A combined method for the analysis of mine ventilation networks

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Synopsis

Solving ventilation networks of natural splitting is a basic problem in mine ventilation. Several mathematical techniques are used to solve this problem. This paper proposes a new method which combines the advantages of both classical Hardy Cross method and unconstrained optimization techniques. The trials carried out on model networks have shown that the method offers some facilities in the analysis of mine ventilation networks.

Keywords: **Ventilation network analysis, Hardy Cross method, Steepest Descent method.**

Introduction

Mine ventilation networks can basically be classified as the networks with natural splitting and the networks with controlled splitting. In case of mine ventilation networks with natural splitting, the resistance factors of all branches, fan locations and coefficients of fan characteristic curves are known. In this type of network, the problem is to determine the air quantities for all branches.

Traditionally, a mine ventilation network is solved by means of iterative techniques using Kirchoff's current and voltage laws. The most popular one of these methods is the Hardy Cross algorithm. A number of computer programs based on this algorithm have been developed (Yalcin[1999]).

Despite their popularity, the Hardy Cross method and its modifications have some disadvantages. They have uncertain convergence characteristics, convergence to optimal solution and solution time are highly dependent on the initial arbitrary air quantities assigned for all branches. In order to overcome these difficulties, some different solution procedures based on several mathematical methods like linear programming, non-linear programming and network analysing techniques have been proposed by several investigators (Ueng and Wang,[1984]; Wang,[1984]; Bhamidipati and Procarione,[1986]).

The purpose of this paper is to introduce a combined method for analysis of mine ventilation networks with natural splitting and to discuss its applicability.

Mathematical model

In ventilation networks, the sum of the pressure drops along branches does not give total pressure drop of the network. However, the total air power consumption in a network is equal to the sum of individual power consumption along branches. Therefore, a mathematical function which describes total power consumption in the network can be taken as objective function (Wang[1984]).

For a ventilation network with (B) branches, (N) nodes and (F) fans, the objective function to be minimized is given by;

$$U = \frac{1}{3}\left[\sum_{i=1}^{M} R_i\left|Q_i^3\right| + \sum_{i=M+1}^{B} R_i\left|\left(\sum_{k=1}^{M} b_{ki}Q_k\right)^3\right|\right] \pm \sum_{i=1}^{F}\left(A_iQ_i + \frac{1}{2}B_iQ_1^2 + \frac{1}{3}C_iQ_1^3\right) \quad [1]$$

where;

- R_i : resistance factor for the branch i, Ns^2/m^8
- Q_i : air quantity for the branch i, m^3/s
- M : number of meshes (M = B – N + 1)
- b_{ki} : an element of the fundamental mesh matrix
- A_i, B_i, C_i : coefficients for the fan i

In this form, it is an unconstrained minimization problem with M unknown variables. The following can be said about the objective function;

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 Paper received May 2000; revised paper received Sep. 2000.

- It is an unconstrained, increasing and non-linear model with multivariable
- It is a convex, continuous and separable function
- It has only a single minimum point
- It is equal to one-third of the power that is consumed in the system
- Its first partial derivative with respect to each of the independent variables and gradient vector for each point can be calculated
- It is a twice continuously differentiable function and the Hessien Matrix can be derived.

There are *M* independent variables in the objective function. At the minimum value of this function, the partial derivative of *U* with respect to each of the independent variables must be zero. If this is realized, *M* simultaneous non-linear equations with *M* unknowns are derived. Since these equations are equal to zero at the optimum point, they satisfy Kirchoff's voltage law. In other words, the determination of the air quantities at the optimal point at which the partial derivatives are sufficiently close to zero gives the solution to the objective function.

Non-linear programming techniques

Several non-linear programming methods can be used to solve the objective function given above. Amongst these methods, Gradient Techniques are accepted as the fastest ones for the optimization of unconstrained models. In these methods, the problem is solved by using the gradient of the objective function at the current point (Himmelblau[1972]).

If a $f(x)$ function is continuously differentiable, this function has a$\nabla f(x)$ gradient at any x point. The gradient at any x point is a vector whose elements are the first partial derivatives of the function at that point and gives the slope of the function at the current point.

The greatest local increase in the function at any point occurs when we move in the direction of the gradient. Adversely, in the opposite direction of gradient vector, the function shows the greatest decrease. In other words, the negative of the gradient vector defines the direction of the most rapid decrease of $f(x)$ and therefore this path is followed for minimization (Fiacco and McCormick[1968]).

Of the non-linear programming techniques which use the gradient of the objective function, the fundamental and the most simple one is the Steepest Descent method. The basic idea of the method is to find successive points in the direction of steepest descent starting from an initial point (Myskis[1975]). The method is an iterative procedure.

