The design of tunnel support in deep hard-rock mines under quasistatic conditions

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Synopsis

The induced elastic stresses in the sidewalls of gold-mining tunnels at a depth of 4000 m in the Witwatersrand exceed the peak compressive strength of even the best-quality quartzite rocks. Laboratory tests measuring post-yield stress versus strain response show that brittle fracture occurs, followed by a loss in strength associated with strain-softening behaviour. Numerical modelling based on an assumed Mohr-Coulomb criterion fitted to the test results shows that large ongoing deformations of the sidewall occur, leading to total tunnel failure without adequate support.

A methodology is presented to enable a quick assessment to be made of support requirements in terms of ground characteristic curves for numerically derived pressure versus displacement. From these curves, estimates can be made of the force levels for cable bolting and of the thickness and strength requirements for shotcrete linings. As an example of its application, the methodology is used in a comparison of the relative stability efficiencies of a square-shaped tunnel and a horizontal ellipse-shaped tunnel.

Introduction

After gradual extraction of the shallower gold-bearing reef, mining in South Africa is being carried out at increased depths (approaching 4000 m). Spearing1 has presented principles relating to the excavation and support of mine tunnels with a desired final cross-sectional area of 14 m² at a depth of 4000 m in Witwatersrand quartzite. Elastic stress analysis of mining tunnels of typical shapes—square and rectangular—showed that there is a high potential for failure of the surrounding rock. This is most prevalent in the sidewalls of tunnels owing to the relatively low horizontal and high vertical field stresses at such depths (k-ratios of 0.5 are common). The extent of the potential failure into the sidewall can be reduced somewhat by reduction of the tunnel height accompanied by an increase in the span to maintain the same cross-sectional area. A horizontal ellipse showed promising results in this regard, and preliminary proposals for the support of such a tunnel shape using cable bolting and shotcrete were presented. This comprised rings of nine 4 m long cables at a longitudinal spacing of 1.25 to 1.5 m and shotcrete of 100 to 300 mm in thickness. This was a significant development in that traditional support comprised far longer bolts (longer than 8.0 m was typical).

Although Spearing identified the potential for failure, he did not quantify the anticipated rock-mass behaviour and the tunnel response to such behaviour. He did, however, implicitly expect non-elastic/plastic behaviour in that he proposed a yielding type of cable bolting.

The objectives of this paper are to expand on the principles raised by Spearing, with greater emphasis on the following specific rock-mechanics aspects:

> the mechanical properties of typical Witwatersrand quartzites, particularly those relating to post-failure behaviour
> the use of numerical modelling techniques to predict the deformation response of tunnel excavations at depths of 4000 m
> the development of a methodology based on numerical modelling in the design of support to control such deformation and to define the degree of safety against tunnel collapse.

The developed methodology is applied in an example presented in this paper.

Post-peak stress versus strain behaviour in Witwatersrand quartzites

Witwatersrand quartzites are typically 'hard', with a uniaxial compressive strength (UCS) of the order of 100 to 250 MPa. Elastic stress analysis of mine tunnels of common shapes at depths of 4000 m has shown that the induced sidewall stresses are between 175 and 200 MPa

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for a square tunnel, and between 150 and 200 MPa for a rectangular tunnel. Maximum stresses in the corners of the excavations are as high as 250 MPa. The same analysis for a horizontal-ellipse tunnel shows sidewall stresses of the same order, but over a smaller area of extent owing to the smaller overall height of the tunnel.

The analysis shows that even the best-quality quartzite will be over-stressed, and the post-peak behaviour is therefore of interest. Traditionally, such post-peak behaviour is not considered in rock-mechanics tests measuring the UCS.

However, with more specialized tests using a servo-controlled machine that is programmed to a constant strain rate, the post-peak stress versus strain in rock specimens can be measured.

Several typical Witwatersrand quartzites have been tested in such a machine under triaxial conditions at varying cell pressures, giving the strength–strain curves shown in Figure 1. From these curves, equivalent Mohr–Coulomb shear-strength parameters of cohesion, friction angle, and dilation angle were derived by a curve-fitting procedure.

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Figure 1—Strain-softening behaviour of typical Witwatersrand quartzites (after Kersten)
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The behaviour shown in Figure 1 depicts brittle fracture followed by the rapid decrease in strength typical of strain-softening behaviour in which the friction angle remains constant after full mobilization whilst the cohesion drops to practically zero at a strain of 1.0 per cent.

