Physical and numerical modelling of rock fracture

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Synopsis

This paper presents an overview of the research that has been carried out to improve the understanding of the fracture processes occurring in rock under high levels of stress. The onset and growth of fractures are studied by means of laboratory model testing, and the results obtained are used in the calibration and validation of the numerical model DIGS, which is based on displacement discontinuity. The results obtained so far have shown that, while there are overall similarities between the experimental results and the numerical simulations, improvements to the experimental procedures and the DIGS are required for a detailed mechanistic understanding of shear-band orientations and extension fractures.

Introduction

The stress levels around deep mine excavations invariably exceed the strength of the rock, resulting in fracturing around these excavations. The safe design of support systems against quasi-static and dynamic loading of the damaged walls requires a knowledge of the degree, extent, and orientation of the fractures. Complementary to the current research effort, which addresses this requirement by means of extensive *in situ* monitoring and observations, laboratory physical modelling tests and numerical modelling studies are being carried out to provide a better understanding of the mechanisms involved during the fracturing of brittle rock around deep mining excavations.

Laboratory testing is often preferred in studies of fundamental rock behaviour since it offers quantifiable results obtained under controlled loading conditions. Models prepared from rocks, and in some cases from artificial mixtures with known material characteristics, can be tested in specially designed test rigs under controlled conditions. The results obtained from these tests can then be used with fair confidence in the evaluation of the validity of numerical simulations. Comparative studies between numerical and physical models can also lead to an improved understanding of the mechanisms involved in fracture formation and this, combined with *in situ* observations, can be implemented in numerical models for design purposes.

This review emphasizes the importance of underpinning a research programme in the area of fundamental rock behaviour by laboratory and numerical modelling. The fundamental research undertaken focuses on the aspects of the onset and propagation of fractures, failure criteria, and back-analysis testing of the model by means of numerical modelling.

The numerical model DIGS

The numerical model[1] used throughout the work described here is known as DIGS. It represents fracturing in the form of 'cracks', which are essentially small strain displacement-discontinuity elements that are joined end to end in a series of growth steps corresponding to the development of the localized fracture zone. Fractures are allowed to initiate from the ends of pre-existing cracks, or can be initiated at designated sites, which are termed 'seed points'. In determining the orientation of the growing fractures, it is recognized that different processes control the formation of tensile fractures and of localized damaged zones in the form of shear bands. Tensile fracturing is assumed to be positive. This is equivalent to asserting that the direction of tensile fractures is perpendicular to the minor, possibly tensile, principal stress component. This assumption has been verified by physical modelling experiments[2,3]. It should be noted that, following the introduction of a given growth increment, all the growth sites are re-evaluated, taking the crack interactions into account, before the subsequent growth directions are determined.

The orientation of shear-band localizations is determined on the postulation that the

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growth direction is such that the strain-energy release rate is at a maximum. This growth rule has been partially confirmed by the physical modelling experiments carried out by Dede *et al.*[4], but further experimental study and theoretical development are required before a robust criterion of shear-band formation can be established.

The DIGS model also assumes that elasto-dynamic effects can be neglected in the simulation of a progressive fracture process near the edges of openings. The validity and limitations of this assumption must also be established by means of physical modelling.

Physical and numerical modelling experiments

Fracture growth around openings in layered strata

Horizontal structures, such as bedding planes, within the close proximity of excavations are known to influence the formation of stress fractures. Many fundamental aspects, such as whether fracturing initiates at the layer interface, the effect of interface friction on fracturing, and the conductivity of interfaces in terms of fracture propagation, present challenges to the numerical modelling of the fracturing from layers. *In situ* monitoring and observations are often difficult to interpret owing to uncertainties regarding the operating stress *in situ*, the rock's structural composition, and the subjectivity introduced by the observer. This motivates the need for modelling tests to be conducted under controlled conditions.

The first set of tests was designed to gain quantitative insight into the influence of layers on the fracturing around mine-like openings in layered strata. Layered models were built from plates prepared from plaster-of-Paris mixtures. The plates measured 15 x 15 x 1 cm, and a stack of 15 layers formed a model sample. Tabular openings were created by openings left in particular layers in the stack. The models were tested in a test rig that was designed to approximate plane-strain loading conditions and to provide horizontal confinement. A schematic section of the test rig is shown in Figure 1. The two steel plates on the sides and a flat jack housed in front of one of the plates provided controlled confinement levels up to 15 MPa. The perspex plates located at the front and the back allowed the fracture formation and growth to be photographed.

