Introduction

Coal mining in India has a history of over 225 years. The industry currently occupies a covetable third ranking in the world league table of hard coal production after China and the USA. Mechanization has made possible major breakthroughs in coal mining technology. The record of production and productivity of Indian underground coal mines over the years is dismal, to say the least. At the time of nationalization (1971) of the coal industry contribution of national coal production by underground and opencast mines was 77.4% and 22.5% respectively. By 2000, the share of coal production from opencast mines increased to 78.65% and from underground mines declined to 21.35%. The share of coal production by different methods may be stated thus: bord and pillar (conventional)—55.98%; mechanized (SDL/LHD)—34.01%; longwall (conventional)—0.6%; mechanized 7.44% and other methods 1.97%. Daily production in blasting gallery panel with an LHD is about 800 t. An average ton per day per machine (LHD capacity—5.5 m³) is about 150 t.

Reliability modelling and performance analyses of an LHD system in mining

by B. Samanta*, B. Sarkar†, and S.K. Mukherjee‡

Synopsis

LHD (load haul dumper) is now used as a loading machine for intermediate mechanization in underground coal mining. For survival in the intense competition in the global business environment in recent years it is essential that LHD machine should be reliable and maintained effectively and efficiently. This paper seeks to study the reliability, availability and maintainability (RAM) of an LHD machine with failure and repair data by Markov modelling. The constraints and reasons for machine unavailability are outlined. The reliability and maintainability of an LHD and its subsystems are evaluated. Possible modification or design alternatives are highlighted. Reliability and maintainability of an LHD system are disappointing. There is room to take decisions on optimal maintenance planning and machine improvement from this analysis.

Keywords: reliability, availability, maintainability, Markov model, LHD

The mechanization trend in the bord and pillar method of coal mining introduced sophisticated automated side discharge loader (SDL) / load haul dumper (LHD) machines for loading of coal in place of manual loading. Among the face loading machines, the electric LHD is now the dominant machine in intermediate technology and plays an important role in district or overall mine production. To achieve targeted coal production and to survive the intense competition in the mining industry in recent years, it is imperative that an LHD machine as a system and its subsystems should be reliable and maintained effectively and efficiently to ensure its maximized availability.

Reliability assessments of repairable machines have been explored in some papers. The basic methodology for reliability modelling to analyse the failure characteristics of a repairable machine is presented by Ascher et al and Samanta et al. (2001(a,b) and 2002). Failures of a repairable machine have been modelled on the basis of a renewal process, a homogenous or a non-homogenous Poisson Process (NHPP), or a proportional hazard process. In the renewal process, the time between failures (TBF) are assumed to be independent and identically distributed (iid) and failure data are characterized for modelling by a suitable probability distribution function. NHPP or the power law process is a time-dependent model. In the homogenous Poisson Process, TBFs are assumed to be exponentially distributed (iid) and failure data are characterized for modelling by a suitable probability distribution function. NHPP or the power law process is a time-dependent model. In the homogenous Poisson Process, TBFs are assumed to be exponentially distributed (iid) and failure data are characterized for modelling by a suitable probability distribution function. NHPP or the power law process is a time-dependent model. In the homogenous Poisson Process, TBFs are assumed to be exponentially distributed (iid) and failure data are characterized for modelling by a suitable probability distribution function. NHPP or the power law process is a time-dependent model. In the homogenous Poisson Process, TBFs are assumed to be exponentially distributed (iid) and failure data are characterized for modelling by a suitable probability distribution function.
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...randomly stochastic. These subsystems can be brought back into serviceable condition after repair or replacement. It is interesting to note that the failure of subsystems and their units can never be predicted precisely as they depend upon the operating conditions, mining environment, and repair policy used in the mine. Again, the performance/availability of an LHD machine depends on the reliability, availability and maintainability characteristics of subsystems, maintenance efficiency, operation process and the technical expertise of the miner etc.

Availability is a function of reliability and maintainability. The return on investment on a piece of equipment can be maximized by optimizing its availability. Information on system behaviour and failure modes is extremely important for taking decisions on maintenance strategy or action. So measuring the effectiveness of an LHD system using reliability modelling and performance analysis by Markov modelling appear to be appropriate. A study, therefore, was conducted for analysis reliability, availability and maintainability (RAM) of an LHD machine in a blasting gallery mine where LHDs are deployed at faces for coal loading. This paper deals with RAM modelling and performance analysis of LHD machine failure and repair data analysis using a Markov model. Appropriate conclusions have been drawn on the basis of this analysis.

