



Estimation of mineral resources using grade domains: critical analysis and a suggested methodology

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Synopsis

Common practice in mineral resource estimation consists of partitioning the orebody into several domains defined by grade intervals, prior to the geostatistical modelling and estimation/simulation at unsampled locations. This paper shows the pitfalls of grade domaining through a case study in which we compare the performance of several estimation schemes and demonstrate that the use of domains defined by grade cut-offs implies a deterioration of the resource estimates, mainly in what refers to precision and conditional bias. Then, several conceptual limitations of the grade domaining approach are stressed, in particular the fact that it does not account for the spatial dependency between adjacent domains and for the uncertainty in the domain boundaries. Also, this approach is shown to be sensitive to the cut-offs that define the domains, to provoke artifacts in the kriging maps, histograms and scattergrams between true and estimated grades, and to lower the kriging variance, a feature that may impact the mineral resource classification. An alternative approach is finally proposed to overcome these limitations, based on a stochastic modelling of the grade domains and a co-kriging of the grades using information from all the domains.

Keywords: geostatistics, kriging, conditional bias, geological control, grade zoning.

Introduction

Geological modelling is a key step prior to geostatistical estimation or simulation of the grades within a mineral deposit. Although alteration, mineralization and lithological aspects should be considered in determining the geological model (domaining) for interpolation, common practice consists of contouring the grades, generating grade shells¹. Within each shell the grades are considered homogeneous and can therefore be interpreted as a realization of a stationary random function, allowing variogram modelling and subsequent kriging or conditional simulation. This 'grade domaining' or 'grade zoning' is often done in association with mineralogical classification, for instance for each mineralogical unit, high-grade, medium-grade and low-grade domains are defined and analysed separately, in an attempt to better forecast the performance of the processed ore

in the mill. Although the definition of geological domains by grade contouring is a common approach in the mineral industry, very few references to this subject can be found in the literature^{2,3}.

In this article, we discuss some of the problems of using grade shells from a practical and theoretical viewpoint and we propose a better way to use them in resource estimation. The work has been divided in three sections. First, we show the consequences of using grade shells for the definition of geological domains and the resource estimation by an application to a porphyry copper deposit. The ideal case of perfectly known boundaries between grade domains, and the case of estimated boundaries are compared through a jackknife with the true values obtained from production blast-holes. This empirical study shows the impoverishment of the results as errors are added in the definition of the shell boundaries, which is always the case in practice. Second, we present a general discussion of the grade domaining approach from a theoretical standpoint. The arbitrary definition of the cut-offs to define the grade domains and the problems arising from the uncertainty in their boundaries are pointed out, with emphasis on the artifacts this approach generates in the resulting estimated or simulated grades. In the last section, we propose a solution to avoid some of these limitations, which consists of weighting the grade estimates obtained for each domain by the probability of occurrence of this domain.

Case study: porphyry copper deposit

Presentation of the data and methodology

The case study concerns a Chilean porphyry copper deposit for which two data-sets are

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available: a set of over two thousand samples from a diamond drill-hole exploration campaign, and a set of more than twenty thousand blast-hole samples that cover approximately the same area as the exploration drill-holes (Figures 1a and 1b). All the data (from drill-holes and blast-holes) represent twelve-meter long composites in which the copper grade has been sampled and assayed. The grade histogram of drill-hole composites is close to a lognormal distribution (Figure 1c) with a mean of 1.05 per cent, whereas the variogram shows an anisotropy with a greater continuity along the vertical direction than in the horizontal plane (Figure 1d). Blast-hole samples are almost regularly spaced over the major part of the area and have a mean grade of 1.17 per cent. The grade distribution has the same character (lognormal histogram) as the composite grades from drill-holes. The difference in the mean grade is due to the absence of blast-holes in some low-grade areas of the deposit, not to accuracy problems in the data; as a matter of fact, the kriging of the grades at blast-hole locations from the drill-hole samples provides an estimated mean grade of 1.16 per cent, which confirms that drill-hole and blast-hole measurements have the same accuracy. All the data belong to a single geological population (in terms of alteration, mineralization and lithology) and will be used to illustrate the consequences of domaining based on grade cut-offs.

The study consisted of using the information from the drill-hole data-set to estimate the grades at blast-hole locations, in order to compare these estimated values with the actual grades known from the blast-hole samples, a procedure known in geostatistics as jackknife⁴. For comparison purposes, two different definitions of the grade shells have been considered. First, since the actual grades are known at blast-hole locations, these locations can be classified as belonging or not to a grade shell based on their

real values. This is an unrealistic case, because in practice one never has the true grades at the locations to estimate when the grade shells are being defined; nonetheless, it is useful considering this ideal case for comparison of the results. Second, blast-hole locations can be assigned as belonging or not to a grade shell based on their estimated grades. This situation is closer to what occurs in practice. Each blast-hole location falls inside or outside a grade shell depending on an interpretation of the geological and grade continuity. To give a more repeatable approach in this study, we have considered defining the grade shell from a kriging of the grades obtained prior to any grade domaining, as explained below.