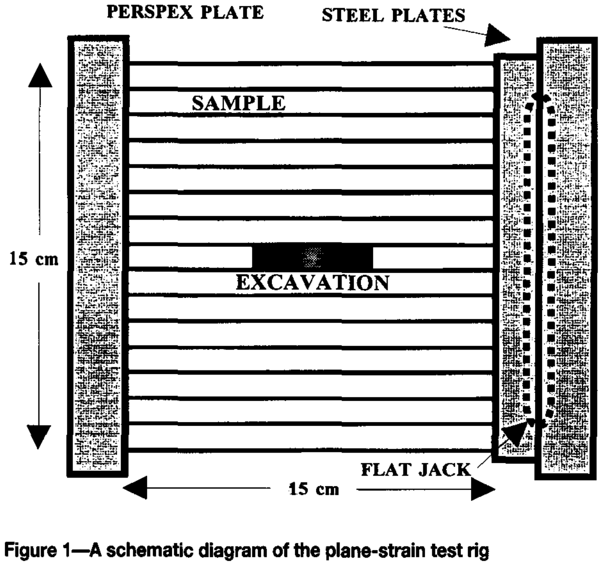

Figure 1—A schematic diagram of the plane-strain test rig

With two rectangular openings at different elevations in the plaster-of-Paris layers, fractures initiated at the corners and tended to slope towards the centre line of the opening (Figure 2). As the load increased, parallel bands of extension fractures started forming in the solid area between the openings. A further increase in loading resulted in intense fracturing in this area, finally forming a localized shear zone along the line joining the closest ends of the openings. These experiments were modelled by use of the computer program DIGS as follows: slip interfaces corresponding to the sample layers were defined and zero stress-boundary conditions were arranged around the excavation surfaces (Figure 2). Tensile seed points were introduced at regular intervals on each layer interface, and fracture growth patterns corresponding to given loading increments were allowed to evolve.

Figure 2 shows generally good agreement between the modelled growth patterns and the physical observations at the early loading stages. However, the coalescence of the initial tensile fractures to form macro shear planes that were observed in the physical models was not reflected in the numerical models.

The work just described was extended to the modelling of mining steps, also in a layered but more brittle material, namely glass[5]. The plates were prepared from glass, and a rectangular opening was introduced by the arrangement of a series of adjacent glass strips, with a suitable gap, at the centre of the layer assembly. The size of the opening could be enlarged in a series of steps by backing off the load, removing one of the centrally placed strips and reloading the assembly. The fractures formed after the first mining step are shown in Figure 3. The numerical simulation of fracture growth around the initial opening involved tension fractures controlled by the tension growth rule, which were allowed to initiate at the layer interfaces and at the seed points within the layers. Although the fractures initiate from the layer interfaces, it is clear that tensile stresses induced parallel to the horizontal surface of the opening cause tension fractures to develop at the centre of the opening.

In general, more pervasive fracturing was observed to occur in the physical model than in the numerical model. Also, the inclination of the shear-driven edge fractures that were simulated numerically was more vertically orientated than the observed edge fractures. These tests illustrate the use of numerical modelling in the formulation of the rules for the simulation of fracture growth. This, in turn, highlights the need to extend the concept of extension fracturing when strong shear-driving forces occur near the corners of the opening. A generalized criterion is, however, still needed to distinguish conditions in which complex shear-band structures are initiated in contrast to the formation of simple tensile fractures.

Punch test

In this series of tests, solid and layered cylindrical sandstone specimens of 54 mm diameter and 60 mm length were subjected to 'punch' loading using a cylindrical steel piece 10 mm in diameter. Figure 4(a) shows the mode of failure, which is signified by the formation of an inverted cone

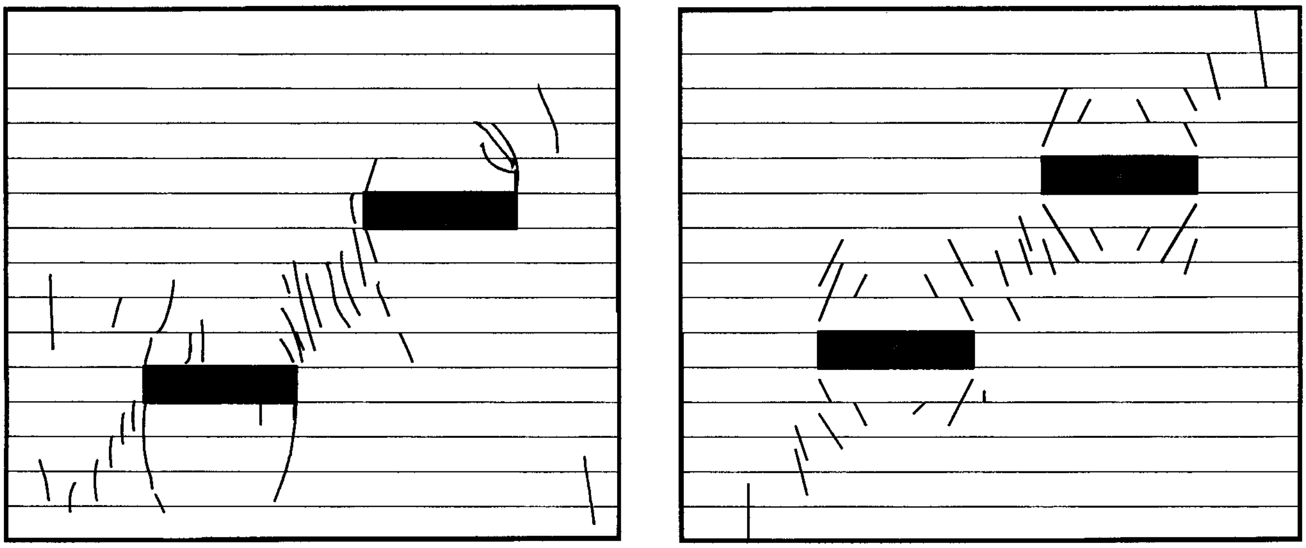

Figure 2—The fracture patterns obtained from plaster-of-Paris modelling (left) and numerical modelling (right)

