Quadratic programming for the multi-variable pre-homogenization and blending problem

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Synopsis

Raw material fed into a processing or refining plant is required to be uniform in composition for several reasons. When the mined ore is highly variable in quality the only way to ensure consistency is to homogenize the ore prior to the process. The problem is more complicated in the case of multiple ore sources. The idea behind the research is that theoretical blending ratios can serve not only to meet predefined specific criteria but also to reduce grade fluctuations of variables under consideration. In this paper, the problem is formulated as a quadratic programming problem, whose objective function is in quadratic form and constraints are linear. A case study was conducted on a data set from an iron orebody to demonstrate the technique. The objective was to minimize the blend variability in terms of each variable (in this study, iron, silica, alumina and lime) grade of ores extracted in three different production faces. The stockpile capacity, lower and upper limits of each variable satisfying operational requirements and non-negativity were constrained to the model. A modified simplex method developed by Wolfe was used for solving the blending and homogenization problem. The promising results can be used as a part of the stacking and reclaiming design.

Keywords: blending and homogenization problem, quadratic programming, Wolfe method.

Introduction

Many industrial operations (e.g., cement, iron-steel and coal fired power generation) that consume mined products require blending and homogenization of the raw material prior to the industrial process. The process efficiency, product quality or environmental compliance depend on the consistency of characteristics of raw material fed into the plant. The homogenization means the minimization of the standard deviation of ore grades fed into the process over time. In mathematical parlance, the homogenization refers to the precision with which consistency is expressed in a set of measurements, regardless of whether the component grades conform to the desired blending specifications. Unless appropriate blending rates are determined from the different sources or production faces, there is no guarantee that the blending specifications will be met. To put it another way, homogenization only reduces grade fluctuations.

On the other hand, the purpose of blending is to prevent, or minimize, the deviation of known mean from predefined grade requirements of the variable under consideration. The blending requirements are dictated by the potential operational, qualitative or environmental reasons in the market and end-user industry. The blending specifications are directly related to the plant operational efficiency, the product quality or environmental regulations. The blending refers to the accuracy with which the deviation from the blending specification is measured. The blending and homogenization problem is especially sophisticated when the ore is extracted from multiple production faces, seams or different orebodies with different qualitative features.

In practice a high- or a low-grade raw material is added at the end of a stockpile so as to meet the predefined grade criteria. However, in the case of multiple variables this will be very difficult. In this research, the blending and homogenization problem is solved using quadratic programming (QP). The problem is to minimize the variances of variable grades of ore fed into the processing or refining unit such that predefined blending specifications are satisfied. Theoretical blending ratios obtained by the QP will help in in designing a bed-blending operation that embodies two stages: stacking and reclaiming. A stacking machine that layers ore in the pile constructs the stockpile. Reclamation is achieved by slicing across the pile orthogonal to the direction of layering[13,8–10]. The stacking process regulates flow of ore fed into the stockpile on the basis of ratios determined by the QP. Dowd developed a technique to infer the output variance from a semi-variogram model of the stockpile input when the stockpile

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geometry is known[3]. Given that the QP achieves input variance minimization, a cheap reclamation process will be adequate. To sum up, the blending and homogenization optimization by the QP can lead to a simple and cheap bed-blending operation.

Method

A non-linear programming problem (NLP), whose objective is quadratic and constraints are linear, is a QP problem:

$$max\, f(x) = \sum_{j=1}^{n} c_j x_j + \sum_{j=1}^{n}\sum_{k=1}^{n} c_{jk} x_j x_k \qquad [1]$$

Subject to:

$$\sum_{j=1}^{n} a_{ij} x_j \le b^* \qquad [2]$$

$$x_j \ge 0 \quad j = 1,...,n \qquad [3]$$

where a_{ij}, b^*, c_j and c_{jk} are any real numbers. The objective function includes all possible quadratic terms, x_j^2 and $x_j x_k$, for all j and k. The QP method is a constrained non-linear programming method representing a transition from the linear programming (LP) to the non-linear programming (NLP). However, an optimal solution in the LP is either an extreme point of the set of all feasible solutions or a convex combination of such extreme points. On the other hand, an optimal solution to the QP can be an interior point or a boundary point[6,11].