In order to better assess the effect of using grade-based domains, several estimation cases are performed:

- 1 *Case 1*—traditional resource estimation is performed via ordinary kriging without grade domaining. This first case will constitute a reference for comparisons. It is also used for estimating the grade shell boundaries for the other cases, that is, at every blast-hole location, the grade estimated by this global ordinary kriging defines to which grade domain this location will be assigned in the following cases (2, 3, and 4).
- 2 Resource estimation is performed via ordinary kriging within grade domains. Three cases are considered:
 - ▶ *Case 2*—three grade domains are defined: a low-grade domain (copper grade between 0 and 0.5 per cent), a medium-grade domain (from 0.5 to 1.0 per cent) and a high-grade domain (above 1.0 per cent).
 - ▶ *Case 3*—again three grade domains are used, but the cut-offs are changed to assess the impact of the choice of these cut-offs. The new geological domains are now defined by cut-offs 0.7 and 1.5 per cent.

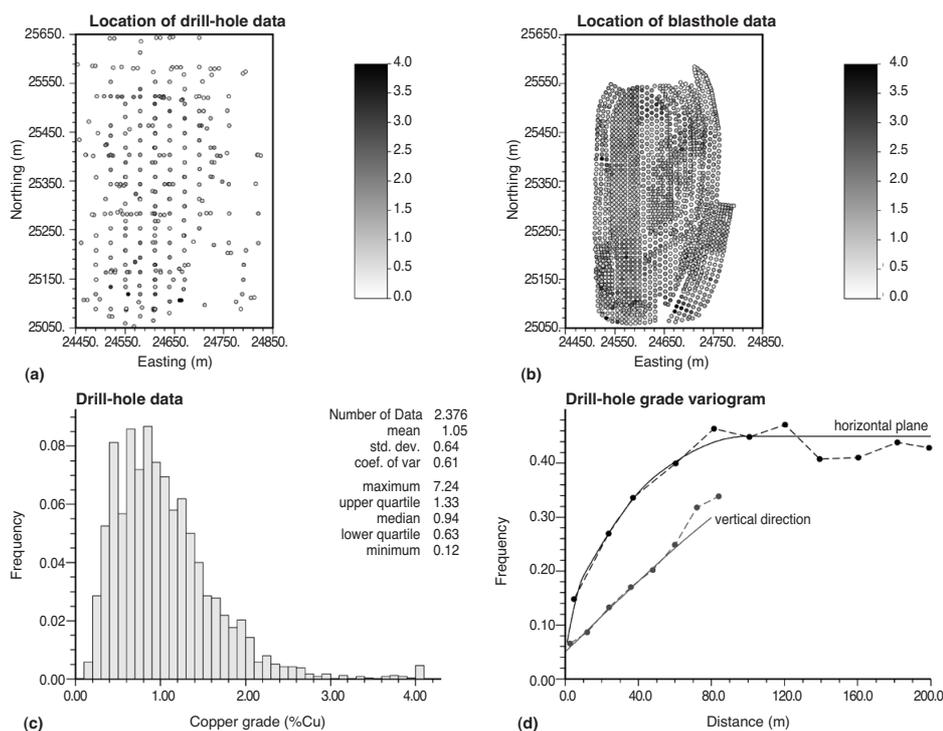


Figure 1—Location maps of (a), drill-hole and (b), blast-hole data (representation of one bench); (c), histogram and (d), variogram of the copper grades

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- *Case 4*—here, four cut-offs are used to define five grade domains: 0.5, 0.7, 1.0 and 1.5 per cent. This case allows assessing the impact of increasing the number of grade shells used to perform resource estimation.

Note that every kriging requires performing variogram analysis per grade domain. For cases 2, 3, and 4, two sub-cases are distinguished:

- *Ideal situation*—the blast-holes are classified in the grade domain corresponding to their true grades (no misclassification), and
- *Effective situation*—the classification is based on the estimated grades obtained in the reference situation (case 1), so that some blast-holes may be misclassified.

As mentioned earlier, this second situation is quite realistic whereas the first one constitutes only a basis for comparison, since it can never be achieved in practice: the grade domaining is always defined according to an interpretation of the deposit from exploration information and is not error-free. In general, grade shells are drawn by hand contouring, using the intersects of drill-hole data and considering the cut-offs that define the grade domains. The use of ordinary kriging for this case study constitutes another way to create the grade shells: unlike hand contouring, this is a repeatable approach and it considers the spatial continuity of the grades in all directions.

Precision of grade estimations

The results obtained for each situation are summarized in Table I, through basic statistics on the estimation errors at the blast-hole locations. The reference case (case 1) corresponds to ordinary kriging without domaining. Cases identified with the letter *a* assume each blast-hole location is assigned exactly to its true grade domain, while cases noted with *b* correspond to the situation where the grade domain assigned to a blast-hole location is inferred from the surrounding drill-hole information.

