Mining grade and sampling theory—the logical way to evaluate ore reserves

by P.I. Page*

Synopsis

In mineral exploration the place that yields the first significant result is marked and two questions are posed immediately. Where should the next sample be drawn and how big should it be? At present the answers are arbitrary and ad hoc. They depend on personal experience and sentiment—that is hunch. The thought processes involved in reaching the decisions are unsatisfactory and unsatisfying for professionals. This paper proposes guidance on choosing sample spacing and sample size. The guidance is obtained from the grades of the samples. The grades also imply the extent of influence of each sample assay. Thus sampling and assay are integrated. The integration is achieved by changing the unit of value in which assays and grades are expressed and by exposing meanings and functions of grade beyond that of measuring rock value. The proposals are extended to guide comminution of particle size in sample size reduction and to deal with the problem of erratic high assays. Also, elucidation becomes possible of the computation of a reliable figure for average grade as a statistical weighted mean.

Introduction

Mining reserves are commonly expressed by tonnage and grade. The former is obtained by measurement of volume and density and the latter by sampling and assay. Problems arise in the selection of sample size and sample spacing and in determining the extent to which each sample assay has influence in the country rock. Exceptionally high assays affect disproportionately the average grade and present acute difficulties. The problems are particularly evident in the mining of dense, obdurate minerals for their precious metal content. The procedure that is here proposed avoids these problems. It is based on parameters that are specific to the location of operations. Pilot investigations are made to provide values for these parameters.

Assay and grade

An assay result and a grade are usually stated as a proportion by mass. An example is 12 g t⁻¹ Au meaning that 12 g of pure gold are likely to be found in 1 t of country rock. The figure with its unit of measurement value the country rock. Together they are a metal—mass grade. If the price that can be realized for the gold content is applied to the grade, the value of the rock can be stated in monetary terms.

However, this unit of measurement is unsuitable for the determination of mining reserves for two reasons. First, sample spacing is measured in the physical dimension of length. Second, tonnage is measured initially as volume, which is the cube of length. Any grade that is measured in the physical dimension of mass is unhelpful for the process of estimating reserves. The mining industry is wedded to weight because grade by mass is the tradition and because legal trade in metals is by weight. But in the determination of reserves, both in exploration and on a mine length area and volume, are more useful than mass.

Metal or mineral

Precious metals are not mined as pure metals. Their minerals are sought and mined, and these minerals occupy more volume than the metals within them. In reserve compilation valuable minerals are more significant than their metals. The metal contents of the minerals are useful only as tracers for the minerals themselves. The economic evaluation of rock may be thought of as a forensic exercise in which metal content is merely one of the clues. Other clues are the geology and mineralogy of the location. These are not considered here.

Mineral-volumetric grade

A suitable way of expressing grade for evaluation purposes is by mineral-volumetric
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proportion. An example is 2.5 cm³ m⁻³ native gold meaning that 2.5 cm³ of native gold are likely to be found in 1 m³ of country rock. For the conversion from metal-mass grade to mineral-volumetric grade knowledge is required of:

➤ the mineral density in g cm⁻³
➤ the fractional pure metal content by mass of the valuable mineral
➤ the density of the country rock in g cm⁻³.

These values may be obtained either from the literature or by established laboratory methods.

As an example of the calculation of mineral-volumetric grade from metal-mass grade suppose the data are:

➤ Density of native gold 16.5 g cm⁻³
➤ Mass fraction of pure gold 0.7975
➤ Density of country rock 2.75 g cm⁻³
➤ Metal mass grade of pure gold 12 g t⁻¹.

Mass of native gold that contains 12 g pure gold is

\[ \frac{12}{0.7975} = 15.05 \text{ g} \]

Therefore mineral-mass grade of native gold is 15.05 g t⁻¹.

Volume of native gold in 1 m³ is

\[ \frac{15.05}{16.5} = 0.91 \text{ cm³} \]

Volume of 1 t = 10⁶ g country rock is

\[ \frac{10^6}{2.75} = 3.63636 \times 10^5 \text{ cm³} \]

Therefore mineral-volumetric grade is 2.5 cm³ m⁻³.