Many algorithms have been prescribed for solution of the QP problems. One of the ways of solving the problem described above is by a general method known as Wolfe's method[16,14]. Note that the QP problem is different from the LP only in that the objective function is quadratic. A QP problem can be transformed into a linear programming problem using the Kuhn-Tucker condition (K-T). The K-T method resembles the Lagrangean Multipliers (LM) method. The only difference between them is in the constraints: the Lagrangean Multipliers use equality constraints, the K-T uses inequality constraints. The objective function should be a convex function so that the absolute minimum of a constrained minimization problem is obtained from the solution of the K-T conditions. Otherwise, it is not certain that an absolute minimum solution will be found because the K-T conditions are necessary but not sufficient. The constraints should be extended so as to satisfy the Lagrangean stationary condition. One way to formulate such an extension is to assign a Lagrange variable, λ, to every constraint. This provide a means of modifying the simplex method for solving a problem expressed in the QP form[15].

Assume a minimization problem:

$$Min\ f(x_1,x_2,...,x_n) \qquad [4]$$

$$\begin{aligned} \text{Subject to}\quad & g_1(x_1,x_2,...,x_n) \le b_1 \\ & g_2(x_1,x_2,...,x_n) \le b_2 \\ & \vdots \qquad\qquad \vdots \\ & g_k(x_1,x_2,...,x_n) \le b_k \end{aligned} \qquad [5]$$

If $\bar{x} = (\bar{x}_1, \bar{x}_2,..., \bar{x}_n)$ is an optimal solution to the problem, this solution must satisfy the k constraints and there must be multipliers $\bar{\lambda}_1,\bar{\lambda}_2,...,\bar{\lambda}_k$ such that

$$\frac{\partial f(\bar{x})}{\partial x_j} + \sum_{i=1}^{k} \bar{\lambda}_i \frac{\partial g_i(\bar{x})}{\partial x_j} = 0 \quad (j = 1,2,...,n) \qquad [6]$$

$$\bar{\lambda}_i \left[b_i - g_i(\bar{x}) \right] = 0 \quad (i = 1,2,...,k) \qquad [7]$$

$$\left[\frac{\partial f(\bar{x})}{\partial x_j} + \sum_{i=1}^{k} \bar{\lambda}_i \frac{\partial g_i(\bar{x})}{\partial x_j} \right] \bar{x}_j = 0 \quad (j = 1,2,...,n) \qquad [8]$$

$$\bar{\lambda}_i \ge 0 \quad (i = 1,2,...,k) \qquad [9]$$

$$\bar{x}_i \ge 0 \quad (j = 1,2,...,n) \qquad [10]$$

Where:

- $f(x)$ is the objective function
- $g(x)$ is the constraints
- n is the number of decision variables (i.e. production faces)
- k is the number of constraints

Solutions $\bar{x}$ and $\bar{\lambda}$ satisfy the K-T conditions for differentiable nonlinear programs if they fulfil complementary slackness conditions, sign restrictions, gradient equation and primal constraints.

Generalized model of the problem

The general model of the multiple variable homogenization and blending problem is given as:

Objective Function

$$\text{Minimize} \left[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{m}\sum_{l=1}^{m} x_i x_j \left(cov\, S_{kl} \right)_{ij} \right] \qquad [11]$$

Subject to:

$$\sum_{i=1}^{n} x_i E(S_{i1}) \le F_{U1}^* \qquad [12]$$

$$-\sum_{i=1}^{n} x_i E(S_{i1}) \le -F_{L1}^* \qquad [13]$$

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$$\sum_{i=1}^{n} x_i E(S_{im}) \le F_{Um}^* \qquad [14]$$

$$-\sum_{i=1}^{n} x_i E(S_{im}) \le -F_{Lm}^* \qquad [15]$$

$$\sum_{i=1}^{n} x_i = 1 \qquad [16]$$

$$x_1, x_2, ..., x_n \ge 0 \qquad [17]$$

Where:

- n is the number of production faces
- m is the number of variables (e.g. iron, silica, alumina and lime for an iron ore)

x_i is blending ratio of production face i

$(cov\ S_{kl})_{ij}$ is covariance between variable k at production face i and variable l at production face j. Note that $Var(S) = Cov(S)$ if $i=j$ and $k=l$ where Var is variance.

