



# Input-output analysis: its potential application to the mining industry

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## Synopsis

Input-output analysis is a major branch of quantitative economics, and is used to analyse the flow of goods and services between production sectors in a national economy. Its purpose is to determine the relative value-adding capacity of each sector. The model has also, in recent years, found increasing use in assessing the environmental and ecological impact of development projects. It has, however, never been applied to individual production facilities, such as large mines, which themselves have complex economic structures, through which there is a flow of goods and services. Conventional cost and accounting systems, as used on mines, treat production sectors as separate cost centres, and do not determine the inter-relationships between them. This can lead to incorrect conclusions on the relative value of sectors and their impact on the total economic performance of the facility. This paper explains the basic principles of input-output analysis, and presents an example of its application to a simplistic model of a mine, which is used in the absence of any real data. The paper concludes that, by forcing definition of the value adding function of each sector, input output analysis will facilitate control of inputs, or costs. This will result in optimal application of resources and exploitation of ore reserves. The authors realise the model needs to be tested on an operating mine, and hope this paper will result in an opportunity to do so.

The authors wish to thank the anonymous referees for their valuable comments.

## Introduction

In 1853 a 'fine diamond was found at Hopetown, near the Orange Free State border' (Welsh<sup>1</sup>). Since then, the mining industry has been regarded as 'a cornerstone of South Africa's economy' (RSA<sup>2</sup>). During the intervening 147 years, the industry has overcome many challenges to become the world leader in deep level mining technology. However, the current environment of rising costs, volatile product prices, labour insecurity and diminishing ore reserves and grades presents the South African mining industry, and particularly the gold mining industry, with perhaps its most serious crisis ever.

Deep-level South African mines are huge business enterprises. The turnover of a single stopping section in a typical gold or platinum mine would dwarf that of most small

businesses and that of an entire mine runs into hundreds of millions of Rands annually. The complex economic structure of a large mine has many dynamic variables, each of which is unique to a specific section of the structure. Each has an impact on the economic performance of the mine as a whole, but it is virtually impossible to determine these impacts using conventional cost and accounting systems. The mass of data produced by these conventional systems is distilled into highly averaged factors, upon which fundamentally important operational and investment decisions are based.

Many countries, including South Africa, use input-output analysis to analyse and manage their national economies. Input-output analysis is generally not well known outside the economics profession, and it has never before been applied to a mining industry. However, research completed in the Department of Mining at the University of the Witwatersrand shows that input-output analysis can be a powerful management tool, ideally suited to large complex mines, with significant benefits.

## An introduction to input-output analysis

Input-output tables, first published by Leontief<sup>3</sup> in 1936, are regarded as one of the twentieth century's major advances in economics. They allow an economic system to be analysed using real numbers instead of complicated algebraic formulae, and are numerical models of the relationships between the production sectors of an economic system. By showing details of the flow of goods and services between industries they provide an

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© The South African Institute of Mining and Metallurgy, 2000. SA ISSN 0038-223X/3.00 + 0.00. Paper received Jun. 2000; revised paper received Oct. 2000.

# Input-output analysis: its potential application to the mining industry

analysis of the process of production, the use of goods and services, and the income generated in production. Leontief<sup>3</sup> (page v) describes input-output analysis as '...a practical extension of the classical theory of general interdependence, which views the whole economy of a region, a country or even of the entire world, as a single system, and sets out to interpret all of its functions in terms of the specific measurable properties of its structure'. The essence of input-output analysis is the compromise between elaborate theoretical structures and over-simplified descriptions of an economy. According to Barna<sup>5</sup> Leontief's input-output method greatly simplified theoretical economic models, but considerably elaborated and refined purely statistical data, allowing theory and empirical implementation to meet.

Since its introduction, input-output analysis has gained widespread acceptance and has become a major branch of quantitative economics. The publication of the first large scale input-output tables came in 1947 when the United States Bureau of Labour Statistics applied input-output analysis in an effort to prevent large scale unemployment at the end of World War II (Rose and Miernyk<sup>6</sup>).

