Summary of present knowledge on the representative sampling of ore in the mining industry

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Synopsis

The paper discusses some theoretical aspects of Gy's sampling theory and provides reasons as to why it has found limited use in the mining industry, particularly for ores with low concentrations of values such as gold. These types of ores are referred to as low-grade ores in the paper. It is shown that the misuse of the *a priori* liberation factor proposed by Gy is the main reason for the implementation limitation of Gy's model for low-grade ores. In addition, results of a case study done at the Gencor Geostatistics Department based on the new implementation of Gy's theory are provided.

Introduction

The subject of ore sampling has received some attention lately in the South African mining industry[1]. Following Lotter's review[1] of evaluation models for the representative sampling of ore and a sampling course presented by the Mineral Resources Development Consultants (USA) in Johannesburg, it is felt that the subject should receive further attention. Significant work relating to a new look at Gy's sampling theory, which is summarized in this paper, has been done by Francois-Bongarcon[2], who has kindly permitted the use of some of his material in this regard. To the author's knowledge, the new and more practical implementation of Gy's theory in South Africa has not been reported in the literature.

Gy's theory of sampling

As correctly reported in Lotter's paper[1], Gy's sampling theory, which was developed in the mid-fifties, is the only sampling theory general enough to be used in mining practice. What Lotter did not mention is that the theory has not attracted many practitioners. The reasons for this are two-fold. The first reason is that the presentation of Gy's work in textbooks and journals has always been complex. Secondly, in spite of its renowned theoretical validity, Gy's theory has been found to have some limitations in its implementation, which are due mainly to the misapplication of the model[2]. The latter requires particular attention. This paper provides some of the main reasons for the implementation limitations mentioned.

Possible sources of error in the sampling of ores

One can possibly classify sampling errors into four main groups[3]:

- fundamental error, which is due to the irregular distribution of ore values in the particles of broken ore to be sampled
- segregation and grouping error, which results from a lack of thorough mixing and the taking of samples
- integration error, which results from the sampling of flowing ore
- operating error, which is due to faulty design or operation of the sampling equipment, or to the negligence or incompetence of personnel.

Gy's theory of the fundamental sampling error

Based on the equiprobable sampling model, Gy's original general model for the variance of the fundamental error (σ^2_R) can be expressed as follows[2-5]:

$$\sigma_R^2 = \left(\frac{1}{M_s} - \frac{1}{M_L}\right) f\, g\, c\, l\, d_n^3. \qquad [1]$$

The parameters are defined as follows:

- d_n is the nominal size of the fragments in the sample. This is the maximum particle size in the lot to be sampled. In practice, d_n is taken as the mesh size that retains 5% of the lot being sampled, and is measured in centimetres.
- f is the particle-shape factor, which is an index varying in most cases between

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0 and 1. In practice, most values are between 0.2 and 0.5, the actual value depending on the shape of the minerals and the degree to which the sample has been comminuted. For most ores, a value of 0.5 is used.

g is the granulometric factor (or grain-size distribution factor), which takes values between 0 and 1; low values of g denote a large range of particle sizes, and high values denote a small range ($g = 1$ denotes that all particles are of identical size). For most ores, g is taken as 0.25.

c is the mineralogical-composition factor[2,5]:

$$c = \frac{1-a}{a}\left[(1-a)\rho_m + a\rho_g\right],$$

where a = decimal proportion of the mineral
ρ_m = density of the valuable constituent
ρ_g = density of the gangue.

For example, for zinc occurring as sphalerite, an assay of 5% would have a decimal proportion of mineral (ZnS) of

$$a = \frac{64+32}{64} \times \frac{5}{100} = 0.075,$$

also, $c \approx \rho_m/t$, where t is the grade of low-grade ores, e.g. gold (Note: the grade should be relative to the same units, e.g. for gold, the grade units should be in grams per gram (g/g) not grams per tonne (g/t).

M_S is the mass of the sample measured in grams.

