Review of evaluation models for the representative sampling of ore

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Synopsis

Evaluation errors and value-frequency distribution models of the gold orebodies in the Witwatersrand and Free State systems are reviewed. It is found that the low-grade sections of the data are prone to the largest evaluation errors, and that an understanding of the shape of the value-frequency distribution is essential to the calculation of mean grade. The evaluation of a mineral process for a particular ore requires attention to mill-feed sampling, sample processing, and monitoring. In particular, the calculation of a representative mass of mill-feed sample (M_S) should take account of size-by-size value variance. Alternative M_S models are discussed.

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Introduction

A study is to be undertaken on the evaluation errors in laboratory-scale flotation tests on samples of Merensky ore. These flotation tests will examine ore in the dimensions of both time and space. It was felt necessary to precede these tests with a summary of the published literature pertaining to the evaluation of precious-metal ores in the contexts of both ore reserves and mineral processing. Such a review would assist in the planning of the later study.

Much of the available literature on the subject is drawn from the reserve estimation of gold ores in the Witwatersrand and Free State deposits. However, the principles reviewed are believed to form a good foundation for the examination of ores from the Bushveld Complex.

Statistical modelling of value distributions

Errors in the estimation of grade

Errors in the estimation of grade based on chip samples of ore from Witwatersrand gold mines were first modelled by Sichel[1], following earlier work by Watermeyer[2]. The latter pointed out that the distribution of gold in a certain block of ore followed a statistical law, and Sichel found that the error in the assay grade was grade-dependent. The percentage error distribution, or PCE, was modelled as q, where the frequency of the error is given by $F(q)$, with

$$F(q) = 1{,}251\,12(1 + (q/100))^{10} \cdot e^{-0{,}1q}. \quad [1]$$

This model was transformed to the transformed percentage error distribution by the substitution of

$$z = (1+(q/100)):$$
$$P(z) = 27\,557{,}3 \cdot z^{10} \cdot e^{-10z}. \quad [2]$$

When t = the obtained assay value of the sample
V = the true sample value
q = the percentage error

$$t = z \cdot V. \quad [3]$$

The probability of an assay value t returning a true value V was then modelled as

$$p(t) = 27\,557{,}3 \cdot ((t)^{10}/(V)^{11}) \cdot e^{(-10\,t/V)}. \quad [4]$$

From Equation [4], when $t = V$,

$$p(t) = 27\,557{,}3/V, \quad V > 0. \quad [5]$$

This would imply that lower grades have a higher probability of being returned with a larger error than do higher grades. This led to the following three observations from the PCE law:

(1) for the low-grade group of assays, the number of probable assays is higher than the respective number of true values
(2) for the middle assay group, the number of probables is less than the number of true values
(3) for the higher assay group, the number of probables is higher than the respective number of true values.

A grade-dependent error function was developed from the PCE and is used, with the group correction factor (GCF), to correct for these biases in the sampling data. The true sample value model, or TSV, was developed to the form

$$\Phi(V) = 488{,}666 \cdot e^{-k}, \quad [6]$$

where

$$k = (-0{,}471\,90 \cdot \ln V + 0{,}761\,55)^2.$$

Subsequently, Sichel modelled the error in the assay grade of a sample as a function of the mass of the sample. This error model showed that the error in the returned assay value of a sample was higher for small samples, of the order of less than 20 oz tr. (or ≈ 622 g) than in samples heavier than 40 oz tr. (≈ 1244 g), and defined the axiomatic nature of reckoning with the sample size towards accurate assay results.

These investigations by Sichel led to two important conclusions.

- The low-grade section of data in an ore reserve misleads the investigator in that the frequency of occurrence of low grades is overstated.
- This error is compounded by the need for low-grade samples to undergo more rigorous evaluation by the use of larger fusion masses for samples and a greater number of check assays.

The quality control of assay data for gold-bearing samples showed that the lower grades of samples tended to display higher variances between original and repeat assays than did higher grades of samples. A method was proposed by Coxon and Sichel[3] in the form of a control chart that established checking limits for the agreement between original and check assays. In their investigation, it was found that the initial sample preparation was susceptible to bias owing to inadequate riffling prior to the reduction of sample size, and that the lower-grade samples were most difficult to assay accurately because of the small mass of the prill obtained from a standard fusion. A summary of their results is shown in Table I and Figure 1.