Input-output tables now occupy a central position in the United Nations system of National Accounts, and virtually all the developed countries in the world, as well as many developing countries, use them. However, compilation of input-output tables requires meticulous data collection and processing capabilities, which are normally not found in undeveloped economies, and exceptionally high demands are placed on the organization that compiles the tables. In South Africa this task is carried out by Statistics South Africa (SSA), the central government agency authorized to compile and publish official statistics in terms of the Statistics Act of 1976.

## Practical applications of input-output analysis

Rose and Miernyk<sup>6</sup> characterized the input-output model by its versatility, and identified it as probably the most widely used method of regional analysis. In particular, this flexibility is ideally suited for the simulation of economic development, and, where data permitted, the construction of tables for the measurement of income and employment multipliers. The input-output model has also, in recent years, proved to be a flexible and versatile tool when examining the environmental and ecological aspects of economic development. For example, Rose<sup>7</sup> identifies several distinguishing features of input-output analysis, which continue to be of lasting value.

These are:

- It is firmly grounded in the technological, measurable relationships of production, which are empirically verifiable
- It bridges the gap between economists, managers and engineers
- The sectoral layout facilitates data collection and data organization
- Input-output tables, and Leontief multipliers, greatly facilitate the analysis of the impact of private-sector decisions and public-sector policies
- Input-output models are the major tool of regional impact analysis in the United States. The US government, through the efforts of Leontief, is

committed to massive data collection for this purpose

- Because the basic formulation of input-output tables is politically neutral, it is a popular modelling tool, especially in developing countries, and can be applied to different economic systems
- It provides a full accounting for all inputs into production, unlike typical approaches, which express quantities only in terms of primary factors of production.

The economic structure of a mine can be described in terms of production sectors, the same as a national economy. Each sector requires an input of intermediate goods and services, in the form of labour, raw materials and equipment, which are then processed to produce the sector's output. Each sector, to be of any worth to the whole system, must add value to its received input. The sum of value added by all sectors in a national economy is called the Gross Domestic Product, or GDP, an important parameter used to describe economic performance of an economy.

Since most of the distinguishing features described by Rose<sup>7</sup> are present in a mine's infrastructure, there is no apparent reason why input output analysis should not be ideally suited to the analysis of a mine's complicated production process. Furthermore it is possible to determine the impact that each sector has on the total economic performance of the mine.

## The basic model

It is in a demand-driven market economy that Adam Smith's famous dictum, made long ago in 1776, that 'the division of labour', finds its most practical application. Complex tasks are broken down into their elements, and each is performed in a sequential manner. A notable application of Smith's principle was Henry Ford's production line, where each of the many tasks required to build a motor car were performed by many different workers, each skilled in a particular task. The division of labour is also manifested on a national scale. No single sector of an economy can exist on its own, and each is dependent on the other sectors, both to supply intermediate input and to consume its output.

Input-output tables show how outputs from each sector of the economy are used as inputs by the other sectors. All the sectors are shown in a column on the left hand side of a matrix, and in a row on the top of the same matrix. Each sector in the left-hand column supplies input to each of the sectors in the top row, including itself. In this way the matrix, or array, which is called a Transaction Table, is developed to show the flow of goods and services, or transactions, as inputs and outputs between every sector in the economy. Since there are as many columns as there are rows, the matrix is always a square. The South African economy, in 1993 for example, had ninety such sectors, so that 8100 different transactions between the various sectors were recorded. A Transaction Table is the heart of input-output analysis.

The basic Leontief<sup>3</sup> input-output model is expressed as a series of accounting identities. Each row is in fact a linear equation with as many variables and coefficients as there are sectors (Rose and Miernyk<sup>6</sup>) i.e.,

# Input-output analysis: its potential application to the mining industry

$$X_i = X_{i1} + X_{i2} + \dots + X_{in} + Y_i (i = 1 \dots n) \quad [1]$$

where:  $X_i$  = total gross output of sector  $i$   
 $Y_i$  = the autonomous final demand for the products of sector  $i$   
 $X_{ij}$  = the sales by sector  $i$  to each of the endogenous sectors  $j$ .