The relationship is

\[ \text{Min-vol grade/cm³ m}^{-3} = \text{Met-mass grade/gt}^{-1} \times \frac{\text{Rock density/g cm}^{-3}}{\text{Mineral density/g cm}^{-3}} \times \frac{\text{Metal fraction}}{\text{Do}} \]

This formula is written in e-arithmetic, as is the arithmetic that follows. The symbol Do means dimensionless.

The formula is not balanced, either for units of measurement or even dimensionally. As written it, is technologically not scientific. The anomaly arises because two terms have been omitted from the right-hand side. These are equal in value but have opposite effects and therefore cancel out numerically.

The right-hand side should be multiplied by the number of cubic centimetres in a cubic metre, that is 10⁶ cm³ m⁻³, and divided by the number of grams in a tonne, that is 10⁶ g t⁻¹. When these two terms, together with their units of measurement, are inserted, the anomaly falls away.

Metal-mass grade is the grade for use in mining commerce. Mineral-volumetric grade is the grade for use within the mining industry.

Conversion factor

In the formula for mineral-volumetric grade the last three terms constitute the conversion factor by which the metal-mass grade should be multiplied to obtain the corresponding mineral-volumetric grade. In the above example

\[ \text{Conversion factor} = \frac{16.5}{0.7975} = 20.92 \]

Index of difficulty

When the conversion factor is inverted, a number with significance is obtained. This number illustrates—indeed measures—the difficulty of finding and evaluating rare, dense mineral deposits.

\[ \text{Difficulty index} = \text{Metal fraction/Do} \times \frac{\text{Mineral density/g cm}^{-3}}{\text{Rock density/g cm}^{-3}} \]

In the example above the index of difficulty is

\[ \frac{0.7975 	imes 16.5}{2.75} = 4.785 \]

The mineral engineer is looking—sight sees size, not mass—for 2 1/2 parts in a million not 12 parts in a million. The difficulty is 4.8 times greater than the metal-mass grade indicates.

Table I lists grade conversion factors and indices of difficulty for selected dense minerals in country rocks with realistic densities.

