J. S. Afr. Inst. Min. Metall., vol. 93, no. 6.
Jun. 1993. pp. 135–141.

# A model of mill power as affected by mill speed, load volume, and liner design

by M.H. Moys*

**SYNOPSIS**

Accurate models of mill power are becoming more necessary as the design, operational, and control requirements for grinding mills become more stringent. Models of mill power are reviewed briefly, and a new semi-phenomenological model is derived. The conventional torque-arm model for mill power is combined with a model of the tendency of the load in a mill to centrifuge at high speeds, to describe mill power that can be related to several design variables (such as mill geometry, mill speed, and liner design) and control variables (such as load volume, slurry rheology, and mill speed in the case of variable-speed milling). The semi-phenomenological nature of the model allows rational conclusions to be reached concerning the effect of various aspects of mill-load behaviour on mill power. The model is shown to be capable of describing reality with much greater accuracy than is achieved by empirical models such as the Bond model.

**SAMEVATTING**

Akkurate modelle van meuldrywing word al hoe belangriker omdat die ontwerp-, bedryfs- en beheervereistes vir meule al hoe strenger word. Modelle vir meuldrywing word kortliks in oënskou geneem en 'n nuwe half-fenomenologiese model word afgelei. Die konvensionele wringarmmodel vir meuldrywing word met 'n model wat die tendens van die meullading om teen hoë snelhede uit te swaai in aanmerking neem, gekombineer om meuldrywing te beskryf wat met verskeie ontwerpveranderlikes (soos meulgeometrie, meulspoed en voeringontwerp) en beheerveranderlikes (soos die volume van die lading, flodderreologie, en meulspoed in die geval van maling met 'n veranderlike spoed) in verband gebring kan word. Die halffenomenologiese aard van die model stel die moontlikheid daar om gevolgtrekkings te maak oor die uitwerking van verskillende aspekte van meulladinggedrag op die meuldrywing. Daar word ook getoon dat die model werklike meulgedrag meer realisties kan beskryf as wat met empiriese modelle, soos die Bond-model, die geval is.

## INTRODUCTION

As tools available for the modelling, design, and control of grinding mills become more sophisticated, the need for accurate models of mill behaviour becomes more acute. In particular, an accurate and flexible model for mill power as affected by important operating variables is required for the following reasons.

(1) The consumption of mill power represents approximately 50 per cent of the operating costs of a mill. Improved mill design (in particular, liner design[1]) can substantially reduce these costs.

(2) The design of mill-control systems remains a formidable challenge after several decades of intensive research. In spite of its importance, mill power is often not regarded as a control variable because of its complex non-linear relationship to manipulated variables. Herbst and his co-workers[2] included it in their model-based control strategy, but were forced to use Bond's empirical formula[3] (discussed below) to relate mill power to operating variables. Bond's formula is still widely used, in spite of its inadequacies (particularly at high mill speeds, variable mill loads and high slurry densities).

(3) Most mills in the South African gold industry run at speeds in the range 90 to 92 per cent of critical speed. It is in this range of speed that power is strongly affected by a wide range of operating variables, and a better understanding of these effects should lead to more effective use of these mills.

(4) In particular, with the advent of variable-speed mills such as that at Leeudoorn, control systems that manipulate mill speed[4,5] will be needed. It is essential that the effect of variable mill speed and its interaction with load volume, liner design, slurry properties, etc. should be understood, particularly in South Africa, where most run-of-mine milling is done at high mill speed.

A model for mill power is developed here that meets the following objectives.

(a) It is based upon a semi-phenomenological understanding of mill behaviour, and is thus likely to be more useful and reliable than purely empirical models.

(b) Its structure reflects the complex, non-linear dependence of mill power on load volume, $J$, and mill speed, $N$ (percentage of critical speed).

(c) The effect of slurry viscosity[6] and liner design can be related to model parameters in a fashion that 'makes sense' in terms of the physics of the process.

## MODELS FOR MILL POWER

### Review of Past Work

It is impossible to do justice to the enormous amount of work done in this area. Rose and Sullivan[7] reviewed much of the earlier work and combined it with their own work, using dimensional analysis to produce a model that incorporated a wide range of variables. It has not found wide acceptance, possibly because of its complexity and the limitations of this approach: dimensional analysis cannot penetrate the non-linear behaviour of high-speed

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© The South African Institute of Mining and Metallurgy, 1993. SA ISSN 0038-223X/3.00 + 0.00. Paper received January 1992; revised paper received February 1992.

grinding mills! Bond[3] used an empirical approach and collated data on mills of widely ranging designs to produce his much-used equation:

$$P = K_1 \rho_L \sin\alpha\, J\,(1-\beta J)\, L\, D^{2,3} N_c \left(1 - \frac{0,1}{2^{\gamma - \zeta N}}\right) W \,, \qquad [1]$$

where $K_1$ is a constant (actually strongly affected by liner design and slurry properties), $\rho_L$ is the bulk density of the load (kg/m³), $\alpha$ is the dynamic angle of repose of the load, $J$ is the fraction of mill volume occupied by the load, $\beta$ is a parameter given a value of 0,937 by Bond (implying that maximum power is drawn at $J = 1/2\beta$, = 0,53), $L$ is mill length (m), $D$ is mill diameter (m), $N_c$ is mill speed (percentage of critical), and $\gamma$ and $\zeta$ are parameters given a value of 9 and 0,1 respectively by Bond. It should be noted that this equation is of the form

$$P = f_\rho(\rho_L)\, f_\alpha(\alpha)\, f_J(J)\, f_L(L)\, f_D(D)\, f_N(N) \,, \qquad [2]$$

and therefore does not allow for the fact that variations in $J$ affect the nature of the dependence of $P$ on $N$, for example. It is shown below that this separation of variables is inappropriate.

