J. S. Afr. Inst. Min. Metall., vol. 92, no. 3.
Mar. 1992. pp. 71–77.

# New developments in borehole valuations for new gold mines and undeveloped sections of existing mines

by D.G. Krige* and W. Assibey-Bonsu*

**SYNOPSIS**

Using a very large number of chip-sample values from a mined-out area of the Hartebeestfontein mine and accepting these as equivalent to borehole values (also areas of 1 km by 1 km as equivalent to 'mines'), the authors apply a series of borehole-valuation procedures to these 'mines'. Large undeveloped sections of existing mines are also regarded as the equivalent of such 'mines'.

These procedures start with the orthodox arithmetic mean and then introduce additional information in the form of the lognormal frequency distribution model for Sichel's 't' estimator. Knowledge of the average log-variance within mines leads to the t″ estimator. The mean grade of the 'family' of mines to which a new mine belongs is then introduced via regression or elementary kriging. Next, further information on the macro spatial structure of the grade is used, together with the grades of adjacent existing mines in ordinary and simple macro-kriging procedures; finally, via the distribution model, the grade is seen as a function of the logs of the grades and their variances. These are introduced as spatial variables, and both are estimated by macro-lognormal ordinary and simple kriging procedures with grades and variances from adjacent mines. Comparisons are also made between macro-kriging on lognormal transformed values versus untransformed values.

Conclusions are drawn on the relative importance of the various items of additional information used, on the procedures followed, and on the significance of the advantages gained.

**SAMEVATTING**

'n Groot aantal kapmonsterwaardes van 'n uitgewerkte area in die Hartebeestfonteinmyn is gebruik en aanvaar as die ekwivalent van boorgatwaardes (1 km by 1 km areas as ekwivalent van 'myne') om 'n series van prosedures vir boorgatwaarderings toe te pas op hierdie 'myne'. Groot onontginde seksies van bestaande myne word ook gesien as die ekwivalent van sulke 'myne'.

Die prosedures begin met die ortodokse rekenkundige gemiddelde en bring dan addisionele inligting in berekening in die vorm van lognormale verdelingsmodel vir Sichel se 't' skatter. Kennis van die gemiddelde logvariansie binne die myne lei dan tot die t″ skatter. Die gemiddelde graad van die 'familie' van myne waarvan 'n nuwe myn 'n lid is word vervolgens gebruik via regressie of elementêre kriging. Daarna word verdere inligting oor die makro ruimte-struktuur van die graad gebruik tesame met die waardes van bestaande aangrensende myne in gewone en eenvoudige makro kriging prosedures. Uiteindelik word die ertsgraad via die verdelingsmodel as 'n funksie van die logaritmes van die waardes en hul variansies gesien. Beide word behandel as ruimte-veranderlikes en word afsonderlik geskat deur makro lognormale gewone en eenvoudige kriging prosedures tesame met waardes en variansies van aangrensende myne. Vergelykings word ook gemaak tussen makro kriging van lognormale getransformeerde waardes en ongetransformeerde waardes. Gevolgtrekkings word gemaak oor die relatiewe belangrikheid van die verskillende items van addisionele inligting wat gebruik is, oor die prosedures gevolg en die beduidenheid van die voordele wat behaal is.

## INTRODUCTION

The first significant introduction of mathematical statistics in South African gold-ore valuation was made by Sichel[1] through the introduction of the lognormal frequency-distribution model. This led to the use of this model for improved estimates of the mean grade via Sichel's 't' estimator[2,3] and to regression estimates for ore-reserve blocks[4,5]. These regression procedures provided the first use of an elementary spatial structure via the correlation of peripheral chip-sample values with those subsequently available from inside the ore blocks; regression can thus properly be accepted as the first elementary kriging application. These procedures also demonstrated that an orebody can be regarded as a series of populations each with its own type and size of support for its members, e.g. chip samples, large ore-reserve blocks, so that an individual ore block is effectively seen as a member of a population of such blocks in an orebody.

The frequency-distribution model was later improved to the three-parameter lognormal model[6], and more general models are at present under investigation[7,8]. More general models for the spatial structures of the grade were developed in the 1960's, e.g. the correlogram[9,10] and the now generally used variogram[11]. These, particularly Matheron's work, led to the kriging procedures that are now used for ore-block valuation throughout the world.