<table>
<thead>
<tr>
<th>Mineral sought</th>
<th>Mineral density g cm⁻³</th>
<th>Metal fraction</th>
<th>Country rock</th>
<th>CR density g cm⁻³</th>
<th>Density ratio</th>
<th>Conversion factor</th>
<th>Difficulty index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Native gold</td>
<td>18.5</td>
<td>0.948</td>
<td>Granite</td>
<td>2.65</td>
<td>7.0</td>
<td>0.15</td>
<td>6.6</td>
</tr>
<tr>
<td>*</td>
<td>17.5</td>
<td>0.877</td>
<td>Granodiorite</td>
<td>2.70</td>
<td>6.5</td>
<td>0.18</td>
<td>5.7</td>
</tr>
<tr>
<td>*</td>
<td>17.5</td>
<td>0.877</td>
<td>Alluvium</td>
<td>1.93</td>
<td>9.1</td>
<td>0.13</td>
<td>8.0</td>
</tr>
<tr>
<td>Native platinum</td>
<td>15.5</td>
<td>0.708</td>
<td>Syenite</td>
<td>2.75</td>
<td>5.6</td>
<td>0.25</td>
<td>4.0</td>
</tr>
<tr>
<td>*</td>
<td>19.0</td>
<td>0.929</td>
<td>Dunite</td>
<td>3.5</td>
<td>5.4</td>
<td>0.20</td>
<td>5.0</td>
</tr>
<tr>
<td>*</td>
<td>17.0</td>
<td>0.856</td>
<td>Dunite</td>
<td>3.5</td>
<td>4.9</td>
<td>0.24</td>
<td>4.2</td>
</tr>
<tr>
<td>*</td>
<td>17.0</td>
<td>0.856</td>
<td>Alluvium</td>
<td>2.0</td>
<td>8.5</td>
<td>0.14</td>
<td>7.3</td>
</tr>
<tr>
<td>Sperrylite</td>
<td>19.0</td>
<td>0.764</td>
<td>Dunite</td>
<td>3.5</td>
<td>4.3</td>
<td>0.30</td>
<td>3.3</td>
</tr>
<tr>
<td>Cassiterite</td>
<td>10.5</td>
<td>0.565</td>
<td>Peridotite</td>
<td>3.2</td>
<td>3.3</td>
<td>0.54</td>
<td>1.9</td>
</tr>
<tr>
<td>*</td>
<td>6.85</td>
<td>0.786</td>
<td>Greisen</td>
<td>2.75</td>
<td>2.5</td>
<td>0.51</td>
<td>2.0</td>
</tr>
<tr>
<td>*</td>
<td>6.85</td>
<td>0.786</td>
<td>Alluvium</td>
<td>1.93</td>
<td>3.5</td>
<td>0.36</td>
<td>2.8</td>
</tr>
<tr>
<td>Columbite</td>
<td>5.2</td>
<td>0.551</td>
<td>Pegmatite</td>
<td>2.65</td>
<td>2.0</td>
<td>0.92</td>
<td>1.1</td>
</tr>
<tr>
<td>Tantalite</td>
<td>7.9</td>
<td>0.705</td>
<td>Pegmatite</td>
<td>2.65</td>
<td>3.0</td>
<td>0.48</td>
<td>2.1</td>
</tr>
<tr>
<td>Diamond</td>
<td>3.5</td>
<td>1.0</td>
<td>Kimberlite</td>
<td>3.2</td>
<td>1.1</td>
<td>0.91</td>
<td>1.1</td>
</tr>
<tr>
<td>*</td>
<td>3.5</td>
<td>1.0</td>
<td>Alluvium</td>
<td>2.0</td>
<td>1.8</td>
<td>0.57</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Notes: a) Density ratio is the ratio of the mineral density to the density of the country rock
b) Conversion factor is the factor by which the metal–mass grade in gt⁻¹ is multiplied to give mineral-volumetric grade in cm³ m⁻³
c) Metal fraction for native gold has been calculated on the assumption that the only contaminant is silver (SG 10.5)
d) Metal fraction for native platinum has been calculated on the assumption that the only contaminant is iron (SG 7.6)
e) Metal fraction for other minerals is calculated from relative atomic and molecular masses
f) Most calculated figures are rounded for quicker comprehension at the expense of precision
Meanings and functions of grade

Grade has three meanings: that of value, that of probability and that of concentration. For the estimation of ore reserves to date only the first of these is employed. In the proposed procedures that follow the other two are shown to possess useful functions.

Value

As already indicated, the usual meaning of grade is that of measuring value, and this is its familiar function.

Probability

The figure and unit for mineral-volumetric grade also measure probability. Thus 2.5 cm³ m⁻³, that is 2.5 cm³ of mineral in 10⁶ cm³ of country rock is the volumetric probability of finding the mineral—namely 1 in 10⁶ ÷ 2.5 = 1 in 400 000. This is the probability 2.5 × 10⁻⁶/D₀.

Fortunately, such a forbidding figure is not required in the theory formed here; but the concept that each grade implies a corresponding probability is needed. (The same probability calculation may be made for the metal-mass grade but the result is not useful, except for comparison.)

Concentration and dilution

The figure and unit for mineral-volumetric grade also measure the volumetric concentration of the mineral in the country rock. Dilution is the inverse of concentration, so the reciprocal of the grade measures the dilution of the mineral by the country rock. Again continuing the example, for the mineral-volumetric grade 2.5 cm³ m⁻³ the corresponding dilution is 10⁶ ÷ 2.5 = 400 000/D₀.

Probability and dilution as measured by mineral-volumetric grade are relied upon in the proposed procedure.

The largest particle of valuable mineral

The minimum size of sample that is needed is related to the size of the largest particle of the valuable mineral. The fear is that the inclusion or exclusion of the largest particle makes an unacceptable difference to the assay value of the sample if the sample size is too small.