$E(S_{ij})$ is expected value of variable j at production face i

F^*_{Um} and F^*_{Lm} are upper and lower limits for variable m

The objective function minimizes the variances of ores from different production faces. The constraints 12–15 guarantee blending specifications.

Calculation of means, variances and covariances

Time series could be used to quantify means and covariances[12,1]. Everett forecasted grades of input ore lots through simple exponential smoothing[5]. Gerstel and Werner used an auto-correlation function that was equivalent to the covariance function in geostatistical structural analysis[7]. Then the grades of ore lots were forecasted by a negative exponential model. This method suffered from the assumption of normal distribution of grades of input lots.

In this paper, the variability of the grades over time in an ore stream is estimated from the semi-variogram model:

$$\gamma(h) = \frac{1}{2N(h)} \sum_{(i,j)|h_{ij} \approx h} (v_i - v_j)^2 \qquad [18]$$

where h is separation vector, N is the number of data pairs separated by h and v's are data values over the $N(h)$. The dispersion variance may thus be regarded as a type of variogram calculation in which pairs of values are accepted in the averaging procedure as long as the separation vector h_{ij} is within the ore stream, S:

$$\sigma^2(o/D) = \frac{1}{2N(S)} \sum_{(i,j)|h_{ij} \in S} (v_i - v_j)^2 \qquad [19]$$

where s^2 (o/D) is the variance of the average value of the attribute (e.g. grade) of sampling sizes within the total deliverable tonnage. Although this could be estimated from a set of sample data, it is usually derived from a semi-variogram model:

$$\bar{\sigma}^2(o/D) \approx \bar{\gamma}(S) \qquad [20]$$

where the right-hand side refers to the semi-variogram model $\bar{\lambda}$ (S) averaged over all possible vectors within S. When a point support semi-variogram is available, the variance calculation is very simple. The average of the structural part of the semi-variogram in a flow can be reduced to a simple equation[4]. In practice, the ore stream is subdivided into n discrete time intervals and the average semi-variogram values can be calculated by approximation of the exhaustive average of the semi-variogram by an average of the n^2 semi-variogram values at the n discrete time intervals:

$$\bar{\sigma}^2(o/D) \approx \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \bar{\gamma}(h_{ij}) \qquad [21]$$

Similarly, the covariances between the variable grades are estimated from the cross-variogram model.

Sequential Gaussian simulation is used to reproduce the characteristics, or behaviour, of the phenomenon as observed in the available data[2]. The ore stream is simulated by weight increments through the sequential Gaussian co-simulation. For each variable a global mean was calculated by averaging the simulated values. Sequential Gaussian co-simulation is illustrated in Figure 1.

Note that unless there is a zero nugget variance cokriging will assign almost all of the weights to the central sample and the immediate neighbour on either side of it. The most important advantage of stochastic simulation over the other approaches is to overcome irregular data and to simulate a different scale of time intervals. Geostatistical simulation can also be used to quantify risk in the interpretation of the available data. It is important to note that the input and output support size should be identical. Otherwise, the procedure will become meaningless.

Case study

A case study was conducted on an iron orebody with three production faces to demonstrate the algorithm developed in this research. Four variables, Fe, Al_2O_3, SiO_2 and CaO, were considered at each production face because of the fundamental quality requirement of the iron market. Initial data were collected from input material entering the stockpile from different production faces on a band conveyor and assayed for Fe, SiO_2, Al_2O_3 and CaO. One realization of each ore stream was generated using sequential Gaussian co-simulation.

Table I shows variance-covariance values for each variable at each production face. These values were calculated from the semi- and cross-variogram of simulated values. The acceptable lower and upper limits for each variable are given in Table II. Means of ore variables under consideration in each source are given in Table III. Global means were calculated by averaging the simulated values.

