Practical issues in the construction of control charts in mining applications

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Synopsis

The Shewhart $\overline{X}$ and R charts are still the most popular control charts in use today for continuous monitoring of quality data. However, for these charts to function properly, the underlying assumption of normality and independence of data must be satisfied. When either independence and/or normality of data are not present, which is a common feature of ore quality data, an application of the conventional Shewhart $\overline{X}$ and R charts may introduce false alarms in the analysis of the data. To address these issues, a guideline is proposed for construction of appropriate univariate and multivariate control charts for a variety of situations. A use of this guideline to a bauxite mine suggested the construction of special cause control chart for the analysis of individual variables namely, Al_2O_3% and SiO_2%; whereas, the multivariate T^2 chart based on residuals is proposed for the bivariate analysis of the variables. The case study clearly revealed that without considering the data correlation, one may be in a state of false impression about an out of control condition while using the Shewhart $\overline{X}$ and R charts for univariate and the Hotelling T^2 chart for multivariate analysis. To compare the effectiveness of the control charts, specifically the Shewhart $\overline{X}$ and the special cause control charts, a simulation study was conducted. It was found that the conventional Shewhart chart provided lower probability of coverage than the special cause control chart. It was also revealed that the quality specification of Al_2O_3% is met as the variation of Al_2O_3% is well within the specification limits; whereas, it is difficult to meet the quality specification of SiO_2% on a regular basis.

Introduction

In recent years the importance of quality has become increasingly apparent due to global competition. According to Banker *et al.* (1998), quality is the most important strategic issue of top management. It is primarily concerned with consumer's view of value desired and value received. In general, the term quality addresses two aspects: quality of design and quality of conformance. In the mineral industry, quality control procedures are primarily concerned with the quality of conformance which aims at meeting consumer desired specification limits. The problem of quality control is of decisive importance in the mining industry due to inhomogeneous formation of ore deposits. Particularly in the cases of ore deposits which are formed under complex geological phenomena, the process of mineralization largely affects the grade distribution. This leads to formation of deposits which are usually characterized by wide variation of ore types and grades.

The task of a quality control practitioner is to homogenize the different ore qualities during extraction in order to satisfy consumer's quality requirements. To facilitate their operations, the use of statistical quality control techniques including control charts has been proved to be a major contributing factor in achieving required quality and improving productivity. Through the application of control charts, various assignable causes of quality variation are identified. The Shewhart $\overline{X}$ and R charts are still the most popular control charts in use today for continuous monitoring of quality variation, due to their simplicity and effectiveness. The main purpose of the application of these charts is to identify the root causes of quality variation based on which corrective actions are taken to remove irregular grade fluctuation. In order for the Shewhart $\overline{X}$ and R charts to function properly, the underlying assumptions are that the sample observations are assumed to be normally distributed and statistically independent. Even though these assumptions are important but their existence is questionable in mining applications. A theoretical justification for normality is based on the Central Limit Theorem. According to the Central Limit Theorem, the distribution of the sample averages of n independent observations will approach normality as the number of observations in a sample increases, even if the individual observations are not normally distributed. In this context, Shewhart observed that many individual observations are nonnormal, although the distribution of sample means of size four will in many cases

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follow the normal curve as predicted by the Central Limit Theorem (Grant and Leavenworth, 1980). However, according to Spedding and Rawlings (1994), more extreme population distribution may require larger sample sizes to achieve a normal distribution of sample means. This situation can usually be encountered in mining applications which may prohibit the application of the conventional Shewhart $\overline{X}$ and R charts. Even though the assumption of the normality can sometimes be met because of the Central Limit Theorem, the need for the sample observations to be statistically independent can be a much more serious problem as autocorrelation among the observations becomes inherent characteristic in mineral deposits where ore grades are spatially related. As a result it is expected that the sample observations X(t) and X(t+1) will be positively autocorrelated. Further, the sample means may tend to drift over time. Any one or combination of these issues pertaining to the behaviour of a quality characteristic in statistical quality control does not mean that the quality characteristic is out of control. They merely represent the inherent quality variation. An application of a conventional Shewhart chart in these cases will result in many false alarms leading to unjustified searches for assignable causes.

Many quality control operations in mining deal with controlling more than one variable for meeting the quality specifications. In such cases, an application of the univariate control charts is unsatisfactory as it fails to consider the problem in multivariate situation ignoring correlation structures amongst the variables. Hence, multivariate control charts should be constructed to identify an out of control condition.

Thus the above discussions clearly revealed that selection of an appropriate control chart depends upon mine-specific situations. In this paper, a guideline is proposed for selecting control charts for a variety of situations. Further, a study is presented to demonstrate the use of the suggested guideline for construction of proper charts for a bauxite mine.

Methodology

Ore quality depends upon a number of factors such as variation of *in situ* grade, extraction method and equipment, production planning, grade control at faces, ore blending, ore dilution and subsequent processing of ore. Depending upon the grade control practices at mines, the ore quality may follow a systematic pattern or it may behave erratically because of occasional presence of some assignable causes. The systematic pattern of variation of a quality characteristic may be due to chance causes of variation such as inherent variation of grade of a block, variation of grades of different blocks, and occurrence of low/high grade zones within a deposit. On the other hand, various assignable causes of quality variations are: improper estimation of *in situ* grade of ore, misclassification of ore and waste blocks, improper blending and processing of ore. As mentioned before, an essential tool for quality control in mines is the control chart. Table I is presented as a guideline for selecting control charts for a variety of situations to identify the chance and assignable causes of quality variation. A discussion of Table I on the application of various control charts is presented here.