## Behaviour of the Mill Load

As mentioned earlier, South African run-of-mine (ROM) mills are often operated at high speed (e.g. 92 per cent of critical speed), at which the liner profile and slurry rheology have a very significant effect on the mill power. A discussion of load behaviour in such mills precedes and lays the foundation for the derivation of a model.

If a large amount of slip occurs between the outer layer of the load and the liner (or if the mill speed is low, e.g. less than 60 per cent of critical speed[8]), the load assumes the orientation in the mill shown in Figure 1, and the torque-arm model for mill power (the basis of Bond's model) is fairly accurate:

$$P = \frac{2\pi N T}{60} \qquad [3]$$

and

$$T = M\, g\, r_c \sin\alpha \,, \qquad [4]$$

where $\alpha$ is the angle of repose, $r_c$ is the radius of the torque arm to the centre of gravity (c.o.g.) of the load of mass $M$, and $N$ is mill speed in revolutions per minute.

The use of a liner design that minimizes slip results in a significant departure from this type of behaviour, particularly as the mill speed is increased. A significant proportion of the grinding medium in contact with the liner is thrown clear of the load, and follows a parabolic trajectory before falling onto the mill load; as the mill speed increases, these particles are thrown clear of the mill load and fall onto the descending shell of the mill, imparting some of their momentum to the mill (with a resultant loss in power). As the mill speed nears 100 per cent of critical speed, centrifuging of the outer layer of the medium starts, resulting in a loss in the active mass of the load and a reduction in the effective mill diameter, both of which contribute substantially to loss of power.

The presence of high lifter bars exacerbates this phenomenon; the medium is lifted away from the *en masse* load on the lifters, and falls away from the lifter only when

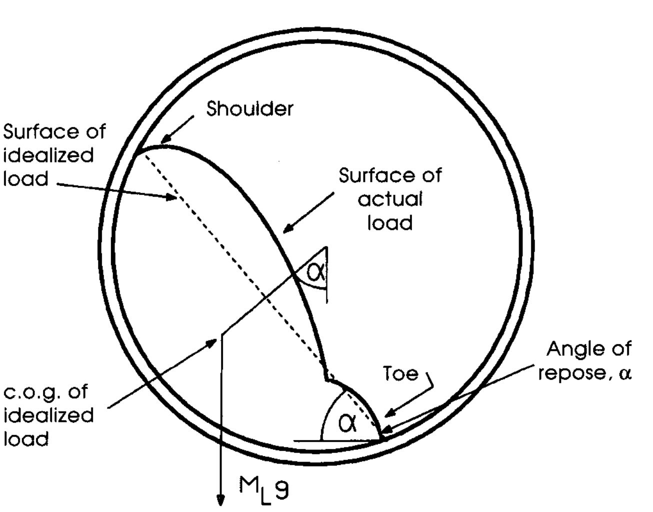


**Figure 1—Orientation of the load in a mill at speeds lower than 60 per cent of critical speed**

gravity overcomes the friction between the liner and the medium and accelerates the medium away from the liner at a rate exceeding the centripetal acceleration imposed by the rotating mill[9]. Complete centrifuging of the outer layer of the load will occur at lower speeds than apply in the absence of lifters. This explains the loss in power that accompanies the insertion of lifters in high-speed mills[10]; it is of interest that, because the grinding is more efficient, this loss of power does not necessarily result in a loss of mill capacity. The extra power consumed in the absence of lifters is probably used to wear the liners down; the insertion of lifters produced a three- to ten-fold increase in liner life in the cases discussed by Powell[10].

Highly viscous slurry will also increase the tendency of the medium to stick to the liners, encouraging premature centrifuging of the mill load[6]. This effect is particularly marked in mills operating at the high speeds typical of South African ROM mills.

## A Semi-phenomenological Model of Mill Power

Accurate modelling of the complex behaviour described above (involving the cascading 'kidney', the cataracting medium, and the centrifuging portions of the load) is a daunting task that will not be attempted here; a less ambitious road is followed. The model described in this section is a compromise between the two extremes of mill behaviour: (i) at low speed (where the torque-arm model applies) and (ii) at high speed (where centrifuging dominates). The resulting model is relatively simple to manipulate and use, and is shown to describe the behaviour of mill power fairly accurately over a wide range of operating parameters.

No attempt is made to describe the energy recovered when an element of the medium is thrown through the air and imparts some of its energy to the descending mill wall on impact. It is assumed for modelling purposes that elements of the medium are either centrifuged completely (thus drawing no power) or are retained in the active, non-centrifuged portion of the load, drawing power according to Bond's power model applied to a mill with a reduced effective diameter. Because of this simplified approach, the model cannot be expected to predict the mill speed at which centrifuging actually starts, nor will it predict the actual thickness of the centrifuged layer. For example, under conditions where the medium has started to cataract, the model will predict a certain amount of centrifuging, even if this is not in fact occurring.