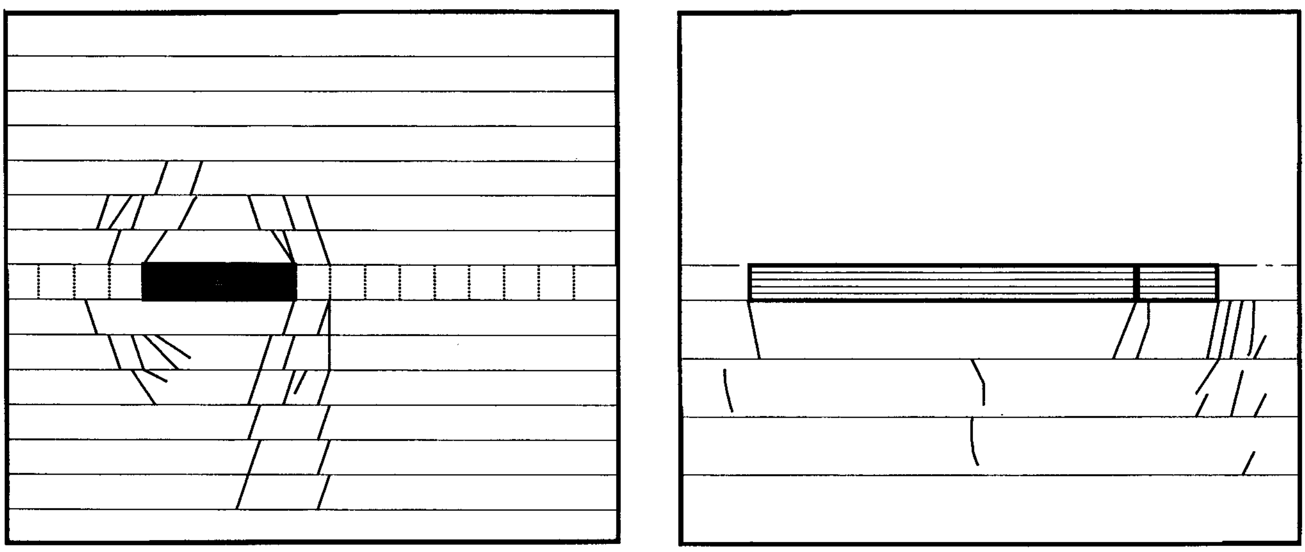

Figure 3—Glass layers (left) and numerical modelling[6] (right)

immediately under the punch and a radial tensile crack along the axis of the specimen. Enclosing the specimens in 4 mm thick steel tubes prevents the formation of a central tensile crack, Figure 4(b). A set of surface parallel extension cracks, initiated around the edge of the punch, is the most significant difference in this series compared with the unconfined tests[4].

Although the tests were carried out using an axisymmetric loading arrangement, some insights could be gained from a comparison of the results observed in the experiments with those of two-dimensional plane-strain simulations of fracturing in test configurations corresponding geometrically to a diametrical section through the samples. For the case of the unconfined sandstone specimen, two shear growth fractures were allowed to initiate below the edges of the region on the sample surface corresponding to the area covered by the punch. This area was loaded uniformly by the stress value corresponding to the maximum observed stress in the physical experiment. The shear fractures are seen to converge in a wedge-shaped structure below the punch, as shown in Figure 4(a). If a seed point is introduced subsequent to the formation of the shear fractures in the region below the punch, it is found to grow below the wedge structure. The final fracture pattern is qualitatively similar to the observed pattern of the physical model, but shows a marked difference in the wedge angle and extent. The sequence of formation of the tensile fracture below the wedge in the numerical model was influenced by the selected position at which the tensile fracture was allowed to initiate. Additional physical evidence is required for identification of the exact sequence of fracturing. The shape of the simulated wedge may differ from the observed shape because of the distinct differences that can arise between an axisymmetric and a plane-strain loading geometry, as well as inadequacy of the shear growth rule.