Study procedure

For the reliability modelling and performance analysis of LHD machines a step-by-step study procedure has been developed and is given in Figure 1. A description of an LHD machine is given at the beginning. In the next section a reliability block diagram (RBD) of the LHD machine has been developed. Markov’s transition diagram has been presented, making some assumptions. From the transition diagram state transition linear differential equations are derived for the Markov process and then the steady state performance of LHD machines has been discussed.

LHD machine

Load haul dumper (LHD) is now a dominant machine for intermediate face mechanization. A LHD plays a very important role for the loading of coal on a chain or belt conveyor. It is typically trackless equipment mounted on four tyres. It has a front-end bucket designed to carry and dump bulk material. Since an LHD is conceived for underground mining, it is compact and of low profile. It has a capacity of 3 m³. It is bi-directional in operation, with powered steering controlled by a driver sitting mostly at right angles to the direction of the vehicle’s movement.

It is driven by a synchronous motor fed with power from a gate end box suitably placed with a flexible cable of about 100 metres in length. It may also be remotely controlled. The speed of the vehicle is controlled mechanically. The transmission is controlled by a hydrostatic drive (Figure 2a). In hydrostatic transmission, the motor drives a variable displacement pump hydraulically connected to a hydro-motor driving the axle via a gearbox. The speed is controlled by changing the displacement volume of the axial pump.

The power train consists of a closed loop hydraulic transmission, parking brakes, two-stage gear box, drive lines front axle with no-spin and rear axle without no-spin (Figure 2b). Four multi-disc service brakes are mounted on the rear and front axles. A 142 litre per minute capacity hydraulic pump caters for service requirements such as hoist, dump, steering, brakes, and reeling and unreeling of cable. An orbital is used for power steering. The cable drum accommodates 150 m of 4 x 35 sq.mm. type trailing cable. An automatic cable reeling device is fitted for smooth operation of the machine. Pumps are driven by a 90 kw/1450 rpm/50 Hz FLP (flame proof) electrical motor. The effective operational range of the machine is 150 m radius. All hydraulic operations are piloted. This facilitates two-point operation of the machine: (a) local from the driver’s seat, and (b) remote from a distance of 15–20 m with the help of a remote control console. This feature helps to recover coal from the goaf area when the operator can stand safely in the supported area. A belt-driven compressor is provided to meet the air need for a pneumatic arrangement. Tyres used are 15.5 x 25 steel cord radial\textsuperscript{15}.

Reliability block diagram (RBD)

It is necessary to construct a reliability block diagram (RBD) of the LHD system for reliability modelling and performance analysis. It is a graphical representation of the components of the system from a reliability viewpoint. The LHD machine is considered to be a system consisting of six major subsystems such as a power generating unit/drive unit, transmission, hydraulic, tyre, brake and others, and a bucket connected in series. The reliability block diagram (RBD) of an LHD has been developed and is presented in Figure 3.

Assumption for modelling

For the purpose of modelling the following assumptions were made:\textsuperscript{10,14,17}:

- Are the random behaviour of LHD system satisfied or assumed as a memoryless process?
- Are there Markov models available?
- Formulate the state transition linear differential equation for Markov process?
- Evaluate LHD performance?

Figure 1—Flowchart for reliability modelling of LHD machine by Markov process
Failure rates and repair rates for all the subsystems of the LHD are constant over time and statistically independent.

Time between failure (TBF) and time to repair (TTR) data are exponentially distributed. So there are no simultaneous failures of subsystems and the probability of more than one failure or repair in a time interval is zero.

The repaired units are as good as new (AGAN) one. Repair or replacement is carried out only in case of failure.

Any subsystem of the LHD remains in either of two
states only: the operating/up state and the non-operating/down state. The machine moves from the up state to down state as a result of a subsystem failure; similarly, the subsystem as well as the machine move at the same time from the down state to the up state as a result of repair. The probability of transition from one state to any other state does not depend on the state that was occupied earlier in the process. Sometimes the machine is in an underrated working capacity, but for simplicity it is taken as operating.

Notation used

In the Markov model, the LHD machine as a system is represented with seven possible states as follows:

- \( P_{S0}(t) \): represents the probability that the LHD machine is in the ‘up’ state (\( S_0 \)) at time \( t \).
- \( P_{Si}(t) \): represents the probability that the subsystems are in the ‘down’ (\( S_i \)) state at time \( t \) (\( i = 1,2,\ldots,6 \)).