Assume that the thickness of the centrifuged layer is $\delta_c D$, so that the mill diameter, $D$, is reduced to an effective diameter, $D_{eff} = (1 - 2\delta_c)D$. The volume of load centrifuged is thus

$$V_c = \frac{\pi}{4}\left(D^2 - D_{eff}^2\right)L = \pi D^2 \delta_c \left(1 - \delta_c\right) L. \qquad [5]$$

The volume of the load is reduced by that amount, i.e.

$$V_{eff} = J\,\frac{\pi D^2}{4} L - V_c, \qquad [6]$$

where $J$ is the fraction of mill volume occupied by the load. It can easily be shown that the active load now occupies a volume fraction $J_{eff}$ of the effective mill volume, where

$$J_{eff} = \frac{V_{eff}}{\pi L_{eff}^2/4} = \frac{J - 4\delta_c(1-\delta_c)}{1 - 2\delta_c^2}$$
$$= 0 \text{ for } \delta \geq 0{,}5\left[1 - (1-J)^{0{,}5}\right]. \qquad [7]$$

This relationship may be unnecessarily complex. For $J = 0{,}5$, it can be shown that the following is a fairly accurate approximation to equation [7] for small $\delta_c$:

$$J_{eff} = J - 2\delta_c$$
$$= 0 \text{ for } \delta_c > J/2. \qquad [8]$$

The implications of the use of this simplification are assessed below.

The reduced mass of active material is assumed to draw power at a rate given by Bond's model (without the empirical term—the last term in equation [1], which Bond introduced to model speeds near 100 per cent of critical speed):

$$P = K_2 D_{eff}^{2,3} \sin\alpha\, \rho_L\, J_{eff}\left(1 - \beta J_{eff}\right) N_{eff}. \qquad [9]$$

All that is now required is a model that relates the thickness of the centrifuged layer to the operating variables. The following empirical model for $\delta_c$ is proposed:

$$\delta_c = J^{\Delta_J} \exp\left[-\frac{N^* - N}{\Delta_N}\right]. \qquad [10]$$

(1) For low mill speeds, it is expected that $\delta_c = 0$. As the mill speed is increased to the point where the load starts to cataract, a rapid increase in $\delta_c$ is expected, motivating the above exponential dependence on $N$. $N^*$ and $\Delta_N$ are parameters that will be strong functions of liner profile and slurry viscosity.

(2) The effect of the load volume on $\delta_c$ is complex. If the liner design is such that substantial slip occurs at low $J$ values, then minimal cataracting and centrifuging of the load will occur and $\delta_c \approx 0$. As $J$ is increased, the load pressure on the liner is increased to the point where slip disappears, so that $\delta_c$ will assume a large value. For a liner design that discourages slip, the behaviour of the load in contact with the liner will be relatively independent of $J$, so $\delta_c$ will be a weak function of $J$. The value of $\Delta_J$ governs the strength of the dependence of $\delta_c$ on $J$, and will be a strong function of liner profile.

## EXPERIMENTAL

A large amount of data was gathered on a versatile milling rig that has been described previously[8]. It is sufficient to say here that the rig allows very precise experimentation to be performed on mills drawing less than 2,5 kW as a function of mill speed, mill dimensions, liner design, load type, and load volume. Data relevant to the present investigation were extracted from a comprehensive database built up in an investigation into the effect of liner design, load volume, and mill speed on mill behaviour.

The four designs of liner investigated are shown in Figure 2. Liner 1 was a smooth liner with 18 lifter bars of 20 x 20 mm welded to it. Liner 2 had the saw-toothed configuration shown. It should be noted that the direction of rotation of the mill ensured that minimal lifting action was imparted to the load; in fact, substantial slip occurred with this lifter. Liner 3 was the same as liner 1 with half the lifters removed. Liner 4 consisted of wire mesh with 12 mm square holes to simulate the effect of the grid liners in use in some high-speed gold mills in South Africa.

The medium consisted of a graded charge of 12 mm, 18 mm, and 25 mm balls with a bulk density $\rho_L$ = 4700 kg/m³. The mill diameter, $D$, was 0,53 m inside the liners, and the mill length, $L$, was 0,3 m.

For each liner, a series of experiments was conducted in which a predetermined mass of medium was added to the mill, and the torque, $T$(Nm), drawn by the mill was measured as a function of the mill speed between 10 and 120 per cent of critical speed. No ore or slurry was added to the mill. The data are summarized in the Addendum, and are plotted in Figures 3 to 6.

### The Measured Results

#### *Liners 1 and 3 (18 and 9 lifter bars respectively)*

For low values of $J$, the torque was approximately constant up to $N$ = 50 per cent. Then it dropped steadily as $N$ approached 100 per cent. Clearly, the lifters reduced the slip of the load, even when $J$ was low; this effect was