The Mohr–Coulomb criterion pre-supposes three conditions:

- that a major shear fracture exists at peak strength
- that the direction of fracturing is at a specific angle to the maximum principal stress (Figure 2)
- that there is a linear relationship between the shear strength and the normal stress on the plane of the fracture.

Observations in the fields have shown that these conditions are not always satisfied, particularly the orientation of fracturing. Stacey found that a criterion based on tensile (extensional) strain fitted the extent and orientation of the observed fractures in areas of low confinement, i.e. close to the surface of the excavation.

The appropriateness of the extensional strain criterion to areas of higher confinement away from the surface of the excavation is open to question, as is also the fundamental basis of the criterion.

It is concluded therefore that no single criterion can or should be used exclusively in the study of a given problem of rock stability.

In this paper, the Mohr–Coulomb criterion is used for the following reason. The problem of support design is not the prediction of initial fracturing (at the surface of the excavation) but, rather, the prediction of the total surface deformation, which itself is a manifestation of the deep-seated plastic strains in the body of the rock under high stress confinements. It is believed that the shear criterion will predict these strains with sufficient engineering accuracy. It is therefore assumed in this paper that the Mohr–Coulomb parameters can be used in numerical modelling to predict tunnel-excavation response in the quartzites shown in Figure 1 at a depth of 4000 m.

This approach will be tested by in situ monitoring at a later stage.

Numerical modelling of tunnel-deformation response

Numerical methods of analysis are based on calculations of stress versus strain, usually in the form of a computer program and, as such, are relevant in cases where the induced stresses around a tunnel excavation are important such as those featuring in this paper.

A number of computer packages are available for use, and the package selected must be related to the requirements of the problem requiring analysis. This study required a program that can model tunnel support in various ways and that can accept, without the occurrence of numerical instability, large deformations due to strain-softening rock behaviour. The program FLAC, which is based on a procedure of finite-difference calculation, is well suited to such an application.

Common mining supports such as cables and shotcrete and their interaction with a rock mass can be modelled by use of FLAC. The cable element allows the user to specify pre-tensioning, grouting, and the length and spacing of each cable, either with or without a faceplate. Shotcrete is modelled as a beam element that is, in effect, totally bonded to the rock. Support pressure can also be modelled as an applied pressure to the periphery of the tunnel excavation or to part of it.

All the plasticity models in FLAC use the same formulation. This consists of a yield function and a plastic-flow function to the deforming model, a shear-yield function, and a non-associated shear-flow rule (i.e. dilation can be modelled), which are based on Mohr–Coulomb elasto-plasticity. The difference, however, in the strain-softening model lies in the ability of the cohesion, friction, and dilation to soften after the onset of plastic yield whereas, in the Mohr–Coulomb plasticity model, the cohesion, friction, and dilation are assumed to remain constant. The user defines the cohesion, friction, and dilation as piecewise linear functions of the plastic strain. The code determines the total plastic strain at each time increment, and causes the cohesion, friction, and dilation to conform to the user-defined functions.

Some initial modelling of tunnels of various shapes without support at a depth of 4000 m, with the (FLAC) Mohr–Coulomb strain-softening constitutive material having pebble quartzite parameters, showed that sidewall deformations of up to 1 m are possible. Such deformations would practically close a tunnel or render it unusable. Such orders of closures have already been measured in existing shallower tunnels. For example, a 3 m by 3 m haulage at a depth of 2300 m in argillaceous quartzite with a UCS of 90 MPa excavated in the 1980s is still in places experiencing closure rates of up to 50 mm per week.
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The large sidewall displacements predicted during this initial modelling show that a yielding type of support would be required, as proposed initially by Spearing1.

Support design based on numerical modelling

Several papers published overseas and locally have described the use of numerical methods in the design of tunnel support10-12. The basic approach followed by the authors of these papers was to choose a trial support system, input its constitutive behaviour into the numerical model, and check if the trial support can bring the excavation into equilibrium without over-stressing the support elements. If not, stronger or denser support is required and remodelled.

This approach, although successful, has proved somewhat time-consuming, particularly since a number of different combinations of support can be used to stabilize the excavation without necessarily indicating the optimum solution. In addition, computing times using FLAC and a PC 486 computer can easily exceed a day.

An alternative approach, one that is presented in this paper, is to derive from the numerical model a relationship between support pressure on the sidewall and displacement of the corresponding sidewall. This relationship is essentially the well-known ground reaction, ground characteristic curve, or required support line4.