M_L is the mass of the material from which M_S is taken, measured in grams.

l is the liberation factor of the mineral constituents. For unliberated particles, Gy assumes that

$$l = \sqrt{\frac{d_o}{d_n}},$$

where d_o is the liberation size for the mineral particles, i.e. the maximum particle diameter that ensures complete liberation of the mineral; d_o is measured in centimetres.

In most practical cases, M_S is much smaller than M_L. Equation [1] then takes the simplified form

$$\sigma_R^2 = \frac{f\,g\,c\,l\,d_n^3}{M_s}. \qquad [2]$$

Gy has proposed an approximation for the estimation of the liberation factor, l. This parameter is one of the key unknown parameters of the model. Gy's empirical estimate for the liberation factor is as follows:

$$l = \sqrt{\frac{d_o}{d_n}}. \qquad [3]$$

However, this empirical liberation formula was calculated experimentally on ores that are **not** low-grade ores[2]. (Examples of typical low-grade ores are those of gold, nickel, and copper.)

Results obtained with Gy's formula, including the corresponding sampling nomograms, are very sensitive to the liberation factor. However, most practitioners, including Lotter[1], use this empirical formula (i.e. equation [3]) as a general rule when applying Gy's model (i.e. equations [1] and [2]). The general use of this *a priori* liberation factor in some cases provides practically meaningless results, and has led to many practitioners abandoning Gy's model[2]. The following demonstrates this important issue.

Example 1

Take, for instance, the use of Gy's model (based on Gy's empirical liberation factor) in the calculation of a minimum sample size for a typical South African gold mine in production. Assume a top particle size of 13 cm. With a gold grade of 5 g/t and a gold density of 19 g/cm^3, the mineralogical factor is

$$c = \frac{density}{grade} = 3.8 \times 10^{+6}\ \text{g/cm}^3.$$

Note that the grade should be in g/g.

For a gold-grain top size of 75 µm (i.e. 7.5×10^{-3} cm), Gy's empirical liberation factor is

$$l = \sqrt{\frac{d_o}{d_n}} = \sqrt{7.5 \times \frac{10^{-3}}{13}} = 2.4 \times 10^{-2}.$$

At a relative precision of 10%, i.e. a variance of 0.01 based on equation [2], the use of the above computed parameters gives a minimum mass of 2507 t. For a typical production rate of between 1000 and 10 000 t per shift, this minimum sample mass is practically unacceptable. In applying Gy's sampling theory based on the empirical liberation factor, Bartlett and Hawkins[6] reported similar unacceptable results for a South African underground gold mine. Although the authors questioned the results, they did not provide any reasons for the unacceptable results (see also Francois-Bongarcon[2]).

The solution: a new effective model for the liberation factor

Current research (Francois-Bongarcon[2,7,8]) has shown that, if one ignores Gy's empirical liberation factor

$$l = \left(\frac{d_o}{d_n}\right)^{0.5}$$

and uses a more general form of model for the liberation factor, better results would be obtained. The proposed model is as follows:

$$l = \left(\frac{d_o}{d_n}\right)^{b}, \qquad [4]$$

where b is an additional parameter of the model that can be 'calibrated' to a particular ore. Exponent b in equation [4] is often found to have a value close to 1.5 in most gold ores[2]. The calculation given earlier, when repeated with b equal to 1.5, gives a minimum sample mass of 1.4 t, which is in full agreement with mining experience (for a lot of 1000 to 10 000 t). In addition, the use of this new model for the liberation factor results in sampling nomograms that are realistic.

General form of Gy's formula

If use is made of the general form of Gy's formula but ignoring his approximation of the liberation factor, his formula can be written as[2,7,8]

$$\sigma_R^2 = \frac{c.f.g.d_o^{3-\alpha} d_n^{\alpha}}{M_s}, \qquad [5]$$

where α is a parameter for specific deposits that can be 'calibrated' to a particular ore. Current research has indicated that α is about 1.5 for most gold ores[2].