The selection and nominal size of the largest particle are usually obtained by wet sieving, but volume rather than nominal size is what is required. Measurement of volume by direct liquid displacement is unlikely in practice to be accurate enough. A more satisfactory method is to weigh the particle on a laboratory balance after its density has been determined using a batch of fragments and thereafter calculate its volume.

The smallest, largest particle that can be handled depends on the sensitivity of the balance. The particle must have a mass greater than the accuracy of the balance. This factor is elaborated in Table II. Mines exploiting invisible gold have a unique problem that the procedure does not solve. However, it does demonstrate the efficacy in these circumstances of numerous small samples at close spacing.

The volume of the largest particle of valuable mineral is an essential factor in the evaluation of ore reserves. Isolation of the largest particle should receive as much attention from samplers as grade determination does from assayers.

Sample size

Previous sampling and assay reveal the largest valuable particle and provide a figure for mineral-volumetric grade that points to the ruling probability and dilution.

Logic demands that the same degree of probability be assigned to finding the largest particle in the next sample. The size of the sample is therefore such that the largest particle is diluted by the bulk of the sample to the same extent. Therefore Volume of sample = Volume of largest particle × Volumetric dilution.

In Table III sample volumes are computed for spherical particles of various nominal sizes all of density 16.5 g cm⁻³ in country rock of density 2.75 g cm⁻³ when the mineral volumetric dilution is 400 000 (all from the example again). The results are related to the place of operations and are practical.

The table is a skeleton for illustration. In practice intermediate particle sizes would be listed for sieve apertures in the root-two series at the least.

Influence of single sample

In the same way as the volume of the largest particle and its dilution in country rock are used to determine sample volume, so also may the volume of country rock influenced by a single sample be computed. The statement may be made with the justification of similitude that in effect the country rock dilutes the sample volume. Hence, the probability of the sample volume (with its attribute of grade) occurring within the country rock is the same when the volume of influence is defined by the dilution. And the probability of finding the grade of the sample within the volume of influence when that volume is limited in this way is also the same. Therefore

Volume of influence = Sample volume × Volumetric dilution

Alternatively by substitution

Volume of influence = Volume of largest particle × (Volumetric dilution)²

Thus the volume of influence for the sample volume 1 676 cm³ (selected from the example—Table III) when the dilution is 400 000, is 1 676 × 1 676 = 6.704 × 10⁸ cm³ = 670.4 m³

Table II

Limitation on the volume of the largest particle by the sensitivity of the laboratory balance

<table>
<thead>
<tr>
<th>Sensitivity of balance and mass of particle</th>
<th>Volume of particle *</th>
<th>Approximate description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mg = 0.001 g</td>
<td>6.06 × 10⁻⁶ cm³</td>
<td>Sphere of half a millimetre</td>
</tr>
<tr>
<td>1/10 mg = 0.0001 g</td>
<td>6.06 × 10⁻⁶ cm³</td>
<td>Sphere of one fifth millimetre</td>
</tr>
<tr>
<td>1/100 mg = 0.00001 g</td>
<td>6.06 × 10⁻⁷ cm³</td>
<td>Sphere of one tenth millimetre</td>
</tr>
</tbody>
</table>

* Density of particle assumed to be 16.5 g cm⁻³
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Table III
Sample size and sample spacing computed for spherical particles of native gold of density 16.5 g cm⁻³ in country rock of density 2.75 g cm⁻³ when the volumetric dilution is 400000