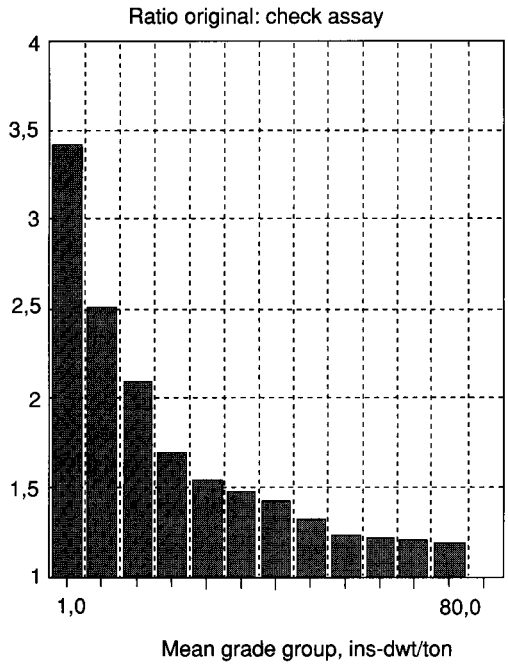

Figure 1—Comparison of original and check assays for routine mine samples (after Coxon and Sichel[3])

Table I

Comparison of original and check assays for routine assays of mine samples (Figure 1)

Value dwt/ton	Assay values, dwt/ton		
	Original	Check	Ratio, O/C
>125	134,80	115,20	1,17
100	108,40	91,60	1,18
80	87,10	72,90	1,20
60	65,60	54,40	1,21
40	44,10	35,80	1,23
20	22,70	17,30	1,31
10	11,75	8,25	1,42
8	9,51	6,49	1,47
6	7,28	4,72	1,54
4	5,02	2,98	1,69
2	2,71	1,30	2,09
1,5	2,15	0,86	2,50
1,0	1,54	0,45	3,42

This bias was later corrected for by the use of a larger mass of sample. A 10 per cent inspection density of mine samples was established.

In the subsequent development of the concept of quality control in underground sampling and attendant assaying, St. J. Rowland and Sichel developed control charts to monitor duplicate assays on one sample and check samples taken from a working place in a stope[4]. They showed that assay values, in dwt/ton, differed more at higher grades than at lower, but the ratio of original to check assay was statistically the more correct unit of adjudication.

The stabilized standard deviation, calculated as the standard deviation derived from the ratio of the original values to the check values, and a distribution of standardized units was plotted after logarithmic transformation. It was shown that this distribution was not normal.

A control graph was constructed from this model, based on upper and lower warning and action limits, to advise an investigator of the reliability of the sampling and assaying system for grade-control purposes. These charts used a logarithmic y-scale.

The quality control of assaying reported by Coxon and Sichel[3] was used by Krige in the form of a fan chart. This is a plot of routine mine assays on the y-axis against the parted check assays for the same samples as the x-axis. The controls are grade-proportional, i.e. more tolerant of differences at higher grades than at lower grades. The correct control unit was found to be the ratio of original assay to check assay, and the limits were calculated from large numbers of observed duplicate assays. However, the grade/error function documented contradicted this tolerance, and the lower-value samples displayed a higher variance between original and check assays.

This led to the conclusion that, generally, original assays over-valued the grade of samples; samples of lower-grade ore particularly showed the largest variance in terms of the true grade, and the error with which this was observed was inversely proportional to the grade.

Two- and three-parameter lognormal distribution models

In a statistical analysis of borehole values in the Free State goldfield, Krige showed that a lognormal frequency curve can be applied to the observed distribution of gold values in the main sector of the goldfield. He demonstrated that the frequency distribution of values could be normalized by the lognormal frequency plot[5]. He also showed[6] that the gold values of a given section of a mine, when collected in the form of a frequency distribution, had a positive skewness, with the mode to the left of the mean and a long drawn-out tail to the right. When such data were plotted on a logarithmic scale, the distribution became more symmetrical and approximated the normal distribution.