With the large deformation capability of FLAC, various sidewall pressures can be inputted, even those that are too low to prevent the finite-difference grid from distorting excessively. The calculated sidewall displacement at stabilization can then be plotted against each applied pressure to provide a curve of pressure versus displacement. Where the pressures are too low, the grid will continue to deform until the iterative solution cannot proceed any further owing to poor geometric considerations. The critical pressure that just stabilizes the grid can be determined quickly, and can be referred to as the pressure at a factor of safety of unity, or the critical pressure.

The critical pressure at which a tunnel 'collapses' is probably dependent to some extent on the fineness of the grid and the boundary conditions. For the study, a quarter-symmetry grid of 2500 zones (50 by 50) was used. The symmetry axes were fixed against normal displacements. The right-hand boundary was placed at a distance of 50 m from the centre of the tunnel, which is 11.8 times the equivalent diameter (4.225 m) of the excavation with an area of 14 m². This boundary was fixed against horizontal displacements. The top boundary was also placed at a distance of 50 m, but a pressure was applied at this level equal to the overburden stress at this level.

The horizontal displacement of the tunnel springline was used to monitor whether the FLAC run was stabilizing, since this part of the tunnel periphery was most highly stressed and showed the highest displacements. The runs, however, were always continued for up to 90 000 timesteps or until the FLAC grid became 'unstable' the message returned being bad geometry. Further analysis is required to assess the effects of the mesh fineness on the results.

Once the curve of pressure versus displacement has been defined, together with the critical pressure, decisions can be made on the allowable displacement, and the corresponding required pressure can be read off the curve. This pressure can be referred to as the design pressure. The ratio of the design pressure to the critical pressure is defined as the factor of safety of the tunnel excavation.

Once the design pressure has been defined, the pressure can be manually converted into equivalent forces for cables or other rock reinforcement. Also, the design pressure can be used in manual computation of the required surface support between the rock reinforcement. For example, shotcrete acting as a beam in bending and shear, or mesh acting as a catenary, can be designed.

The development of pressure-versus-displacement curves is illustrated in the following case study.

Case study: Horizontal ellipse versus square tunnel profiles

As part of a study13, the relative performances, as regards support requirements, of a square tunnel and a horizontal ellipse had to be compared. Each had a cross-sectional area of 14 m² and was excavated at a depth of 4000 m (therefore at assumed field stresses of 104 MPa vertical and 52 MPa horizontal) in a strain-softening pebble quartzite having the properties given in Figure 1 and shear and bulk moduli of 29 and 33 GPa respectively.

The following relationships were used.

Shear modulus, \( G = \frac{E}{2(1 + \nu)} \)

Bulk modulus \( = \frac{E}{3(1 - 2\nu)} \)

where \( E \) is Young’s modulus and \( \nu \) is Poisson’s ratio.

The rock properties used in the FLAC runs were as follows:

- Unconfined compressive strength 57.7 MPa
- Bulk modulus 6.7 GPa
- Shear modulus 4.0 GPa.

The moduli values are lower than the laboratory values in order to reflect the highly stressed rock mass at depth, and correspond closely to typical underground displacements. More reliable estimates of these parameters will be obtained by in situ monitoring at a later stage.

By use of the FLAC program (an input file is given in the Addendum), initially without any applied support pressure to the inside of the tunnel excavations, it was possible to identify the portion of the tunnel periphery that required significant support to prevent collapse of the model mesh. Various pressures were then applied, and the sidewall displacement at the springline level was computed (the springline is defined as the mid-height of the tunnel sidewall).

The required support lines for each profile in terms of sidewall pressure versus horizontal displacement (dilation) of the springline are shown in Figure 3. The following can be seen.

- If the sidewall pressure drops below 250 kPa, the square tunnel will not stabilize. If the sidewall pressure drops below 460 kPa, the elliptical tunnel will not stabilize. These pressures are the critical pressures at which the factor of safety of the excavation is assumed to be unity.
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Table I
Comparison of support pressures, showing the effect of tunnel shape

<table>
<thead>
<tr>
<th>Sidewall displacement mm</th>
<th>Pressure kPa</th>
<th>Total force kN/m (square)</th>
<th>Individual force* kN/m (square)</th>
<th>Pressure kPa</th>
<th>Total force kN/m (ellipse)</th>
<th>Individual force* kN/m (ellipse)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>2050</td>
<td>7671</td>
<td>1279</td>
<td>1800</td>
<td>4399</td>
<td>1100</td>
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<tr>
<td>400</td>
<td>1600</td>
<td>5987</td>
<td>998</td>
<td>1000</td>
<td>2444</td>
<td>611</td>
</tr>
<tr>
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<td>1150</td>
<td>4333</td>
<td>717</td>
<td>700</td>
<td>1711</td>
<td>426</td>
</tr>
<tr>
<td>600</td>
<td>870</td>
<td>3296</td>
<td>543</td>
<td>600</td>
<td>1124</td>
<td>281</td>
</tr>
</tbody>
</table>

* Pressure-versus-displacement relationships are shown in Figure 3

Total horizontal force = Pressure X 3.74 m (square)
= Pressure X 2.44 m (ellipse)
Six individual horizontal forces are required for the square to prevent collapse of the tunnel between the forces in the absence of surface support. For the ellipse, four at least are required.