A computer program was written in FORTRAN. The following input parameters were used for the case study:

- The number of production faces (n)
- The number of variables (m)
- The number of constraints (k)
- $n.m * n.m$ coefficient matrix in the objective function from variance-covariance values (Table I)
- k vector of constraints limits (Table II)
- $n * k$ coefficients matrix of variable means (Table III)

The objective function and constraints are presented as follows:

Objective Function

$$\text{Minimize } S_{min} = var\left(\sum_{i=1}^{m} \sum_{j=1}^{n} x_j Z_{ij} \right) \qquad [22]$$

since [23]

$$E(kZ_i) = kE(Z_i)$$
$$var(kZ_i) = k^2 var(Z_i) \qquad [24]$$
$$cov(aZ_i, bZ_j) = ab\ cov(Z_i, Z_j)$$

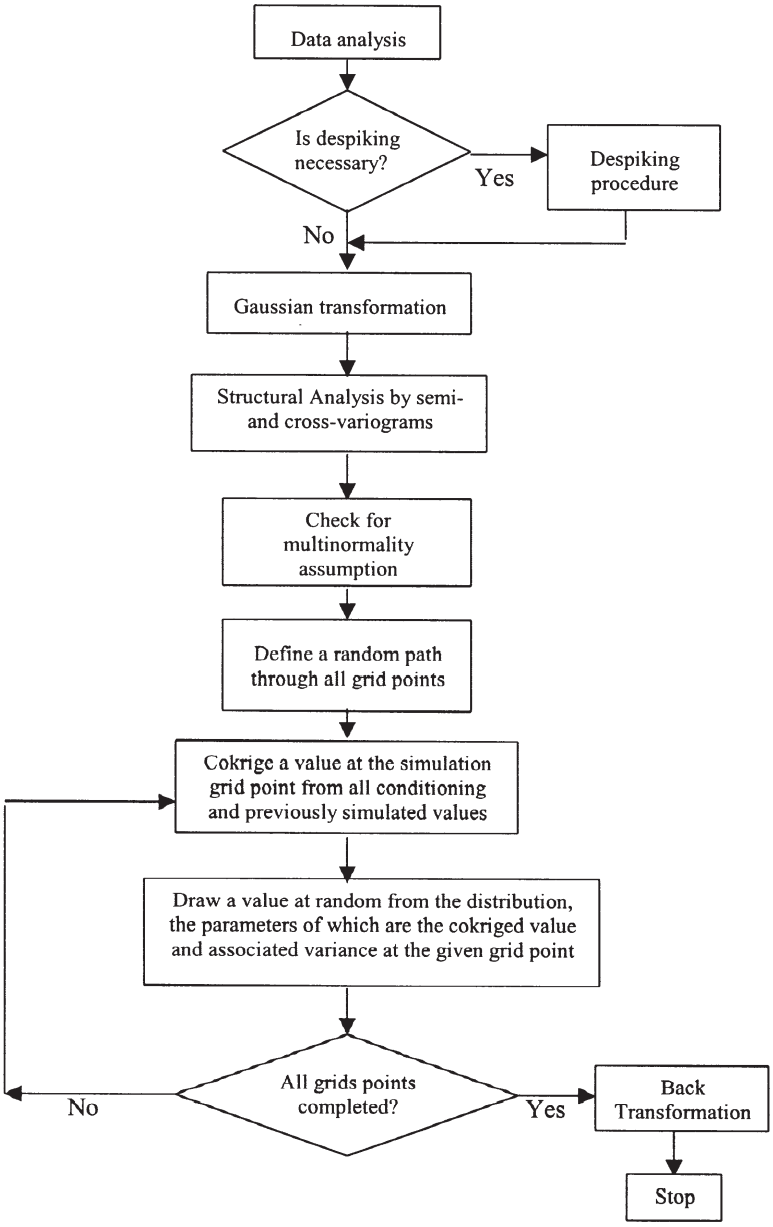

Figure 1—Sequential Gaussian co-simulation

Table I

Variance-covariance values

		Production Face 1				Production Face 2				Production Face 3			
		Fe	Al_2O_3	SiO_2	CaO	Fe	Al_2O_3	SiO_2	CaO	Fe	Al_2O_3	SiO_2	CaO
Production Face 1	Fe	197.93	4.94	-237.82	17.77	-15.07	6.10	16.88	-4.32	-0.61	-0.99	4.36	2.33
	Al_2O_3	4.94	5.61	-13.83	5.65	0.10	0.3532	-0.88	3.43	-1.06	0.41	0.144	0.56
	SiO_2	-237.82	-13.83	360.64	-11.34	18.52	-8.73	-19.16	0.34	7.62	1.35	-12.93	1.34
	CaO	17.77	5.65	-11.34	63.45	1.21	-0.43	1.25	3.55	2.43	-0.44	2.13	5.74
Production Face 2	Fe	-15.07	0.10	18.52	1.21	107.74	-11.46	-96.10	6.56	-0.70	2.63	-0.88	0.98
	Al_2O_3	6.10	0.3532	-8.73	-0.43	-11.46	7.344	1.139	0.57	-11.46	-0.70	-0.25	-0.08
	SiO_2	16.88	-0.88	-19.16	1.25	-96.10	1.139	128.14	-2.35	-0.90	-1.75	5.05	1.11
	CaO	-4.32	3.43	0.34	3.55	6.56	0.57	-2.35	110.23	2.31	0.46	-1.23	4.36
Production Face 3	Fe	-0.61	-1.06	7.62	2.43	-0.70	-11.46	-0.90	2.31	74.99	-6.73	-62.00	3.78
	Al_2O_3	-0.99	0.41	1.35	-0.44	2.63	-0.70	-1.75	0.46	-6.7	5.56	3.76	0.79
	SiO_2	4.36	0.144	-12.93	2.13	-0.88	-0.25	5.05	-1.23	-62.00	3.76	114.06	-0.67
	CaO	2.33	0.56	1.34	5.74	0.98	-0.08	1.11	4.36	3.78	0.79	-0.67	88.97