It was also shown that the same lognormal data can be plotted as a straight line on probability paper on which the x-scale is probability and the y scale is logarithmic. The straightness of the plot gives an indication of the degree to which the data fit a lognormal model.

A further development in this model was made for certain orebodies that did not produce a straight-line plot but showed a curvilinear form, with departure from linearity in the low-grade area of the data. In such cases, Krige proposed that a third term be added to the lognormal model:

$$\Phi(x) = \left[\sigma \cdot (2\pi)^{0,5}\right]^{-1} \cdot \exp\left[-\left(1/2\sigma^2\right)(x-\alpha)^2\right], \quad [7]$$

where

$x = \ln(z + a)$
α = mean of $\ln(z + a)$
a = additive constant
σ^2 = variance of $\ln(z + a)$.

In the sampling of lognormal populations, the arithmetic mean is an unbiased estimate of the population mean if the population is regarded as an infinite number of individuals obtainable from repeated sampling using the same procedures. However, the physical sampling can introduce a bias in relation to the population values; this is often the case in underground sampling. No mathematical treatment can correct for such a bias.

Having taken the error distribution into account, Krige concluded that the limits for the sampling mean and other estimators would be significantly skew, and this should be borne in mind in all ore-evaluation procedures in which lognormality applies.

Where small data samples are taken from such a lognormal distribution, the likelihood of such sampled data including a value from the high-grade tail of the distribution, and thus influencing the estimate of the mean, should be considered. In this regard, it was concluded that the geometric mean, after correction, should be used.

The Sichel t-estimator

Sichel[7] reviewed the issue of sampling from lognormal populations by estimating the population from small samples.

The estimation of an orebody from a few borehole cores raised the problem of small data sets using arithmetic means instead of lognormal, yet having to account for the likelihood that the small data set included an observation from the right-hand side of the skew distribution.

In larger populations, this is not as much of a problem since the data set is robust. In the case of small data sets, the arithmetic mean is misled by such high grades, and the inclusion of the latter leads to over-estimation of the mean value of the ore grade.

In a case study of the Harmony Gold Mining Company, a database of 5170 values obtained from individual sample sections of underground faces drew an arithmetic mean of 618 ins-dwt, and the data were subdivided on a random basis to 1034 subsets of 5 data. In an extreme case, subset no. 637 yielded the data shown in Table II.

Table II

Subset no. 637 of 1034 values at Harmony Gold Mining Company

Observation no.	Assay value, ins-dwt
1	329
2	277
3	111
4	189
5	5071
Mean	1195

The arithmetic mean of this subset did not agree well with the population mean of 618 ins-dwt; yet the value of 5071 ins-dwt was valid and had to be included in the data. If small data sets are used in the estimation of the mean grade of an orebody, neither the exclusion of the high value nor its inclusion provides an accurate estimate of the grade. The other 1033 subsets were not, however, reviewed.

The Sichel t-estimate of the mean grade in such a case is calculated as the corrected geometric mean:

$$t = e^{\bar{x}} \cdot \tau_n(V), \qquad [8]$$

where

t = Sichel t, or corrected geometric mean
$\bar{x}$ = $(1/n) \cdot \sum x_i$ for $i = 1$ to n
x = $\ln(z)$
z = raw grade, e.g. ins-dwt/t, or as cm-g/t,
V = $(1/n) \cdot \sum (x_i - \bar{x})^2$ for $i = 1$ to n
$\tau_n(V)$ = $1 + \sum (n-1)^r \cdot V^r/(2r \cdot r!(n-1) \cdot (n+1) \ldots (n+2r-3)$
x_i = $\ln(z_i + \alpha)$.

The use of Sichel's t-estimator reduces this estimate to 894 ins-dwt and, although closer to the population mean, is still +44,6 per cent overstated. Therefore, both the arithmetic mean and the Sichel t-formulae do not derive accurate grade statements from a small population in this case.

Some advances on this theory were reported later by Sichel[8], who indicated that, for the typical case of 5 to 20 boreholes drilled for a particular orebody, the lognormal probability plot was not applicable since much larger quantities of data are necessary. This limits one from calculating the third lognormal parameter, α, in borehole data to correct for the under-estimation of low-grade sections of the orebody.

