



Control of strong ground motion of mining-induced earthquakes by the strength of the seismogenic rock mass

by A. McGarr*

Synopsis

The shear stress τ that can be sustained by the rock mass in the environs of a mining-induced earthquake controls the near-fault peak ground velocity v of that event according to $v \leq 0.25(\beta/G)\tau$, where β is the shear wave speed and G is the modulus of rigidity. To estimate τ at mining depths, I review the results of four studies involving Witwatersrand tremors that relate to the bulk shear strength. The first and most general analysis uses the common assumptions that the seismogenic crust is pervasively faulted, has hydrostatic pore pressure before mining, and an extensional stress state that is close to failure. Mining operations reduce the pore pressure to zero within the mine and redistribute the stresses such that, in localized regions, the state of stress is again at the point of failure. Laboratory friction experiments can be used to estimate τ in the zero-pore-pressure regime. Second, model calculations of states of stress in the vicinity of mining at about 3 km depth indicated the shear stress available to cause faulting near the centre of a distribution of induced earthquakes. Third, laboratory experiments combined with microscopic analyses of fault gouge from the rupture zone of a mining-induced event provided an estimate of the average shear stress acting on the fault to cause this earthquake at a depth of 2 km. Fourth, moment tensors determined for mining-induced earthquakes usually show substantial implosive components, from which it is straightforward to estimate τ . These four different analyses yield estimates of τ that fall in the range 30 to 61 MPa which implies that near-fault particle velocities could be as high as about 1.5 m/s. To the extent that the causative fault ruptures previously intact rock, both τ and v , in localized regions, could be several times higher than 61 MPa and 1.5 m/s.

Introduction

The idea that the strength of the rock mass that ruptures during an earthquake limits the level of high-frequency ground motion from that event is not new. It is well known that earthquakes in weak formations tend to be 'slow', that is, with extended source time histories that give rise to only low levels of ground acceleration or velocity. In contrast, the strong quartzitic strata of the deep gold mines in South Africa are thought to play an important role in the high levels of damaging ground motion associated with many of the Witwatersrand tremors. The objective of this study is to propose and test a more specific

and quantitative relationship between the bulk strength of the rock mass and the peak ground motion due to earthquake rupture there.

The seismic source parameter apparent stress provides the means of relating bulk strength to peak ground motion. A recent study¹ has indicated that apparent stress τ_a is related to the shear stress τ causing fault slip according to

$$\tau_a \leq 0.06\tau \quad [1]$$

where τ is the average shear stress acting on the fault during the seismic slip and is close to the static shear strength of the rock mass, as will be explained.

For purposes of determining how τ might constrain the peak ground motion from an earthquake, [1] is useful because apparent stress is a function of the time-dependent fault slip. McGarr and Fletcher² argued that

$$\tau_a = \frac{f(v_R)}{2} \frac{\rho\beta}{D} \int \dot{D}(t)^2 dt \quad [2]$$

where f is a function of the rupture velocity v_R that is determined empirically from dynamic rupture models of the earthquake source²⁻⁴, ρ is density, β is the shear wave speed, D is the fault slip and $\dot{D}(t)$ is the slip rate. If, as proposed by McGarr⁵ the time dependence of the fault slip rate is given, to a good approximation, by $\dot{D}(t) \propto 1 - \cos\omega t$, for $0 \leq t \leq \frac{2\pi}{\omega}$, then it is easy to show, using [1] and [2], that, adjacent to the causative fault, the peak particle velocity v , which is half of the peak slip rate, is limited according to

$$v \leq 0.08\beta\tau / (f(v_R)G) \quad [3]$$

For rupture velocities in the range 0.6β to 0.95β , $f(v_R)$ ranges from 0.11 to 0.4, whereas laboratory evidence suggests that typically $f=0.32$ ². For the present purpose I take the

* U.S. Geological Survey, Menlo Park, CA, USA.

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laboratory estimate of f which leads to

$$v \leq 0.25\beta\tau / G \quad [4]$$

Because the ground motion most damaging to mine production usually occurs in the near-field of a mining-induced earthquake⁵, [4] is a useful relationship between the shear strength of the rock mass and the near-fault peak velocity. Moreover, support capability is often expressed in terms of peak ground velocity^{6,7}.

[4] is only useful, however, to the extent that we can estimate the shear strength of the rock mass. Accordingly, most of this report reviews four studies bearing on the shear strength in the deep mines of the Witwatersrand. We shall see that the four independent analyses give shear strengths that are in surprisingly good agreement.