In the meantime, the problem of improving grade estimates for new South African mines based on a small number of borehole results did not advance beyond Sichel's 't' estimator until it was suggested that a new mining property should, like an ore block, be regarded as a member of a population of such units. Where this is feasible and suitable information is available on the average variance within such mines, this can be introduced as *a priori* information for the t″ estimator[9]. Where additional information is available on the spatial characteristics of the

---

* University of the Witwatersrand, P.O. Wits, 2050 Transvaal.

SA ISSN 0038-223X/3.00+0.00. Paper received 13th May, 1991; modified paper received September 1991.

population of mines, a 'macro'-kriging estimate can be made for a mine[12], which is similar in principle to a Bayesian approach. For this purpose, the estimates of borehole grade used in macro-kriging can be the arithmetic mean, or 't' or $t''$ estimates.

However, it was not until very recently that the role of variance in borehole valuations was analysed[7], including the concept of a spatial structure for the variance. The present paper endeavours to extend this analysis, and to bring into focus the relative roles of all the items of additional information outlined briefly above.

The introduction of variance as a spatial variable should, of course, logically be extended to routine ore-reserve and face (panel) valuations for mines, and work is proceeding on this problem[13]. The valuation of large undeveloped sections of existing mines for longterm mine-planning purposes is usually based on a few borehole values and the known grade pattern in adjacent developed and/or mined-out mine sections. This also presents a common problem, which is in principle the same as that of the valuation of a new mine. References to 'mines' can therefore also be taken to cover such mine sections.

## DATABASE FOR ANALYSES

The chip-sample data from a large mined-out section of the Hartebeestfontein gold mine in the Klerksdorp goldfield provided gold values (in centimetre·grams per tonne) for a total of 72 767 sampling sections on the Vaal Reef. These were accepted as equivalent to individual borehole-core values, and were divided into areas of 1 km by 1 km of simulated 'mines'. On average, some 3000 values were available in each of 20 'mines', and these values provided the follow-up 'actual' mean grade and the logarithmic variance for each mine. From each of these 'mines', 9 values were drawn on a stratified random basis. These were accepted in each case as the equivalent of 9 single borehole values, i.e. without deflections. The process was executed three times to yield a total of 60 borehole sets.

The presence of spatial structures for grade and variance is demonstrated in Figure 1 by the semivariogram for the 'actual' grades and variances of the population of mines. All the analyses were done on the three-parameter lognormal transformed values by use of ln(cm·g/t + 50). The semivariograms for both grade and variance are therefore, on a logarithmic basis with a third parameter ($\beta$) of 50. The suitability of this model is demonstrated in Figure 2, which shows that the distribution of the 72 767 borehole values, as well as that of the 20 actual mine grades, can be accepted as normal. A test for the suitability of this model for the 20 mines showed

(1) a low correlation between the 'actual' logmeans and the logvariances ($r^2 = 16$ per cent)

(2) a difference between the overall means for the 20 mines, based on the arithmetic means, and the lognormal mean of 1,5 per cent , and a correlation level between the two sets of means of 99,8 per cent.

The sets of simulated borehole values were subjected to a series of estimation procedures covering all the approaches outlined earlier. The estimates were analysed statistically and correlated with the 'actual' mine values in order to provide comparisons of the relative improvements shown by the various procedures. Improvement was measured by the observed error variances of the estimates (inclusive of any small global biases); also by the level of correlation between estimates and actuals and by the extent, if any, of the remaining conditional biases as reflected by the slope of the regression of actuals on the estimates. For conditional unbiasedness, this slope should be unity.

Conditional biases, such as were demonstrated[3,4] for ore blocks in 1951/1952, were also found on a 'mine' scale for