For the confined sample depicted in Figure 4(b), it was assumed that the vertical edges of the sample were not able to move laterally and that no shear traction was imposed on these edges. Shear bands were allowed to initiate from the edges of the punch, and tension fractures were allowed to

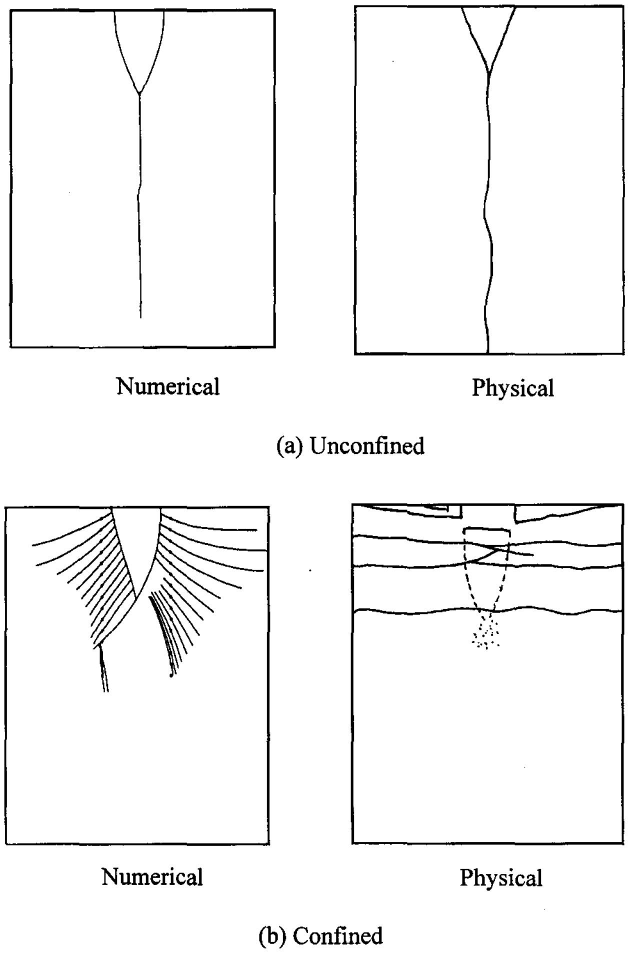

Figure 4—Fracture patterns obtained in the punch test

initiate from 'seed' points arranged in vertical columns on either side of the punch. Although there are some qualitative similarities to the observed results, it is clear that the orientation of the tensile fractures is not sub-parallel to the free surface as observed in the unconfined tests, Figure 4(a).

Biaxial cell-strip punch tests

The stress conditions to which excavations are subjected are normally three-dimensional, and the fracturing under three-dimensional loading can differ significantly from that in two dimensional or axisymmetric cases. In order to carry out three-dimensional tests on prismatic rock samples, the biaxial load cell shown in Figure 5(a) was built. The cell can accommodate cubic specimens of 70 to 100 mm under confining pressures of up to 70 MPa. Models approximating the shapes and loading of strip pillars, Figure 5(b), square pillars, and stopes can be simulated in studies of fracturing and failure.

In a study of the initiation and development of fracturing, strip-pillar models Figure 5(b), were loaded incrementally, and sections were cut for fracture observations. Figure 6(a) shows typical results at three stages of a test where the initial tensile fracture formed vertically under the punch at a low confining stress of 9 MPa. This was followed by shear fracturing starting immediately below the punch abutments and developing into a wedge with a height equal to twice the punch width.

The results from the numerical modelling of the tests using DIGS are illustrated in Figure 6(b). The overall similarity of the DIGS results to those of the physical tests could be achieved only when the shear and tensile seed points were placed in a particular configuration and the material properties of the quartzite had been modified. The wedge under the punch area could be initiated by the insertion of two shear points at the edges of the pillars. As shear fractures grew at these edges, additional tensile failure points located in the middle