Leontief's analysis deals with the question:

'What level of output should each of the  $n$  industries in an economy produce in order that it will just be sufficient to satisfy the total demand for that product'<sup>11</sup>.

The SSA table shows, in quadrant 1, the value of transactions between the various industrial sectors of the economy. The rows in quadrant I show the distribution of the output of a particular industry to other industries, and each column shows the value of each industry's usage of raw materials, semi-finished goods and services.

## Structure of the I-O tables

The format shown in Table I is essentially the same as that used by Statistics South Africa. Quadrant II shows the various components of final demand, i.e.

- ▶ Consumption expenditure by general government
- ▶ Gross domestic fixed capital formation
- ▶ Change to inventories
- ▶ Exports.

The rows in quadrant II show the distribution of intermediate output to these final demand sectors, and the columns show the value consumed by each final demand sector.

Value added to output from the various industrial sectors, and value added in the consumption of intermediate output by the various final demand sectors, is shown in sectors III and IV, respectively.

Direct coefficients are calculated from quadrants I and III, and these in turn produce the Leontief inverses and the output multipliers.

Input-output tables also contain the three elements of the gross domestic product, or GDP, i.e. remuneration of employees, gross operating surplus and net indirect taxes. When households (or remuneration of employees) are excluded from the transaction table, these three elements are all found in quadrant III, and the model is said to be Open

with respect to households. When households are included in the transaction table, i.e. quadrant I, and gross operating surplus and net indirect taxes are in quadrant III, the model is said to be Closed with respect to households.

## Absorption or transaction matrices

This is the first of the three matrices published by Statistics South Africa. The values of these transactions are shown in a matrix of  $n$  rows and columns, and are denoted by  $a_{ij}$ , where both  $i$  and  $j$  range from 1 to  $n$ .

If all the industries shown at the beginning of each row are generally denoted by the subscript  $i$ , and all the industries at the head of each column are generally denoted by the subscript  $j$ , then the absorption of goods from  $i$  to  $j$ , i.e., the input to  $j$  from  $i$ , or the output from  $i$  to  $j$ , is given by:

$$X_{ij}$$

The total absorption of goods and services by  $j$  from all the industries in the economy, i.e., all the industries shown at the beginning of the rows (in other words, all the  $i$  industries), is given by the sum of all the inputs to a given 'column' industry from all the 'row' industries, i.e.;

$$\sum_i X_{ij}$$

The Total Input or Total Cost of an industrial sector is given by the sum of all the inputs to the industry, i.e.;

$$X_j = \sum_i x_{ij} + V_j + M_j \quad [2]$$

where  $V_j$  and  $M_j$  are the value added and imports to sector  $j$ , respectively. Similarly, the sum of any row,  $i$ , is given by;

$$X_i = \sum_j x_{ij} + F_i \quad [3]$$

which is the total use made of the output of a sector  $i$  industry (a 'row' industry) by all the other productive sectors,  $\sum_j x_{ij}$ , (the column industries) plus the final demand for that sector,  $F_i$ .

Because each row is a linear balance equation, the total outputs from all the row industries must equal the total inputs to all the column industries, so that;

$$X_j = X_i \quad [4]$$

## Direct coefficient matrices

Direct coefficient matrices, the second matrix in the Statistics

Table I			
A simplified transaction table			
	Intermediate demand	Final demand	Total output
Intermediate inputs Value adding Primary inputs, Imports and GDP components Total input	Quadrant I: Intermediate production and Consumption Quadrant III: Primary inputs to Production	Quadrant II: Final output of Production sectors Quadrant IV: Primary inputs to Final demand	Sum of; Intermediate input and Final demand

(Source: Clements and Greig<sup>9</sup>)

\*The components of GDP are:  
 Remuneration of Employees, ROM;  
 Net Independent Taxes, NIT;  
 Gross Operating Surplus, GOS.