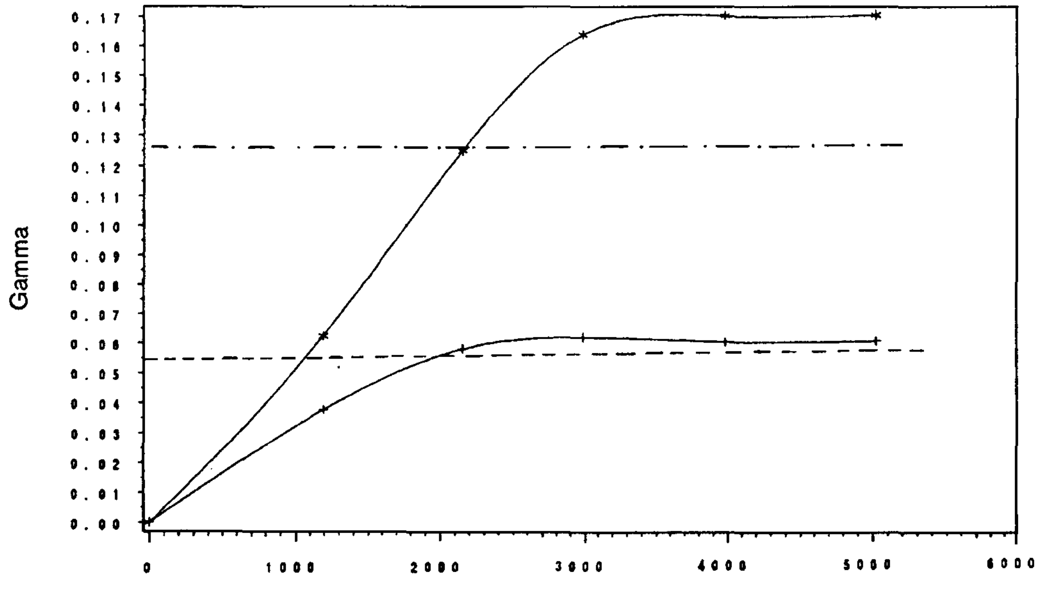

**Figure 1—Semivariograms for the means and variances of the transformed grades within twenty 'mines' of 1 km by 1 km**

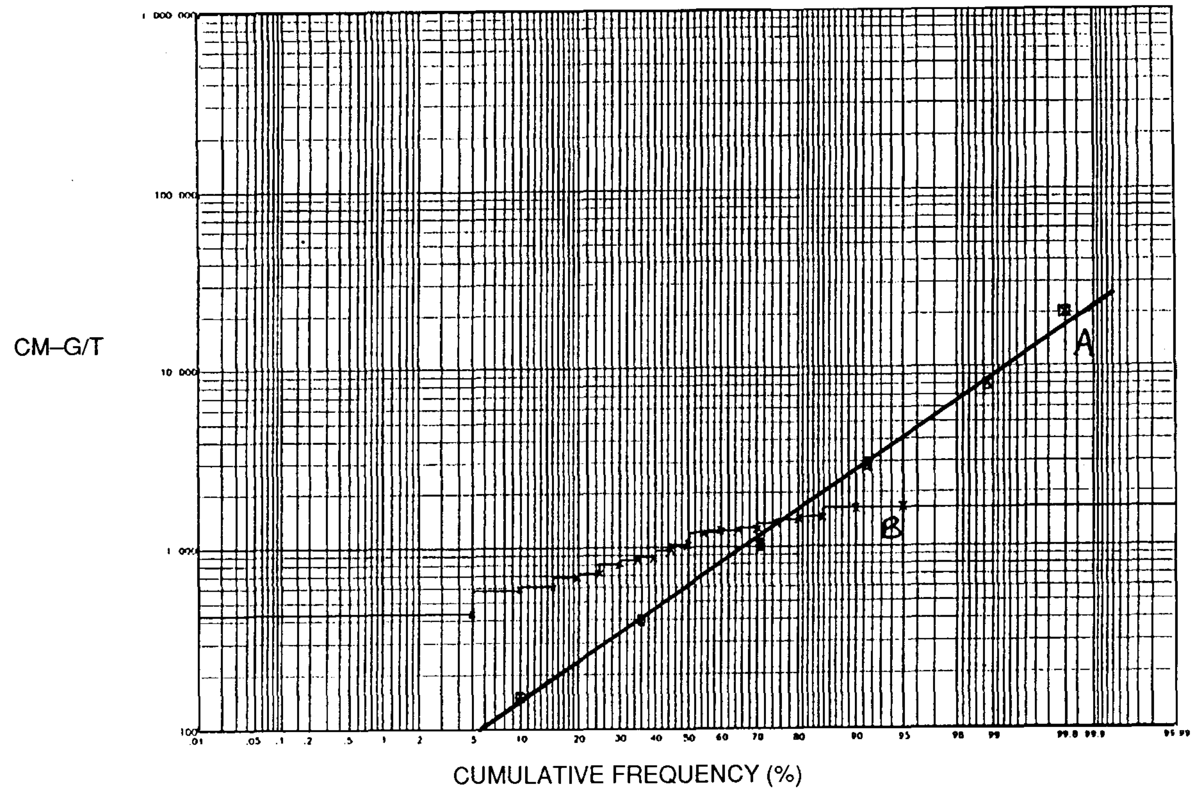

**Figure 2— Cumulative frequencies of the cm.g/t values (+50)**
**A for 72 767 individual sampling sections; B for 20 actual-mine mean grades**