<table>
<thead>
<tr>
<th>ASTM sieve</th>
<th>Aperture</th>
<th>Particle volume cm³</th>
<th>Particle mass g</th>
<th>Sample volume cm³</th>
<th>Sample mass g</th>
<th>Sphere of influence m³</th>
<th>Sample spacing m</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2.83 mm</td>
<td>0.0119</td>
<td>0.1958</td>
<td>4747</td>
<td>13054</td>
<td>1899</td>
<td>15.4</td>
</tr>
<tr>
<td>10</td>
<td>2.00 mm</td>
<td>4.1888 x 10⁻³</td>
<td>0.0691</td>
<td>1676</td>
<td>4608</td>
<td>670</td>
<td>10.9</td>
</tr>
<tr>
<td>18</td>
<td>1.00 mm</td>
<td>5.2360 x 10⁻⁴</td>
<td>8.6394 x 10⁻³</td>
<td>209</td>
<td>576</td>
<td>83.8</td>
<td>5.4</td>
</tr>
<tr>
<td>35</td>
<td>500 µm</td>
<td>6.5450 x 10⁻⁵</td>
<td>1.0799 x 10⁻³</td>
<td>26.2</td>
<td>72</td>
<td>10.5</td>
<td>2.7</td>
</tr>
<tr>
<td>60</td>
<td>250 µm</td>
<td>8.1812 x 10⁻⁶</td>
<td>1.3499 x 10⁻⁴</td>
<td>3.27</td>
<td>9</td>
<td>1.3</td>
<td>1.4</td>
</tr>
<tr>
<td>120</td>
<td>125 µm</td>
<td>1.0227 x 10⁻⁶</td>
<td>1.6874 x 10⁻⁵</td>
<td>0.41</td>
<td>1.12</td>
<td>0.16</td>
<td>0.68</td>
</tr>
<tr>
<td>230</td>
<td>63 µm</td>
<td>1.3092 x 10⁻⁷</td>
<td>2.1603 x 10⁻⁶</td>
<td>0.052</td>
<td>0.144</td>
<td>0.02</td>
<td>0.34</td>
</tr>
<tr>
<td>400</td>
<td>37 µm</td>
<td>2.6522 x 10⁻⁸</td>
<td>4.3761 x 10⁻⁷</td>
<td>0.011</td>
<td>0.029</td>
<td>0.004</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Note: The last two particles may be too small to be weighed individually (See Table II)

Sample spacing

If the sample can be considered to be drawn from a point, the form of influence is spherical. The diameter of such a sphere with the sample at its centre is \((6 \times 670.4 \div 11)\) \(\frac{1}{2} = 10.86\), say 11 m.

The sample spacing is therefore 11 m and the influence of the sample grade (in whatever units one may choose to use) extends for 5.5 m in all directions from the sample position.

If the sample is drawn from a channel, groove or drill core, the centreline of the sample void may be respected and the 5.5 m of influence measured in all directions normal to the centre line.

Again the result is based on local data and is practical. In Table III the last two columns list spheres of influence and sample spacings for the same range of particles.

A second example of the application of this approach is given in the Appendix for a vein tin deposit. In this example the grade is greater than that in the text example by three orders of magnitude (1 000 times), yet the results for sample volume and sample spacing are of the same order. This occurs because in the computational procedure the volume of the largest particle is three orders greater while the volumetric dilution is three orders smaller.

Sampling control

Early in a campaign of exploration or reserve compilation the mineral and country rock densities and the metal mass fraction are determined. The conversion factor is calculated and as metal-mass grades become available the mineral-volumetric grade is computed and hence the volumetric dilution. The volume of the largest particle of valuable mineral is obtained and thereafter the sample size and sample spacing.

As more samples are collected and more metal-mass grades become available, the procedure is repeated. Densities and the metal fraction are checked. The conversion factor and the mineral-volumetric grade are recalculated, as is the volumetric dilution. A larger, largest particle when found is used to revise the sample size and spacing. In this way sampling control is improved periodically with due respect for local conditions.

The procedure provides a check of mining reserves already compiled by empirical methods.

Sample reduction for laboratory assay

Most field and mine samples need to be reduced in size for laboratory assay. This is sampling a sample. It is a process of compression maintaining the original volumetric probability and dilution in order to preserve the original assay value. It is achieved by comminution to ensure that the reduced volume of the largest valuable particle, if included in or excluded from the reduced sample size, does not cause an unacceptable difference in the assay result.

Since this procedure uses the material of the original sample, it is governed by the table of data compiled to dictate the volume of that sample. The sieve size to which comminution must proceed before the sample is split is found from the data in this table.

The table may be extended upwards for larger particles as a guide to the reduction of bulk samples for transport. Thus, from an environment having the parameter values envisaged in Table III and using this skeleton table a large field sample should be ground to pass sieve 7 before coning and quartering to leave a minimum of about 13 kg or 4.75 dm³ for further reduction.