# Input-output analysis: its potential application to the mining industry

South Africa set of Input-Output Tables, are derived from absorption, or transaction, matrices, and show the input coefficients, or direct requirements of each industry in relation to the output of the industry. Direct coefficients are the ratio of each basic value in the absorption matrix to the total output of each industry. They show the inputs necessary to a given industry required to produce one unit of final demand from that industry, and are generally denoted thus;

$$a_{ij} = x_{ij} / X_j \quad [5]$$

or,

$$x_{ij} = a_{ij} \cdot X_j \quad [6]$$

therefore, from [3]:

$$X_i = \sum_j a_{ij} \cdot X_j + F_i \quad [7]$$

This is the linear balance equation to which reference has already been made. The set of all the linear balance equations in the direct coefficient matrix is written as;

$$X = A_{NN} + F \quad [8]$$

where  $A_{NN}$  is the matrix of the technical coefficients derived from the basic absorption matrix, and  $F$  is the final demand of each industry. Because technical coefficients are dimensionless, they allow comparisons between different economies, and, more importantly, allow the derivation of inverse coefficients.

## Inverse coefficients

The third matrix produced by SSA contains inverse coefficients. Inverse coefficients are the direct and indirect requirements (in the case of an open model), and direct, indirect and induced requirements (in the case of a closed model) per money unit of final demand. They trace the impact of one extra unit of demand, from any particular industry, back through the whole economy.

Inverse coefficients allow the impact of a change in final demand from each industry to be calculated, and to analyse the inter-dependence of the production system. The Direct Coefficient Matrix consists of simultaneous linear balance equations derived from the Absorption Matrix. Matrix algebra facilitates solution of these equations. Inverse coefficients can be defined as the inverse of the difference between the Identity Matrix and the matrix of direct coefficients,  $A$ , and are known as Leontief Inverses, i.e.;

$$[I - A]^{-1}$$

Kuhn and Jansen<sup>10</sup> illustrate this linkage effect as follows:

If final demand for output from, say, the agricultural sector increased by R1 million, then in addition to this amount agricultural output would increase by the technical coefficient, say 0.06, so that the output would need to be R1.06 million. Each other sector in the economy would in turn be affected by the requirement to increase intermediate input to the agricultural sector according to their own technical coefficients.

This is a tedious reiterative process using only technical coefficients, but a far more efficient method is to use the inverse coefficient matrix, which contains all the inter-

industry linkage effects. This will show the total effect that an increase in demand for any one sector's output will have on the all the other sectors.

The whole system is a general equation:

$$F - (I - A) \cdot X \quad [9]$$

Where:  $I$  is an identity matrix, i.e. a square null matrix with 1's on the principal axis

$A$  is the technical coefficient matrix.

Therefore:  $X = (I - A)^{-1} \cdot F$

Where:  $(I - A)^{-1}$  is the Leontief Inverse.

Multipliers can be calculated for variables such as GDP and employment from this equation where:  $X$  is the relevant multiplier and  $Y$  is either the vector or the diagonal matrix of the variable of which the multiplier is required, e.g. GDP or employment

The value of the multiplier  $X$  depends on the correctness of the two components,  $Y$  and  $[I - A]^{-1}$ . These in turn depend on the accuracy with which the transaction table has been compiled. (For a detailed discussion on matrix algebra and the derivation of Leontief Inverses, refer to Chiang<sup>11</sup>).

## Application of input-output analysis to a mine

A mine can be described in terms of production sectors, the same as a national economy. Each sector requires input, in the form of labour, materials and services, which are then processed to produce an output, and which may vary in form from sector to sector. Output is also in the form of intermediate and final demand, the same as in a national economy.