the orthodox arithmetic mean, for Sichel's 't' and t″, and for some of the kriged estimates (particularly for ordinary kriging). A clear motivation for the introduction of 'outside' information on the variance, as well as the grades in any procedure to be used, was provided by the correlation analyses of the errors of the borehole estimates with the corresponding variance estimates. Figure 3 shows one of these analyses, i.e. for the Sichel 't' estimates. Similar patterns were found for the orthodox mean and t″. Figure 3 demonstrates clearly that estimates based on a small set of values drawn from such skew distributions, even the more sophisticated 't', will result in serious conditional biases. Where such a set gives a low-grade and/or variance estimate, the tendency will be to undervalue the mine grade and, in the case of high estimates, overvaluation will tend to occur.

## ESTIMATION PROCEDURES USED

The *first* procedure is that of the orthodox arithmetic mean, i.e. where the 9 borehole grades per mine are used at face value without the introduction of any 'additional' information.

The *second* procedure introduces the knowledge of the three-parameter lognormal distribution pattern through the use of Sichel's 't' estimator. The substitution of the average mine variance for the variances as estimated from the borehole sets yields the t″ estimates as an alternative.

*Thirdly*, the concept of spatial structure is introduced on the elementary basis of regression. The additional information used is that of a parent distribution of mine grades with known mean and variance, plus the knowledge of the error variances of the arithmetic mean, 't' or t″ borehole-grade estimates. With this additional information, the required correlation models can be used[14] to provide regressed estimates for the arithmetic mean, Sichel's 't', and t″.

*Fourthly*, the spatial structures for the population of mine grades (and variances) are introduced using the semivariograms observed. Accepting the lognormal model, the borehole estimates and mine grades can be treated as spatial variables directly, or grade can be seen as a function of the mean of the transformed grades and the corresponding variances.

Kriging, ordinary and simple and with the customary assumptions of stationarity, can therefore be done on the grades directly, or on the transformed grades ($\xi$) and variances ($\sigma^2$) separately, to give the corresponding grade estimates via the appropriate function. For a population, this function is defined as follows[14]:

Mean grade = $\exp(\xi + \sigma^2/2) - \beta$.

When estimates are dealt with, the relationship will be approached only as the error variances of the estimates become small. Therefore, the correct relationship for small $n$ values, similar to that for Sichel's 't' estimator, is required. As a first approximation, the concept of the 'equivalent' number ($n$) of boreholes can be used if the estimated (regressed or kriged) error variances for $\xi$ and $\sigma^2$ in each case are determined, and these are compared with those corresponding to the orthodox estimates normally used for Sichel's 't' estimator[15]. However, for the range of estimated variances and the equivalent $n$ values applicable in this investigation, the differences between the use of the