The process begins at a pre-selected starting point X^o and the gradient of the objective function at this point is calculated. The next point is determined such that;

$$X^{k+1} = X^k - r^k \nabla f\left(x^k\right) \qquad [2]$$

where, r^k is a parameter called optimal step size. This parameter defines the length of movement in the direction of steepest descent without increasing the function value. In other words, if a function is defined such that;

$$h(r) = f\left(X^k - r^k \nabla f\left(x^k\right)\right) \qquad [3]$$

r^k is the value of r minimizing $h(r)$. Since $h(r)$ is a single variable function, any one-dimensional search method may be used to find the value of r^k (Kowalik and Osborne[1968]).

After the determination of the values of X^{k+1} set, the gradient vector at this point is evaluated and the process is then repeated. When the elements of gradient vector are sufficiently close to zero, the iterative procedure is terminated. This satisfies the optimality condition and gives the optimum solution set.

In the analysis of mine ventilation networks by using the Steepest Descent method, the following algorithm can be proceeded;

Step 1 : Select the meshes ($M = B - N + 1$)
Step 2 : Assign Q^o initial point, ε tolerance level, $k = 0$
Step 3 : Evaluate $\nabla U(Q_i^k)$ ($i = 1,2,......,M$)
Assign $P_i^k = - \nabla U(Q_i^k)$
Step 4 : Calculate P_j^k for all branches, ($J = 1,2,.....,N$)
Step 5 : Evaluate the optimal step size ${}_r^k$ such that minimize U ($Q_j^k + {}_r^k P_j^k$)
Use any one-dimensional method for this search
Step 6 : Find the successive point from $Q_j^{k+1} = Q_j^k + {}_r^k P_j^k$
Step 7 : Do $k = k+1$, if $P_i^k < \varepsilon$ then stop. Otherwise, return to Step 3.

When the procedure is terminated, the obtained Q^k set gives the air distribution which minimizes the power consumption in the network.

In addition to the Steepest Descent method, Fletcher-Reeves, Newton and Davidon-Fletcher-Powell methods were also applied to the objective function. Computer programs based on these algorithms were written and tried on various model networks. The results obtained can be summarized as follows;

- Ventilation networks can be solved by means of non-linear programming techniques, as well as by Hardy Cross method. The air quantities calculated by these two approaches are reasonably close to each other.
- The optimization techniques, except the Steepest Descent method, need less iterations than that of Hardy Cross method. Nevertheless, this superiority disappears for larger networks.
- None of the non-linear programming techniques is superior to Hardy Cross method with regard to total solution time, since each iteration needs time consuming procedures.
- In Hardy Cross method, the assignment of initial air quantities as zero considerably increases the solution time and the number of iterations. Therefore, non zero initial air quantities should be assigned for all branches in the network, such that Kirchoff's current law is satisfied. This is also tedious, complicated and time consuming process. However, in the application of non-linear programming techniques, the assignment of initial air quantities as zero does not significantly affect the total solution time and the number of iterations. This feature can be regarded as an important advantage for the optimization techniques.
- Although the obtained point at the first iteration falls far away from the optimum point in Hardy Cross method, a rapid convergence is observed within later iterations. Excessive fluctuations do not occur near the

optimum point. However, an adverse behaviour is seen in the application of the optimization methods. They nearly converge to the optimum point even within the first few iterations. Nevertheless, their later convergence rates, especially near the optimum point are too slow that leads to increased total solution time.

The proposed combined method

In order to benefit from the advantages of both Hardy Cross method and the unconstrained optimization techniques, a combined method has been developed. The combined method uses the Steepest Descent algorithm for the first few iterations, and all air quantities nearly approach to the optimum point at the end of this procedure. The method follows the Hardy Cross algorithm within the later iterations so that the convergence near the optimum point can be accelerated.

The proposed algorithm can be given as follows;

Step 1 : Select the meshes ($M = B - N + 1$)
Step 2 : Assign ε tolerance level, $k = 0$, $Q^o = 0$
Step 3 : Evaluate $\nabla U(Q_i^k)$, ($i = 1,2,......,M$)
Assign $P_i^k = -\nabla U(Q_i^k)$
Step 4 : Calculate P_j^k for all branches ($J = 1,2,.....,N$)
Step 5 : Evaluate the optimal step size r^k to minimize $U(|Q_j^k + r^k P_j^k|)$. Use quadratic interpolation method for this one-dimensional search
Step 6 : Find the successive point $Q_j^{k+1} = Q_j^k + r^k P_j^k$
Step 7 : Do $k = k+1$, if $k<2$ return to Step 3
Step 8 : Calculate the correction (ΔQ) in the Hardy Cross method for each mesh
Step 9 : Correct the air quantities
Step 10 : Do $k = k+1$, if $|\Delta Q_{max}| < \varepsilon$ then stop, otherwise return to Step 8.