For a laboratory sample of 65 g (enough for two metric assay tonnes) to represent the conditions of Table III the minimum sample of 72 g for sieve 35 is in theory too large. Grinding should therefore be carried to pass sieve 60. If the splitting rejects 50 per cent, not less than 130 g must be ground.

In practice grinding would be continued to pass sieve 80 for fire assay and even finer when chemical dissolution is involved. These are grain sizes to satisfy the assaying not the sampling in the ambient circumstances.

Erratic high assays

Erratic high assays have always plagued the industry. They distort rational, arithmetic ways of computing averages. Various rule–of–thumb methods of dealing with the problem have been tried and some are used. An arbitrary cut belittles the work of field and laboratory staff to the point of insult. An exceptionally high assay value after checking is a fact, not to be ignored but to be made use of. To tinker with the value is no way to confront the problem.
Exceptionally high assays occur at places of intense deposition where the particle sizes of the valuable mineral are greatly enhanced. The effects of finding a much greater largest particle are illustrated in the following, second example of an underground gold mine. The importance of obtaining an accurate figure for the volume of the largest particle is stressed. This is the parameter that must be checked when an exceptionally high assay crops up.

For the example, the average grade of the reserves is assumed to be 8 \( \text{gt}^{-1} \) Au. High assays are commonly 10 to 20 times the average and exceptionally 50 times or more. Table IV compares sample volume, volume of influence and sample spacing for the average grade to assays of 120 \( \text{gt}^{-1} \) (15 times the average) and 400 \( \text{gt}^{-1} \) (50 times the average).

In each case other assumptions are:

- Density of native gold \( 16.5 \text{ g cm}^{-3} \)
- Mass fraction of pure gold content \( 0.7975 \)
- Density of country rock \( 2.75 \text{ g cm}^{-3} \).

Therefore Conversion factor \( 2.75 \div 16.5 \div 0.7975 = 0.209 \)

Gold price USD 400 per troy oz of 31.10348 g \( \equiv \) USD 12.86 \( \text{g}^{-1} \)

In Table IV attention is drawn to the combined effect of decreasing dilution due to increasing assay and increasing particle size on:

- the sample sizes, which are of the same order
- the extents of influence, which are greatly reduced
- the sample spacings, which remain practical.

The logical approach to high assays is to realize that the answer lies in particle size and the assay itself. The high assay measures not only high value but also high probability and high concentration and therefore low dilution. The low dilution limits the influence of the high assay and hence the tonnage to which it is applicable. Do not cut arbitrarily the high assay value. Cut after investigation of particle size the tonnage it affects.

### Average grade

**Definition**

Average mineral-volumetric grade from first principles may be defined as the proportion of the total volume of valuable mineral to the total volume of payable rock that contains it. The total volume of valuable mineral is obtained by summing the products of the relevant mineral-volumetric grades and their corresponding volumes of influence. The total volume of rock under consideration is obtained by summing the relevant volumes of influence.

### Sampling geometry

Both the above summations involve volumes of influence. This highlights the interdependence of grade and sampling geometry. Evidently the problem of determining a reliable figure for average grade lies not only in the grades themselves but also in the sizes of their parent samples and the positions from which these samples are drawn. This is a significant finding for the valuation of mines for financial investment and blocks of ore for development. Sampling geometry needs as close attention as assay procedure.

### Alternative approaches

Each mineral-volumetric grade has an equivalent metal-mass grade from which it is derived. The determination of average grade may therefore be done either through the mineral-volumetric grades or through the metal-mass grades or through both as a check.