In every case, the grade estimation is almost unbiased since the average error is close to zero: unbiasedness is a general property of ordinary kriging and should always be fulfilled, unless strong mistakes are made in the definition of the domains, for instance if the spatial extension of the high-grade domain is overstated. Henceforth, we focus on the two other criteria (mean absolute and root of the mean squared errors), which measure the average amplitude of the error and therefore indicate the precision of the estimation.

The *ideal cases*, for which the delineation of the grade shells is error-free, always improve the results of the reference case. This observation proves that the knowledge of the true grade contours is valuable information for mineral

resource estimation. This can be illustrated by considering the scattergram between true and estimated grades (Figures 2a and 2b): when grade domaining is used, the cloud of points is constrained to the sectors along the first bisector that are delimited by the cut-offs used for domaining, hence it has a lower dispersion around the first bisector. Furthermore, if this ideal case could be achieved, the grade estimation would necessarily improve as more domains were added, and in the extreme case where a series of very tight grade intervals are used, estimation could solely be based on the grade domaining.

However, the *effective cases*, for which the grade shell delineation is estimated, lead to poorer results than the reference situation. This is explained by the misclassifications when assigning each blast-hole to a grade domain. Indeed, although a blast-hole is classified as low graded, in reality it can belong to another grade domain: the scattergram between true and estimated grades presents stripes (Figure 2c) and has a greater dispersion around the first bisector than the reference. From now on, only the effective cases will be discussed, since the ideal cases are unrealistic: a perfect definition of the grade domains is never met in practice.

Conditional bias

An important feature of the scattergrams between true and estimated grades displayed in Figure 2 is to determine whether conditional bias exists, that is, the expected value of the true grade given the estimated grade at the same location is not equal to the true grade⁵. This property has a great impact on the assessment of recoverable reserves above a given cut-off. Indeed, at the time of exploitation, the selective mining units are sent to mill or dump depending on whether their estimated grade (not their true grade, which is unknown) is above or below this cut-off. In case of conditional bias, the average estimated grade of the mining units sent to mill does not match their true average grade.

In the example, the estimations performed without grade zoning (case 1) are almost conditionally unbiased, except maybe for high cut-offs. Instead, a conditional bias appears when resorting to grade domaining, as shown in Table II. This can be explained by considering that once the high and intermediate grade domains are defined, they are filled with high or intermediate estimated grades, but in reality they may contain sectors with lower grades, hence an overestimation (bias) is made. This effect always occurs if a non-zero cut-off is applied (bold figures in Table II).

In the next section, we discuss some other pitfalls of the grade zoning approach from a theoretical point of view. The case study is also used to illustrate some of the conceptual problems.

Table I

Basic statistics on the estimation errors at the blast-hole locations, for the four cases under study: no grade domaining (case 1), three grade domains (cases 2 and 3) and five grade domains (case 4)

	Case 1	Case 2a	Case 3a	Case 4a	Case 2b	Case 3b	Case 4d
Mean error (% Cu)	0.009	0.017	0.028	0.020	-0.022	-0.017	-0.026
Mean absolute error (% Cu)	0.387	0.265	0.232	0.184	0.432	0.435	0.431
Root of the mean squared error (% Cu)	0.623	0.511	0.450	0.430	0.690	0.721	0.715

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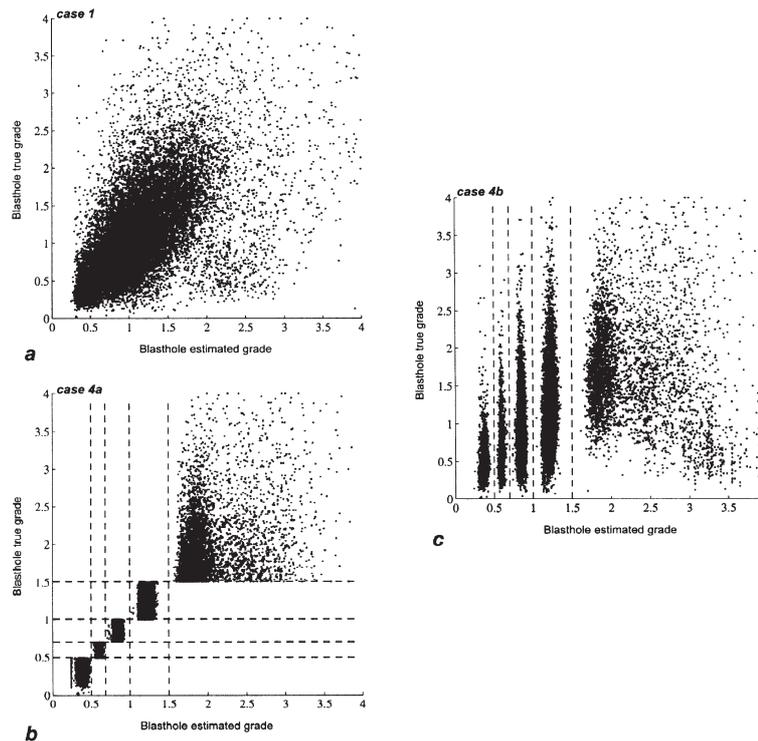