\( \lambda_i \) is the failure rate of the subsystem (\( i = 1,2,\ldots,6 \)).

\( \mu_i \) is the repair rate of the subsystem.

Transition diagram

The transition diagram for an LHD machine is presented in Figure 4. Based on the above RBD, assumption and failure rates and repair rate etc. are shown in Table I. The operating state is denoted by the number ‘0’ and the non-operating or fail state is denoted as ‘i’ (\( i = 1,2,\ldots,6 \)). At the beginning (i.e., at \( t = 0 \)), the machine is in the operational state and once the subsystems of the machine enter the non-operating state, it may return only to an operating state and vice versa, i.e., transition occurs only between the up state and down state. Here subsystems reside in a discrete state and are continuous in time. So from the above discussion the problem can be modelled as the Markov Process\(^{10-14,17} \). Here machine and subsystems are in a communicating state. The different equations related to the transition diagram are formed. The steady state availability of the machine, as well as different subsystems, are derived from these equations and the reliabilities of the machine, as well as its different subsystems, are estimated with different mission times.

### RAM modelling of the LHD system

From the transition diagram presented in Figure 4 and the Markov equations\(^{17} \), the steady state availability of the LHD is found as

\[
P_0 = \frac{1}{1 + \frac{\sum \lambda_i}{\mu_i}} = \frac{1}{1 + D},
\]

where \( D = \frac{\sum \lambda_i}{\mu_i} \) (See Appendix 1).

---

**Table I**

<table>
<thead>
<tr>
<th>Transition diagram</th>
<th>Working state</th>
<th>Failure rate (( \lambda_i ))/h</th>
<th>Repair rate (( \mu_i ))/h</th>
<th>Probability of working state</th>
<th>Probability of failed state</th>
<th>Failed state</th>
<th>( \Sigma \lambda_i / \mu_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 4</td>
<td>L (drive unit)</td>
<td>.00375</td>
<td>.0496</td>
<td>( P_0 )</td>
<td>( P_1 )</td>
<td>1</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>H (transmission)</td>
<td>.00693</td>
<td>.0732</td>
<td></td>
<td>( P_2 )</td>
<td>h</td>
<td></td>
</tr>
<tr>
<td></td>
<td>D (hydraulic)</td>
<td>.00931</td>
<td>.144</td>
<td></td>
<td>( P_3 )</td>
<td>d</td>
<td></td>
</tr>
<tr>
<td></td>
<td>R (tyre)</td>
<td>.00458</td>
<td>.135</td>
<td></td>
<td>( P_4 )</td>
<td>r</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A (brake)</td>
<td>.00543</td>
<td>.102</td>
<td></td>
<td>( P_5 )</td>
<td>a</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M (bucket)</td>
<td>.00491</td>
<td>.122</td>
<td></td>
<td>( P_6 )</td>
<td>m</td>
<td></td>
</tr>
</tbody>
</table>

---

![Figure 4—Transition diagram of an LHD](image-url)
Reliability modelling and performance analyses of an LHD system in mining

The reliability of different subsystems and machine

\[ R_1(t) = e^{-\lambda_1 t}, \quad R_2(t) = e^{-\lambda_2 t}, \quad R_3(t) = e^{-\lambda_3 t}, \quad R_4(t) = e^{-\lambda_4 t}, \quad R_5(t) = e^{-\lambda_5 t}, \quad R_6(t) = e^{-\lambda_6 t} \]

As subsystems are connected in series, so the reliability of the LHD system will be the product of the individual subsystem reliabilities:

\[ R_{LHD}(t) = R_1 R_2 R_3 R_4 R_5 R_6 = e^{-\sum \lambda_i t} \] where \( F = \sum \lambda_i \), \( i = 1, 2, \ldots, 6 \).

The maintainability of different subsystems and machine is as follows:

\[ M_1(t) = 1 - e^{-\mu_1 t}, \quad M_2(t) = 1 - e^{-\mu_2 t}, \quad M_3(t) = 1 - e^{-\mu_3 t}, \quad M_4(t) = 1 - e^{-\mu_4 t}, \quad M_5(t) = 1 - e^{-\mu_5 t}, \quad M_6(t) = 1 - e^{-\mu_6 t} \]

As a system machine failure rate is \( \Sigma \lambda_i \), So MTTF = \( \Sigma / \lambda \)

It is known that for steady state availability (A) = MTTF/(MTTF + MTTR).