Because there is a flow of goods and services through the mine a transaction table can be constructed, and inverse coefficients and multipliers can be calculated. This transaction table will be a dynamic model of the flow of goods and services through the whole organization, presenting a total picture of the operation. It will allow this flow of goods and services to be monitored and controlled, and will identify those sectors of the mine that add the greatest value to input received and which have the greatest impact on the economic performance of the mine. It will also force the identification and definition of each sector's production function.

## A simple example

A simple transaction table for a mine, using illustrative values, is shown in Table II. This assumes, for example, that the Administration section required an input of its own goods and services with a value of R2000. The other sections, i.e., Engineering, Metallurgical Plant, U/G Section 1 and U/G Section 2, required inputs of the Administration section's goods and services with values of R4000, R4000, R5000 and R6000 respectively. The total intermediate output of the Administration section was therefore R21 000.

Likewise, U/G Section 2 supplied goods and services with values of R3000, R5000, R33 000 and R6000 to the Administration, Engineering, Metallurgical Plant and U/G Section 1 respectively, and required R3000 of its own goods and services. Its total intermediate output was therefore R50 000.



## Input-output analysis: its potential application to the mining industry

Table II also assumes that the Administration section required intermediate inputs from Engineering, Metallurgical Plant, U/G Section 1 and U/G Section 2 worth R1000, R0, R3000 and R3000 respectively. It purchased external stores worth R2000. Its total intermediate input, including its own goods and services, was therefore R11 000.

The total intermediate input received by each sector from the other sectors, plus input required from external sources, 'Imports' or 'Stores', is the total intermediate input required by each sector in order to produce its own output. This is shown as R11 000, R38 000, R97 000, R55 000 and R63 000 for the Administration, Engineering, Metallurgical Plant, U/G Section 1 and U/G Section 2, respectively.

### Added value

Every industry, or sector of an economy, requires an input of a product or service, to which it then adds value in order to generate its own output.

Adding value to input, of whatever nature, is the prime function of any business. If a business fails to do this, it merely consumes input and becomes a worthless expense. This concept can be expressed as a basic business equation, i.e.:

$$\text{Input Value} + \text{Added Value} = \text{Output Value} \quad [10]$$

Value Added by all the sectors in an economy is known as the Gross Domestic Product. It has three components (SSA, page xx), and can also be expressed as an equation, i.e.:

$$\text{GDP} = \text{Net Independent Taxes} + \text{Gross Operating Surplus} + \text{Remuneration of Employees} \quad [11]$$

The three components of GDP can also be found in the transaction matrix describing the flow of goods and services through a mine, as shown in Table I. The value adding power of each section in the mine can thus be defined, the same as it is in a national economy.

The Value Added by the Administration section to its intermediate input, (its 'GDP'), is shown as R2000 minus R1000 plus R5000, i.e., R6000. The total input to the Administration section was therefore R11 000 plus R6000, or

R17 000. U/G Section 2 had a 'GDP' of R71 000, i.e., it added value worth R71 000 to its intermediate input, and required total input worth R134 00.

The total 'GDP', or value added to intermediate input, of this illustrative mine was R248 000, i.e., R46 000 plus R111 000 plus R91 000. The total inputs received by the various sectors as intermediate input totalled R180 000. Stores purchased amounted to R84 000, so the total input was R512 000. Theoretically, total input must equal total output, but in practice this hardly ever happens, due to inherent inaccuracy in the collection of data.

### Calculation of multipliers

Using the data shown in Table I, GDP and Employment multipliers are calculated using the Leontief model. These are shown in Table III.

### Interpretation of multipliers

The multipliers are calculated for an 'open' model, i.e., Remuneration of Employees, also known as 'Households', is excluded from the Transaction matrix. This is because the induced effects of increased wages will not impact on the mine, as these wages will not be spent within the mine's economic structure. The multipliers therefore represent the Direct and Indirect impacts of increased output from each section in the mine.