Figure 2—Scattergrams between true and estimated grades at the blast-hole locations, for the reference case (no grade domaining) and the ideal and effective cases associated with five grade domains

Table II

Statistics for conditional bias for the reference case (no grade domaining) and for the estimation within grade domains (cases 2 to 4). The statistics consist of the true and estimated mean grades of the blast-holes with estimated grades greater than a given cut-off

	Cut-off (% Cu)	0.0	0.5	1.0	1.5
Case 1	True	1.17	1.22	1.47	1.90
	Estimated	1.16	1.22	1.51	2.08
Case 2b	True	1.17	1.22	1.47	1.72
	Estimated	1.19	1.29	1.67	1.99
Case 3b	True	1.17	1.26	1.43	1.90
	Estimated	1.19	1.32	1.49	2.33
Case 4b	True	1.17	1.22	1.47	1.90
	Estimated	1.20	1.27	1.61	2.33

Conceptual limitations of grade domaining

Dependency between grade domains

A first theoretical limitation of the grade zoning approach stems from the dependency between the different domains caused by the spatial continuity of the grades within the deposit. In general, boundaries defined by grade domaining are not hard, that is, there is spatial correlation between the grades at both sides of the boundary. However, the estimation within a domain usually omits information from adjacent domains, hence it loses precision, especially along boundaries of the target domain. Estimating the grades in each domain separately means that the domains are considered as independent entities: this creates a boundary that does not exist geologically and contradicts the assumption of spatial continuity in the grade distribution.

The spatial dependency between domains (defined by grade shells or by mineralogical considerations) could be taken into account in the mineral resource estimation, for example through a co-kriging approach, as will be proposed in the last section of this work.

Uncertainty in the domain boundaries

In practice, grade shells are determined by hand contouring intersects of the drill-holes with low, medium, and high grade zones in cross-sections. Then wireframing is used to define the volumes (grade shells), with the consequent possibility of inconsistent volumes—grade shells may cross each other—and the little repeatability of the process. Another approach to define the domains is to use a quick interpolation of the available samples (in general, drill-hole cores) and then define the domains from its result. In any case, the domain boundaries may not be accurate.

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Now, in general, the grade zoning approach does not allow accounting for the uncertainty in the shell boundaries and for possible misclassifications of the unsampled locations. This can have far-reaching consequences in the estimated ore tonnages and, therefore, in the economic appraisal of the mining project. For instance, let us consider a domain defined by the grade interval $[a, b]$. The estimations at the unsampled locations in this domain usually lie in $[a, b]$ since they only use data that belong to such interval. As a consequence, the tonnage of material whose grade belongs to $[a, b]$ practically does not depend on the variogram or the kriging parameters defined by the user. If the grade domains are numerous and defined by a series of very tight grade intervals, the same reasoning proves that the whole grade-tonnage curve is predetermined by the partitioning into grade shells, before performing the geostatistical modelling and kriging. This approach is clearly not advisable for ore reserve estimation: for instance, in case the spatial extension of the high-grade domain is overestimated (e.g. waste wrongly classified as ore), the total amount of recoverable reserves is likely to be overstated.

The delineation of the grade domains must be performed with extreme care. In general, the true contours are much more irregular than the interpreted contours, since the interpolation of the unsampled grades (by kriging or inverse distance weighting) on which the interpretation is based is always smoother than reality⁵ (Figure 3). We strongly recommend the practitioner to perform a sensitivity study and compare several domaining schemes in order to assess the potential impact of the uncertainty in the spatial extensions of the different domains.

Artifacts in maps, histograms and scattergrams

To illustrate the artifacts generated by using grade domains, consider the previous application, in which a domain is defined by a grade interval $[a, b]$. Because of the smoothing effect of kriging, the estimated grade will approximate the average grade in this domain, say $(a+b)/2$. Hence, a concentration of grades near this average value will be observed, with a resulting decrease in the amount of grades near both cut-offs a and b . This can be seen as steps on a cumulative

histogram. As a consequence, the map of the estimated grades will present abrupt transitions when crossing the boundary between two grade domains; their histogram will be multimodal (Figure 4), whereas the scattergram between true and estimated values will show stripes, as presented in Figure 2c. Such artifacts are more pronounced when more cut-offs (more domains, like in case 4) are used in the partitioning of the deposit and may be dangerous for the valuation of the orebody, particularly if one of these cut-offs has an economic significance, e.g. for distinguishing ore and waste.

When conditional simulation is used, discontinuities will exist at the boundary between two grade domains, since only the information belonging to the geological domain of the simulated location is considered and all other information (from different grade intervals) is disregarded. The post-processing of such simulated models will again reflect the effect of this modelling approach. This could be partially solved by integrating the information from all the grade shells via a multivariate approach (co-kriging or co-simulation).