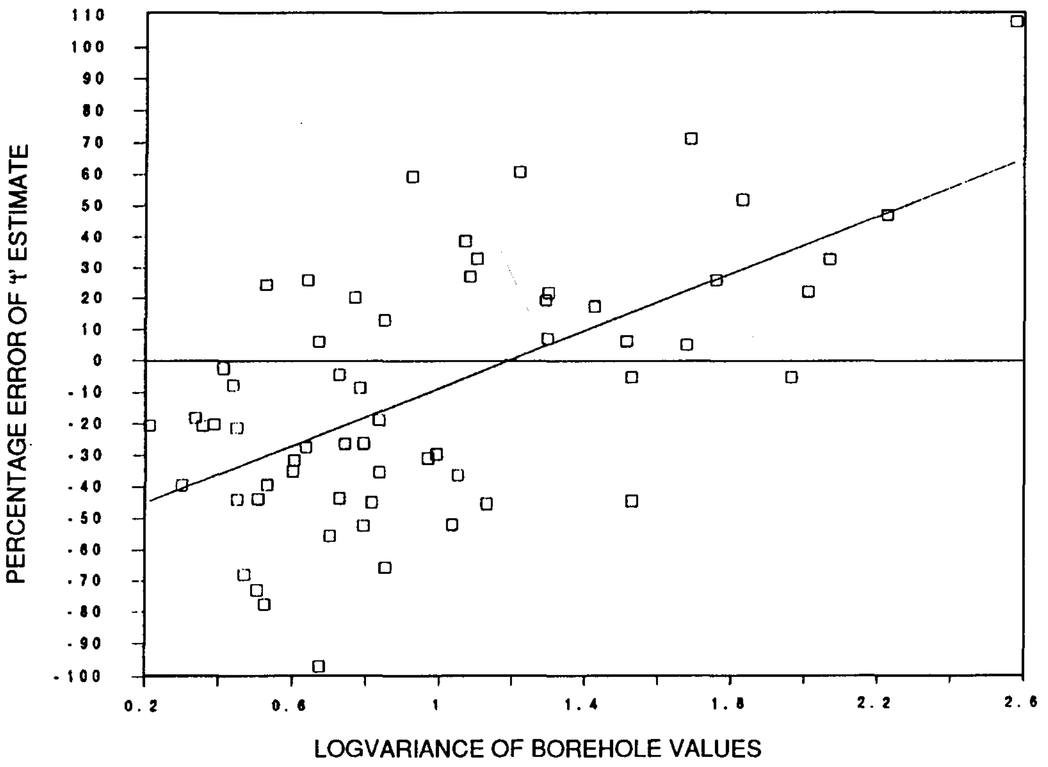

**Figure 3—Correlation of the errors in borehole estimates with the variances in borehole values**

population relationship as distinct from the equivalent *n* approach are less than 1 per cent, and the population relationship was therefore used. For the estimation of confidence limits, the equivalent *n* approach is, of course, essential.

## RESULTS

Table I highlights the main problems underlying the orthodox approach using the arithmetic mean.

**Table I**

**Some basic statistics for the twenty mines analysed**

| | Actual mine means | Borehole means | |
|---|---|---|---|
| | Log variance | Log variance | Log error variance |
| Grade – arithmetic mean | 0,157 | 0,282 | 0,178 |
| –mean of log values | 0,120 | 0,173 | 0,095 |
| Variances of log values within mines | 0,052 | 0,290 | 0,226 |

It is evident that the variance of the 'actual' mine grades at 0,157 is lower than the error variance of the borehole means at 0,178; thus, the regional mean grade, if accepted as an estimate of an individual mine's grade, is in this sense a slightly better estimate than the borehole means. However, for the variance, the corresponding two figures are 0,052 and 0,226, indicating that the regional mean of the 'actual' within-mine variances is a far superior estimate for a new mine than that shown by the 9 borehole values. Also, on log-transformation, the variance of the actual values reduces somewhat from 0,157 to 0,120, but the error variance of the borehole estimates is nearly halved from 0,178 to 0,095; this is clearly the reason why Sichel's 't' will be an improvement on the arithmetic mean, as will be seen later. However, this improvement is severely hampered by the very high error variance of the variance (0,226) as estimated from the 9 boreholes and used in Sichel's 't' estimator together with the mean of the 9 log-grades.

These results already stress the importance of outside information in the form of regional averages for both grade and variance. This was confirmed by the series of estimates carried out on the 60 sets of borehole values, the results of which are tabulated in Table II.

Compared with the arithmetic mean (no. 1 in Table II), Sichel's 't' (no. 2) shows an improvement on all three measures of correlation with actual grades, conditional biases (regression slopes), and error variances. The t″ estimates (no. 3), which are based on the regional average log-variance (and ignore the borehole variance estimate), show a very substantial further gain in efficiency over the 't' estimator.

After the introduction of further information on the parameters for the distribution of actual mine grades in the region via regression and elementary simple kriging, further improvements are obtained as shown (no. 4/6), mainly because this process, if applied efficiently, eliminates the conditional biases and thus reduces the error variances. Up to this stage, the error variance has been reduced progressively from 0,18 (arithmetic mean), to 0,16 ('t'), to 0,11 (t″), and 0,08/0,10 (regression).

Further improvements are shown when the macro spatial structure for grades is introduced, together with the known grades of existing adjacent mines. If the regional mean grade is ignored as in ordinary kriging, and only one