### Statistics

Every grade of either type has a corresponding influence measured either in cubic metres or as tonnage. To accord with the definition of average grade, every individual grade is weighted by its influence. Average grade is therefore a

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**Table IV**

Affect of high assay value on sample size and sample spacing and extent of influence in the country rock

<table>
<thead>
<tr>
<th>Example</th>
<th>Metal-mass assay</th>
<th>( \text{gt}^{-1} \text{ Au} )</th>
<th>8</th>
<th>120</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross value of ore</td>
<td>( \text{USD} \text{ t}^{-1} )</td>
<td>103</td>
<td>1,543</td>
<td>5,144</td>
<td></td>
</tr>
<tr>
<td>Mineral–volumetric grade</td>
<td>( \text{cm}^3 \text{ m}^{-3} )</td>
<td>1.67</td>
<td>25.08</td>
<td>83.6</td>
<td></td>
</tr>
<tr>
<td>Volumetric dilution</td>
<td>( D_0 )</td>
<td>600,000</td>
<td>39,672</td>
<td>11,962</td>
<td></td>
</tr>
<tr>
<td>Volumetric probability</td>
<td>( 1 \text{ in} )</td>
<td>600,000</td>
<td>39,672</td>
<td>11,962</td>
<td></td>
</tr>
<tr>
<td>Largest particle volume–sphere of diameter that is</td>
<td>( \text{mm} )</td>
<td>5.2360 ( \times 10^{-4} )</td>
<td>4.1888 ( \times 10^{-3} )</td>
<td>0.0141</td>
<td></td>
</tr>
<tr>
<td>Mineral particle mass</td>
<td>( \text{g} )</td>
<td>8.6394 ( \times 10^{-3} )</td>
<td>0.0691</td>
<td>0.2333</td>
<td></td>
</tr>
<tr>
<td>Pure gold content</td>
<td>( \text{g} )</td>
<td>6.8899 ( \times 10^{-3} )</td>
<td>0.0551</td>
<td>0.1861</td>
<td></td>
</tr>
<tr>
<td>Gross value of particle</td>
<td>( \text{US cents} )</td>
<td>&lt; 9</td>
<td>70</td>
<td>239</td>
<td></td>
</tr>
<tr>
<td>Sample volume</td>
<td>( \text{cm}^3 )</td>
<td>314</td>
<td>167</td>
<td>169</td>
<td></td>
</tr>
<tr>
<td>Sample mass</td>
<td>( \text{g} )</td>
<td>864</td>
<td>459</td>
<td>465</td>
<td></td>
</tr>
<tr>
<td>Volume of influence</td>
<td>( \text{m}^3 )</td>
<td>188</td>
<td>6.66</td>
<td>2.02</td>
<td></td>
</tr>
<tr>
<td>Tonnage influenced</td>
<td>( \text{t} )</td>
<td>518</td>
<td>18.3</td>
<td>5.56</td>
<td></td>
</tr>
<tr>
<td>Gross value of tonnage</td>
<td>( \text{USD} )</td>
<td>53,290</td>
<td>28,240</td>
<td>28,600</td>
<td></td>
</tr>
<tr>
<td>Sample spacing</td>
<td>( \text{m} )</td>
<td>7.11</td>
<td>2.33</td>
<td>1.57</td>
<td></td>
</tr>
</tbody>
</table>
Mining grade and sampling theory—the logical way to evaluate ore reserves

weighted mean and is obtained directly in this manner. Average grade is a statistical property of a particular body of rock and should be as representative as the samples that give rise to it. The determination of average grade is itself a statistical computation and no further statistical adjustment is called for.

Theory and practice

Engineering practice that is based on theory is stuffed with corrective coefficients and factors of safety. Hydraulics and strength of materials are subjects that come to mind. The theory of sampling should be approached with some caution. One reason is that for small particles their form is assumed to be spherical with their diameters equal to the sieve apertures. This controls the theoretical sample size and the sampling geometry. Clearly it is artificial.

Experience in application of the theory may show that a coefficient of unreality needs to be superimposed. At any given location where a particular combination of valuable mineral and country rock is present, the coefficient should be constant.

Summary

The twentieth century brought technologies and instruments that facilitate exploration for orebodies. By contrast, evaluation of an orebody remains dependent on empirical decisions based on experience to control the sampling. That experience may have been gained in other localities where the geological and mineralogical scenarios were different, leading to the use of faulty methods. In an operating mine sampling for the blocking out of future ore proceeds by rules of thumb established by what has proved satisfactory in the past.