So MTTR = MTTF \times D, where D = \( \Sigma \lambda_i / \mu_i \)

System repair rate (\( \mu \)) = 1/MTTR = 1/MTTF \times D

LHD machine maintainability \( M_{LHD}(t) = 1 - e^{-\mu t} \).

Again, RLHD(t) = e^{-\mu t} \cdot t = \ln RLHD(t)/\mu.

Thus, from different expected reliability of the LHD system the maintenance interval can be estimated.

Case study

The LHD remains a major constraint in achieving desired mine output. Reasons for low availability15,18–19, derated working performance and modification done at the mine to improve machine performance are given below. Problems faced in the colliery can be grouped into two categories

➤ Environmental and
➤ Technical.

Environmental

Seepage water

No problem was faced in operating the LHD on the dry floor at the case study mine. But during the rainy season, the floor becomes mucky with the movement of the LHD and other machinery, mainly due to the presence of an aquifer bed just below the original floor. Wheels formed deep groves in the floor and the machine chassis would rub the floor while tramming. This resulted in a extra load on the machine. The result was quick heating up of the pump and subsequent power loss. This caused early failure of almost all hydrostatic pumps/motors of the LHD and increased the repair time of the machine due to non-availability of a proper maintenance place. This problem has been partly solved with a water collecting ditch/sump in the rise side level, just outside the panel.

Temperature

It was observed that the hydrostatic unit of the machine would get overheated in continuous operation. It was found that when oil temperature rose beyond 60°C, the machine would start losing output power, causing difficulties in coal loading, hauling load on the up gradient, etc. So it appears that these machines were designed with the ambient temperature far below 60°C in mind. Hence, the oil cooling system is quite inadequate to keep down the oil temperature to the desired level. One cooler each is provided in hydrostatic and hydraulic circuits. For the above problem, both the coolers were put in series in the hydrostatic unit. The hydraulic unit suffered by this arrangement but no problem was faced due to the huge capacity of the hydraulic tank (270 litres). It was found that the hydrostatic pump performance improved considerably.

Dust

LHD remote control is a pneumatic control system. Coal dust at the face creates problems for remote operation due to the frequent choking of the shuttle valve, pipes etc.

Technical

Cable reeling

The cable reeling device was driven by a series of 1.27 cm pitch simple chains. These would break very frequently, causing cable damage. These chains were replaced by 1.9 cm duplex chains. Bearing type guide rollers were replaced by bush type rollers.

Articulation

It was observed that the bottom articulation chassis bolts broke frequently. This results in strain on the chassis and hairline cracks developing along the row of bolt holes.

Gear box

The gear box is suspended from the chassis by 4 16 mm bolts through rubber mountings. Bolts work loose frequently, causing damage to threads in the gearbox.

Collection of accurate and sufficient failure and repair data is necessary in machine reliability and performance analysis for achieving accurate results that are really helpful for mine management in decision making. Data collected from the field are assumed to be the best. Field data are, however, expensive and time consuming to collect and subject to error. Again, data are required to be collected over a period of time for providing satisfactory representation of the true operational characterization of the machine. The first author has experience of working in the case study mine for about a decade and he was closely involved with LHD operation. Failure and repair data are recorded on the operation sheet and maintenance log book at each shift. For this study, three years of reliable and complete failure and repair data of the LHD machine, as well as its subsystems, were taken. The failure and repair rate of LHD and its different subsystems are given in Table I.

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Unavailability</th>
<th>Failure Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive unit</td>
<td>P1 = 0.0555</td>
<td>( \lambda_1 )</td>
</tr>
<tr>
<td>Transmission</td>
<td>P2 = 0.0475</td>
<td>( \lambda_2 )</td>
</tr>
<tr>
<td>Hydraulic</td>
<td>P3 = 0.0749</td>
<td>( \lambda_3 )</td>
</tr>
<tr>
<td>Bucket</td>
<td>P4 = 0.0391</td>
<td>( \lambda_4 )</td>
</tr>
<tr>
<td>Tyre</td>
<td>P5 = 0.0449</td>
<td>( \lambda_5 )</td>
</tr>
<tr>
<td>Brake/others</td>
<td>P6 = 0.0249</td>
<td>( \lambda_6 )</td>
</tr>
<tr>
<td>( \Sigma P_i )</td>
<td>P0 = 0.7339</td>
<td>( \mu_0 )</td>
</tr>
</tbody>
</table>