Univariate charts

Most commonly used univariate control charts are the Shewhart $\overline{X}$ and R charts (Grant and Leavenworth, 1980; Porter and Caulcutt, 1992; Bissell, 1991; Wetherill and Brown, 1991; Mitra, 1993). The literature review revealed that the Shewhart $\overline{X}$ and R charts have been extensively used in other industries; however, a limited literature is available on the application of quality control charts in mining (Coxon and Sichel, 1959; Khuntia, 1991; Samanta *et al*., 1998). In order for these charts to function properly, the assumptions of normality and statistical independence of sample observations must be tested through various statistical tests. Basically, the Shewhart $\overline{X}$ chart is used to monitor the average of a quality characteristic and the R chart is used to monitor the variability. In both the $\overline{X}$ and R charts, three lines are drawn which are called the centre line (CL), upper control limit (UCL) and lower control limit (LCL). In the $\overline{X}$ chart, the CL is set at the process average; whereas, the UCL and LCL are drawn at $\overline{X}\pm3\sigma_{\overline{X}}$.

Table I

Suggested approach for selection of control charts

	Chart type	Purpose/usage	Remarks
UNIVARIATE	Shewhart $\overline{X}$ chart	Monitoring the average of a quality characteristic across time	Sample observations are normally distributed and statistically independent
	Shewhart R Chart	Monitoring the variability of a quality characteristic across time	Sample observations are normally distributed and statistically independent
	Cumulative Sum chart	Monitoring shifts in the average of a quality characteristic when conventional Shewhart charts are not sensitive enough	Sensitive to smaller shifts but slower to react to large shifts
	Weighted variance (W.V.) and Johnson control charts	Similar to Shewhart $\overline{X}$ and R charts	Sample observations are skewed and not normally distributed, however, they are statistically independent
	Special cause control chart.	Monitoring the average of a quality characteristic across time when sample observations are auto-correlated	Sample observations are not statistically independent and they are auto-correlated
MULTIVARIATE	Hotelling T^2 chart	Monitoring several related quality characteristics	Quality characteristics are jointly distributed according to multivariate normal distribution and observations are statistically independent
	Box-Cox transformed data applied to T^2 chart	Similar to Hotelling T^2 chart.	Quality characteristics are not multivariate normal but statistically independent
	Modified T^2 chart based on residuals	Similar to Hotelling T^2 charts	Quality characteristics are not statistically independent

Similarly, in the R chart the CL is set at the mean range of the process and the UCL and LCL are drawn at $\overline{R} \pm 3\sigma_R$.

The Shewhart $\overline{X}$ chart is used to detect large shifts from a target value. In places where it is necessary to detect very small shifts from target values, the CUSUM control chart is useful. The CUSUM control chart is very powerful in detecting sudden and persistent changes as opposed to intermittent assignable causes of the Shewhart chart. The CUSUM chart is based on the idea that while a small change of quality characteristic may not lead to a single point outside the control limits, the presence of this change may be detected if the effect accumulates over several samples (Ewan, 1963; Johnson, 1961; Lucas, 1973). Thus, it takes into account the cumulative sum of past observations as well as current observation in detecting an out of control condition.

As previously mentioned the construction of Shewhart charts is based on the normality assumption of sample observations. However, if a characteristic is not normally distributed, but normal based techniques like the Shewhart charts are used, serious errors can be made. Yourstone and Zimmer (1992) showed that significant departures from normality can have serious effects on the error probability and interpretation associated with such charts. Among the various approaches dealing with nonnormality while constructing the control charts, the most popular methods used are the Weighted Variance (W.V.) method (Choobineh and Ballard, 1987; Bai and Choi, 1995) and the Johnson transformation (Spedding and Rawlings, 1994; Chou *et al.*, 1998). The W.V. method is used for a skewed population. This method is based on the idea that a skewed distribution can be split into two segments at its mean depending upon its skewness and each segment can be used for creating a new symmetric distribution (Choobineh and Ballard, 1987). The two new distributions created from the original skewed distribution have different standard deviations. The W.V. method uses these two distributions for setting up the limits of a control chart, one for computing standard deviation for the upper control limit (UCL) and the other for estimating standard deviation for the lower control limit (LCL). If the population is skewed to the right, then the distance of the UCL from the mean is larger than that of the LCL. Similarly, if the population is skewed to the left, then the distance of the LCL from the mean is larger than that of the UCL. Detailed discussion for formulating the control limits will be found in Bai and Choi (1995). On the other hand, using the Johnson transformation the nonnormal data is transformed to the normal data using the three families of the Johnson's distribution and the Shewhart control charts are applied on the transformed data as they follow the normal distribution (Chou *et al.*, 1998).

In addition to the normality assumption, constructions of the Shewhart charts are based on the assumption that observations are independent. As mentioned before, the assumption of independence of observations of a quality characteristic in mining is questionable as autocorrelation among the observations becomes an inherent characteristic in quality control data. Johnson and Bagshaw (1974), Harris and Ross (1991), Alwan (1992), Padgett *et al.* (1992), and Maragah and Woodball (1992) had shown that the operating characteristics of traditional control charts are adversely affected by the violation of the assumption of independence. Hence, for correlated observations the approaches for construction of control charts based on the independence of observations are unsatisfactory as they fail to capture the actual natural variability due to presence of autocorrelation of data. Even a very low level of serial correlation produces dramatic disturbances in the control chart properties. These disturbances lead to erroneous conclusions about the state of control of a quality characteristic (Montgomery and Mastrangelo, 1991). One of the most widely used approaches to address this problem is the application of time series models (Ermer *et al.*, 1979; Alwan and Roberts, 1988; Montgomery and Friedmen, 1989; Alwan, 1992; Wardell *et al.*, 1992). The method involves a two-stage process in which one first fits the data with a time series model and then monitors the one-step-ahead forecast residuals over time using traditional control charts. The essence of this approach is to use an appropriate time series model to account for autocorrelation and presence of trend in the data set. Once the modelling is completed, a chart namely Special Cause Control chart termed by Alwan and Roberts (1988) based on residuals or one-step ahead forecast errors can be used to monitor a quality characteristic. Barring any special causes, the residuals should be independent and identically distributed and hence all the assumptions of traditional quality control chart hold. In case of any disturbance, the residuals will begin to show a departure from statistical control.