Since c, f, g, d_o, and α are constant for a particular ore, equation [5] can be simplified as follows:

$$K = c.f.g.d_o^{3-\alpha}.$$

Thus,

$$\sigma_R^2 = \frac{K\, d_n^{\alpha}}{M_s}. \qquad [6]$$

By use of equation [6], the following three basic problems of sampling can be solved effectively.

- What weight of sample should be taken from a larger mass of given ore characterized by the constants K (and α), if the maximum size of the particles present (dimension d_n) is known, in order that the sampling error will not exceed a specified variance σ^2_R?
- What is the possible error introduced when a sample of given weight, M_s, is obtained from a given ore having a constant K and a known maximum particle size d_n?
- Alternatively, before a sample of given weight, M_s, is obtained from a given bulk of broken ore characterized by a constant K, what is the degree of crushing or grinding required to lower the error to a specified value of variance σ^2_R?

Experimental calibration of Gy's formula to avoid the liberation factor approximation

From equation [6],

$$\sigma_R^2 = \frac{K\, d_n^{\alpha}}{M_s}. \qquad [7]$$

If one takes the logarithm of both sides of equation [7] after transposing,

$$\ln\left(\sigma_R^2 M_s\right) = \alpha \ln d_n + \ln(K). \qquad [8]$$

The experimental plot of the left-hand side of equation [8] against $\ln(d_n)$ can be fitted to a straight line (in the form of $y = mx + c$) on log-log paper, as shown in Figure 1 (along with the model for $d_n < d_o$, i.e. below liberation). This shows the line of slope α and intercept $\ln(K)$.

Therefore, α and K can be 'calibrated', thus providing estimates of these parameters for a particular ore (rather than the use of Gy's approximated liberation factor).

Once $\ln(K)$ has been obtained, d_o can be computed since

$$K = c.f.g.d_o^{3-\alpha}.$$

The practical calibration exercise can be done by use of the sample-tree experiment[2,8]. This involves a series of ore splitting and assays at different top sizes.

Nomograms based on 'calibrated' constants

Sampling protocol charts or nomograms based on 'calibrated' constants for a particular ore are built as follows.

As shown previously,

$$\sigma_R^2 = \frac{K\, d_n^{\alpha}}{M_s}. \qquad [9]$$

If the logarithm of both sides is taken,

$$\ln\left(\sigma_R^2\right) = \ln\left(\frac{1}{M_s}\right) + \ln\left(K d_n^{\alpha}\right) \text{ and} \qquad [10]$$

$$\ln\left(\sigma_R^2\right) = (-1)\ln\left(M_s\right) + \left[\alpha \ln\left(d_n\right) + \ln(K)\right]. \qquad [11]$$

Equation [11] shows that, for a given stage of comminution (i.e. a fixed value of particle size d_n), the term $\ln(k) + \alpha \ln(d_n)$ is a constant, say $C(d)$, and