In such a case, the robustness of the Sichel t-estimate of the variance of the lognormal distribution forms the subject of a sensitivity analysis. It was concluded that, for large variance and large shape factors, the t-estimator was not robust.

Subsequently, Wainstein[9] applied the typical lognormal theory to risk-analysis models for new mining ventures. He stated that the lognormal model remained the most popular for the modelling of ore values, and that the t-distribution, a central part of the theory, was dependent on the sample size, n, which, for borehole data on an undeveloped orebody, is usually less than 10. It was found that the t-distribution was robust for values of the nuisance parameter σ^2 between 0,1 and 2,5. However, $\sigma^2 = 0,7$ can be used with an error of ± 1 per cent, which is acceptable for such calculations.

Later, Clark[10], using personal computers instead of mainframe computers, reported progress with the data processing of the Sichel t-estimator. She concluded[10] that tables of Sichel's t could be calculated accurately, obviating the need for linear interpolation between intervals of V.

Clark preferred the use of the natural logarithm to that of the base-10 logarithm, since the latter introduces an unnecessary constant. The overall conclusion of this work[10] was that more-accurate calculations resulted from the computed values of Sichel's t than from the older table-and-graph method.

New developments in borehole evaluation were reported by Krige and Assibey-Bonsu[11] using a very large number of chip samples from a mined-out area of the Hartebeesfontein Mine. The data were subdivided into smaller units, and were evaluated in the form of simulated borehole data to represent the overall distribution of data. They concluded that, wherever possible, the three-parameter lognormal model should be used when individual borehole values are available, leading to the Sichel t-estimator. However, in the case of very large databases for an existing mine, there is little or no advantage in the use of the lognormal model instead of the normal model for the calculation of mean grade.

Sichel *et al.*[12] pointed out the limitations of the lognormal model for a specific case of Ventersdorp Contact Reef (VCR). It is likely that the placer theory for the Witwatersrand System should be modified to account for the distribution of gold values in the VCR. As distances from the source of transportation increased, the ability to carry coarse gold was reduced, resulting in the deposition of coarse gold nearer to the source. Thus, there should be a relationship between the log-means and the log-variances of the gold variable as a function of distance from the stream source.

Further, the erosional features of certain types of reefs invite the re-location of high-grade values, leaving sub-lognormality in the parent zone. The use of this approach reduced the bias of the gold-grade estimate from less than 5 per cent for the three-parameter lognormal model to less than 2,5 per cent for the compound lognormal model. It was concluded that the plot of the empirical cumulative frequency distribution is very necessary as a test of departure from lognormality.

Evaluation of a mineral process

Gathering of plant performance data

In a summary of process evaluation in the dimension of time, Bartlett and Hawkins[13] addressed principally the gathering of long-term data on process performance for statistical and analytical purposes. These data are historical, and are generated from the sampling and analysis of production process streams.

The principal components of such an evaluation system are as follows:

- sampling of process streams to produce representative samples
- preparation of the samples, usually involving dewatering, drying, crushing, and pulverization
- capturing of mass-balance data
- analysis of samples
- analysis of data for the production of metallurgical balances.

Bartlett and Hawkins[13] used conventional statistics to calculate the mean and standard deviations, and then calculated the coefficient of variation, as follows:

$$s^2 = \left(\sum x_i^2 - n\bar{x}^2\right)/(n-1), \quad [9]$$

where

$\bar{x}$ = the mean value of the observations
x_i = the ith observation of x
n = the number of observations
s^2 = the variance of the mean
s = the standard deviation of the mean.

The coefficient of variance, CV, is then given by

$$CV = (s/\bar{x})\cdot 100. \quad [10]$$

This is also referred to as the relative standard deviation, or RSD.

The standard error of the mean is then given by $s/\sqrt{n}$.

The upper and lower 95 per cent confidence intervals are $\pm 2s/\sqrt{n}$. It was observed that, for sample sizes of of 30 or more, 68 per cent of the observations would fall within $\bar{x} \pm s$, and 95 per cent of the observations would fall within $\bar{x} \pm 2s$.