Multivariate charts

Many quality control operations in mining deal with controlling more than one quality characteristic. For a multivariate situation, if the variables are not related to each other then univariate charts should be constructed. However, if the variables are correlated, then a standard multivariate control chart should be used in monitoring the quality characteristics. Many researchers investigated multivariate control charts and successfully implemented them in other industries (Pignatiello and Runger, 1990; Tracy *et al.*, 1992; Hawkins, 1993). The multivariate approach to quality control was first widely used in late 1940s by Hotelling in the testing of bombsights (Hotelling, 1947 and Hotelling, 1950). Since then, the Hotelling T^2 chart is popularly used in the multivariate quality control problems (Alt, 1985; Jackson, 1985; Rius, *et al.*, 1997; Morud, 1996; Samanta and Bhattacherjee, 1999). The Hotelling T^2 chart takes into consideration the deviation of the observed vectors from the mean vector as well as the variance-covariance matrix of the data sets. A theoretical background for formulating this chart will be found in Mitra (1993).

The Hotelling T^2 is basically the multivariate counterpart of the Shewhart univariate chart. Therefore, all the assumptions of univariate Shewhart charts are also valid here; that is, the variables under study should be statistically independent and they will follow multivariate normal distribution. In mining applications, even though individual variables of ore quality characteristics sometimes follow normal distribution; however, they may not follow multivariate normal distribution. One of the remedies in this situation is to transform the data using the Box-Cox transformation to achieve the multivariate normality and then apply multivariate T^2 chart of the transferred data (Tang and Tham, 1999). The effects of statistically independence of the data on T^2 are less severe than the univariate case in some situations (Mastrangelo *et al.*, 1996); however, in presence of autocorrelated data each variable may be fitted by time series models to remove autocorrelation from each variable's respective data set. The residuals that remain can then be used to construct the T^2 chart (Slutter *et al.*, 1998).

Case study

The study was conducted in a metal mine supplying bauxite ore to an aluminium refinery plant. The morphology of the deposit and its irregular thickness made it impossible to adopt conventional bench method of open pit mining. Instead a modified trench method consisting of development of roughly parallel trenches has been adopted for efficient quality control. Further, a computer-aided mine planning system is practised by the mine management for progressive development of the area for mining which takes into account the geological features of the deposit and the variability of ore quality. Specifically, in the trenches bauxite is extracted in two distinct phases. In the first phase, a slice of around 8–12 metres thickness is extracted using normal drilling and blasting procedure; while in the second phase, bauxite is extracted selectively in 4 metres thick slices. This process minimizes the ore dilution effectively. Large size wheel loaders or hydraulic excavators have been found suitable for loading operations in the first phase; whereas, in the second phase backhoe hydraulic excavators are used for selective mining. Normally, the width of a trench is more than 70 metres for efficient use of heavy machinery, and level differences between adjacent trenches are usually maintained at less than 4 meters. The material extracted from a face by shovel is loaded into dumpers which, in turn, dispatch the material into a crusher. From the crusher, the material is sent out to a stockpile for blending through a single flight conveyor of 14 metres length. From there the material is sent to a refinery plant.

The main constituents of the bauxite ore are Al_2O_3% and SiO_2%. For conducting this study, samples collected for a period of six months from the despatch section of the mine were used. The average grade information of the samples were recorded on a daily basis by the quality control personnel of the mine. The quality norms set for the supplying of ore from the mine to the refinery plant are the following: Al_2O_3%–42.5% ± 2%, and SiO_2%–max 3%.

Selection of control charts

Before constructing control charts, a preliminary analysis of the original data sets was conducted. Figures 1 and 2 show the day-to-day grade variation for Al_2O_3% and SiO_2% in a chronological order. An examination of the figures revealed that there was no trend in the observed data. This fact was also justified by comparing the means and variances of the data sets at different time intervals (Table II). The results indicated that there was no trend in the mean values of Al_2O_3% and SiO_2%.

Initially univariate control charts were investigated. For constructing the control charts, a subgroup size of five was found to be most appropriate to capture the quality variation of the grade of ore. The subgroup size was chosen as five as it represented a rational subgroup size to capture the weekly quality variation of the grade of ore. As a result, thirty samples were generated based on which control charts were prepared.

As the primary consideration for the selection of the Shewhart $\bar{X}$ and R charts is the validity of normality assumption and statistical independence of the observations, at first, these two assumptions were tested for the entire data sets. For verifying the normality assumption, histograms of the data sets of both the variables Al_2O_3% and SiO_2% were first investigated. The histogram plots of the variables showed the initial evidences for normality of the data sets. However, a more reliable approach for normality testing is the normal probability plot of data. The normal probability plots of Al_2O_3% and SiO_2% are shown in Figures 3 and 4 respectively. The plots showed that the data were more or less following the diagonal lines. In addition to this, the Kolmogorov-Smirnov (K-S) test for normality was conducted for the data sets. The K-S test showed that the estimated probability values (α) for the variables Al_2O_3% and SiO_2% were 0.771 and 0.752 respectively. Thus, it advocated the normality of the data sets.

For testing the statistical independence of the data sets, the run test was conducted and it was found that the assumption of statistical independence of the observations failed for both the data sets. Further, the autocorrelation at lag 1 for the variables Al_2O_3% and SiO_2% were 0.688 and 0.649 respectively, which showed that the observations were moderately autocorrelated.