Sensitivity to the cut-off definition

The definition of the grade domains is sometimes made on the basis of the grade histogram, via the analysis of statistical tools such as log-probability plots. Some practitioners interpret a change of slope in the log-probability plot as a mixture of populations and, from this interpretation, define a set of cut-offs that are deemed relevant for grade zoning. This approach is questionable since, in general, a nonlinear log-probability plot indicates a departure from lognormality rather than the presence of several populations⁶, hence the grade domains have no physical meaning. Furthermore, the number and values of the cut-offs used to define the grade domains have an important impact on the estimation results. For instance, in the previous case study, the estimations obtained in case 1b are poorly related with the ones obtained in case 2b (Figure 5), although the kriging neighborhood is unchanged and the variograms are not so different between both cases.

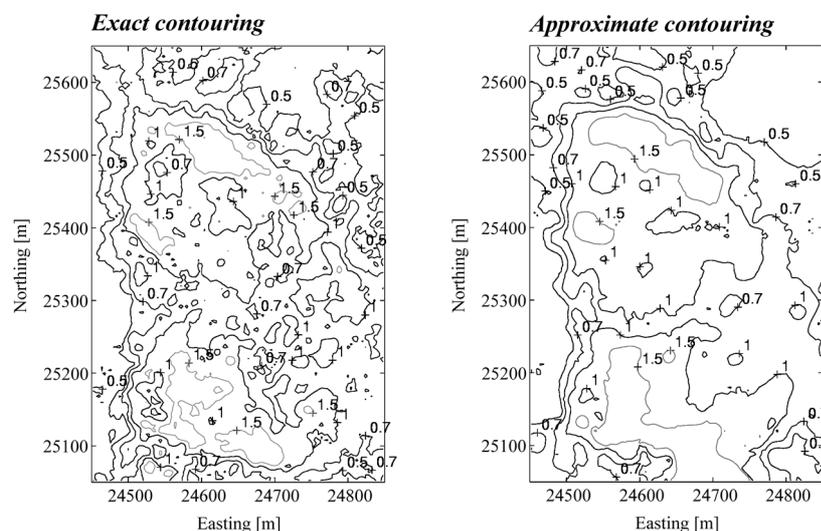


Figure 3—Comparison of the true and estimated contours associated with cut-offs 0.5, 0.7, 1.0 and 1.5 per cent (representation of one bench)

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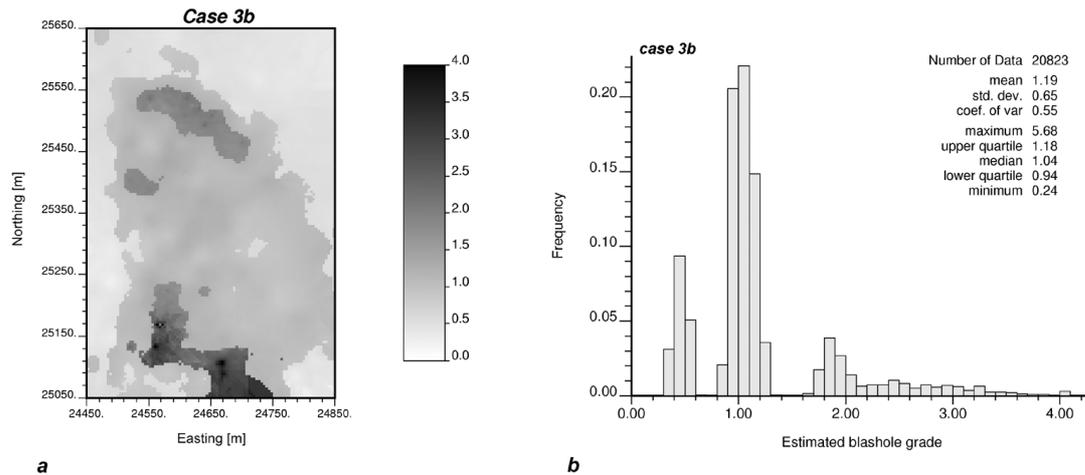


Figure 4—a, map (representation of one bench) and b, histogram of the grades estimated with three grade domains defined by cut-offs 0.7 and 1.5 per cent