Bartlett and Hawkins[13] studied the limitation of the normal distribution to describe the distribution of gold values in a run-of-mine ore stream prior to any separation of the gold, concluding that the lognormal distribution described the data more effectively.

As the normal distribution is symmetrical, the two key parameters in describing this symmetry are skewness and kurtosis, given by

$$\text{skewness} = m3/(s^3) \quad [11]$$
$$\text{kurtosis} = m4/(s^4), \quad [12]$$

where $m3$ and $m4$ are the third and fourth moments of the distribution respectively. These are calculated as

$$m3 = (1/n)(\sum x_i - \bar{x})^3 \quad [13]$$
$$m4 = (1/n)(\sum x_i - \bar{x})^4. \quad [14]$$

For normally distributed data, kurtosis = 3 and skewness = 0.

Table III

Types of sampling errors and their origins

Type of error	Origin
Fundamental error	Particulate nature of ore
Group and segregation error	Inhomogeneous mixing of ore
Weighting error	Uneven flow of ore
Long-range quality-fluctuation error	Natural variability
Periodic fluctuation error	The quantities to be measured
Increment delimitation error	Incorrect cutter design
Increment extraction error	Incorrect cutter speed

Sampling of mill feed

The best known of the theoretical methods to determine the precision of sampling are reported by Gy[14]. Sampling errors as listed by Bartlett and Hawkins[13] are given in Table III.

The representative sample size (Ms) was calculated as follows:

$$Ms = c\cdot l\cdot f\cdot g\cdot d^3/(s^2), \quad [15]$$

where

$$c = \tau/a \quad [16]$$

τ = relative density of the precious mineral
a = the grade of the precious mineral expressed as grams per tonne of ore
s^2 = the sampling variance

$$f = V/(d^3) = 0{,}5 \text{ for most cases} \quad [17]$$

d = mean diameter of the ore particle
V = volume of the ore particle

$$g = \left(l/\left(d^3M\right)\right)\cdot \sum_{i=1}^{n} M_i d_i^3, \text{ where} \quad [18]$$

d = mean particle diameter in cm
M = lot mass in g
d_i = the ith size fraction, expressed as particle diameter
M_i = the ith fractional mass corresponding to d_i

$$l = \sqrt{(d_l/d)} \quad [19]$$

d_l = liberation diameter of the precious mineral.

Bartlett and Hawkins[13] proposed further that the sampling variance was indirectly proportional to the cube of the top particle size in the distribution of the sizes being sampled.

As a cross-check, they reviewed Gy's fifty-piece experiment, in which fifty or more specimens of rock are taken from a conveyor belt carrying run-of-mine ore. The solids density and assay value of each are then determined.

The results of this experiment are transposed with equation [20] to give the relationship between the fundamental variance and the sample mass:

$$f_v = (l/M_s)\cdot\left(gv/a^2M\right)\cdot \sum_{i=1}^{n}\left(\left(a_i - a\right)^2/V_i\right)\cdot M_i^2. \quad [20]$$

The acceptable fundamental variance, f_v, is defined by the investigator, e.g. 8 per cent, and equation [19] reduces to

$$M_S = K/f_v, \quad [21]$$

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where K = a real number derived from the calculation of g, v, a, and M.

The resulting value of M_S is sensitive to observations of grade with high values. This results in increasing the M_S value by large amounts. The value of M_S is also sensitive to size-dependent value concentration into fine fractions. Thus, if only large specimens of ore are taken for the experiment, the lower grades found in the coarser sizes will under-state the value of M_S. The combined effect of grade and particle size were reported by Laplante[15] to be very significant in the calculation of M_S.

He suggested a further enhancement in the M_S model for alluvial tin deposits, where the M_S should be divided by the fractional mass of tin finer than the topsize. This indicates that the model would under-estimate the M_S unless the size-related distribution of values were taken into account. Laplante suggested a similar approach for gold and silver in solid solution or finely disseminated in sulphides.