Univariate chart

An application of the conventional Shewhart chart was

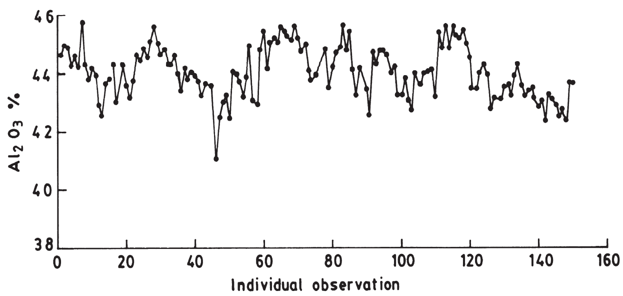

Figure 1—Grade variation of Al_2O_3% for the sample mine

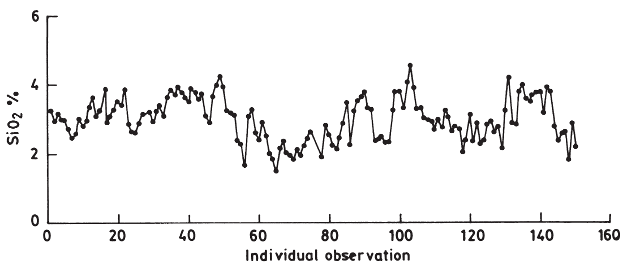

Figure 2—Grade variation of SIO_2% for the sample mine

Table II

Comparison of mean and variance values of the data sets

Observations of data sets in chronological order	Al_2O_3%		SiO_2%	
	Mean	Variance	Mean	Variance
1–50	43.97	0.73	3.31	0.25
51–100	44.37	0.66	2.69	0.40
101–150	43.78	0.75	3.04	0.35

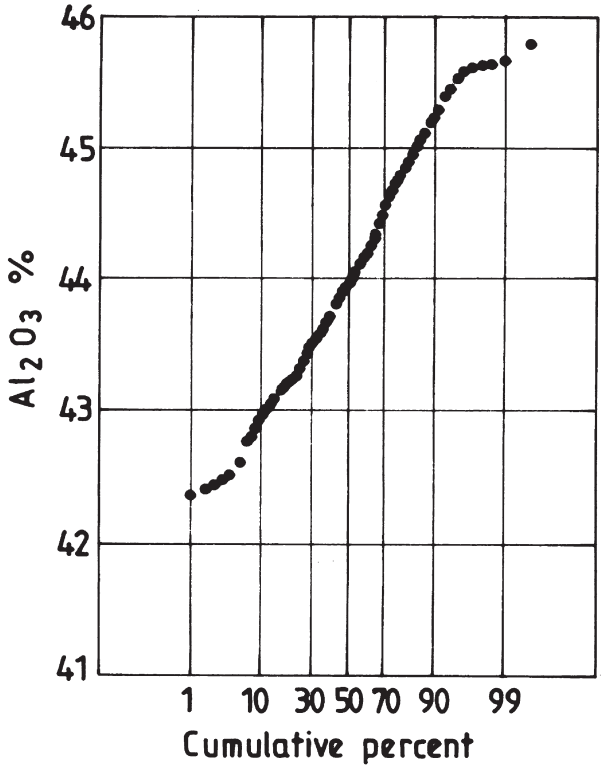

Figure 3—Normal probability plot of Al_2O_3% for the sample mine

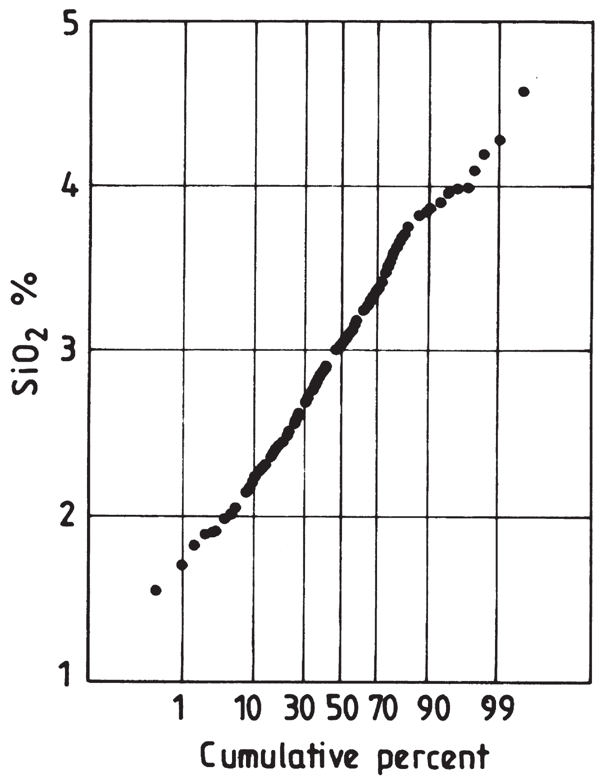

Figure 4—Normal probability plot of SIO_2% for the sample mine

prohibited as the statistical independence of the data sets failed. In this case, the most suitable chart will be the special cause control chart. In order to use the special cause control chart a suitable time series model was fitted. Prior to fitting an appropriate time series model, the stationarity condition was checked. By studying the general appearance of the estimated autocorrelation and partial autocorrelation functions of each series, clues were obtained about the stationarity condition and it was inferred that there was no trend in the data sets.

As the construction of control charts was based on the subgroup size of five, the subgroup means of sample size five were fitted to an appropriate time series model which was found to be the first order autoregressive (AR (1)) model for both the data sets. The estimated AR parameters for the variables Al_2O_3% and SiO_2% were .486 and 0.487 respectively. The t statistic values of these parameters and their associated p values revealed that the parameters are statistically significant. The model results revealed that subgroup means of the data sets were positively correlated. In addition, presence of autocorrelation for the range values for constructing R chart was also tested for both the data sets. The estimated autocorrelation values at lag 1 revealed that the range values for the data sets were free from autocorrelation. Therefore, R charts were constructed using the Shewhart method.