**Table II**

**Estimates based on 60 sets of borehole values**

Codes for additional information used:
Three-parameter lognormal model
For borehole values ................................ 1a
For mine grades....................................... 1b

| *Parameters for population of mines:* | *20 'mines'* | *Klerksdorp field* |
|---|---|---|
| Mean | 2a | 2b |
| Mean logvariance within mines | 3a | 3b |
| Logvariance of mine mean grades | 4a | 4b |
| *Spatial structures:* | | |
| Mine mean grades—untransformed | 5a | 5b |
| Mine mean grades—transformed | 6a | |
| Within-mine variances—transformed | 7a | |

| Technique | Additional information codes | Correlation coefficient | Reg. slope actual / estimate | Observed log error variance of estimate |
|---|---|---|---|---|
| 1. Arithmetic mean of boreholes | – | 0,56 | 0,37 | 0,178 |
| 2. Sichel's 't' | 1a | 0,65 | 0,49 | 0,162 |
| 3. t″ | 1a, 3a | 0,69 | 0,65 | 0,110 |
| *Regression:* | | | | |
| 4. Of arithmetic mean | 1a, 2a, 4a | 0,63 | 1,00 | 0,095 |
| 5. Of Sichel's 't' | 1a, 1b, 2a, 4a | 0,65 | 1,00 | 0,091 |
| 6. Of t″ | 1a, 1b, 2a, 3a, 4a | 0,69 | 1,00 | 0,083 |
| *Macro lognormal kriged estimates—with mean grade of one adjacent mine:* | | | | |
| 7. Ordinary—b/h arithmetic mean | 1b, 4a, 5a | 0,65 | 0,81 | 0,097 |
| 8. Simple—b/h arithmetic mean | 1b, 2a, 4a, 5a | 0,67 | 1,16 | 0,088 |
| 9. Simple—b/h 't' | 1a, 1b, 2a, 4a, 5a | 0,69 | 1,09 | 0,082 |
| 10. Simple—b/h t″ | 1a, 1b, 2a, 3a, 4a, 5a | 0,73 | 1,01 | 0,067 |
| *Macro lognormal kriged estimates—with mean grades of two adjacent mines:* | | | | |
| 11. Simple—b/h t″ | As for 10 | 0,73 | 1,01 | 0,066 |
| *Macro kriged estimates with mean grades of 3 adjacent mines and b/h t″* | | | | |
| 12. Simple lognormal | As for 10 | 0,77 | 1,03 | 0,061 |
| 13. Simple normal | As for 10 | 0,77 | 1,05 | 0,060 |
| 14. Simple normal (field parameters) | 1a, 1b, 2b, 3b, 4b, 5b | 0,78 | 1,11 | 0,059 |
| *Estimates via macro kriged log grades and log variances:* | | | | |
| 15. Simple—with 1 mine | 1a, 1b, 3a, 6a, 7a | 0,78 | 0,99 | 0,070 |
| 16. Simple—with 3 mines | 1a, 1b, 3a, 6a, 7a | 0,82 | 1,08 | 0,049 |

outside mine is used, the error variance remains at 0,10 (no.7), and substantial conditional biases are still present; these disappear only when a substantial number of outside mines are introduced (not shown in Table II), i.e. when the regional mean is used effectively, or with simple kriging. The remaining analyses were therefore confined to simple kriging.

Simple kriging with one mine and sequentially using the borehole mean (no.8), 't' (no. 9), and t″ (no. 10) show a progressive increase in correlation, negligible remaining conditional biases, and a progressive decrease in the error variance from 0,09 (borehole means), through 0,08 (borehole 't's) to 0,07 (borehole t″s). The introduction of two adjacent mines with the borehole t" shows very small further improvements (no. 11) but, when three mines are used with the borehole t″ (no.12), the correlation improves to 0,77 and the error variance to 0,06. The configurations for adjacent mines are shown in Figure 4.

In a comparison of the macro-kriging procedure on the lognormal basis (i.e. on transformed values) with the normal approach (i.e. on untransformed values), kriging with the borehole t″ and three adjacent mines (no. 12 in Table II) was repeated with the same weights on untransformed values (no. 13). Almost identical results were obtained, with a correlation coefficient between the two sets of estimates of over 99 per cent. This was to be expected because the semivariogram of the transformed and untransformed grades when scaled in units of the

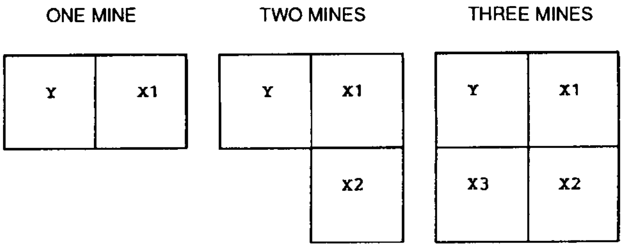

**Figure 4—Configurations of adjacent mines**
**Y New mine X1, X2, X3 Adjacent mines**

population variance will be identical if the third parameter, $\beta$, equals 0, and almost identical if $\beta$ is small relative to the population mean.