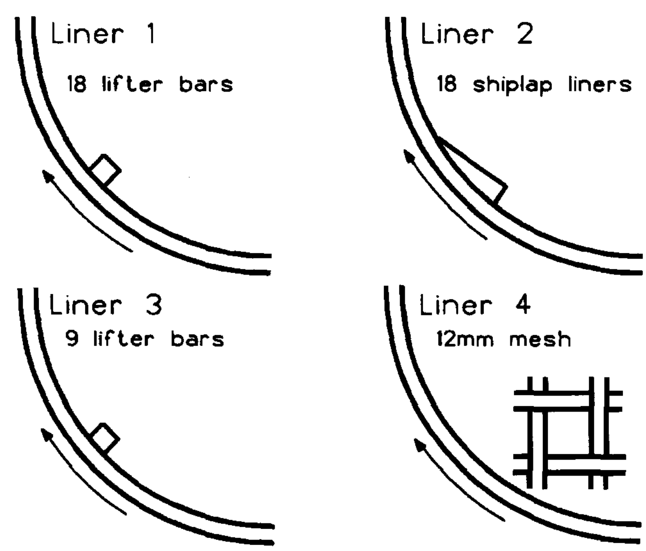


**Figure 2—Liners used in the experimental programme**

strongest for liner 1, which had the most lifters. For higher values of $J$, the torque increased with $N$ between 10 and approximately 60 per cent. This has been shown[11] (by the analysis of measurements not discussed here) to be caused by an increase in the angle of repose of the load in the mill. As $N$ increased to 120 per cent, the torque dropped steadily; significant torque occurred at $N$ = 120 per cent. (Bond's equation implies that torque is zero for all $J$ at $N$ = 123 per cent.)

### *Liner 2 (smooth shiplap)*

The torque was independent of $N$ for $J$ < 0,2, because the load was slipping on the liner and was not centrifuged, even at $N$ = 120 per cent. For $J$ > 0,2, a dramatic change occurred: the profile suddenly assumed that typical of the other liners, indicating that the load was now keyed into the liners because the charge pressure between the load and the liners had exceeded a critical value.

### *Liner 4 (12 mm mesh)*

For low $J$, slip occurred until $N$ = 100 per cent, and then the torque dropped to lower values, indicating that the rough liner was reducing the slip. The transition to a behaviour similar to that of liners 1 and 3 occurred smoothly as $J$ increased.

## Analysis of the Results

The data measured in the experimental programme involved measurements at values of $J$ and $N$ that will never be used in practice, e.g. $J$ = 0,128 and lower values, $N$ = 10 and 120 per cent. The purpose of this investigation was to assess models to be used in normal ranges of operation for these variables; the data extracted from the database thus obeyed $0{,}18 \leq J \leq 0{,}7$ and $60 \leq N \leq 100$.

Powell's non-derivative regression technique[12] was used in the estimation of the parameters in the following models:

(a) Bond's model as given by equation [1], involving the parameters $K$, $\beta$, $\gamma$, and $\zeta$

(b) the semi-phenomenological model (the SP model) defined by equations [7], [9], and [10], involving the parameters $K$, $\beta$, $\Delta_J$ and $\Delta_N$

(c) the simplified semi-phenomenological model (the SSP model) defined by equations [8], [9], and [10]), involving the same parameters as (b).

The following relative standard deviation was used in an assessment of the goodness of fit of the model to the torque data:

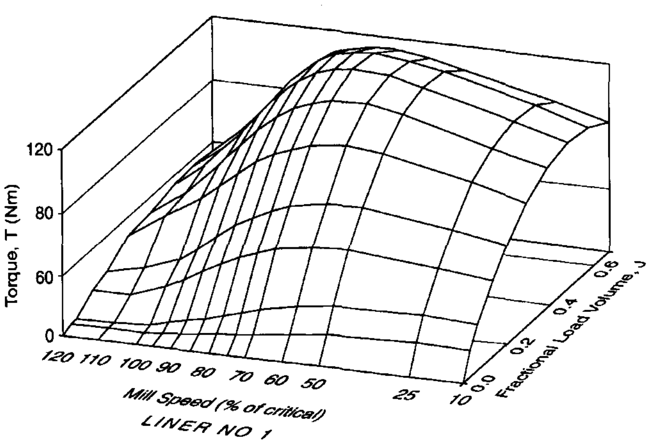


**Figure 3—The effect of fractional load volume and mill speed on torque for liner no. 1**

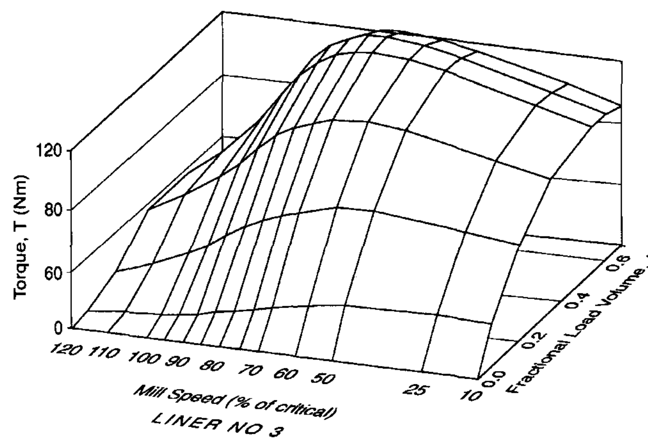


**Figure 5—The effect of fractional load volume and mill speed on torque for liner no. 3**

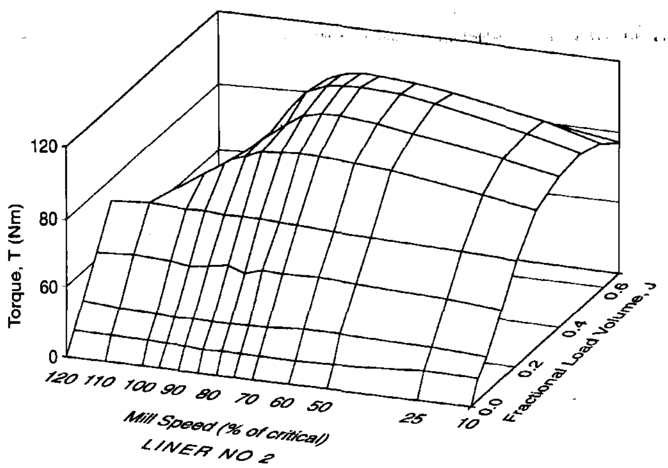


**Figure 4—The effect of fractional load volume and mill speed for liner no. 2**

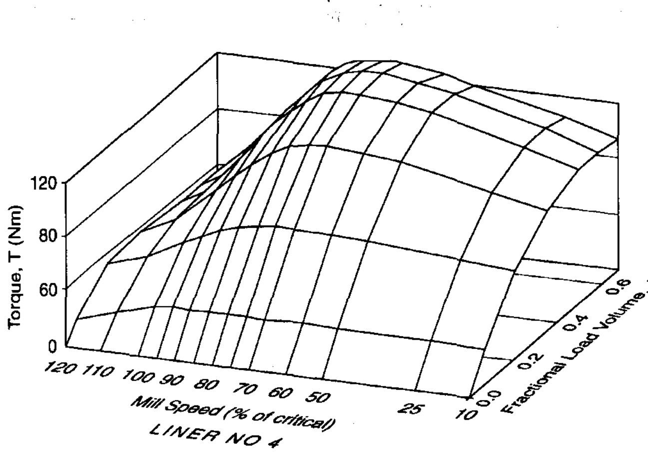


**Figure 6—The effect of fractional load volume and mill speed for liner no. 4**

$$\bar{\sigma}_T(\%) = \frac{\text{Std dev of } T \text{ between model and expt}}{\text{Maximum value of } T} \cdot \frac{100}{1}$$

$$= \frac{\left[\Sigma\left(T_{\text{model}} - T_{\text{expt}}\right)^2 / (°F)\right]^{0,5}}{T_{\max}} \cdot \frac{100}{1}, \qquad [11]$$

where $°F$ = no. of degrees of freedom = no. of experiments – no. of parameters.

The validity of the Bond model was assessed first. The parameters were estimated under the following hypotheses:

$H_{B1}$: The Bond model as discussed earlier, with $\beta = 0{,}937$, $\gamma = 9$, and $\zeta = 0{,}1$ is adequate; only $K$ is estimated.

$H_{B2}$: All the parameters are estimated.

The results of this analysis are given in Table I. It is clear that the published values for the Bond model (hypothesis $H_{B1}$) result in large errors between theory and practice. The estimation based on the other hypothesis reveals that significant improvements in fit are possible for this purely empirical model. The values of $\beta$ and $J_{max}$ (= $1/2\beta$, the value of $J$ at which maximum torque occurs) for $H_{B2}$ are given in Table I. The liner design certainly had a marked effect on this variable.

The validity of the simplification made in the SP model (to provide the SSP model) was then assessed for the various hypotheses concerning $\beta$ given in Table II. (All the other parameters—$\Delta_J$, $\Delta_N$, $N^*$, and $K$—were estimated for each case.) An inspection of the data reveals that the SSP model is in general at least as good as (and in some cases slightly better than) the SP model.

Several other hypotheses were investigated by the use of these models.

**Table I**
**Assessment of the Bond model**

| Liner no. | $\bar{\sigma}$ and parameters for hypothesis | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $H_{B1}$ | | $H_{B2}$ | | | | | |
| | $K$ | $\bar{\sigma}$ | $\gamma$ | $\zeta$ | $K$ | $\beta$ | $\bar{\sigma}$ | $J_{max}$ |
| 1 | 0,101 | 8,82 | 7,7 | 0,10 | 0,106 | 0,87 | 5,29 | 0,57 |
| 2 | 0,095 | 7,62 | 16,0 | 0,17 | 0,107 | 1,05 | 4,70 | 0,48 |
| 3 | 0,105 | 9,03 | 9,5 | 0,11 | 0,104 | 0,84 | 4,07 | 0,60 |
| 4 | 0,114 | 6,64 | 11,7 | 0,14 | 0,113 | 0,89 | 2,12 | 0,56 |

**Table II**
**Comparison of two semi-phenomenological models**

| Model | Liner no. | $\bar{\sigma}$ for values of Bond parameters | | | |
|---|---|---|---|---|---|
| | | $\beta = 0{,}937$ | $\beta = 1$ | $\beta$ estd ($\beta$) | |
| SP | 1 | 3,07 | 2,99 | 2,85 | (,975) |
| | 2 | 4,32 | 4,15 | 4,04 | (1,01) |
| | 3 | 3,58 | 4,64 | 3,41 | (,906) |
| | 4 | 3,45 | 4,64 | 3,23 | (,890) |
| SSP | 1 | 2,94 | 2,56 | 2,54 | (,985) |
| | 2 | 4,67 | 4,25 | 4,24 | (1,03) |
| | 3 | 3,22 | 4,25 | 3,39 | (,925) |
| | 4 | 3,10 | 4,02 | 2,77 | (,909) |

(1) In the analyses given so far, the angle of repose, $\alpha$, was assumed to be 45 degrees. Hypotheses that attempted to relate $\alpha$ to $N$ and $J$ were tested without revealing significant results. This was probably because the data for $N < 60$ per cent were excluded from the analysis; as discussed earlier, the variation of torque with $N$ at low values of $N$ was shown[11] to be related to variations in $\alpha$. As the data for $N \geq 60$ per cent do not contain enough information to discriminate between the centrifuging models discussed above and a model for $\alpha$, it was necessary to assume that $\alpha$ has a fixed value.