Using the forecasted residuals, the special cause control charts of Al_2O_3% and SiO_2% were constructed. In addition, the Shewhart $\overline{X}$ charts for both the variables were also constructed to observe the effect of autocorrelation on the control charting procedure. Figure 5 shows the special cause control chart of Al_2O_3%. The chart shows that the upper and lower control limits are somewhat wider than the original Shewhart $\overline{X}$ chart. As a result, no sample has gone out of control in this chart; whereas, in the original Shewhart $\overline{X}$ chart seven samples have gone out of control condition (Figure 6). Thus, this observation strongly reveals that the serial correlation of observations has a pronounced effect on the performance of the Shewhart control chart. Figure 7 shows the special cause control chart of SiO_2% for correlated observations. This chart also shows that the upper and lower control limits are placed somewhat wider than the original Shewhart $\overline{X}$ (Figure 8). As a consequence, no sample falls out of control in the special cause control chart compared to twelve samples of the Shewhart chart. Hence, it can be inferred for the case study mine that the use of a conventional Shewhart chart instead of special cause control chart for both the variables will result in many false alarms. This case study also revealed that without considering the data correlation one may be in a state of false impression about an out of control condition. However, it should be mentioned here that the special cause control chart is based on time series residuals which are random. As a result, the special cause control chart may not pick up a trend in the data which

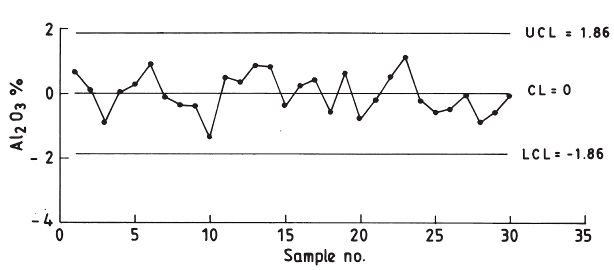

Figure 5—Special cause control chart of Al_2O_3% for the sample mine

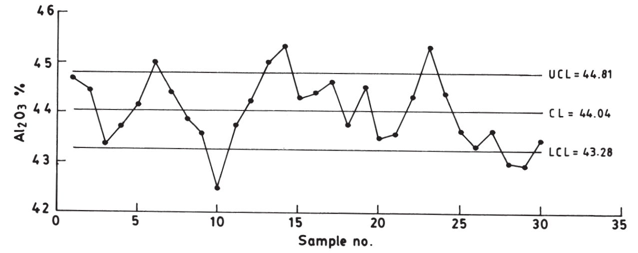

Figure 6—$\overline{X}$ chart of Al_2O_3% for the sample mine

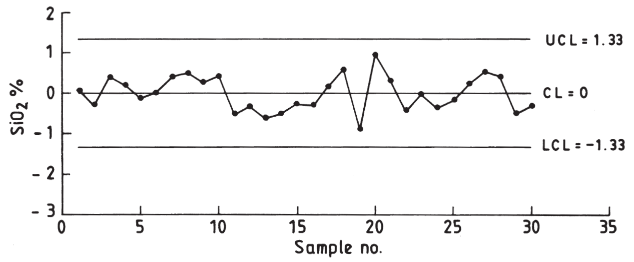

Figure 7—Special cause control chart of SiO_2% for the sample mine

may be observed in the Shewhart chart which is constructed based on the original data. For example, the special cause control chart of SiO_2% presented in Figure 7 does not pick up the trend in the mean values of the original data, which was reflected in the Shewhart $\overline{X}$ chart shown in Figure 8. Figures 9 and 10 show the Shewhart R charts for the variables Al_2O_3% and SiO_2%. The Figures reveal that no sample falls out of control condition in the R chart of Al_2O_3%, whereas only one sample falls in the out of control of the R chart of SiO_2%.

The revealing fact that a large number of sample points were out of control in the Shewhart $\overline{X}$ chart compared to the special cause control chart led to a simulation study to observe the effect of autocorrelation on control charts. In the simulation study, the AR (1) model was chosen to generate different time-series with varying autoregressive parameter values (ϕ). A wide range of ϕ values varying from 0.1 to 0.9 were considered for an extensive study of measuring the influence of autocorrelation on the performance of the two control charting procedures. The effectiveness of the control charts was measured by the probability of coverage. The probability of coverage computes the probability of the samples falling between the chart limits. A control chart will show improved performance if it provides a higher probability of coverage over other methods (Choobineh and Ballard, 1987; Samanta and Bhattacherjee, 2001). Table III presents the results of the simulation study, which indicate that as ϕ value increases, the probability of coverage in the Shewhart chart decreases, whereas the probability of coverage in the special cause control chart remains constant. Thus, the simulation study clearly revealed the fact that with increased autocorrelation the performance of the Shewhart $\overline{X}$ chart drastically deteriorates. It is also revealed that even though a large number of samples were out of control in the Shewhart $\overline{X}$ chart compared to the special cause control chart for the case study mine, this phenomenon was not reflected in the simulation study. For example, for the variable Al_2O_3%, 23.3% (7 out of 30) of the samples were out of control in the Shewhart $\overline{X}$ chart; whereas, in the simulation study 6.6% of the samples were out of control in the Shewhart $\overline{X}$ chart.