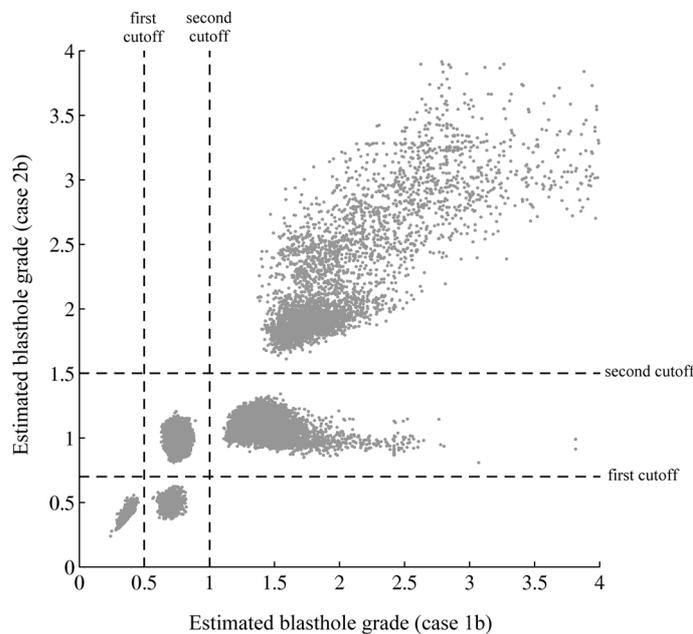


Figure 5—Comparison of the grade estimates at the blast-hole locations obtained by varying the domain definition. The estimates associated with three grade domains defined by cut-offs 0.7 and 1.5 per cent (ordinate) are plotted against the estimates associated with three grade domains defined by cut-offs 0.5 and 1.0 per cent (abscissa)

Self-justificatory approach

The partitioning of a deposit in several geological domains is often validated by comparing the statistical characteristics of the grades in the different domains, in particular their histogram and variogram. Now, in case of using grade shells, this statistical validation is often misleading. Concerning the histogram, by definition of the grade zoning approach, a hierarchy will be observed between the different domains, for instance in what refers to their mean grade. The following result might be less obvious to the practitioner: strong differences are also expected when comparing the grade variograms in different domains. Indeed, in many deposits (copper, uranium, precious metals, etc.), the spatial distribution of grades is ruled by a property known as destructuring of extreme values^{7,8}, which states that the extreme values do not cluster in space, i.e. the occurrence of extreme

values is purely random. In mathematical terms, the destructuring of extreme values means that the simple and cross variograms of indicator variables come close to a pure nugget effect when the indicator thresholds are very low or very high. As a consequence, the extreme-grade domains are expected to have a variogram with a higher relative nugget effect and a shorter range than the intermediate-grade domains. An illustration of this statement is given in Figure 6.

In summary, although the spatial distribution of the grades within the deposit is homogeneous, the comparison of the histograms and variograms between grade domains will always lead to the conclusion that these domains have very different statistical behavior, hence that grade zoning is justified. Geological knowledge should always mandate whether different domains can be considered or not: this decision should not be based solely on statistical analyses.

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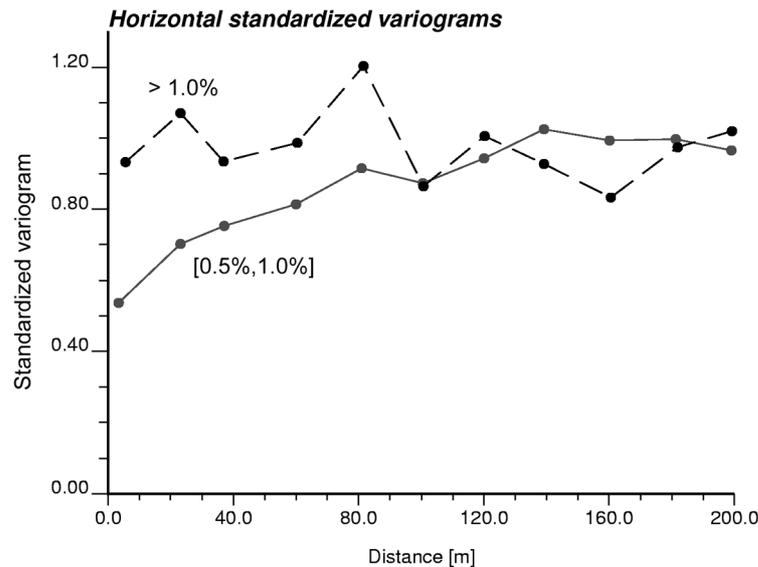


Figure 6—Standardized variograms of the drill-hole copper grades for two grade domains: 0.5-1.0 per cent (continuous line) and > 1.0 per cent (dashed line)

Bias in the kriging variance

A last drawback of partitioning the deposit by grade cut-offs concerns the kriging variance. In each domain, the grades are restrained to a limiting interval, hence their dispersion is lower than the one of the global histogram and is closely related to the choice of the cut-offs used for grade zoning. For instance, if a domain is defined by a tight grade interval, the corresponding grades will have a small variance: their variogram takes small values, and so does the kriging variance; this may lead to the wrong idea that the unsampled grades are well estimated. The problem comes from not incorporating the uncertainty on the grade shells in the estimation variance, while this can be the main source of uncertainty for grade estimation.