Minimise

$$x_1^2\,var\,F_1 + 2x_1^2\,cov(F_1,A_1) + 2x_1^2\,cov(F_1,S_1) + 2x_1^2\,cov(F_1,C_1) + x_1^2\,var\,A_1 +$$
$$2x_1^2\,cov(A_1,S_1) + 2x_1^2\,cov(A_1,C_1) + x_1^2\,var\,S_1 + 2x_1^2\,cov(S_1,C_1) + x_1^2\,var\,C_1 +$$
$$2x_1x_2\,cov(F_1,F_2) + 2x_1x_2\,cov(F_1,A_2) + 2x_1x_2\,cov(F_1,S_2) + 2x_1x_2\,cov(F_1,C_2) +$$
$$2x_1x_2\,cov(A_1,F_2) + 2x_1x_2\,cov(A_1,A_2) + 2x_1x_2\,cov(A_1,S_2) + 2x_1x_2\,cov(A_1,C_2) +$$
$$2x_1x_2\,cov(S_1,F_2) + 2x_1x_2\,cov(S_1,A_2) + 2x_1x_2\,cov(S_1,S_2) + 2x_1x_2\,cov(S_1,C_2) +$$
$$2x_1x_2\,cov(C_1,F_2) + 2x_1x_2\,cov(C_1,A_2) + 2x_1x_2\,cov(C_1,S_2) + 2x_1x_2\,cov(C_1,C_2) +$$
$$2x_1x_3\,cov(F_1,F_3) + 2x_1x_3\,cov(F_1,A_3) + 2x_1x_3\,cov(F_1,S_3) + 2x_1x_3\,cov(F_1,C_3) +$$
$$2x_1x_3\,cov(A_1,F_3) + 2x_1x_3\,cov(A_1,A_3) + 2x_1x_3\,cov(A_1,S_3) + 2x_1x_3\,cov(A_1,C_3) +$$
$$2x_1x_3\,cov(S_1,F_3) + 2x_1x_3\,cov(S_1,A_3) + 2x_1x_3\,cov(S_1,S_3) + 2x_1x_3\,cov(S_1,C_3) +$$
$$2x_1x_3\,cov(C_1,F_3) + 2x_1x_3\,cov(C_1,A_3) + 2x_1x_3\,cov(C_1,S_3) + 2x_1x_3\,cov(C_1,C_3) +$$
$$x_2^2\,var\,F_2 + 2x_2^2\,cov(F_2,A_2) + 2x_2^2\,cov(F_2,S_2) + 2x_2^2\,cov(F_2,C_2) + x_2^2\,var\,A_2 +$$
$$2x_2^2\,cov(A_2,S_2) + 2x_2^2\,cov(A_2,C_2) + x_2^2\,var\,S_2 + 2x_2^2\,cov(S_2,C_2) + x_2^2\,var\,C_2 +$$
$$2x_2x_3\,cov(F_2,F_3) + 2x_2x_3\,cov(F_2,A_3) + 2x_2x_3\,cov(F_2,S_3) + 2x_2x_3\,cov(F_2,C_3) +$$
$$2x_2x_3\,cov(A_2,F_3) + 2x_2x_3\,cov(A_2,A_3) + 2x_2x_3\,cov(A_2,S_3) + 2x_2x_3\,cov(A_2,C_3) +$$
$$2x_2x_3\,cov(S_2,F_3) + 2x_2x_3\,cov(S_2,A_3) + 2x_2x_3\,cov(S_2,S_3) + 2x_2x_3\,cov(S_2,C_3) +$$
$$2x_2x_3\,cov(C_2,F_3) + 2x_2x_3\,cov(C_2,A_3) + 2x_2x_3\,cov(C_2,S_3) + 2x_2x_3\,cov(C_2,C_3) +$$
$$x_3^2\,var\,F_3 + 2x_3^2\,cov(F_3,A_3) + 2x_3^2\,cov(F_3,S_3) + 2x_3^2\,cov(F_3,C_3) + x_3^2\,var\,A_3 +$$
$$2x_3^2\,cov(A_3,S_3) + 2x_3^2\,cov(A_3,C_3) + x_3^2\,var\,S_3 + 2x_3^2\,cov(S_3,C_3) + x_3^2\,var\,C_3 \qquad [25]$$