Multivariate control chart

It is clear that construction of univariate control charts for both the variables will result in losses of information of the inter-correlation of the variables. Therefore, standard multivariate control charts should be investigated in addition to the univariate control charts. As previously mentioned, the most popular multivariate control chart is the Hotelling T^2 chart. However, an application of the Hotelling T^2 chart requires the assumption of multivariate normality and the

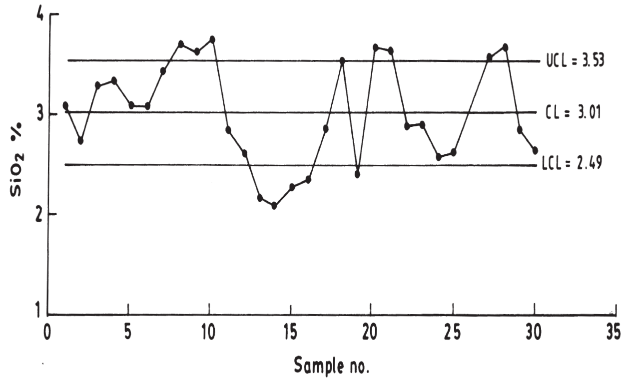

Figure 8—$\overline{X}$ chart of SiO_2% for the sample mine

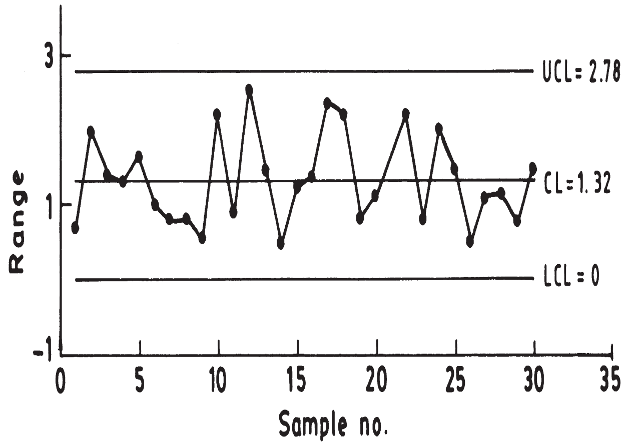

Figure 9—Shewhart R chart of Al_2O_3% for the sample mine

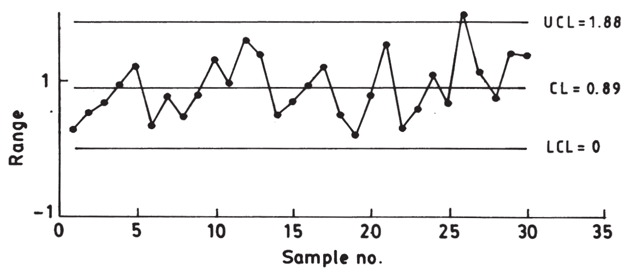

Figure 10—Shewhart R chart of SiO_2% for the sample mine

Table III

Probability of coverage for special cause control chart and Shewhart $\overline{X}$ chart

Autoregressive Parameter (ϕ)	Special cause control chart Probability of coverage	Shewhart $\overline{X}$ chart Probability of coverage
0.1	0.9966	.9966
0.2	0.9966	.9936
0.3	0.9966	.9840
0.4	0.9966	.9680
0.5	0.9966	.9363
0.6	0.9966	.8816
0.7	0.9966	.8046
0.8	0.9966	.6903
0.9	0.9966	.5320

statistical independence of the observations. Therefore, the multivariate normality of the data sets was first verified before applying this chart. Although the univariate test statistics showed that the variables were normality distributed, however, there is no guarantee that the variables will jointly follow multivariate normal distribution. In order to test the multivariate normality the criteria given by Mardia (1970, 1974, 1983,1985) were used, which are mainly based on measurement of multivariate skewness and kurtosis values. The results showed that estimated multivariate skewness and kurtosis values for the variables were 0.397 and 2.85, and their corresponding percentage points (p) for normality were 0.08 and 0.91 respectively. Therefore, the assumption of multivariate normality for the data sets was satisfied. However, the statistical independence of the data sets was not satisfied as previously tested. Therefore, the Hotelling T^2 chart based on original data cannot be an appropriate choice. As mentioned in Table I, the modified T^2 chart based on forecasted residuals of the time series models of the individual variables would be constructed.

Figure 11 shows the multivariate T^2 chart constructed based on residuals from the first order autoregressive model. In addition, the Hotelling T^2 chart based on original raw data sets was also constructed to see the performance of the two charts (Figure 12). It was found that the two charts have the same upper control limit because the computation of the upper control limit of Hotelling T^2 control chart does not depend on the data values; and it is a function of (i) number of subgroups, (ii) subgroup size, (iii) number of quality characteristics, and (iv) specified type I error. The essential difference in the above two multivariate charts is that the T^2 values differ in the two methods. Figure 11 shows that no sample goes beyond the control limits; whereas, Figure 12 reveals that fifteen samples are out of control. Therefore, the effect of serial correlation of the observations was also reflected in the Hotelling T^2 chart.

Discussion

The study clearly revealed that no sample was detected to be out of control by the appropriate use of univariate or multivariate control chart. It was also revealed that the quality specification of Al_2O_3% is met as the variation of Al_2O_3% is well within the specification limits. On the other hand, it is difficult to meet the quality specification of SiO_2% as the grand mean is very close to the specification limit. In this case, even though the quality characteristic is in statistical control, it is not possible to meet the specification limit of SiO_2% on a regular basis. Therefore, the specification limit of SiO_2% may be reviewed. Otherwise, some other means may be explored to reduce the average value of SiO_2%.

In order to get an insight about the outcome of this investigation, a detailed discussion was made with the quality control personnel of the mine. The discussion revealed that an exhaustive exploration work was carried out at the mine based on which grade estimation of the blocks was performed. As a result, a reliable estimate of grade was obtained which was supported by regular blast hole sampling of the production blocks. Based on this information, the mine management developed an extraction schedule along with a grade control plan to meet the quality requirement. In addition, blending was performed at the stockpile on a regular basis for reducing the variability of ore. All these factors led to systematic behaviour of ore grade fluctuation which was indicated in the control charts.