Bulk strength of Witwatersrand strata

Although laboratory strength tests have been performed on the various rock types found in the Witwatersrand mines⁸, the results of these experiments are of limited value for estimating the strength of the rock mass at the scale of a mining-induced earthquake (often several hundred metres) because the small laboratory samples used in these tests are generally composed of intact, unfaulted rock. Indeed, one would expect the strengths of the laboratory samples to be much greater than the large-scale strengths of the rock mass, which shows pervasive faulting at a variety of scales. Thus, the challenge here is to estimate large-scale shear strengths that are consistent with laboratory results and yet take the actual nature of the Witwatersrand strata into account. To this end, I review four analyses whose results bear on the large-scale shear strength of the Witwatersrand strata.

(1) Crustal strength altered by mining. This argument pursues a suggestion made by McGarr⁹ by considering first the ambient state of the seismogenic crust and then the state as altered by mining operations for purposes of estimating the crustal shear strength at the focus of a mining-induced earthquake. Throughout most of the Witwatersrand basin the crustal state of stress is extensional⁹⁻¹¹ and so the vertical stress S_v , due to the weight of the overburden, is the largest of the principal stresses, being typically twice as large as S_3 , the minimum, horizontally-oriented, principal stress. If the water table is near the surface (hydrostatic pore pressure), then this state of stress is close to the failure state, assuming a pervasively-faulted crust¹². Indeed, this near-failure crustal state may be the general rule worldwide^{13,14}.

Following Brace and Kohlstedt¹², if the crust is assumed to be at the point of failure, then in the upper part of the crust

$$S_v - P = \frac{A(S_3 - P)}{\sqrt{1 + \mu^2}} \quad [5]$$

where μ is the coefficient of friction, and

P is the pore pressure. For hydrostatic pore pressure $P=9.8(\text{MPa/km})z$, where z is depth in km. For an average density of 2700 kg/m^3 , the vertically-oriented stress is given by $S_v=26.5(\text{MPa/km})z$. If $\mu=0.75$ ¹⁵ then $A=4$ and [5] can be used to solve for S_3 giving

$$S_3 = 14(\text{MPa/km})z. \quad [6]$$

Mining changes the state of the crust in several ways. First, pumping operations stabilize the crust by reducing the pore pressure to zero so that it is further from a failure state. Second, the mine workings, especially the extensive tabular stopes, enhance the state of deviatoric stress to the point of failure in localized regions, giving rise to mining-induced earthquakes. The most important change in the stress state due to mining, at least in the vicinity of the production faces, is to augment the vertically-oriented principal stress^{16,17} whereas S_3 remains less affected. Accordingly, the maximum (near vertically-oriented) principal stress S_1 can be estimated by setting $P=0$ in [5] and replacing S_v by S_1 to obtain $S_1=4S_3=56(\text{MPa/km})z$. Because the favoured fault plane is at an angle v with the direction of S_1 , which is nearly vertical, and

$$v = 45^\circ - (1/2)\tan^{-1} \mu \quad [7]$$

where μ is the coefficient of static friction, and

$$\tau = (1/2)(S_1 - S_3)\sin(2v) \quad [8]$$

(e.g., Jaeger¹⁸, p. 8), then for $\mu=0.75$ and $v=26.6^\circ$, the shear stress available to cause faulting on optimally-oriented faults is

$$\tau = 16.8(\text{MPa/km})z. \quad [9]$$

The depth range for most of the mining-induced earthquakes of the Witwatersrand is 2 to 3 km. Evaluating [9] at these two depths gives estimates of τ of 33.6 and 50.4 MPa.

Repeating calculations [5] – [8] for $\mu=0.6$ and 0.9 yields $\tau=13.8 (\text{MPa/km})z$ and $19.7 (\text{MPa/km})z$, respectively.

(2) Relationship of mine tremors to mining-enhanced deviatoric stresses. Since the rockburst investigations of Cook¹⁶, we have known that mining-induced earthquakes generally show a tight correlation with active mining such that the hypocentres are located within several hundred metres of the nearest advancing production face. Cook also estimated, using the techniques of Muskhelishvili¹⁹, the altered state of stress in the vicinity of a deep mine excavation.