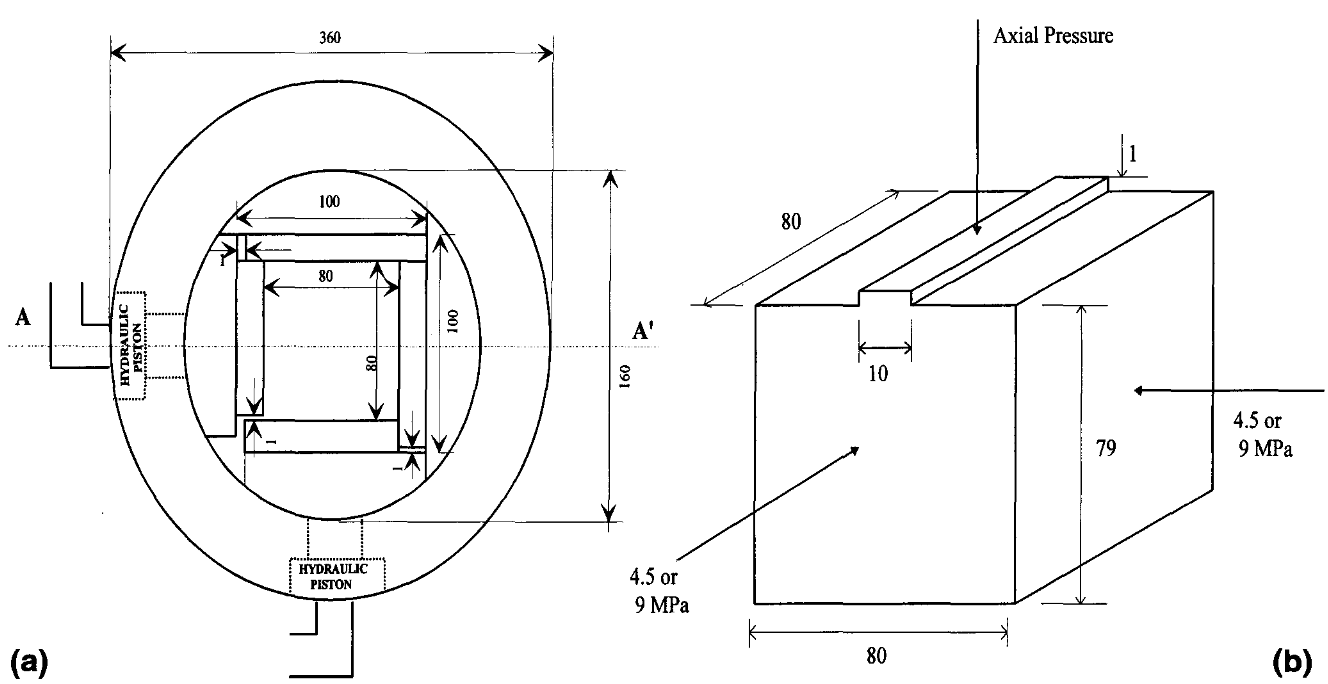

Figure 5—The poly-axial test frame (a) and a typical test specimen (b)

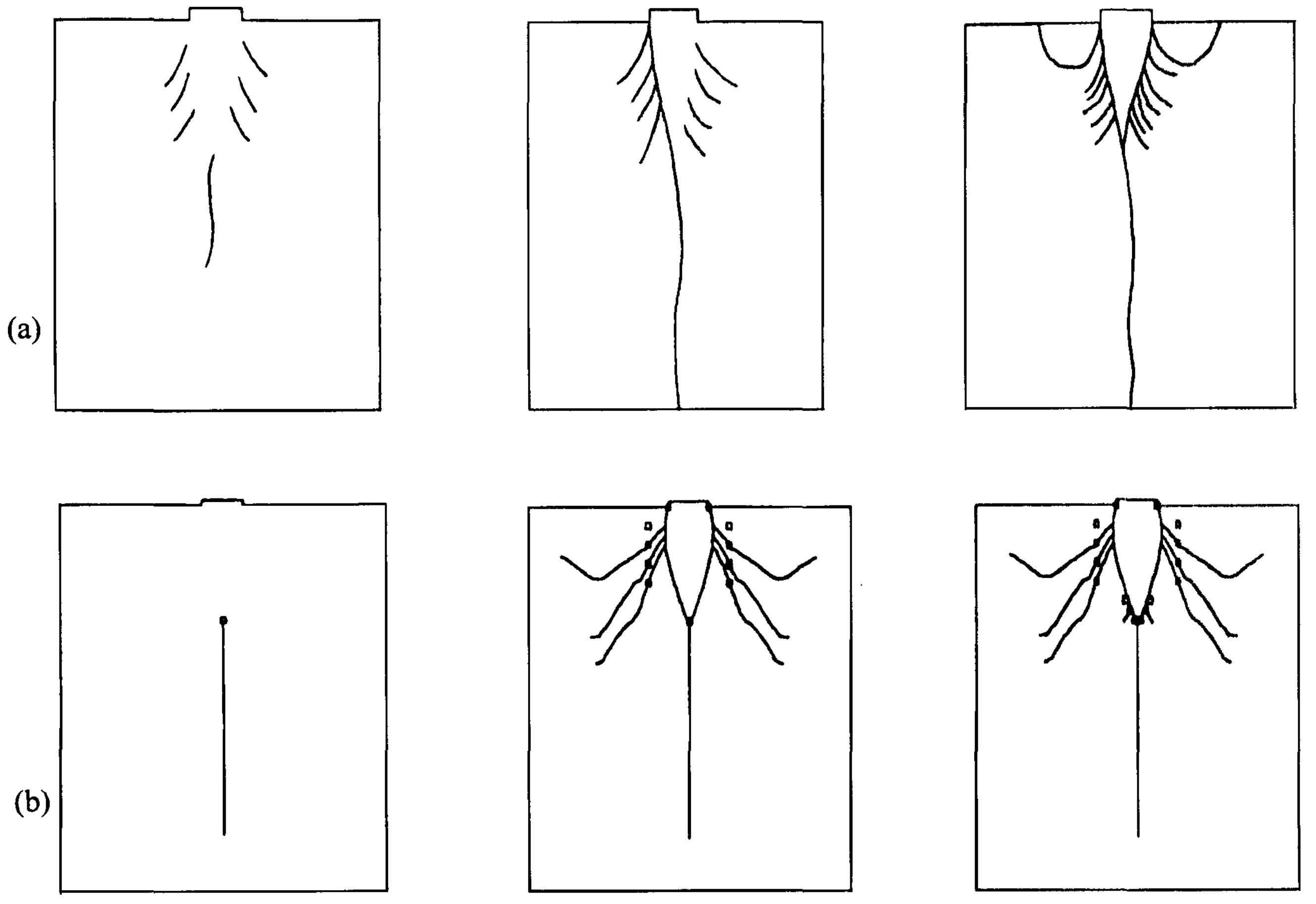

Figure 6—Fracture patterns under increased loading as observed in strip-punch tests: (a) physical modelling and (b) numerical modelling