The robustness of such a macro-kriging estimator was also demonstrated by substitution, for the parameters of the distribution of 20 'mines', the parameters for the Vaal Reef in the Klerksdorp field obtained from various other analyses[16]; these parameters covered the regional grade distributions and spatial structures for areas of 1 km by 1 km. On these regional parameters, simple macro-kriging with the borehole $t''$ and three mines (no. 14 in Table II) again produced results that were virtually identical to those of the previous two analyses (nos. 12 and 13).

The results of the analyses done on simple macro-kriging of the means of the transformed grades and their log-variances are shown in Table II under nos. 15 and 16 for the borehole values with one and three adjacent mines respectively. It is evident that, with one adjacent mine, there is no significant difference between this approach (no. 15) and the $t''$ approach (no.10). However, with three mines, the error variance further improved from 0,06 (no. 10) to 0,05 (no. 16). This seems logical since the macro-kriged variance estimate used in no. 16 should be superior to the regional average variance used for the $t''$ estimate on which no. 10 is based.

Figure 5 demonstrates graphically the overall advantage of estimating the grade of a new mine by macro-kriging of the transformed grades and their variances separately (using the borehole values, the values for three adjacent mines, the regional mean values, and the regional spatial structures) over the orthodox borehole arithmetic mean. The two sets of estimates are shown in terms of units of the actual mean grades of the mines, and thus provide a visual indication of the narrowing of the observed confidence limits. The central 80 per cent confidence limits are reduced from about -50/+60 per cent to ± 30 per cent. In the practical case of a new mine to be opened, such an improvement in the confidence of the grade estimate will be of substantial significance.

## CONCLUSIONS

From these results, the following conclusions can be drawn for borehole valuations of skew grade data with variabilities such as for the Hartebeestfontein gold mine.

### Distribution Model

Wherever applicable, the three-parameter lognormal model should be used when individual borehole values are available. For this purpose, the 't' estimator is preferred to the arithmetic mean but, given an average of actual variances within mines in the region, the $t''$ estimator can be better. Where the departures from this model are significant, the more general sub- or hyper-lognormal model should be considered[8]. However, small sampling

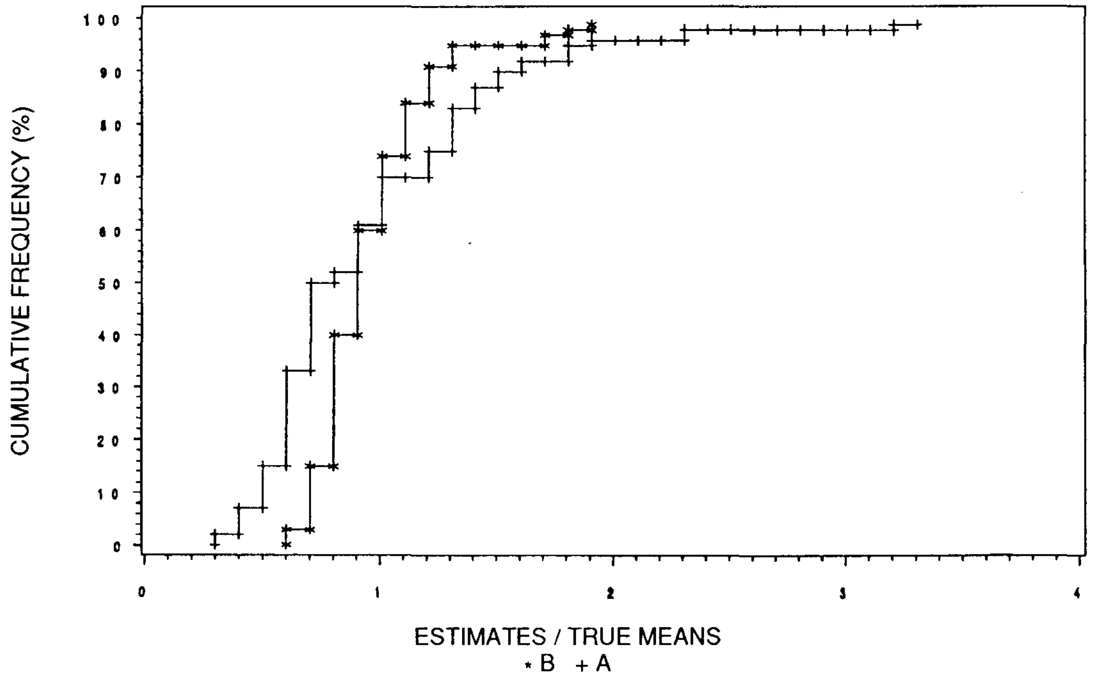

**Figure 5—Cumulative frequency distribution of borehole arithmetic means (+) and macro-kriged estimates with three mines (*) —nos. 1 and 16 in Table II**

theory for these models is yet to be developed. When actual mean grades of large mining units, e.g. existing mines, are available, there seems to be little or no advantage in the use of the lognormal model in preference to the normal, except for the estimation of confidence limits where some skewness is still present.