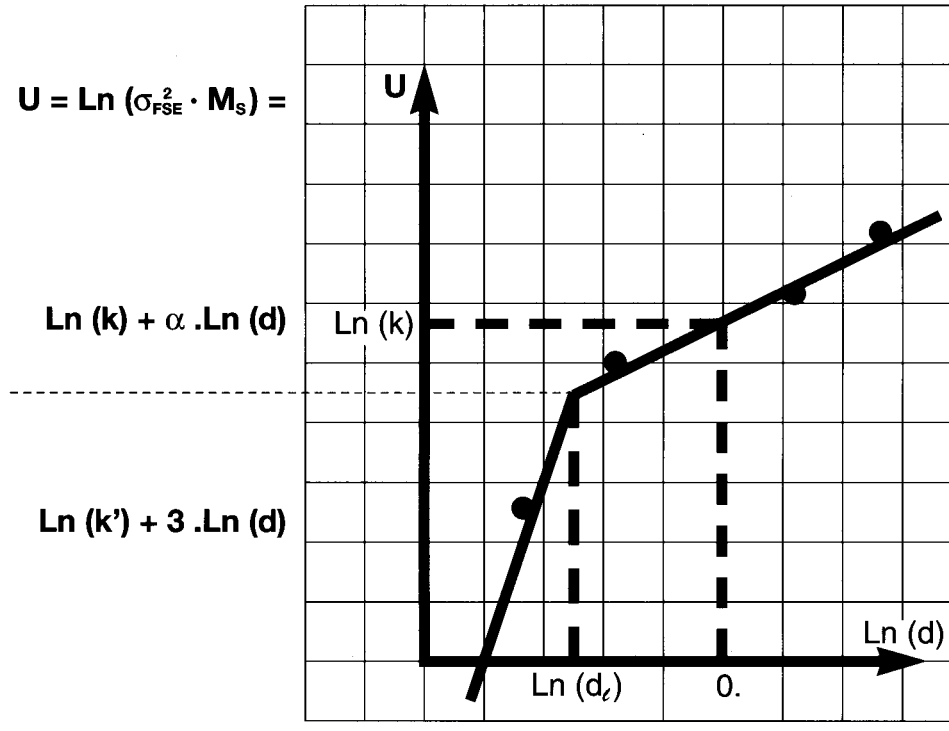

Figure 1—Fitting a model to the fundamental sampling error

$$\ln\left(\sigma_R^2\right) = -\ln(M_s) + C(d). \qquad [11a]$$

Equation [11a] is such that, on log–log paper, the graph of the left-hand side against $\ln(M_s)$ can be fitted to a straight line, i.e. for a given size d_n, this is a line of slope –1.

It should be noted that Gy's formula changes for $d_n < d_o$ (i.e. below liberation size[2]).

Importance of sample nomograms

With regard to equation [11a], the operation at each stage of any sampling operation (i.e. mass reduction at constant rock-particle size d_n) can be plotted on a chart as a path along a straight line of slope –1. There is one unique line for each comminution size. In Figure 2, (A)–(B) demonstrates such a path. Point (A) represents the mass of the initial crushed material. Point (B), however, corresponds to the mass of the crushed material split out for the next stage of comminution (i.e. for grinding). The difference between the respective variances at points (B) and (A) represents the relative variance for the stage. The latter assumes an absence of segregation error. The vertical lines in Figure 2 represent the crushing and grinding stages, which are not supposed to contribute to the variance. The entire sampling-preparation protocol can thus be displayed graphically, showing the contribution of respective stages to the overall precision variance. **Such charts based on 'calibrated' parameters of respective deposits constitute useful tools for the design, assessment, control, and improvement of sampling processes.**

Case study based on the new approach

Based on the new approach to the implementation of Gy's sampling theory as summarized in this paper, the calibration experiment described earlier was carried out for a gold orebody by the Gencor Geostatistics Department. Chip samples from reverse-circulation drilling were used. The aim was to calibrate the parameters α and k for the orebody (α and k being as defined earlier).

Based on these calibrated parameters and Gy's model, a suitable sampling protocol was recommended for the evaluation drilling (a theoretical description is given earlier). So far the results have been very encouraging.

Table I shows the calibrated parameters, as well as the gold-liberation size estimates, based on the model for three prospecting areas; the α values ranged from 0.76 to 1.15. These α values can be compared with the research values of about 1.5 as observed by Francois-Bongarcon[2] for gold ores. Gy's empirical estimate[2] of α is 2.5. This *a priori* α value of 2.5 gave significantly different results (including the corresponding sampling nomograms) for the gold orebody that featured in the case study.

Conclusions

The earlier theoretical discussion and the case study show that the implementation limitations of Gy's useful model for ores with low concentrations of values such as gold is due to the unfortunate generalization of his empirical liberation factor made by many practitioners.