- With decreasing pressures below 500 kPa, the sidewall displacement of the square profile increases so rapidly that the tunnel becomes dysfunctional.
- For a given pressure above the highest critical pressure, the sidewall displacement of the elliptical profile is less than that for the square.
- The sidewall pressure for the elliptical tunnel has to be applied over a height of 2.4 m on the sidewall, compared with a height of 3.6 m for the square tunnel.
- If a design criterion of 500 mm maximum allowable sidewall displacement is adopted, then an applied pressure of 700 kPa is required for the elliptical tunnel, compared with 1150 kPa for the square tunnel. These pressures are the design pressures. In terms of total horizontal sidewall force, the elliptical tunnel requires a force of 1700 kN/m longitudinally, compared with 4300 kN/m longitudinally for the square tunnel.

Table I gives the required sidewall support pressures and total horizontal forces for various allowable sidewall displacements. Also given are the number and value of individual

Figure 3—Support lines required in terms of sidewall pressure versus sidewall displacement
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forces required to provide the total horizontal force and to prevent collapse of the tunnel between these forces, in the absence or ineffectiveness of surface support such as mesh and lacing or shotcrete. It can be seen from Table I that the total horizontal support force is higher for the square tunnel. Since the cost of tunnel support is related directly to the required force capacity, it can be inferred that the cost of supporting the square tunnel could be approximately 2.5 times the cost of supporting the ellipse in order to realize the same level of sidewall deformation.

Design methodology

Based on the case study, the following site-specific methodology is recommended.

(1) By direct measurement, extrapolation from similar cases, or three-dimensional elastic modelling, obtain an estimate of the field stresses acting prior to the tunnel excavation.

(2) Obtain the strain-softening behaviour of the specific rock by carrying out triaxial compression tests in a servo-controlled machine that tests at a constant strain rate, and then fit equivalent Mohr–Coulomb parameters to the laboratory-obtained curves of stress versus strain.

(3) Model the tunnel excavation using the two-dimensional FLAC program, and run it initially without any support in the large strain mode. This procedure will allow identification of that part of the excavation periphery that will require support to prevent the FLAC grid from collapsing through excessive distortion.

(4) Apply various pressures to the part of the excavation periphery identified in (3), which gives the corresponding wall displacement at stabilization. In this way, a pressure-versus-wall displacement can be developed that is the numerically derived ground characteristic curve.

(5) Choosing an allowable maximum deformation level and using this curve, calculate equivalent force levels in various chosen directions and at various longitudinal spacings in the tunnel. These forces represent the required working loads of the rock-reinforcement elements being considered. It should be noted that the installed support must be able to accommodate the deformation level selected without failing (i.e., the factors of safety of the individual support elements must all be greater than unity).

(6) In the same way, compute the shotcrete thicknesses and mesh strengths, using the design pressure derived from the characteristic curve and the spacings of the rock-reinforcement elements.

(7) Undertake final FLAC runs in the usual manner, using cables and shotcrete dimensioned on the basis of the characteristic curve. Such runs will be far fewer than otherwise required since the required support pressure/force levels have already been determined, and more optimum cost-effective solutions can be obtained.

Conclusions

Based on this paper, the following conclusions can be drawn.

(1) The use of numerical stress analysis in the design of tunnel support has traditionally involved a trial-and-error basis, which may not necessarily give an optimum design.

(2) The approach presented in this paper requires the derivation of the support line or the ground reaction/characteristic curve in the form of sidewall pressure versus horizontal displacement on the tunnel sidewall. The pressure required to control the displacement to a predetermined level can be read off the derived curve, from which the equivalent required cable forces can be calculated, as well as the shotcrete thickness and mesh strength.

(3) The numerical model is based on uniform, elastic, strain-softening rock behaviour in which large deformations due to elastic over-stressing are expected. A comparison should be made with field results before conclusions are drawn as to the general applicability of such an approach.