Conclusions

The construction of conventional quality control charts is based on the assumption of existence of normality and independence of data. However, common features of ore quality data are serial correlation and nonnormal distribution. To address these issues, a guideline is suggested in this paper for evaluating various control charts for mine-specific situation. The case study clearly revealed that the application of conventional Shewhart charts in the bauxite mine will result in false alarms leading to unjustified searches for assignable causes. As the quality data sets are

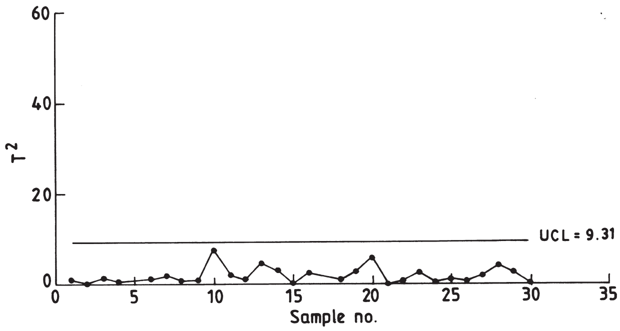

Figure 11—Modified multivariate T2 chart of SiO_2% and Al_2O_3% for the sample mine

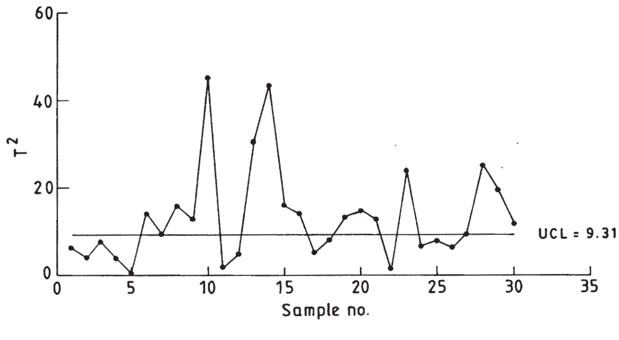

Figure 12—Multivariate control chart of SiO_2% and Al_2O_3% for the sample mine

serially correlated, it is proposed that the special cause control chart should be constructed for monitoring the quality of individual variables. Further, a simulation study revealed that in an auto-correlated environment conventional Shewhart chart provided lower probability of coverage than the special cause control chart. For the multivariate case, the modified multivariate T^2 chart based on residuals is recommended. The application of control charts clearly revealed that it is difficult to meet the quality specifications of SiO_2% on a regular basis; whereas, the quality specification of Al_2O_3% can easily be met as the variation of Al_2O_3% is well within the specification limits.

It is further recommended that whenever the control charts are constructed, the quality control personnel should understand the assumptions underlying the charts and they should ensure that the assumptions are satisfied. It is fortunate that even when the assumptions are violated and the Shewhart $\overline{X}$ and R charts can no longer be successfully used for quality monitoring purposes; nevertheless, they may still provide useful qualitative information.

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References

ALT, F.B. 'Multivariate quality control', *Encyclopedia of Statistical Sciences 6*, edited by Kotz, S. and Johnson, N.L., John Wiley and Sons, New York, NY. 1985.

ALWAN, L.C. 'Effect of autocorrelation on control chart performance', *Communications in statistics- Theory and Methods*, vol. 21, 1992. pp. 1025–1049.

ALWAN, L.C. and ROBERTS, H.V. 'Time series modeling for statistical process control', *Journal of Business and Economic Statistics*, vol. 6, 1988. pp. 87–95.

BAI, D.S. and CHOI, I.S. '$\overline{X}$ and R Control Charts for Skewed Populations', *Journal of Quality Technology*, vol. 27, no. 2 , 1995. pp. 120–131.

BANKER, R.D., KHOSLA, I., and SINHA, K.K. 'Quality and Competition', *Management Science*, vol. 44, no. 9, 1998. pp. 1179–1192.

BISSEL, A.F. 'Getting more from control chart data-Part I', *Total Quality Management*, vol. 2, 1991. pp. 45–55.

CHOOBINEH, F. and BALLARD, J.L. 'Control—Limits of QC charts for skewed distribution using Weighted Variance', *IEEE Transactions on Reliability*, vol. R-36, no. 4, 1987. pp. 473–477.

CHOU, Y., POLANSKY, A.M., and MASON, R.L. 'Transforming nonnormal data to normality in statistical process control', *Journal of Quality Technology*, vol. 30, no. 2, 1998. pp. 133–141.

COXON, C.H. and SICHEL, H.S. 'Quality control of routine mine assaying and its influence of underground valuation', *South African Institute of Mining and Metallurgy*, 1959. pp. 489–517.

ERMER, D.S., CHOW, M.C., and WU, S.M. 'A time series control chart for a nuclear reactor', *Proceedings 1979 annual reliability and maintainability symposium; Institute of Electrical and Electronic Engineering*, New York, 1979, pp. 92–98.

EWAN, W.D. 'When and how to use CUSUM charts', *Technometrics*, vol. 5, 1963. pp. 1–22.

GRANT, E.L. and LEAVENWORTH, R.S. '*Statistical quality control*', Fifth edition, McGraw-Hill Book Company. 1980.

HARRIS, T.J. and ROSS, W.H. 'Statistical process control procedures for correlated observations', *The Canadian Journal of Chemical Engineering*, vol. 69, 1991. pp. 48–57.

HAWKINS, D.M. 'Regression adjustment for variable in multivariate quality control model', *Journal of Quality Technology*, vol. 25, no. 3, 1993. pp. 170–182.

Hotelling, H. 'Multivariate quality control', *Techniques of statistical analysis*, edited by Eisenhart, C., Hastay, H., and Wallis, W.A., McGraw-Hill, New York, 1947. pp. 111–184.