Laplante[15] further interpreted Gy's equation as a form of Poisson distribution in which events have a low probability of success but a large number of trials. One property of this law is that, if an average of n successes is calculated given a certain sample size, then the standard deviation of the average will be $\sqrt{n}$. Thus, if a sample contains 100 chalcopyrite particles on average, 100 ± 10 particles will be encountered, yielding an RSD of 10 per cent. Gy's model can therefore provide an estimate of the order of minimum sample size, but it has its limitations. A cross-check was suggested by use of the d_{95} of a particle-size distribution in the case of a mill feed. Here the Poisson distribution is used, since an investigator, by calculating the mass per topsize particle in the distribution, and requiring that 100 particles of topsize are polled, can calculate the representative mass of topsize.

Thus, the total mass of sample required, i.e. inclusive of the whole size distribution, is the representative sample mass calculated as M, where

s = density of the ore, t/m^3
S = topsize in particle-size distribution, cm
m = mass of topsize particle, g
r = radius of topsize particle, cm

$$m = s \cdot ((4/3) \cdot \pi \cdot r^3). \qquad [22]$$

For 100 topsize particles to be included, the mass of topsize to be sampled will be 100 m, and the total mass of sample, M, will then be $(100/5) \cdot 100\ m$.

Use of the semi-variogram

Clark[16] reviewed the construction and use of the spherical semivariogram after Matheron, initially in the spatial dimension for the analysis of relationships between sample values in a borehole drilling programme. Although there are several versions of the semivariogram for special cases, the spherical model is regarded as the simplest and most convenient in a first analysis.

The semivariogram value $\tau(h)$ is calculated for different spacings of data, i.e. $\tau(1)$ is calculated as

$$\tau(1) = (1/2N) * \sum_{i=1}^{N} (g_i - g_j)^2, \qquad [23]$$

where

i and j are data separated by 1 distance unit
g_i = the grade at location i
N = the number of data.

Equally, $\tau(2)$ is calculated from equation [23] but with 2 distance units between i and j. The successive results of $\tau(h)$ are plotted on the y-axis, and the corresponding h value on the x-axis. A typical format of this version is shown in Figure 2, with the following construction:

$$\begin{aligned} &\tau(h) = 0 && h = 0 \\ &\tau(h) = C\left[(1{,}5 \cdot h/a) - \left(0{,}5 \cdot \left(h^3/a^3\right)\right)\right] + C_o \\ & && 0 < h \le a; \\ &\tau(h) = C + C_o && h > a. \end{aligned} \qquad [24]$$

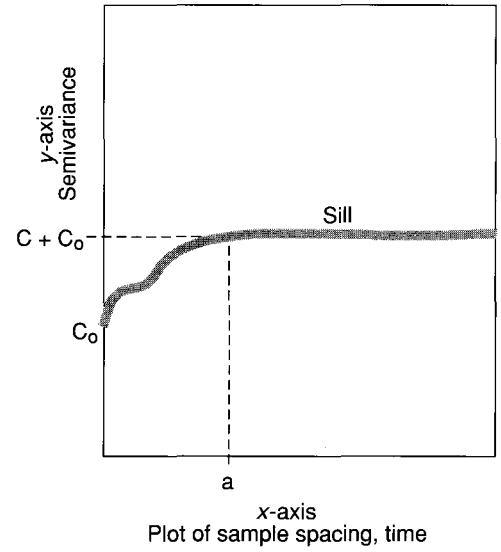

Figure 2— The semi-variogram

The overall semivariance in the sample data should be equivalent to the sill value $C + C_o$.

Semivariograms can also be used in the time dimension, as in a production mineral process in which a crushed mill feed is conveyed to a primary milling step. Apart from the necessary rules of the probability of inclusion for any particle to enter the sample increment, the semivariogram informs of the minimum time between sample increments, as well as the overall variance in the sample data.

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It is possible, therefore, for an investigator, in setting up an evaluation system for a mineral process, to determine the fineness to which a mill feed should be crushed prior to assaying. He can do this by using a series of intermediate crushing steps with semivariograms at each stage in the dimension of replicates to determine the minimum number of assay determinations required for that size distribution.

In such a case, the recommended method is to take a complete belt cut per increment, i.e. to stop the conveyor belt and place a metre-long frame over the belt, and then scoop the entire contents of the conveyor belt between the two frame limits into a sample bucket. This includes all the material on the belt between the limits of the frame.