Acknowledgements

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References

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Addendum: FLAC simulation data file (for reference)

tit
Optimal tunnel shape: Run 4-Ellipse, a=2.55m b=1.75m
gr 50,50
model 'ss

* Generate geometry
* gen -40000 -3950 -3950 50, -3998 50, -4000 1.05 1.05 i=1,1 j=1,1
* ellipse
* table 1.0.0 -3998.2500 0.25 -3998.2584 0.5 -3998.2840 0.75 -3998.3274
* table 1.0.0 -3998.392 2.25 -3998.4747 2.5 -3999.1765 2.4 -3999.4086 2.5 -3999.6552
* table 1.255, -4000.0
* gen table 1
* correct coordinates of start and end points
* in i=0.0 j=-3998.25 i=11 j=11
* in i=2.55 j=-4000.0 i=10 j=1
* mark i=11 j=11
* mark i=10 j=1
* windo -1.6 -4001, -3994
* ; set boundary conditions
* fix x i=1
* fix x i=51
* fix y j=1
* apply syy=-106.65e6 j=51
* set grav=10
* ; set properties
* prop bu=6.666e9 sh=4e9 dens=2700 ctab=2 dtab=3 dtab=4 tens=16e6
* tab 2 0.175e6 0.001, 16.0e6 0.002, 13.5e6 0.004, 11.0e6 0.006, 75e6
* tab 2 0.008, 4.0e6 0.01, 0.5e6 0.015, 0.55e6 0.1, 0.0001
* tab 3 0.275, 0.001, 32.5 0.002, 32.5 0.004, 32.5 0.006, 32.5
* tab 3 0.009, 32.5 0.01, 32.5 0.015, 32.5 0.1, 32.5
* tab 4 0.0, 0.001, 20 0.002, 20 0.004, 20 0.006, 20
* tab 4 0.008, 6.5 0.01, 5.5 0.015, 5.5 0.1, 5.5
* ; initialise stress regime
* ini syy=-108.0e6 var=0.1.35e6
* ini sxx=-54.0e6 var=0.0.675e6
* ini szz=-54.0e6 var=0.0.675e6
* ; set histories
* his nsteps=10
* his unbal
* ; displacements and velocities in h/w, f/w and s/w
* hist yd i=1 j=7
* hist yv i=1 j=7
* hist xd i=10 j=1
* hist xv i=10 j=1
* ; vertical and horizontal stresses in h/w, f/w and s/w
* hist sxx i=1 j=7
* hist syy i=1 j=7
* hist sxy i=10 j=1
* hist syy i=10 j=1
* hist sxy i=10 j=1

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; Solve to equilibrium
solve step=30 force=100 clock=1000

; Initialise displacements
set large
ini xdis=0 ydis=0
mo null reg=1,1

; Run 200 steps plastically to simulate failure before support is effective
solve s=200
pl hi 4 hold
pl hi 2 hold
pl gr red plas hold
*solve s=10000 f=100 c=1000
*tit
*Ellipt:unsupported
*ret
*apply pressure=2.0e6 from 8,6 to 10,1
apply xforce=1379e3 i=8 j=5
apply yforce=1157e3 i=8 j=5
apply xforce=1739e3 i=10 j=3
apply yforce=466e3 i=10 j=3
tit
Ellipse:4 s/w normal forces each 1800kN
pl gr red apply vel hold
*solve s=10000 f=100 c=1000
save ellip48.sav
*ret
*Put in cables
str c b g 8.5 e 4.29,-3996.98 se=12 p=2
str c b g 10.3 e 5.38,-3998.834 se=12 p=2
*str p=1 a=804e-6 e=200e9 kb=1e9 sb=1382e3 yi=362e3 *Y32 yield,483kN ult
*str p=2 a=491e-6 e=200e9 kb=1e9 sb=1394e3 yi=221e3 *Y25 yield,221kN,295kN ult
*str p=2 a=442.2e-6 e=198e9 kb=1e9 sb=1392e3 yi=795e3 *3 strands 15,7mm
str p=2 a=1031.8e-6 e=198e9 kb=1e9 sb=3142e3 yi=1855e3 *7 strands 15,7mm
*str p=1 a=201e-6 e=200e9 kb=1e9 sb=1194e3 yi=91e3 *Y16 yield 91kN,121kN ult
tit
Ellipse:4 s/w reinforcing (7/15,7mm strands), each 1855kN capacity
pl hold grid red cab yell
*solve s=9000 f=100 c=1000
save ellip47.sav
*ret

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