The kriging variance is sometimes used as a ranking index for classifying the estimated grades by decreasing order of confidence, e.g. for classifying the mineral resources into measured, indicated or inferred categories⁹⁻¹¹. In consequence, the classification becomes an artifact of the set of cut-offs used to define the grade domaining: for instance, if the high-grade areas are partitioned into many domains corresponding to small grade intervals, these areas are likely to be classified as measured resources, even if they are under sampled.

A suggested approach for improving the resource estimation

Methodology

In this section, we propose an approach to estimation of mineral resources using grade shells. For clarity, it will be presented using the porphyry copper case study, for which three grade domains are defined: a low-grade domain (between 0 and 0.5 per cent), a medium-grade domain (from 0.5 to 1.0 per cent) and a high-grade domain (above 1.0 per cent). This is referred to as 'case 5'. Henceforth, the grade at location x will be denoted as $Z(x)$ (in the geostatistical formalism, this is a random variable).

The proposed methodology consists of the following steps:

- (a) Estimate the probabilities for the unsampled locations to belong to each grade domain, i.e.

$$\begin{cases} P_1(\mathbf{x}) = \text{Pr ob}[0.0\% \leq Z(\mathbf{x}) < 0.5\%] \\ P_2(\mathbf{x}) = \text{Pr ob}[0.5\% \leq Z(\mathbf{x}) < 1.0\%] \\ P_3(\mathbf{x}) = \text{Pr ob}[1.0\% \leq Z(\mathbf{x})] \end{cases} \quad [1]$$

To perform this estimation, an indicator kriging or indicator co-kriging approach can be used; a post-processing step is then necessary to ensure the consistency of the estimated probabilities⁸, i.e. make sure that $P_1(x)$, $P_2(x)$ and $P_3(x)$ are nonnegative and sum to one at any location x . Alternatively, one may resort to conditional simulation of random sets to draw multiple realizations of the grade domains and derive the previous probabilities. This option is more demanding since it requires a spatial distribution model for the random sets, for instance a truncated Gaussian or a plurigaussian model¹².

- (b) Consider the grade in each domain as a particular regionalized variable, which defines three different random fields:

$$\begin{cases} Z_1(\mathbf{x}) = [Z(\mathbf{x}) | 0.0\% \leq Z(\mathbf{x}) < 0.5\%] \\ Z_2(\mathbf{x}) = [Z(\mathbf{x}) | 0.5\% \leq Z(\mathbf{x}) < 1.0\%] \\ Z_3(\mathbf{x}) = [Z(\mathbf{x}) | 1.0\% \leq Z(\mathbf{x})] \end{cases} \quad [2]$$

- (c) Perform a variogram analysis of the coregionalization $\{Z_1, Z_2, Z_3\}$. Because these random fields are defined on non-overlapping domains (case of total heterotopy), the assessment of the cross-structures is not easy. One may use the cross-covariance as a structural tool instead of the cross-variogram¹³. The nugget effects of the cross-structures remain unknown; however they

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are not required in the co-kriging system (following step)¹⁴.

- (d) Perform a co-kriging of the coregionalization. At each blast-hole location, one obtains a set of estimates (one per grade domain), say Z_1^* , Z_2^* and Z_3^* .
- (e) The final grade estimation is obtained by weighting the previous estimates by the probability of belonging to the corresponding domain:

$$Z^*(\mathbf{x}) = P_1(\mathbf{x})Z_1^*(\mathbf{x}) + P_2(\mathbf{x})Z_2^*(\mathbf{x}) + P_3(\mathbf{x})Z_3^*(\mathbf{x}) \quad [3]$$

Application and discussion

The previous methodology is applied to the copper case study, by using an ordinary indicator co-kriging for step (a) and an ordinary co-kriging for step (d). Figure 7 displays the

scattergram between true and estimated grades at the blast-hole locations, which is quite similar to the one obtained in the reference case (Figure 2a). The statistics for measuring the precision and conditional bias are summarized in Tables III and IV and are quite satisfactory (the mean absolute and mean squared errors are even less than in the reference case).

The proposed approach has its pros and cons. On the one hand, the uncertainty in the domain boundaries and the dependency between grade domains are taken into account, via a probabilistic modelling of the domains (step a) and a co-kriging of the grades (step d), respectively. Moreover, one has a data-charged model for describing the spatial distribution of grades, by using a different variogram for each grade domain; this allows incorporating structural changes in the grade spatial distribution, e.g. a change in the anisotropy orientation when the grade increases.