of the sample were necessary for the initiation and growth of tensile fracturing as observed in the physical model. Further modelling studies are being carried out to provide a better understanding of the fracturing process and so enable DIGS to be less dependent on the pre-defined fracture modes and location of seed points.

Strength of brittle rock in plane-strain loading

Many excavations in rock are treated as two-dimensional, and the state of stress around these excavations approximates that of plane strain. Linear elastic analysis of these excavations is usually carried out with two-dimensional models incorporating plane-strain loading conditions. However, the non-linear modelling of two-dimensional excavations, is more complicated, and the selection of the constitutive laws and parameters governing rock behaviour are not always straightforward.

One of the common constitutive laws used for non-linear rock behaviour is the strain softening model proposed by Vermeer and de Borst[7], which is based on the Mohr–Coulomb failure criterion. Traditionally, input parameters for two-dimensional modelling with strain softening (typically using FLAC[8] are obtained from triaxial tests, and the validity of this approach is still to be verified. Yumlu and Ozbay[9] aimed at identifying the differences in rock behaviour when tested under plane-strain and triaxial loading conditions. A test rig with a stiff frame was developed and used, mainly because of cost considerations. Figure 7(a) shows a schematic representation of the test rig. The rig comprises two steel U-frames, which are connected via two left/right threaded bolts to restrict displacements in the x_2 direction (plane-strain direction). The U-frames are made of steel pieces 30 mm thick to minimize bending at the specimen–steel contact, and it is assumed that the U-frames remain 'rigid' during tests. When the steel bolts of the U-frame are turned simultaneously, the specimen is pre-stressed along the x_2 direction.

Higher strength values were observed under plane-strain loading, and this was believed to be due to the strengthening effect of the intermediate principal stress. The most significant differences occurred between the tests under unconfined axisymmetric conditions (i.e. conventional uniaxial-compressive strength) and those under unconfined plane-strain conditions (prismatic samples loaded $\sigma_1 > \sigma_2 \neq 0$, $\sigma 3 = 0$). Increasing confinement caused a slightly higher friction angle for quartzite. The largest strength difference between the two different types of tests occurred with coal (39 and 42% respectively), in which the loading was closer to true plane strain than in the tests on the other two rock types. One of the most significant effects of plane-strain loading on the stress–strain curve was the suppression of the non-linear region preceding the failure as compared with triaxial compression. As can be seen in Figure 8, in triaxial loading, the deviation from the linear behaviour at the onset of 'yielding' took place at stress levels that were about two-thirds of the peak stress attained by the sample. The rate of strength increase with increasing strain decreased continuously,