An increase of R1 in production from each separate section (Administration, Engineering, Metallurgical Plant, U/G Section 1 and U/G Section 2) will, in addition, lead to an increase in value added, or GDP, by the whole mine of

Table III

### GDP and employment multipliers

	Admin.	Engineer.	Met. Plant	U/G Sect. 1	U/G Sect. 2
GDP multipliers	0.7338	0.5859	0.7623	0.7526	0.7210
Employment multipliers	0.0064	0.0082	0.0033	0.0052	0.0056

Table II

### A simple transaction table for a mine

Sectors producing intermediate output	Sectors receiving intermediate input					Total intermediate Output
	Admin.	Engineer.	Met. Plant	U/G Sect. 1	U/G Sect. 2	
Administration	2000	4000	4000	5000	6000	21000
Engineering	1000	4000	10000	15000	17000	47000
Metallurgical plant	0	6000	4000	3000	6000	19000
U/G Section 1	3000	4000	25000	5000	6000	43000
U/G Section 2	3000	5000	33000	6000	3000	50000
Sub-total Intermediate input	9000	23000	76000	34000	38000	180000
Imports (ext. stores)	2000	15000	21000	21000	25000	84000
<b>Total intermediate input</b>	<b>11000</b>	<b>38000</b>	<b>97000</b>	<b>55000</b>	<b>63000</b>	<b>264000</b>
Net independent taxes	2000	4000	12000	13000	15000	46000
Gross surplus	-1000	-2000	50000	35000	29000	111000
Remuneration	5000	12000	20000	27000	27000	91000
Value Added (GDP)	6000	14000	82000	75000	71000	248000
<b>TOTAL INPUT</b>	<b>17000</b>	<b>52000</b>	<b>179000</b>	<b>130000</b>	<b>134000</b>	<b>512000</b>
Employees (Actual)	20	35	75	102	106	338

# Input-output analysis: its potential application to the mining industry

R0.7338, R0.5859, R0.7623, R0.7526 and R0.7210, respectively. Likewise, the corresponding increase in employment in the whole mine, caused by a R1 increase in Output from each sector, would be 0.0064, 0.0082, 0.0033, 0.0052 and 0.0056 jobs, respectively.

The conclusion is reached that the Metallurgical Plant has the most potential to impact on the 'GDP' of the mine, and also that it will do so at the minimum cost in extra jobs.

## Conclusions

Conventional cost and accounting systems:

- ▶ Record the direct inputs and outputs to and from each sector
- ▶ Do not show either the direct or indirect coefficients
- ▶ Do not show the impact, on the entire economic structure, of an increase or decrease in output from any one sector.

Unlike conventional cost and accounting systems, input-output analysis will determine the value adding power of each sector of a mine with a high degree of accuracy, because each is treated as a wholly integrated element in the structure, instead of an isolated cost centre. This means that costs are controlled by identifying expensive consumers of resources, and capital resources are applied to those sectors of the mine which have the greatest impact on value added.

There is virtually no risk attached to the use of input output analysis. It is used throughout the world and is well understood. One of the major objections many economists have with IOA is that values for transactions between sectors are averaged over three- or four- year periods. This problem can be overcome if topical data on the flow of goods and services through a mine is used, a concept which is entirely possible given available data capturing systems. The risk therefore lies in setting up and maintaining such a system, because the accuracy and effectiveness of IOA depends entirely on the quality and extent of data used to build the transaction matrix.

Input output analysis is a powerful tool, and there is no apparent reason why it cannot be used within the mining industry. As was stated at the beginning of this paper, value of goods and services that flow through the industry is very large and almost impossible to control. Input output analysis can do this.

In order to test the validity of the model, the authors, who readily concede its theoretical nature, approached two large mining companies to conduct field trials on a working mine. Unfortunately the response was not encouraging. It is hoped that this paper will result in an opportunity to do these trials.

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## OBITUARY

		<u>Date of Election</u>	<u>Date Deceased</u>
T Chitsike	Associate	19 May 1995	7 September 2000
F Leiding	Retired Fellow	5 May 1965	13 May 2000
R G Tarr	Fellow	17 June 1968	3 November 2000