(2) The need to include the parameter $\Delta_J$ in the model for $\delta$ was assessed by means of regressions on the data with $\Delta_J = 0{,}0$ (i.e. $J$ had no effect on $\delta$) and $\Delta_J = 1{,}0$. Both these cases produced significantly poorer fits of the model to the data for several of the sets of data analysed, and it was concluded that the $J^{\Delta J}$ term was necessary.

Two parameters ($\Delta_N$ and $N^*$) are used to quantify the effect of $N$ on load behaviour, and are thus difficult to relate to liner design parameters. However, it was noted that $N^*$ for the SP model was virtually independent of liner design, ranging between 134 and 138. It was therefore fixed at a value of 136, and the remaining parameters for the SP model were estimated and are reported in Table III.

The following observations are made:

- $\Delta_J$ is a clear indication of the effect of liner design on the tendency of the load to slip. Small values of this parameter indicate that $J$ has a small effect on $\delta$, implying that the pressure between the load and the liners has little effect on load behaviour. This applies to liners 1 and 3, involving 18 and 9 lifter bars respectively. The rough grid liner no. 4 has $\Delta_J = 0{,}77$, while the smooth shiplap liner has $\Delta_J = 1{,}53$. If $\Delta_J = a/n_L$, where $n_L$ is the number of lifters, then, by use of the data for liners 1 and 3 (for which $n_L = 18$ and 9 respectively), it can be shown that $a \approx 3$. This relationship can then be used in the calculation of the 'equivalent number of 20 mm x 20 mm lifters' $\hat{n}_L = a/\Delta_J$ for liners 2 and 4 (Table III).
- $\Delta_N$ varies in a monotonic fashion with $\hat{n}_L$ as shown in Figure 7. The regressed equation relating the two variables is

$$\hat{\Delta}_N = 20{,}8 - 2{,}4 \log_e\left(\hat{n}_L\right). \qquad [12]$$

There is substantial agreement between $\Delta_N$ and its estimate. The impact of this estimate of $\Delta_N$ on the relative variance $\bar{\sigma}$ was calculated and was shown to be entirely negligible for all four liners, i.e. $\hat{\Delta}_N$ is statistically indistinguishable from $\Delta_N$.

**Table III**
**Analysis of simplified semi-phenomenological (SSP) model ($N_{100}$ = 136)**

| Liner no. | $\Delta_J$ | $\Delta_N$ | $\beta$ | $K$ | $\bar{\sigma}$ | $J_{max}$ | $\hat{n}_L$ | $\hat{\Delta}_N$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0,16 | 14,2 | 0,99 | 0,120 | 2,45 | 0,51 | 18,7(18*) | 13,9 |
| 2 | 1,53 | 19,6 | 1,03 | 0,111 | 4,24 | 0,49 | 2 | 19,1 |
| 3 | 0,33 | 15,3 | 0,92 | 0,115 | 3,19 | 0,54 | 9(9*) | 15,5 |
| 4 | 0,77 | 16,9 | 0,91 | 0,122 | 2,98 | 0,55 | 4 | 17,5 |

* Actual number of lifters

## DISCUSSION

It has been shown here that some of the parameters of the SSP model can be related to liner design as follows.

(a) $\Delta_J = a/n_L$; this allows the calculation of $\Delta_J$ for a liner with well-defined lifter bars or, alternatively, the calculation of the 'equivalent number of lifter bars', $\hat{n}_L$, for a liner without lifter bars in situations where an estimate of $\Delta_J$ is available.

(b) An equation such as [12] can be used in the estimation of parameter $\Delta_N$.

Unfortunately, the other parameters do not appear to be related to liner design in as 'neat' a fashion. It is tempting to fix $\beta$ at a value of 1 to reduce the number of parameters, but this produces a significant degradation in the fit of the model to the data. Both $\beta$ (which fixes the value of $J$ at which maximum power is drawn) and $K$ (which governs the value of maximum power) are essential parameters in the model.

Space does not permit the analysis of the effect of slurry properties on the power equations. However, the semi-phenomenological nature of the model lends itself to the interpretation of slurry-viscosity effects in high-speed mills, where a high viscosity produces significant losses in power. It is postulated that this is due to the increased thickness of the layer of slurry or medium that is centrifuged in the mill. This phenomenon would be modelled by (for example) the proposal of a relationship between the parameter $N^*$ and slurry viscosity. This will be explored in future work.

## CONCLUSIONS

(1) The Bond model for power is inadequate for describing the behaviour of high-speed mills of arbitrary liner design.

(2) The semi-phenomenological model proposed combines two extremes of load behaviour in the mill: the tendency of the load to centrifuge at high speeds, high lifter profiles, and high slurry viscosities, and the torque-arm model for power at low speeds.

(3) When this model was simplified substantially, no reduction in its ability to describe the experimental results was observed.

(4) Some of the parameters of the model can be related to the parameters of liner design. Further work is required to relate the remaining parameters to liner design.

(5) The model has the correct form for the exploration of slurry-viscosity effects in high-speed mills.