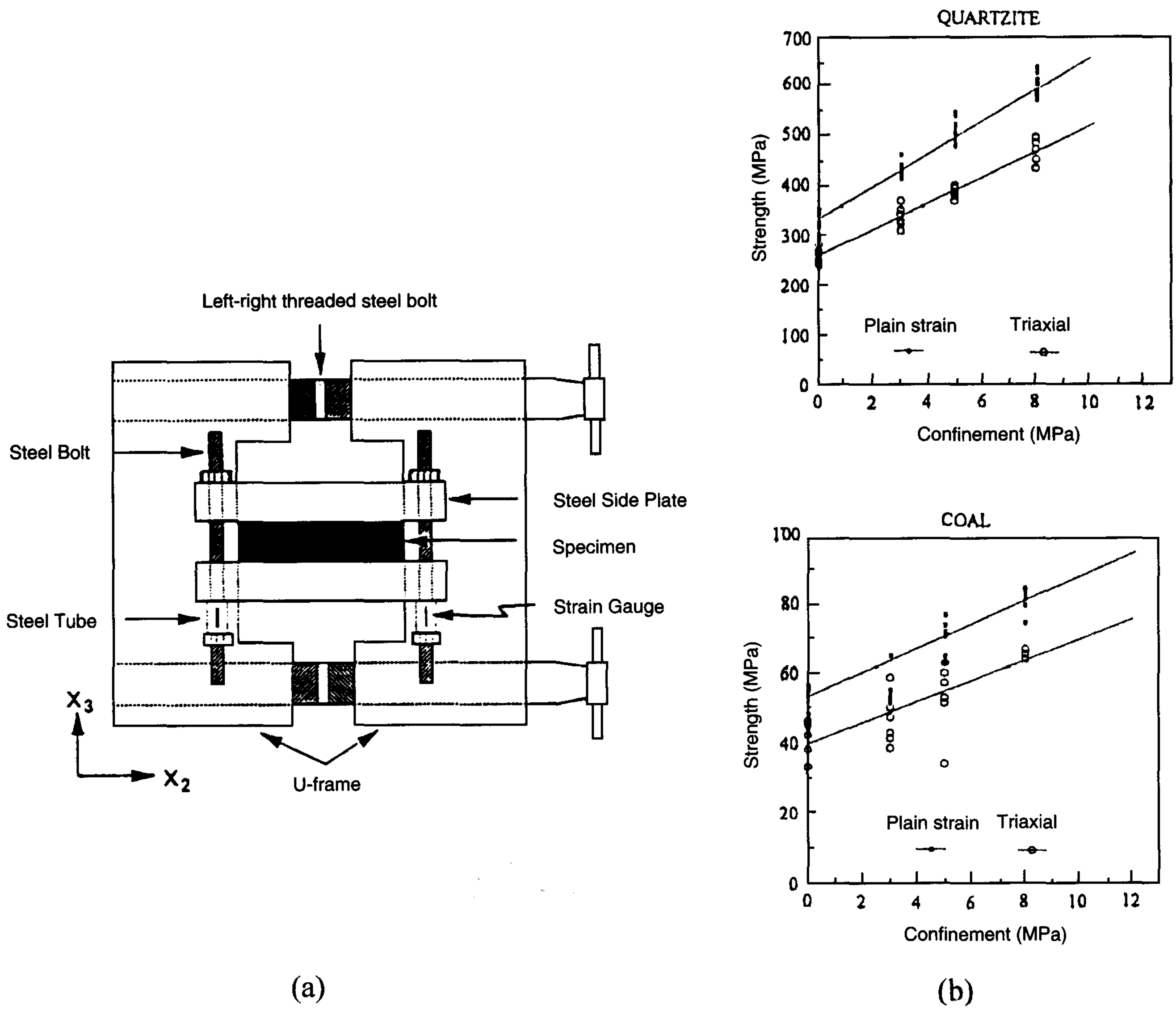

Figure 7—Plan view of the plane-strain test rig (a), and the strength characteristics obtained from axisymmetric and plane-strain testing (b)

finally becoming zero at the peak stress value. Also, the strain range over which the yielding took place was larger at increased confinement than in the plane-strain tests. Under plane-strain loading, the yield zone was suppressed considerably. The stress–strain curve remained mostly linear in the pre-peak regime, and the failure occurred at much smaller strain values than it did in the triaxial tests. In the post-peak regime, the residual strength was greater under plane-strain loading. The plane-strain tests showed 'strain hardening' in the post-peak regime. This is probably due to the stiffer confinement applied by the steel plates, and also to the increasing confining stresses as the bolts reacted to the dilation of the specimen.

Conclusion

It was shown that the numerical code DIGS, which is based on displacement discontinuity, can be set to give fracture patterns similar to those obtained from physical modelling experiments conducted under controlled conditions in a laboratory. The locations of the seed points and failure criteria used in the initiation and growth of fractures in DIGS appear to have a strong influence in reproducing the fracture patterns obtained in physical modelling. Continued calibration and validation of DIGS against physical models, by variation of the seed point configurations, modification of the failure criteria, and the use of more realistic modelling of rock structures, can potentially provide an improved understanding of fracture and failure mechanisms in brittle rock. Work of this nature, supported by *in situ* observations, can lead to the development of a more realistic numerical model that can be used by practising rock engineers in designing support and layouts for deep mining excavations.

References

1. Napier, J.A.L., and Hildyard, M.W. Simulation of fracture growth around openings in highly stressed brittle rock. *J. S. Afr. Inst. Min. Metall.*, vol. 92, no. 6. pp. 159–168.

2. Thomas, A.L., and Pollard, D.D. The geometry of echelon fractures in rock: implications from laboratory and numerical experiments. *J. Struct. Geol.*, vol. 15. 1993. pp. 323–334.