Substituting the value of \( \lambda_1 \) and \( \mu_0 \) in Equation [5] of Appendix 1, the steady state availability of LHD

\[ P_0 = 0.7339, \quad \text{where} \quad D = .362603 \]

Steady state unavailability of different subsystems of LHD from Equation [6] (from Appendix 1) are given below

<table>
<thead>
<tr>
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<tr>
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</tr>
<tr>
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<td>( \lambda_6 )</td>
</tr>
<tr>
<td>( \Sigma P_i )</td>
<td>P0 = 0.7339</td>
<td>( \mu_0 )</td>
</tr>
</tbody>
</table>

Failure rate of LHD = \( \lambda_i \) (as units are connected in series) = 0.0349099/h

Hence mean time to failures (MTTF) of LHD = 28.645 h.

LHD reliability with time

\[ RLHD(t) = e^{-\sum \lambda_i t} = e^{-0.03491 t}, \quad t = ln RLHD(t)/0.0349099 \]
Subsystems' reliabilities are given below:

\[
R_1(t) = e^{-0.00375t}, R_2(t) = e^{-0.00693t}, R_3(t) = e^{-0.00931t},
R_4(t) = e^{-0.04586t}, R_5(t) = e^{-0.005431t}, R_6(t) = e^{-0.004941t},
\]

MTTR = MTTF = \(28.645 \times 3.562603\) hr = 10.387 hr

LHD reliability at different times:

\[
R_1(t) = e^{-0.00375t}, R_2(t) = e^{-0.00693t}, R_3(t) = e^{-0.00931t},
R_4(t) = e^{-0.04586t}, R_5(t) = e^{-0.005431t}, R_6(t) = e^{-0.004941t},
\]

MTTR = MTTF = \(28.645 \times 3.562603\) hr = 10.387 hr

LHD maintainability at different times:

![LHD Availability](image)

**Discussion of analysis**

Factors responsible for machine unavailability are presented in Figure 5. From the figure it is observed that significant causes are the transmission, drive unit and hydraulic subsystems. The reliability and maintainability of the LHD with different mission times are presented in Tables II and III, and graphically shown in Figures 6a and 6b. From Tables II, III and Figure 6, it is found that the reliabilities of the different subsystems are different as well as decreasing with time. It is also seen that R6(20) = .50, which means that machines will not fail for 20 hours of operation with only a 50% probability. It can also be seen that there is an 85% chance that any failure in the machine will be repaired within 20 h of repaired time M6(20) = .85. There is a 100% chance that any failure in the LHD machines will not fail for 20 hours of operation with only a 50% probability. It can also be seen that there is an 85% chance that any failure in the machine will be repaired within 20 h of repaired time M6(20) = .85.

For example, the LHD must be maintained every 15 h for a 60% machine reliability.

**Conclusion**

Different parameters of machine performance, such as reliability, availability, maintainability and factors responsible for unavailability are evaluated. It is found that the steady state availability of the LHD is 73.39%. A few

![Figure 5—System and subsystems, performance](image)
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significant causes of machine unavailability have been detected and they demand special attention. The main reasons transmission, drive and hydraulic subsystems with 7%, 6% and 5% of working hours respectively. It is seen that the reliability of the hydraulic, transmission with time is not satisfactory. The overall reliability of the LHD system drops significantly with time. For improvement, the reliability of those subsystems/machines requires strengthening the maintenance efforts, which can result in decreasing their failure rate or in increasing their time to failure (TTF). Maintainability of the drive unit and transmission are low. Maintenance time may also be reduced by proper planning and spare parts management for increased availability of the machine. The constraints and reasons for low reliability and maintainability suggested that possible modification and design alternatives of the machine should be considered.

From the outcome of this analysis, it was clear that there is room for better maintenance planning and for improving the RAM of the machine from this type of modelling and quantitative analysis by the Markov processes. The case study provides data for predicting the control needs in maintenance or repair processes and potential design modification to ensure a desirable level of the LHD system’s reliability, availability and maintainability.

Acknowledgements

The authors are thankful to the mine management and former colleagues of first author for providing necessary help for preparing this paper.

References

1. SAMANTA, B. Challenge and strategies for performance improvement of a large public sector coal company in the face of changing business scenario in India. Unpublished MBA. project, 2000, IGNOU, New Delhi, India.
Appendix 1

From transition diagram presented in Fig 4 and the Markov equations\textsuperscript{[10-21]} can be derived.