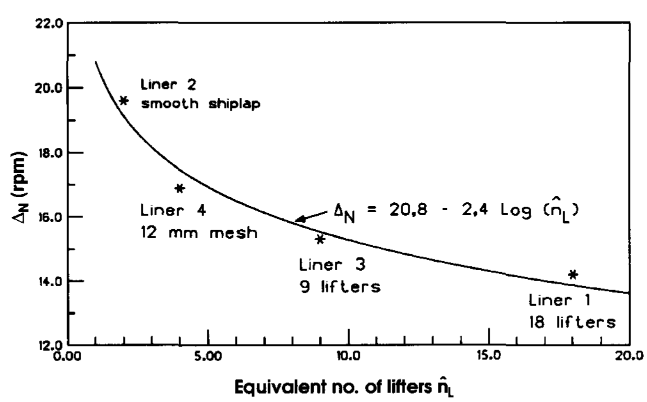


**Figure 7—Correlation between parameter $\Delta_N$ and the equivalent number of lifters for each liner**


## ACKNOWLEDGEMENTS

This paper is a slightly updated version of a paper presented at the 7th European Symposium on Comminution in Ljubljana (Yugoslavia) in June 1990. Permission from the organizers to publish it elsewhere is gratefully acknowledged. Acknowledgement is also made to W. Pate of Control International, Inc., Salt Lake City, for assistance with some of the regression work; to D. Dortmund for graphics (Figures 3 to 6); to Anglo American Research Laboratories for funding part of the experimental work; and to Mintek for providing the experimental equipment.


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## ADDENDUM: TORQUE MEASURED AS A FUNCTION OF LINER DESIGN, LOAD VOLUME, AND MILL SPEED

| J | N(r/min) and T(Nm) for liner no. 1 | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0,128 | 60,0 | 54,6 | 70,0 | 51,1 | 70,8 | 47,1 | 75,5 | 42,7 |
| | 80,2 | 37,4 | 85,0 | 30,9 | 89,7 | 24,3 | 94,5 | 18,3 |
| 0,196 | 60,9 | 73,7 | 70,8 | 69,4 | 75,0 | 65,2 | 80,7 | 59,6 |
| | 85,7 | 53,8 | 90,6 | 45,3 | 95,6 | 35,6 | 100,5 | 27,6 |
| 0,289 | 60,9 | 99,6 | 70,8 | 96,6 | 75,8 | 93,3 | 80,7 | 88,4 |
| | 85,7 | 81,7 | 90,6 | 72,4 | 95,6 | 63,1 | 100,5 | 54,0 |
| 0,385 | 60,9 | 115,2 | 70,8 | 114,0 | 75,8 | 111,1 | 80,7 | 106,4 |
| | 85,7 | 99,3 | 90,6 | 89,2 | 95,6 | 77,7 | 100,5 | 65,8 |
| 0,479 | 60,9 | 121,2 | 70,8 | 122,7 | 75,8 | 120,5 | 80,7 | 116,9 |
| | 85,7 | 110,2 | 90,6 | 100,1 | 95,6 | 87,0 | 100,0 | 73,0 |
| 0,574 | 60,9 | 118,1 | 70,8 | 120,6 | 75,8 | 119,6 | 80,7 | 116,9 |
| | 85,7 | 111,0 | 90,6 | 102,1 | 95,6 | 90,4 | 100,5 | 75,6 |
| 0,670 | 60,4 | 107,8 | 70,3 | 110,7 | 75,3 | 110,4 | 80,2 | 109,3 |
| | 85,2 | 105,7 | 90,2 | 99,1 | 95,1 | 90,1 | 100,1 | 76,8 |

| J | N(r/min) and T(Nm) for liner no. 2 | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0,128 | 60,7 | 45,8 | 70,7 | 46,0 | 75,7 | 46,8 | 80,0 | 45,0 |
| | 85,7 | 46,4 | 90,7 | 45,4 | 95,7 | 44,4 | 100,7 | 45,1 |
| 0,193 | 60,2 | 64,4 | 70,2 | 65,4 | 75,2 | 65,7 | 80,2 | 66,1 |
| | 85,2 | 66,4 | 90,2 | 66,6 | 95,2 | 66,9 | 100,2 | 67,1 |
| 0,289 | 61,6 | 91,5 | 71,6 | 92,8 | 76,5 | 93,1 | 81,5 | 92,9 |
| | 86,5 | 91,4 | 91,5 | 87,8 | 96,5 | 82,9 | 101,5 | 76,6 |
| 0,385 | 61,6 | 96,3 | 71,6 | 97,9 | 76,5 | 97,8 | 81,5 | 96,1 |
| | 86,5 | 91,6 | 91,5 | 86,8 | 96,5 | 80,8 | 101,5 | 72,7 |
| 0,481 | 62,2 | 108,2 | 72,3 | 110,0 | 77,3 | 109,0 | 82,4 | 107,0 |
| | 87,4 | 101,9 | 92,5 | 91,1 | 97,6 | 77,8 | 102,6 | 67,4 |
| 0,575 | 61,8 | 103,1 | 71,9 | 105,1 | 77,0 | 105,1 | 82,1 | 103,5 |
| | 87,2 | 99,5 | 92,2 | 90,4 | 97,3 | 76,4 | 102,4 | 60,3 |
| 0,670 | 61,8 | 88,0 | 71,9 | 89,7 | 77,0 | 90,3 | 82,1 | 89,0 |
| | 87,2 | 85,9 | 92,2 | 80,3 | 97,3 | 71,2 | 102,4 | 58,1 |