Processing of the mill-feed sample

The crushing of the sample prior to subdivision is limited by the reduction ratio of the sample-crushing process. The percentage cut into the subsample is related to the topsize of the original sample, d_1, and the subsample topsize, d_2, by

$$M_1/M_2 = (d_1^3/d_2^3), \qquad [25]$$

where

M_1 = mass of sample, kg
M_2 = mass of subsample, kg
d is quoted in mm.

The crushing and blending of a representative ore sample have the advantage of reducing whatever skewness may exist in the primary sample-value distribution[17].

It is likely that, for the time dimension (viz samples of production mill feed from month to month), many of the spatial constraints of skew distributions mentioned earlier will have a minimal effect. The critical parameter in time would then be a structured method of systematic crushing, blending, and subsampling followed by the inspection of replicate head samples for scatter in terms of RSD. This is a procedure for which control graphs should be applicable.

Monitoring of a mineral process

Subsequently, Krige[17] documented the statistical quality controls for mine valuation, listing three areas amenable to cross-checking: mill values, assaying, and mine sampling.

Initially, a plot of daily mill feed plus values over a 99-day period of production yielded a lognormal plot of frequencies. The distribution was cut off on either side at the 2,5 per cent level, thus excluding only 5 per cent of the values in the distribution. The warning limits at the 5 per cent level were then overlain on the plot of daily values, and the equivalent 1 per cent level limits were also calculated and shown. The latter refer to action limits. The population mean was then estimated from a geometric formula. Such a graph provides an objective means for the interpretation of whether the significance of the nth mill feed value is within normal scatter.

Krige[17] demonstrated a different form of graph for use with daily mill-feed values plotted in a progressive form with 5 per cent limits, overlain on the plot of daily values.

Conclusions

This review has examined the available literature on the characteristics of ore evaluation, mainly for gold-bearing ores.

Sampling of ore in time

In the sampling of a production mill feed from a conveyor belt to provide a flotation laboratory with representative material, four checks are necessary:

- the fifty-piece experiment, taking note of the consequence to sample size of mill-feed topsize and of variance between the grades of metal in the topsize
- the fundamental sample-size equation of Gy as a cross-check of the sample size
- the possibility of Laplante's proposed method to poll 100 pieces of topsize in the sample based on an interpretation of the Poisson distribution, thus giving a sample size through knowledge of the d_{95} of the particle-size distribution
- the semi-variogram as a check on the sampling interval between increments so as to minimize short-term scatter effects in the sampling variance.

Further checks on the scatter between replicate subsamples of crushed and blended ore prior to flotation are advisable. This is to confirm whether significant skewness in the value distribution between replicates has, in fact, been reduced to inconsequential levels.

Sampling of ore in space

In the extension of flotation tests to support an exploration programme for a new orebody, many of the problems identified by Sichel, Krige, and others are encountered, in that small sample populations from a borehole drillcore programme of the order of n ±5–10 may be drawn from a lognormally distributed population. In the present case, certain exercises in Merensky and UG-2 ore will be conducted in the form reported by Sichel[7] and Krige *et al.*[6]. If lognormality is proved in such an exercise, the use of Sichel's t-estimator for recoverable grades, as well as assay head grades, may be applicable; further, the three-parameter distribution model of Krige may be valid. Should lognormality not apply, the procedures for sampling in time for the projected programme will be re-examined for use in the spatial dimension. ◆

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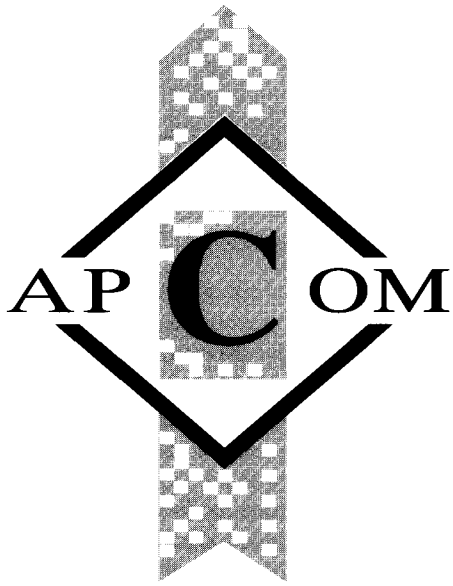

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