Since it is simple and needs less period for each iteration than other non-linear programming techniques, the Steepest Descent method has been used in the first two iterations of this algorithm. A computer program based on this algorithm was developed and tried on various model networks. Operation performance of the method was compared with that of Hardy Cross algorithm.

The combined method reaches to the optimum solution in less iterations and period. Furthermore, it does not need the assignment of non-zero initial air quantities. All air quantities can be initially assigned as zero. This does not greatly affect the solution time.

Conclusion

The classical Hardy Cross method and the Steepest Descent method which is the simplest one of the non-linear programming techniques were combined in the same algorithm. A computer program based on this algorithm was written. Computational experience demonstrates that the proposed algorithm converges the optimum solution very rapidly, requires less number of iterations and needs almost half of the total solution time when compared with the Hardy Cross method. In addition to these advantages, the proposed method eliminates the need for initial estimates of airflow quantity.

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Mintek researches cheaper route for magnesium*

Minerals researcher, Mintek, with partners from government (The Department of Arts, Culture, Science and Technology) (DACST) and industry, has embarked on a major development programme aimed at reducing the costs of magnesium production substantially.

World demand for magnesium is expected to rise sharply in the next five years, with the market for die-cast automotive components, the largest growth sector, increasing by some 15 per cent per annum.

The Mintek route, incorporating DC arc furnace technology, aims to achieve lower operating costs, coupled with enhanced furnace productivity, and a reduction in energy consumption and waste generation.

A paper on the process was presented at the recent South Australasian Magnesium Conference in Sydney by Dr Nic Barcza, Mintek's General Manager: Business Development and Technology Commercialization.

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'Thumbs up' for world's longest project

The world's longest running mineral processing research project, known as 'P9', has received a significant 'thumbs up' from leading North American mining and service companies following a recent six-monthly sponsor's review meeting in Australia.

P9 started in 1962 as a University of Queensland-based mill control project, brokered through the Australian Mineral Industries Research Association (AMIRA).

These days the project deals mainly with optimizing existing mineral processing plants and designing new ones.

The work is led from Brisbane, Australia, by the Julius Kruttschnitt Mineral Research Centre, with support from the University of Cape Town in South Africa and McGill University in Montreal.

JKMRC Project Leader Professor J-P Franzidis said a tour of Canada and the USA late in 1999 led to a sponsorship base of seven North American companies, including Inco, Noranda, Cominco, Phelps Dodge, ME International, Cleveland Cliffs and Baker Process, bringing the total number of P9 sponsoring companies to 38 world-wide.

Accompanying Professor Franzidis on the tour was AMIRA International P9 Project Coordinator Mr David Stribley.

'Most of the people we spoke to in North America, other than our existing sponsors, weren't aware of P9 and what it was doing,' Professor Franzidis said.

Growing Canadian and USA support for the project has coincided with the announcement of formal research collaboration with McGill University in Montreal.

'The McGill mineral processing team brings a wealth of experience in flotation to the project,' Professor Franzidis said.

ME International Inc International Sales vice-president, Mr Art Reith, said his company joined P9 after assessing their current technical capabilities in semi-autogenous (SAG) milling.

'SAG milling has moved into more complex applications compared to previous ball mill operations,' Mr Reith said.

'It's essential that we became familiar with global developments in mill processing so as to design and produce mill liner products to meet current and future requirements from operators, metallurgists and maintenance people.'

Mr Reith said JKMRC research conducted through P9 was relevant to ME International's business objectives.

In terms of why other North American mining or supply companies would come to the project, Mr Reith says P9 could easily become a *de facto* extension of their own technical departments.

He said the globalization of the mining industry and a trend towards outsourcing R & D made it easier for US-based companies to join transnational research projects such as P9.

'Most major mining companies operating in the US would also operate outside, such as Phelps Dodge who are significant within America, but also have operations in Chile.'

Phelps Dodge Technology vice-president Mr John Marsden said many processors in the US or Canada who stood to benefit from the project may not be aware of its existence.

He said P9 presented a number of areas of interest to Phelps Dodge, particularly fine grinding in the project's comminution (rock breakage) module.

The JKMRC's work in flotation and liberation modelling using JKSimFloat, a software simulation package developed from the P9 project, is seen as an essential tool for the industry, Mr Marsden said.

'Phelps Dodge has had a connection with the JKMRC for quite some time through the use of their JKSimMet software in a number of our operations, but we've now decided to join the project.

'We've been through a down cycle in our copper business as we've been building up our technology group and R&D activity over the past three years, which makes this an appropriate time to be supporting this effort.'

Professor Franzidis said he hoped that other American companies would follow Phelps Dodge and ME International in joining the P9 project.

For more information about P9 contact:

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