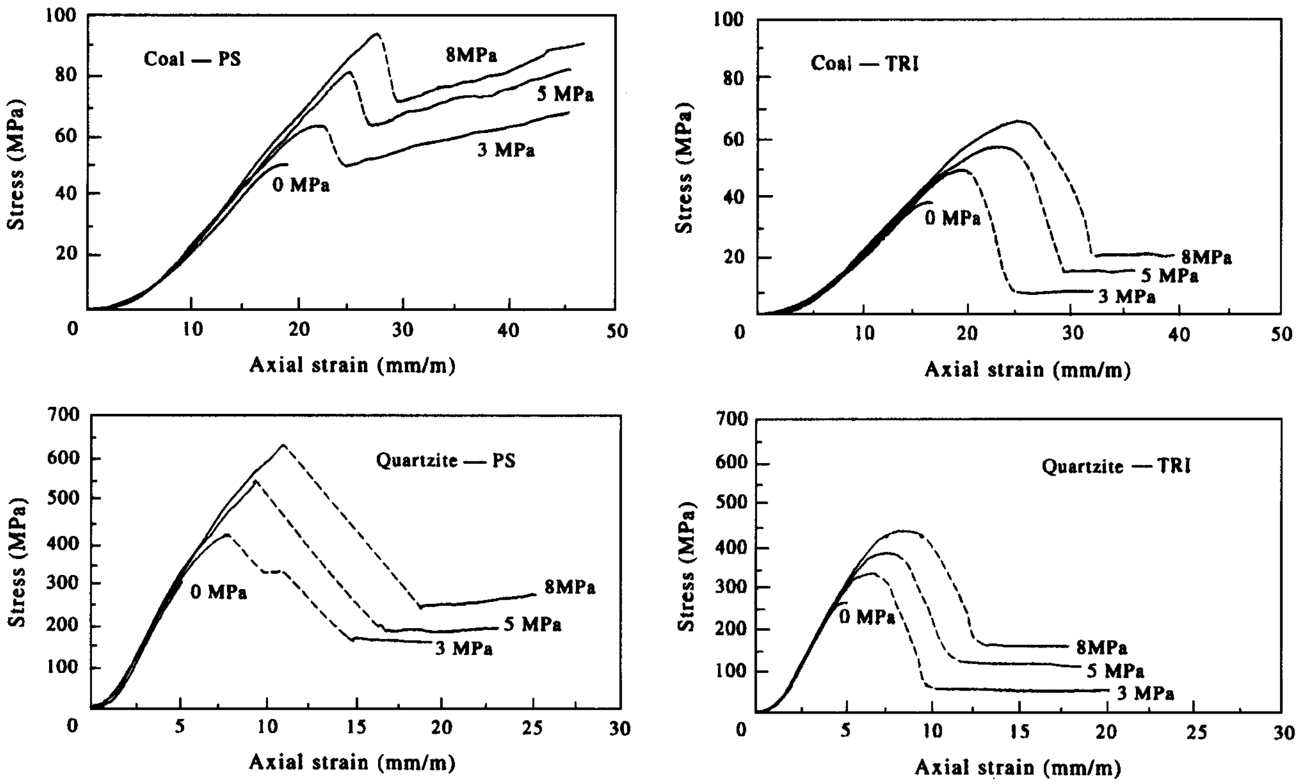

Figure 8—The differences in the stress–strain and strength characteristics of quartzite and coal tested under triaxial and plane-strain conditions

3. MALAN, D.F., NAPIER, J.A.L., and WATSON, B.P. Propagation of fractures from an interface in a Brazilian test specimen. *Int. J. Rock Mech. Min. Sci. and Geomech. Abstracts*, vol. 31. 1994. pp. 581–596.
4. DEDE,T., OZBAY. M.U., and NAPIER, J.A.L. Fracture onset and propagation through layered rock. *2nd International Conference on Mechanics of Jointed and Faulted Rock.* Vienna, sup. vol., 1995. pp. 9–13.
5. NAPIER, J.A.L., and OZBAY, M.U. The role of physical modelling in the calibration of numerical models. Application of Numerical Modelling in Geotechnical Engineering, SANGORM Symposium, Pretoria, 1994.
6. NAPIER, J.A.L., and OZBAY, M.U. Application of the displacement discontinuity method to the modelling of crack growth around openings in layered media. Assessment and Prevention of Failure Phenomena in Rock Engineering, ISRM, Turkey, 1993. pp. 947–956.
7. VERMEER, P.A., and DE BORST, D.E. Non-associated plasticity for soils, concrete and rock. *Heron* 29. 1984. pp. 1–64.
8. CUNDALL. P.A. *FLAC users's manual.* ITASCA Consulting Co., 1987.
9. YUMLU, M., and OZBAY, M.U. Plane strain testing of brittle rock. *Integral approach to applied rock mechanics.* Santiago (Chile), 1994. pp. 1–7. ◆

East Dagga pays debenture interest of R20,4 million*

The first debenture interest accrual since the re-structuring of East Daggafontein Mines Limited (East Dagga) earlier this year has been fixed at 20,4 million rands, which is the equivalent of 120 cents per linked unit for the six months to the end of September 1996. East Dagga chairman, Peter Bieber, notes that this translates to 240 cents on an annualized basis. Debenture interest and an interim ordinary dividend will be declared in November for payment in December.

Operationally, East Dagga reports satisfactory results for the quarter to the end of September, with income before debenture interest and taxation of 13 million rands, as against 18,5 million rands for the previous three months. This, according to Peter Bieber, is in line with the expectations published during the June quarterly report.

Strike action and a shut-down at the Impala smelter adversely affected production at the 6L13 platinum project, and 409 kg of platinum and other platinum-group metals were produced, with a revenue of R1 343 000 for the period. The mined reserve from 6L13 is currently being drilled for grade- and tonnage-control purposes.

In addition, a number of other projects which might prove suitable for acquisition by East Dagga are being evaluated. ◆

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