The probability that the machine is in the operating state after time interval $dt$ i.e. at time $(t+dt)$ is given by

\[
P_S(t + dt) = \left[ (\text{Probability of being in operating state at time } t) \text{ AND } (\text{Probability of not failing between } t \text{ and } t + dt) \right] + \left[ (\text{Probability of being failed states at time } t) \text{ AND } (\text{Probability of being repaired between } t \text{ and } t + dt) \right].
\]

Probabilities of failure between $t$ and $dt$ are $\lambda dt$ and the probabilities of not failing are $(1-\lambda dt)$. Similarly, the probabilities of repair are $\mu dt$. Using the addition and multiplication rule for probabilities gives

\[
P_S(t+dt) = P_S(t) \left[ (1 - \lambda_1 dt) + (1 - \lambda_2 dt) + (1 - \lambda_3 dt) + (1 - \lambda_4 dt) + (1 - \lambda_5 dt) + (1 - \lambda_6 dt) \right]
\]

\[
+ \mu_1 dt \ P_S(t) + \mu_2 dt \ P_S(t) + \mu_3 dt \ P_S(t) + \mu_4 dt \ P_S(t) + \mu_5 dt \ P_S(t) + \mu_6 dt \ P_S(t).
\]

\[
P_S(t+dt) = P_S(t) = (\lambda_1 dt + \lambda_2 dt + \lambda_3 dt + \lambda_4 dt + \lambda_5 dt + \lambda_6 dt) \ P_S(t)
\]

\[
+ \mu_1 dt \ P_S(t) + \mu_2 dt \ P_S(t) + \mu_3 dt \ P_S(t) + \mu_4 dt \ P_S(t) + \mu_5 dt \ P_S(t) + \mu_6 dt \ P_S(t).
\]

\[
\text{As } dt \to 0, \quad \frac{dP_S(t)}{dt} = (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6) \ P_S(t)
\]

\[
+ \mu_1 P_S(t) + \mu_2 P_S(t) + \mu_3 P_S(t) + \mu_4 P_S(t) + \mu_5 P_S(t) + \mu_6 P_S(t).
\]

\[
\frac{dP_S(t)}{dt} = \Sigma \lambda_i P_S(t) - \Sigma \mu_i P_S(t) \quad \text{----------------------------------- [1]}
\]

Similarly for other states will be

\[
\frac{dP_1(t)}{dt} = -\lambda_1 P_1(t) - \mu_1 P_1(t)
\]

\[
\frac{dP_2(t)}{dt} = -\lambda_2 P_2(t) - \mu_2 P_2(t)
\]

\[
\frac{dP_3(t)}{dt} = -\lambda_3 P_3(t) - \mu_3 P_3(t) \quad \text{-----------------------------------[2]}
\]

Equating first order derivative to zero for a steady state, these equations will take the following forms

\[
P_0(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6) = \mu_1 P_1 + \mu_2 P_2 + \mu_3 P_3 + \mu_4 P_4 + \mu_3 P_5 + \mu_6 P_6 \quad \text{[i.e taking } P_S(t) = P_i, i=0,1,2..6]\]

\[
P_1 \mu_1 = P_0 \lambda_1 \quad \text{so, } P_1 = \lambda_1 / \mu_1 P_0
\]

\[
P_2 \mu_2 = P_0 \lambda_2
\]

\[
P_3 \mu_3 = P_0 \lambda_3 \quad P_4 \mu_4 = P_0 \lambda_4 \quad P_5 \mu_5 = P_0 \lambda_5
\]

\[
P_6 \mu_6 = P_0 \lambda_6 \quad \text{------------------------[3]}
\]

\[
P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = 1 \quad \text{------------------------[4]}
\]

Substituting value of $P_1, P_2, P_3, P_4, P_5$ and $P_6$ in equation 4

the steady state availability of LHD is found as

\[
P_S = 1/ (1 + \Sigma \lambda_i / \mu_i) - 1 / (1 + D), \quad \text{where } D = \Sigma \lambda_i / \mu_i \quad \text{------------------------ [5]}
\]

Machine residing in other subsystems is as

\[
P_1 = (\lambda_1 / \mu_1) / (1 + D)
\]

\[
P_2 = (\lambda_2 / \mu_2) / (1 + D)
\]

\[
P_3 = (\lambda_3 / \mu_3) / (1 + D)
\]

\[
P_4 = (\lambda_4 / \mu_4) / (1 + D)
\]

\[
P_5 = (\lambda_5 / \mu_5) / (1 + D)
\]

\[
P_6 = (\lambda_6 / \mu_6) / (1 + D)
\]