| J | N(r/min) and T(Nm) for liner no. 3 | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0,195 | 61,3 | 70,6 | 71,4 | 65,0 | 76,5 | 61,0 | 81,5 | 56,6 |
| | 86,5 | 51,2 | 91,6 | 44,1 | 96,7 | 35,9 | 101,7 | 30,0 |
| 0,339 | 61,3 | 108,0 | 71,4 | 105,6 | 76,5 | 102,4 | 81,5 | 98,5 |
| | 86,5 | 92,0 | 91,6 | 82,5 | 96,7 | 70,7 | 101,7 | 61,5 |
| 0,481 | 60,0 | 123,4 | 70,0 | 124,9 | 75,0 | 123,2 | 80,0 | 120,3 |
| | 85,0 | 114,0 | 90,0 | 103,6 | 95,0 | 89,4 | 100,0 | 74,6 |
| 0,578 | 60,0 | 123,7 | 70,0 | 126,7 | 75,0 | 125,7 | 80,0 | 124,1 |
| | 85,0 | lla,3 | 90,0 | 108,6 | 95,0 | 94,1 | 100,0 | 77,6 |
| 0,672 | 61,4 | 115,9 | 71,3 | 117,8 | 76,3 | 117,8 | 81,2 | 115,9 |
| | 86,0 | 110,0 | 91,0 | 102,1 | 96,0 | 88,4 | 101,0 | 73,6 |

| J | N(r/min) and T(Nm) for liner no. 4 | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0,195 | 61,4 | 71,7 | 71,3 | 72,7 | 76,3 | 72,7 | 81,2 | 71,7 |
| | 86,2 | 68,7 | 91,1 | 63,8 | 96,0 | 57,9 | 101,0 | 51,5 |
| 0,338 | 61,4 | 110,2 | 71,3 | 111,9 | 76,3 | 111,0 | 81,2 | 108,0 |
| | 86,0 | 102,1 | 91,0 | 93,3 | 96,0 | 82,8 | 101,0 | 71,3 |
| 0,482 | 61,4 | 127,7 | 71,3 | 130,1 | 76,3 | 129,9 | 81,0 | 126,7 |
| | 86,0 | 118,8 | 91,0 | 108,0 | 96,0 | 94,3 | 101,0 | 77,6 |
| 0,578 | 61,4 | 128,6 | 71,3 | 131,6 | 76,0 | 131,6 | 81,0 | 128,6 |
| | 86,0 | 121,8 | 91,0 | 110,0 | 96,0 | 95,3 | 101,0 | 79,3 |
| 0,674 | 61,4 | 121,8 | 71,3 | 124,7 | 76,3 | 125,7 | 81,0 | 123,7 |
| | 86,0 | 118,8 | 91,0 | 109,0 | 96,0 | 96,2 | 101,0 | 78,6 |

ENVIRONMENTAL Feature

# *Columbus's commitment to environmentally friendly development*

A decision by joint-venture partners Anglo American, Gencor, and the Industrial Development Corporation to proceed with the expansion of the Columbus Stainless Steel plant in Middelburg will contribute significantly towards the economic growth that South Africa needs so desperately. The expanded facility, expected to come on-line in 1995, will be the largest single-plant producer of stainless steel in the world, and the company will rank sixth overall in world production.

Middelburg, the eastern Transvaal, and the country as a whole will undoubtedly benefit from the accelerated industrial and commercial activities generated by the expansion, but the partners recognize that this activity must be undertaken in an environmentally sound manner. Columbus's policy and approach to sound environmental management are guided by several key tenets, one of which is the belief that legislative requirements *vis-a-vis* environmental matters are merely a minimum requirement that should be met as the company becomes a significant player in the stainless steel markets of the world.

## THE MISSION

The importance of the environment to Columbus is clearly demonstrated by the fact that environmental policy is guided by the first clause in the company's mission statement. In this guiding document, it is stated that the company 'will mobilize people and other resources to create wealth' by becoming the preferred and competitive supplier in the stainless steel industry in a manner that takes account of 'respect for and commitment to the individual and the environment'.

This statement, enunciated in the document that drives the fundamentals of the company, gives rise to a clear approach to the operation of the existing facility, and the development of the expansion project. Columbus is unique in that it shares its site with a sister, but nonetheless separate, company, Samancor's Middelburg Ferrochrome: this means that any approach to site environmental policy must be tackled jointly, since the environmental impact of one company's actions is inextricably tied in with the other, sited side-by-side as they are. The overall control of environmental aspects for the site and the two companies resides in the Joint Environmental Council, chaired by the Chief Executive of Columbus, Fred Boshoff, and composed of senior staff from both firms, as well as Denis Bruckmann, an environmental consultant with over 25 years of experience in dealing with steelmaking and the environmental matters entailed in its production.

Involved with Highveld Steel since its conceptual stage in the 1960's, and latterly as their director in charge of engineering, Denis has once again been involved in bringing a world-class steel facility from concept to reality, paying special attention to its interaction with the environment. He notes that, in this time when South Africa and her products are once again being welcomed into overseas markets, Columbus must 'do the right thing' from an environmental perspective, both because of international concern over the production of industrial goods in an environmentally friendly manner, and because of industry's responsibility to have a genuine concern for the environment. In his opinion, 'few companies appreciate the importance of grass-roots involvement with the community as Columbus does'. This commitment to the community in which Columbus is the largest employer has been demonstrated by the company's involvement of the community in environmental forums in various stages of project planning. All this adds up to Columbus's pro-active environmental policy.

## PRO-ACTIVE POLICY

Columbus has grasped the nettle as far as the environment is concerned, and has been working with the Environmental Evaluation Unit of the University of Cape Town to