Following up on this early work, McGarr *et al.*¹⁷ compared precise hypocentral locations of tremors at depths near 3 km within the ERPM Gold Mine with the state of stress in the same region. The mining, which results in extensive tabular stopes, perturbs the ambient state of stress considerably to cause the mining-induced earthquakes and, in particular, McGarr *et al.*¹⁷ found that at the centre of the distribution of hypocentres, at a depth of 3.2 km, the perturbed state of stress^{16,19} is one for which the maximum principal stress is about 180 MPa and the minimum about 41.5 MPa. Inserting these two principal stresses into [8] yields a shear stress of 55.4 MPa on an optimally-oriented fault at this depth.

This argument differs from the first one in that, instead of estimating the change in shear strength due to a reduction in pore pressure, we considered the enhanced deviatoric stress associated with an extensive tabular mine stope (Cook¹⁶). Both arguments, however, require that the ambient state of stress be known.

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(3) Analysis of fault gouge. A mining-induced earthquake involves slip across a shear zone of highly comminuted rock (Figure 1). Motivated by the similarity between features seen in laboratory samples of quartzite loaded to failure under confining stress and those exemplified in Figure 1, Spottiswoode²⁰ (see also McGarr *et al.*²¹) measured total surface areas of gouge formed in the laboratory samples to conclude that the energy consumed in creating the surface area is proportional to the loading shear stress. Specifically

$$\tau = \zeta G_c \rho_g w / D \quad [10]$$

where ζ is the surface area per unit mass, G_c is the crushing energy, determined by Spottiswoode²⁰ as $G_c=450$ J/m², ρ_g is the density of the gouge (about 2000 kg/m³, w is the width of the shear zone, and D is the fault slip. (To avoid confusion, I note that G_c is not the same as surface energy (e.g., Brace and Walsh²²), but, instead, represents the energy required to produce the gouge, whose surface area was estimated, at stress conditions intended to approximate those *in situ*. In fact, much of G_c is expended in overcoming friction and ends up as heat).

Spottiswoode²⁰ then applied these laboratory results to analyse samples of gouge extracted from three regions of a shear zone exhumed at a depth of 2 km in the western region of the ERPM Gold Mine^{23,24}. The three samples yielded estimates of τ of 47, 36, and 40 MPa (see McGarr *et al.*²¹, Table 2).

(4) Implosive components of seismic moment tensors.

The fourth technique for estimating τ is based on the determination of seismic moment tensors for 16 mining-induced earthquakes, located in several mining districts of the Witwatersrand²⁵. Of the 16 events, 5 showed moment tensors with no volumetric component whereas the other 11 had implosive components comparable in magnitude to their corresponding shear components. For these 11 events, the shear stress causing fault slip can be estimated from²⁶

$$\tau = GW / M_0 \quad [11]$$

where G is the modulus of rigidity, M_0 is the deviatoric component of the seismic moment and W is the coseismic energy released. W is estimated from $W=-S_v\Delta V$, where S_v is the overburden stress, as before, and ΔV is the coseismic volume change estimated from the trace of the moment tensor²⁶. Table I gives the essential results for the 11 events with significant volume changes.

From Table I we see that shear stresses estimated from the moment tensors range from 30 to 61 MPa for events in the depth range 1.9 to 3.2 km.

Summary of shear strength estimates. Considering the diversity of the four approaches used here to estimate the large-scale shear stresses that cause fault slip, the agreement between them is remarkably good. Because all of these estimates fall in the range 30 to 61 MPa, it seems clear that we have a reasonably good idea of the bulk strength of the Witwatersrand strata at the scale of several hundred metres, or so.

Several qualifications should be noted. First, these various types of estimates are not necessarily equivalent. In the first two approaches, I have estimated the peak shear stress, or yield stress whereas the second two give the

average stress acting on the fault during slip. In principle, these results should differ by half the stress drop of a mining-induced earthquake, but in view of the uncertainties, which greatly exceed a typical stress drop²⁷, accounting for the small difference between yield stress and average stress is not warranted here.

The second qualification involves the common observation underground (e.g., Figure 1) that at least over localized regions, the rupture traverses intact rock. If so, then the coefficient of friction is effectively much higher with values perhaps in the range 1.3 to 1.6²⁸ and the resulting bulk strength, estimated using [5]–[8], would be correspondingly higher, a point to which I return.