Table II
Blending specifications

	Iron	Silica	Alumina	Lime
Lower limit	55.75 %	5.50 %	3.50 %	0.60%
Upper limit	57.30 %	5.85 %	3.75 %	0.75%

Table III
Means of variables in production faces

Production Face	Iron	Silica	Alumina	Lime
1	51.21	5.32	2.95	0.49
2	58.71	5.83	4.48	0.81
3	54.23	6.54	2.25	0.92

Subject to:

$$1.\ x_1 + x_2 + x_3 = 1 \qquad [26]$$

$$2.\ x_1E(F_1) + x_2E(F_2) + x_3E(F_3) \le F_U^* \qquad [27]$$

$$3.\ \pm x_1E(F_1) - x_2E(F_2) - x_3E(F_3) \le -F_L^* \qquad [28]$$

$$4.\ x_1E(A_1) + x_2E(A_2) + x_3E(A_3) \le A_U^* \qquad [29]$$

$$5.\ -x_1E(A_1) - x_2E(A_2) - x_3E(A_3) \le -A_L^* \qquad [30]$$

$$6.\ x_1E(S_1) + x_2E(S_2) + x_3E(S_3) \le S_U^* \qquad [31]$$

$$7.\ \pm x_1E(S_1) - x_2E(S_2) - x_3E(S_3) \le -S_L^* \qquad [32]$$

$$8.\ x_1E(C_1) + x_2E(C_2) + x_3E(C_3) \le C_U^* \qquad [33]$$

$$9.\ \pm x_1E(C_1) - x_2E(C_2) - x_3E(C_3) \le -C_L^* \qquad [34]$$

$$10.\ x_1, x_2, x_3 \ge 0 \qquad [35]$$

where:

x_i is the blending ratio from production face i

$var\,Z_i$ is the variance of variable Z at production face i where Z can denote F, A, S or C variables

$cov(Z_i, Z_j)$ is the covariance between variable Z's at production face i and j where Z can denote F, A, S or C variables

$E(Z_i)$ is the expected value of variable Z at production face i where Z can denote F, A, S or C variables