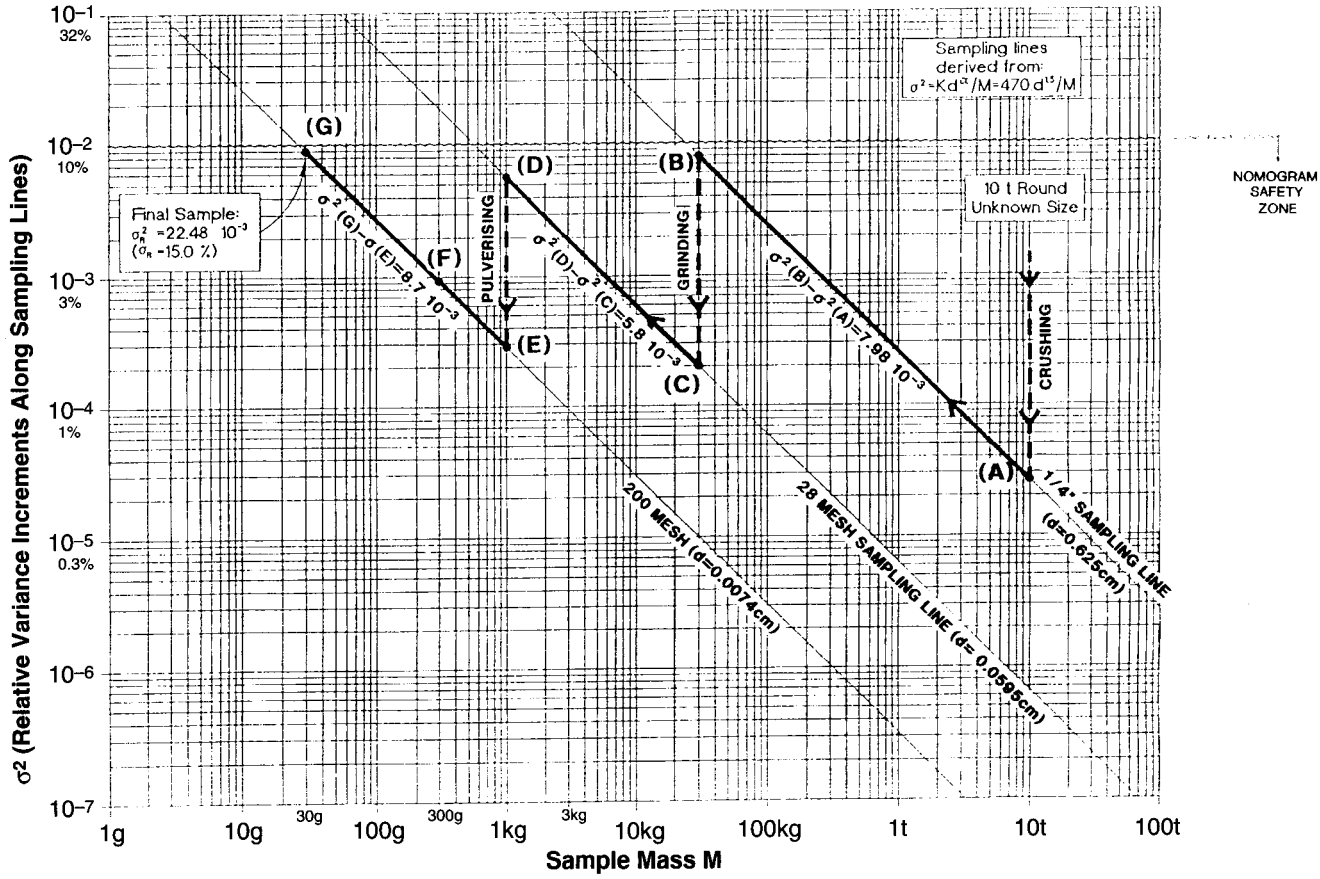

Figure 2—Example of the use of a sample protocol chart (after Francois-Bongarcon[8])

Table I

Summary of calibration parameters (α and *k*) and gold-liberation size estimates for a gold orebody based on a modified form of Gy's sampling theory

Target area	α	k	Gold-liberation size µm
Area 1	0.86	40	59
Area 2	1.15	117	46
Area 3	0.76	44	77

The paper shows that the effective application of Gy's model, particularly for low-grade ores, is to 'calibrate' the parameters of the model for individual deposits.

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