HOTELLING, H. 'A generalized T^2 test and measure of multivariate dispersion', *Proceedings of the Second Berkeley Symposium in Mathematical Statistics and Probability*, University of California Press, 1950. pp. 23–41.

JACKSON, J.E. 'Multivariate quality control', *Communications in Statistics—Theory and Methods*, vol. 14, 1985. pp. 2657–2688.

JOHNSON, N.L. 'A simple theoretical approach to cumulative sum charts', *Journal of American Statistical Association*, vol. 56, 1961. pp. 835–840.

JOHNSON, R.A. and BAGSHAW, M. 'The effect of serial correlation on the performance of CUSUM Tests', *Technometrics*, vol. 16, 1974. pp. 103–122.

KHUNTIA, G.S., MISHRA, P.K., and KHARE, A.K. 'Control charts—Statistical tool for quality control', *Transactions of the Mining, Geological and Metallurgical Institute of India*, vol. 88, no. 2, Oct. 1991. pp. 152–161.

LUCAS, J.M. 'A modified V-mask control scheme', *Technometrics*, vol. 15, 1973. pp. 833–837.

MARAGAH, H.D. and WOODBALL, W.H. 'The effect of autocorrelation on the retrospective X-charts', *Journal of Statistical Computation and Simulation*, vol. 40, 1992. pp. 29–42.

MARDIA, K.V. 'Measures of multivariate skewness and kurtosis with applications', *Biometrika*, vol. 57, 1970. pp. 519–530.

MARDIA, K.V. 'Applications of some measures of multivariate skewness and kurtosis in testing normality and robustness studies', *Sankhya*, vol. B36, 1974. pp. 115–128.

MARDIA, K.V. 'Omnibus tests of multinormality based on skewness and kurtosis', *Communication in statistics*, vol. 12, 1983. pp. 207–221.

MARDIA, K.V. 'Mardia's test of multinormality', *Encyclopaedia of statistical sciences*, vol. 5, 1985. pp. 217–221.

MASTRANGELO, C.M., RUNGER, G.C., and MONTGOMERY, D.C. 'Statistical process monitoring with principal components', *Quality and Reliability Engineering International*, vol. 12, 1996. pp. 203–210.

MITRA, A. '*Fundamentals of quality improvement and control*', NY, Macmillan. 1993.

MONTGOMERY, D.C., and FRIEDMAN, D.J. 'Statistical process control in a computer-integrated manufacturing environment', *Statistical process control in automated manufacturing*. Marcel Dekkar, New York, NY, 1989. pp. 67–87

MONTGOMERY, D.C. and MASTRANGELO, C.M. 'Some statistical process control methods for autocorrelated data', *Journal of Quality Technology*, vol. 23, 1991. pp. 179–193

MORUD, T.E. 'Multivariate statistical process control; Example from chemical process industry', *Journal of Chemometrics*, vol. 10, 1996. pp. 669–675.

PADGETT, C.S., THOMBS, L.A., and PADGETT, W.J. 'On the α risks for Shewhart control charts', *Communications in statistics—Simulation and Computation*', vol. 21, 1992. pp. 1125–1147.

PIGNATIELLO, J.J. and RUNGER, G.C. 'Comparisons of multivariate cusum charts', *Journal of Quality Technology*, vol. 22, 1990. pp. 173–186.

PORTER, L.J. and CAULCUTT, R. 'Control chart design; a review of standard practice', *Quality Reliability Engineering International*, vol. 8, 1992. pp. 113–122.

RIUS, A., CALLAO, M.P. and RIUS, F.X. 'Multivariate statistical process control applied to sulphate determination by sequential injection analysis', *Analyst*, vol. 22, 1997. pp. 737–741.

SAMANTA, B., BHATTACHERJEE, A., and RAY, S.C. 'A statistical quality control model for surface mine operations', *Proceedings of the International Conference on Surface Mining*, 9–11th July, 1998, Calcutta, 1998. pp. 306–320.

SAMANTA, B. and BHATTACHERJEE, A. 'Statistical quality control: A multivariate approach', *Mineral Resources Engineering*, vol. 8, no. 2, 1999. pp. 227–238.

SAMANTA, B. and BHATTACHERJEE, A. 'An investigation of quality control charts for autocorrelated data', *Mineral Resources Engineering*, vol. 10, no. 1, 2001. pp. 53–69.

SLUTTER, B.R., MASTRANGELO, C.M., BAXLEY, R.V., and KING, C.J. 'Selection of a multivariate monitor for an autocorrelated and cross-correlated data', Technical Journal, University of Virginia, Charlottesville, VA. 1998.

SPEDDING, T.A. and RAWLINGS, P.L. 'Nonnormality in Statistical Process Control measurements', *International Journal of Quality and Reliability Management*, vol. 11 , no. 6 , 1994. pp. 27–37.

TANG, L.C. and THAM, S.E. 'Computing process capability indices for nonnormal data: A review and comparative study', *Quality and Reliability Engineering International,* vol. 15, 1999. pp. 339–353.

TRACY, N.D., YOUNG, J.C., and MASON, R.L. 'Multivariate control charts for individual observations', *Journal of Quality Technology*, vol. 25, 1992. pp. 88–95.

WARDELL, D.G., MOSKOWITZ, H., and PLANTE, R.D. 'Control charts in the presence of data correlation', *Management Science*, vol. 38, no. 8, 1992. pp. 1084–1105.

WETHERILL, G.B. and BROWN, D.W. '*Statistical process control: Theory and practice*', London. Chapman and Hall. 1991.

YOURSTONE, S.A. and ZIMMER, W.J. 'Nonnormality and the design of control charts for averages', *Decision Sciences,* vol. 23, 1992. pp. 1099–1113. ◆