## Macro-kriging

Simple macro-kriging shows definite advantages over ordinary kriging. The most efficient method appears to be simple macro-kriging of the log-transformed borehole and mine grade and variances separately, and the combination of these to give the grade estimate for a new mine.

## ACKNOWLEDGEMENTS

Gratitude is expressed to Hartebeestfontein mine, which provided the database, and to the Anglovaal group for permission to publish this paper. Acknowledgement is also made to Mrs. G. Knox, who prepared the database for the analysis, and to the Anglo American/De Beers Ore Evaluation Division, who assisted with discussions and some computer facilities.

## REFERENCES

1. Sichel, H.S. An experimental and theoretical investigation of bias error in mine sampling with special reference to narrow gold reefs. *Trans. Instn Min. Metall.*, Lond., vol. 56. 1947. pp. 403-473.
2. Sichel, H.S. New methods in the statistical evaluation of mine sampling data. *Bull. Instn Min. Metall.*, Lond., Jun. 1952. pp. 261-288.
3. Krige, D.G. A statistical analysis of some of the borehole values in the Orange Free State goldfields. *J. Chem. Metall. Min. Soc. S. Afr.*, Sep. 1952. pp. 47-64.
4. Krige, D.G. A statistical approach to some mine valuation and allied problems on the Witwatersrand. M.Sc. (Eng.) Thesis, University of the Witwatersrand, Johannesburg, 1951.
5. Krige, D.G. A statistical approach to some basic mine valuation problems on the Witwatersrand. *J. Chem. Metall. Min. Soc. S. Afr.*, Dec. 1951. pp. 119-139.
6. Krige, D.G. On the departure of ore value distributions from the lognormal model in South African gold mines. *J.S. Afr. Inst. Min. Metall.*, vol. 61. 1960. pp. 231-244.
7. Krige, D.G., Kleingeld, W.J., and Oosterveld, M.M. *A priori* parameter distribution patterns for gold grades in the Witwatersrand basin to be applied in borehole evaluations of potential new mines using Bayesian geostatistical technique. *APCOM 90, Berlin.* 1990. pp. 715-726.
8. Sichel, H.S., Kleingeld, W.J., and Assibey-Bonsu, W. Comparative study of three distributional models: 3-parameter lognormal, compound lognormal and log-generalized inversed Gaussian distributions. Paper submitted for publication to *J.S. Afr. Inst. Min. Metall.*, 1991.
9. Krige, D.G. Statistical applications in mine valuation. *J. Inst. Mine Survey. S. Afr.*, vol. 12, (2). 1962. pp. 95-136.
10. Krige, D.G., and Ueckermann, H.J. Value contours and improved regression techniques for ore-reserve valuations. *J.S. Afr. Inst. Min. Metall.*, May 1963. pp. 429-452.
11. Matheron, G. *Traité de geostatistique appliquée*, tome 1. Memoires de Bureau de Recherches Geologiques et Minières, no. 14. Editions Technip, Paris. English translation: *Treatise on applied geostatistics*, Kennecott Copper Corporation, Salt Lake City, Utah, 1960.
12. Krige, D.G. Geostatistics and definition of uncertainty. *Trans. Instn Min. Metall. (Sec. A: Min. industry)*, vol. 93. Jul. 1984.
13. Assibey-Bonsu, W. Research work shortly to be submitted for Ph.D. (Eng.) degree, University of the Witwatersrand, Johannesburg.
14. Krige, D.G. *Lognormal-de Wijsian geostatistics for ore evaluation.* Johannesburg, South African Institute of Mining and Metallurgy, Monograph series, Geostatistics 1, 1978.
15. Kleingeld, W.J. *La geostatistique pour des variables discretes.* Ph.D. thesis, L'Ecole Nationale Supérieure des Mines de Paris. Fontainebleau (France), 1987.
16. Krige, D.G. Unpublished research still in progress, 1990/1992.