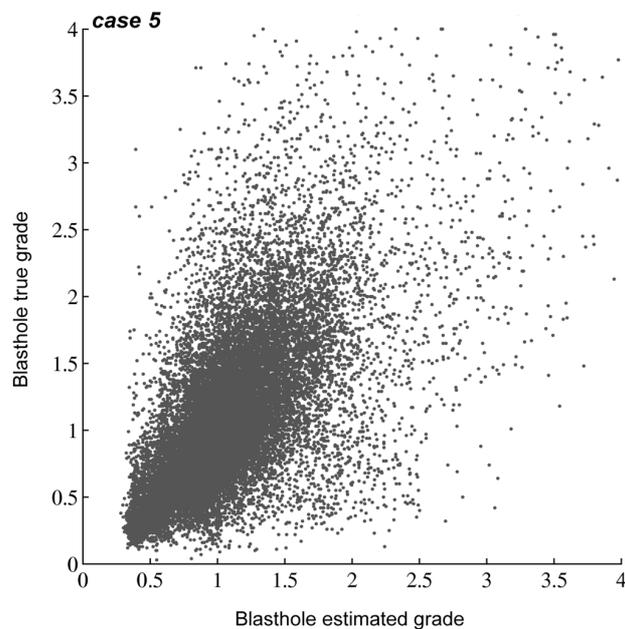


Figure 7—Scattergrams between the true blast-hole grades and the grades estimated using the proposed methodology

Table III

Basic statistics on the estimation errors at the blast-hole locations when the estimates are obtained with the proposed methodology

	Mean error (% Cu)	Mean absolute error (% Cu)	Root of the mean squared error (% Cu)
Case 5	0.044	0.374	0.602

Table IV

Statistics for conditional bias for the proposed methodology. The statistics consist of the true and estimated mean grades of the blast-holes with estimated grades greater than a given cut-off

	Cut-off (% Cu)	0.0	0.5	1.0	1.5
Case 5	True	1.17	1.21	1.48	1.97
	Estimated	1.13	1.17	1.43	1.96

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On the other hand, the approach has several drawbacks:

- ▶ First, it requires more work than a classical kriging, especially in what refers to variogram analysis; hence it is not suitable when many grade domains are defined. In such cases, there may even be too few data per domain to perform variogram inference.
- ▶ Second, the user must pay attention to the neighborhood search radii in order to find enough data for co-kriging (step d); for instance, the high-grade area contains few Z_1 -data, which makes difficult estimating Z_1 .
- ▶ Third, the method implicitly assumes that there exists a hard boundary between the grade domains: ordinary co-kriging supposes that the mean grades are unknown but differ from one domain to another, hence the transition is not continuous.
- ▶ Finally, the calculation of the estimation variance is quite complicated; it requires expressing the final estimator (Equation [3]) as a weighted average of the data:

$$Z^*(\mathbf{x}) = \sum_{\alpha=1}^{n_1} \lambda_{\alpha} Z_1(\mathbf{u}_{\alpha}) + \sum_{\alpha=1}^{n_2} \mu_{\alpha} Z_2(\mathbf{v}_{\alpha}) + \sum_{\alpha=1}^{n_3} \nu_{\alpha} Z_3(\mathbf{w}_{\alpha}) \quad [4]$$

and expanding the estimation variance as follows:

$$\begin{aligned} & \text{var}[Z(\mathbf{x}) - Z^*(\mathbf{x})] \\ &= P_1(\mathbf{x}) \text{var}[Z_1(\mathbf{x}) - Z^*(\mathbf{x})] + \\ & P_2(\mathbf{x}) \text{var}[Z_2(\mathbf{x}) - Z^*(\mathbf{x})] + \\ & P_3(\mathbf{x}) \text{var}[Z_3(\mathbf{x}) - Z^*(\mathbf{x})] \end{aligned} \quad [5]$$

The different terms of this equation can be calculated by using the variogram model of the coregionalization.

Conclusions

Although it constitutes a common practice in the mineral industry, grade domaining may increase the dispersion of the estimation errors and provoke a conditional bias in the resource estimation, a deterioration that can be explained mainly by the uncertainty in the domain boundaries and the resulting misclassifications of unsampled locations (ore wrongly considered as waste, or vice versa). This approach also has consequences on the estimated ore tonnages and on the kriging variance, with possible implications on the resource classification.

Statistical validations of the domaining are misleading, since the grade histograms and variograms are expected to differ strongly from one grade domain to another. Grade zoning should always be confirmed by geological considerations. For instance, it may be useful for vein-type deposits or for separating an overburden or a non-mineralized area from the ore zone. But even in these cases, the practitioner must pay attention to two issues:

- ▶ The definition of the domains and the uncertainty on their spatial extent (unknown boundaries).
- ▶ Although a grade shell is used to define the ore area, the resource/reserve estimation should account for all the samples, even the ones located outside this area, unless there exists a clear-cut discontinuity between ore and waste.

To avoid the previous limitations, a new approach is proposed, which combines two improvements with respect to traditional grade domaining. On the one hand, the uncertainty in the domain boundaries is quantified by calculating the probabilities for the unsampled locations to belong to each grade domain. On the other hand, the mineral resources in each domain are estimated with data from all the domains, via co-kriging, which accounts for the spatial correlations of the grades between adjacent domains. Finally, the grade estimates corresponding to each domain are weighted by the probability of occurrence of this domain. When applied to the porphyry copper deposit study, such an approach led to better results than traditional grade domaining, with the same precision and conditional bias as the reference kriging with no domaining.

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