails integration of the product of the carbon-loading distribution function and the rate expression to account for the effect of this distribution on the overall rate of adsorption. In general, this would require measurement of the loading-distribution functions. Although possible, this is not generally practical.

This convenient method of parameter estimation is difficult to apply to complex models such as that proposed by Van Deventer<sup>5</sup>. In that model, the carbon is divided into macropore and micropore regions. In order to perform parameter estimation as discussed above, it would be necessary to measure the gold loadings in the macropores and the micropores, and no techniques for the taking of these measurements have been published. Even if it were possible to make these measurements, they would have to be made as soon as the sample was taken. This is because the mechanism of surface diffusion would result in diffusion of gold from the macropores to the micropores (or *vice versa*) until an equilibrium situation, in which the loadings in both regions are equal, is reached.

Thus, the use of a rate expression that is linear in the loading terms has several advantages. It allows the method of moments to be used, reducing the complexity of the equations that must be solved in the derivation of a realistic model for the CIP or CIL adsorption process. It also allows a simple scheme to be used for the estimation of parameters from full-scale plants. Such rate expressions provide a good description of adsorption in

many batch systems as described in the previous paper. The next paper will show that such expressions are also capable of accurately predicting full-scale plant operation. In situations where linear rate expressions are not suitable, the complexities associated with the distribution of loadings on the carbon must be addressed as described earlier in this paper. This issue was considered in detail in previous work<sup>3</sup>.

#### Use of the Model

The model presented above can be used in simulation studies during design, as well as in the optimization of existing plants. To facilitate the use of the techniques described here in an industrial situation, the adsorption and leaching models are included in a simulation package for mineral-processing operations. The package is known as MicroSim<sup>6</sup> and runs on MS-DOS microcomputers. MicroSim can simulate most ore-dressing and coal-preparation flowsheets, as well as leaching and the CIP adsorption process. This allows integrated flowsheets, with their complex interactions between unit processes, to be simulated. The capabilities of MicroSim have been described in more detail elsewhere<sup>6,14</sup>.

#### Conclusions

In summary, a flexible model for the simulation of CIP or CIL circuits has been implemented within the framework of a user-friendly mineral-processing simulator. This was achieved by use of the population-balance method to derive the general equations for the carbon mass balance. These equations were then simplified by the method of moments. Although this simplification holds for any rate expression that is linear in the loading terms, it allows the model to be used effectively on a microcomputer. This makes the simulation of the process practical for any engineer with access to an IBM personal computer or compatible microcomputer.

Convenient techniques of parameter estimation, if properly utilized, will facilitate the use of modelling and simulation for both the design and the operation of CIP or CIL plants, leading to more efficient plants.

In the next paper in the series, the validation of the simulator described here will be discussed based on data obtained from full-scale plants. In addition, the versatility of the model in quantifying the effects of various operating phenomena will be illustrated.

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## Materials World '91

MW '91, the Materials World Exhibition and Conference to be held in the Brussels Exhibition Centre, Belgium, from 16th to 18th April, 1991, now has co-sponsorship from the Plastic & Rubber Institute (UK), the Federation of European Materials Suppliers, AMS International (USA), and TMS (USA).

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