This approach is not satisfying intellectually. The theory presented here seeks to remedy the situation. It is based on data peculiar to the location. It is based on the concept of measuring grade by mineral-volumetric proportion and the concepts that grade, in addition to measuring value also measures, probability and concentration/dilution. Figures for these parameters are obtained from and for the location. From the location also the largest particle of the valuable mineral is captured and its volume measured. This figure is essential to the procedure.

The theory develops formulae for determining sample size, the influence of a single sample and sample spacing. Values for these parameters are then calculated using the data obtained from the locality. The theory and the numerical procedure are extended to provide a guide for the reduction of sample size and a method for handling erratic high assays. Arbitrary acts are abolished. Philosophical reason permeates the procedures.

Appendix

Example of a tin deposit of cassiterite veins

Density of cassiterite 6.85 gcm^{-3}
Mass fraction of tin in cassiterite 0.786
Density of country rock 2.75 gcm^{-3}
Density ratio 6.85 ÷ 2.75 = 2.5
Difficulty index 6.85 × 0.786 ÷ 2.75 = 1.958, say 2.0
Conversion factor 2.75 ÷ 0.786 ÷ 6.85 = 0.51
Metal–mass grade needed for mining to be viable 1.5 wt% Sn ≡ 15000 g t^{-1} Sn
Mineral–volumetric grade 15000 ÷ 2.75 ÷ 6.85 ÷ 0.786 = 7661 cm^3 m^{-3} cassiterite
(For better comparison to the metal-mass grade this may be expressed as, say, 0.766 vol % cassiterite)
Volumetric dilution 10^6 ÷ 7661 = 150.5/ D^0
Information on the volume of the largest particles in these deposits is not available. Volumetric dilution of this order indicates that the volume of the largest particle is likely to be measured as a whole number of cubic centimetres. The range of illustrative volumes in Table V below has been chosen in cognizance of the fact that dressing plants at several deposits of this type employ pre-concentration by dense medium separation with a nominal feed size of 2.5 to 3.0 cm.

Table V

<table>
<thead>
<tr>
<th>Particle volume cm^3</th>
<th>Particle mass g</th>
<th>Sample volume cm^3</th>
<th>Sample mass</th>
<th>Sphere of influence</th>
<th>Sample spacing cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>128</td>
<td>876.8</td>
<td>16 704</td>
<td>45.94 kg</td>
<td>2.18 m^3</td>
<td>161</td>
</tr>
<tr>
<td>64</td>
<td>438.4</td>
<td>8 352</td>
<td>22.97 kg</td>
<td>1.09 m^3</td>
<td>128</td>
</tr>
<tr>
<td>32</td>
<td>219.2</td>
<td>4 176</td>
<td>11.48 kg</td>
<td>0.545 m^3</td>
<td>101</td>
</tr>
<tr>
<td>16</td>
<td>109.6</td>
<td>2 088</td>
<td>5.74 kg</td>
<td>0.272 m^3</td>
<td>80</td>
</tr>
<tr>
<td>8</td>
<td>54.8</td>
<td>1 044</td>
<td>2.87 kg</td>
<td>0.136 m^3</td>
<td>64</td>
</tr>
<tr>
<td>4</td>
<td>27.4</td>
<td>522</td>
<td>1.44 kg</td>
<td>68.1 dm^3</td>
<td>51</td>
</tr>
<tr>
<td>2</td>
<td>13.7</td>
<td>261</td>
<td>718 g</td>
<td>34.1 dm^3</td>
<td>40</td>
</tr>
<tr>
<td>1</td>
<td>6.85</td>
<td>130.5</td>
<td>359 g</td>
<td>70.0 dm^3</td>
<td>32</td>
</tr>
<tr>
<td>0.5</td>
<td>3.425</td>
<td>65.25</td>
<td>179 g</td>
<td>8.5 dm^3</td>
<td>25</td>
</tr>
<tr>
<td>0.25</td>
<td>1.713</td>
<td>32.6</td>
<td>89.7 g</td>
<td>4.26 dm^3</td>
<td>20</td>
</tr>
</tbody>
</table>