As seen in Figure 2, which shows the various estimates of τ plotted as functions of depth, the different types of estimates seem fairly consistent with one another. Moreover, the shear stresses generally seem to increase with depth, although whatever depth dependence there might be is somewhat overwhelmed by the broad distribution of stresses

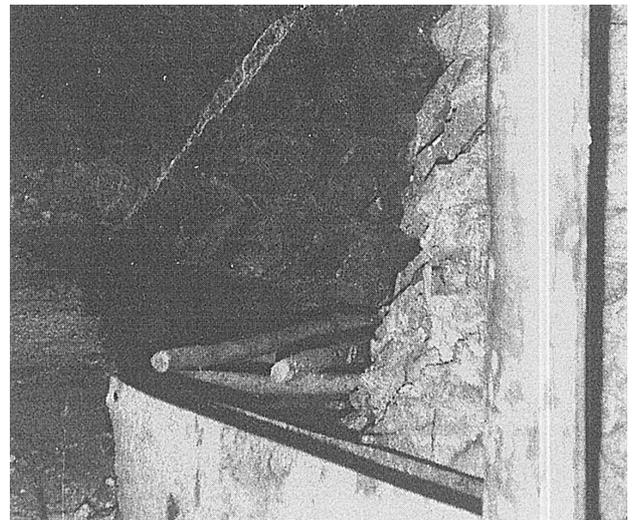


Figure 1—Shear zone observed in the sidewall of a footwall drive at a depth of about 2 km in the ERPM Gold Mine, South Africa. (Photograph by W.D. Ortlepp.)

Table I

Moment tensor estimates of τ (Extracted from Table 2 of McGarr²⁶)

Event	Depth, km	M_0 , N-m	$-\Delta V$, m ³	W , J	τ , MPa
3102038	2.04	3.65×10^{11}	5.0	2.98×10^8	30.
3121332	2.68	6.14×10^{11}	10.3	7.86×10^8	48.
3151552	2.04	1.98×10^{12}	55.0	3.20×10^9	61.
3151554	2.20	4.85×10^{12}	74.5	4.67×10^9	36.
3241523	3.24	1.75×10^{12}	29.4	2.71×10^9	58.
3241624	2.18	5.64×10^{12}	90.0	5.57×10^9	37.
3251605	2.17	2.95×10^{12}	66.0	4.07×10^9	52.
0301411	2.12	8.1×10^{13}	1980.0	1.20×10^{11}	56.
0301411a	2.06	2.75×10^{13}	426.0	2.49×10^{10}	34.
0341528	2.06	4.40×10^{12}	88.0	5.16×10^9	44.
0271046	1.92	6.19×10^{12}	150.0	8.19×10^9	50.

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at depths near 2 km. At least one can see that these data are not inconsistent with the linear depth dependence, Equation [9], expected on the basis of laboratory friction experiments¹²; that is, the symbols corresponding to [9], and its depth dependence, are within the range of the other types of data in Figure 2.

Implications for near-fault ground velocity

Having shown that the bulk shear strengths of the Witwatersrand strata are most likely in the range 30 to 61 MPa for the depth range of about 2 to 3 km (Figure 2), then what is the corresponding constraint on ground motion and is this constraint consistent with other types of evidence? If we take 61 MPa (Figure 2) as an upper bound for τ then from [4] (with $G=3.7 \times 10^4$ MPa and $\beta=3.7$ km/s)

$$v \leq 1.52 \text{ m/s} \quad [12]$$

which is consistent with the general observation that the rapid-yielding hydraulic props, commonly used as near-face support in the deep mines of the Witwatersrand, are adequate in terms of their ability to accommodate rapid closure rates without failing; that is, this type of support can accommodate at least several m/s of closure rate^{6,7} without loss of integrity.

Direct measurements of near-fault ground velocities at depth are quite scarce, but one such estimate can be gleaned from information given by McGarr⁵. Figure 3 shows ground velocity from a mining-induced earthquake of M4.4 that occurred almost directly beneath the surface recording site. The simple S-wave pulse, especially clear on the vertical and east components, indicates that the fault slip had a rise time of 0.16 s. Underground measurements at the causative fault, near the Five shaft of the Vaal Reefs Gold Mine²⁹ showed seismic slips of up to 0.2 m at depths near 2 km. Dividing this slip by the rise time gives an average slip rate of 1.25 m/s and from the sinusoidal shape of the S-pulse, the peak slip rate was about 2.5 m/s. This yields a peak ground velocity adjacent to the fault of about 1.25 m/s, which is consistent with [12].

There are several reasons, however, to suppose that

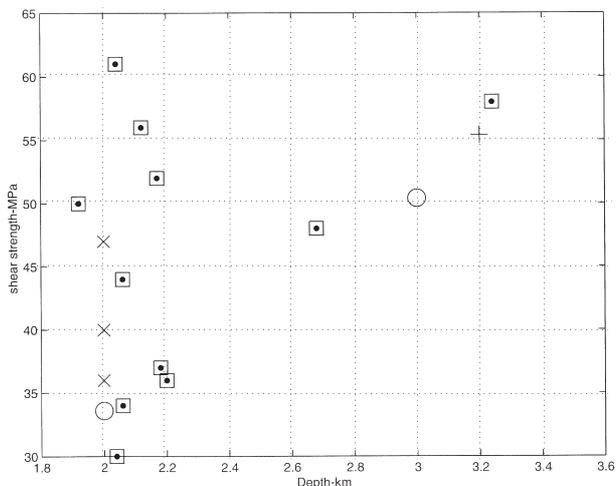


Figure 2—Large-scale shear strength of the Witwatersrand strata as a function of depth. The circles are estimates from [9]; the plus sign is an estimate based on results from McGarr *et al.*¹⁷ and [8]; the x's are estimates from [10], involving the analysis of fault gouge; finally, the squares are estimates (Table I) derived from moment tensors of mining-induced earthquakes

ground velocities may occasionally exceed the limit given by [12]. First, hydraulic yielding props have been known to fail on occasion, suggesting that ground velocities may have exceeded several m/s. Second, the analysis leading to [12] is consistent with a pervasively-faulted seismogenic crust, whereas, as mentioned before, exhumed shear zones associated with mining-induced earthquakes (e.g., Figure 1) often entail the fracture of previously-intact rock; that is, the shear zones do not appear to follow pre-existing faults very often, at least where exhumed³⁰.

Indeed, experiments on Witwatersrand quartzite in triaxial compression³¹ show that failure occurs when

$$S_1 - 6S_3 = S \quad [13]$$

where S is the strength in uniaxial compression, commonly about 200 MPa for Witwatersrand quartzites. Assuming that S_3 , the minimum horizontal stress, can be estimated from [6], then at a depth of 3 km $S_3=42$ MPa and from [13], with $S=200$ MPa, $S_1=452$ MPa, and from (8), $\tau=164$ MPa, which vastly exceeds the bound of 61 MPa (Figure 2). If τ were as high as 164 MPa then from [4], the corresponding ground velocity could be as high as 4.1 m/s. Ground velocities this high might cause prop failure in the zones where the rupture traverses previously intact rock.

Conclusions

The idea that the strength of rock through which a mining-induced earthquake ruptures determines the level of ground motion from that event is tested here and found to be useful. Essentially, the seismic source parameter apparent stress serves to relate the shear strength of the rock mass to the near-fault ground motion (Equations [1]–[4]). The main challenge here was to estimate the strength of the rock mass at a scale appropriate to a mining-induced earthquake. Four different analyses of the large-scale shear strength of the strata in the deep Witwatersrand gold mines give results that are in surprisingly good agreement, ranging from 30 to 61 MPa for strata in the depth range of about 2 to 3 km. A maximum strength of 61 MPa would limit the near-fault ground velocity to about 1.5 m/s. The main caveat to this is

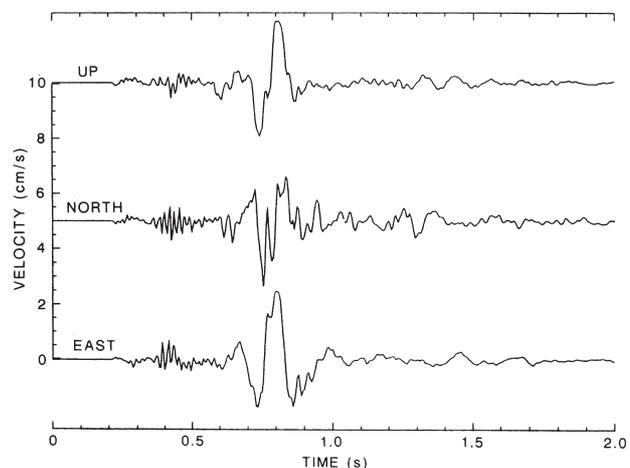


Figure 3—Ground velocity recorded at a surface site approximately 2.7 km from a mining-induced earthquake of M4.4 (from McGarr⁵)

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the possibility, supported by many observations underground³⁰, that the shear zone of a mining-induced earthquake ruptures previously intact rock, at least over small regions. If so, then near-fault ground velocities several times greater than 1.5 m/s would be anticipated locally in these higher strength zones.

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