F_L^* and F_U^* are the lower and upper blending specifications for iron

A_L^* and A_U^* are the lower and upper blending specifications for alumina

S_L^* and S_U^* are the lower and upper blending specifications for silica

C_L^* and C_U^* are the lower and upper blending specifications for lime

F, A, S and C denote iron, alumina, silica and lime respectively

The variance-covariance values (Table I) were substituted in Equation [25]. Similarly, blending specifications (Table II) and the mean of variable under consideration (Table III) at each production face were substituted in the constraints [2–9]. The algorithm checked whether the objective function was convex or not. Since the quadratic form and its symmetric matrix were positive definite, the function was convex. Using the Kuhn—Tucker condition, the problem was transformed in such a way that it could be solved with linear programming. The execution of the program yielded the following blending ratio and joint variance:

$x_1 = 0.2618$
$x_2 = 0.5219$
$x_3 = 0.2163$
$S_{min} = 61.3377$

The obtained means and variances of each variable in the stockpile input are given in Table IV. Note that these variances can be reduced further by the bed-blending operation.

Table IV

Means and variances of variables in the stockpile input

	Iron	Silica	Alumina	Lime
Mean	55.75	5.85	3.59	0.75
Variance	42.075	59.397	2.629	41.140

Conclusions

The research focused on use of the QP to solve the mineral homogenization and blending problem in the case of multiple production faces and variables. The case study yielded promising results. The method can both minimize variances of ore grades of the variables under consideration and satisfy predefined grade specifications simultaneously. The blending rates determined by the QP provide useful information for the stacking and reclaiming process design. Given that bed-blending is a very expensive process, the determination of blending rates serves to reduce the design costs. In addition, this alleviates environmental concerns, and increases operational efficiency and product quality. Use of stochastic simulation provides a means of assessing risk. Many alternate models are generated and processed to construct a distribution of possible values for specified attributes. This distribution is used to assess the risk associated with the random nature of the variables.

References

1. Cheng, B.H., Woodcock, B., Sargent, D., and Gleit, A. Time series analysis of coal data from preparation plants, *Journal of Air Pollution*, 1982, 32/11, pp. 1137–1141.
2. Deutsch, C.V. and Journel, A. GSLIB: Geostatistical Software Library and User's Guide, Oxford University Press, Oxford, 1997.
3. Dowd, P.A. The design of a rock homogenising stockpile, *Mineral Processing in the UK*, IMM, London, 1989, pp. 63–82.
4. Dowd, P.A. Basic Geostatistics' Course notes for the MSc. in Mineral Resources and Environmental Geostatistics, University of Leeds, UK, 1992.
5. Everett, J.E. Simulation to reduce variability in iron ore stockpiles, *Mathematics and Computer in Simulation*, 1997, vol. 43, pp. 563–568.
6. Frank, M. and Wolfe, P. An algorithm for quadratic programmin, *Naval Research Logistics Quarterly*, 1956, vo. 3, pp. 95–110.
7. Gerstel, A.W. and Werner, J.W. Computer simulation program for blending piles, *Bulk Solids Handling*, 1996, vol. 16, pp. 49–58.
8. Gy, P. A new theory of bed-blending derived from the theory of sampling —development and full-scale experimental check, *International Journal of Mineral Processing*, 1981, vol. 8, pp. 201–238.
9. Gy, P. Optimizing the operational strategy of a mine-metallurgy or quarry-cement works complex, *Canadian Metallurgical Quarterly*, 1999, vol. 38, pp. 3157–3163.
10. Osborne, D.G. Blending and Homogenization, *Coal Preparation Technology*, Graham and Trotman Pub., 1989.
11. Rardin, R.L. Optimization in Operations Research, Prentice-Hall Int. Inc., New Jersey, 1998.
12. Schofield, C.G. *Homogenisation/Blending System Design and Control for Mineral Processing*, Trans Tech Pub., 1980.
13. Serra, J., Huijbregets, Ch., and Ivanier, L. Laws of linear homogenization in ore stockyards, *11. International Mineral Processing Congress*, Cagliari, 1975, pp. 263–291.
14. Simmons. D.M. *Nonlinear Programming for Operations Research*, Prentice-Hall Inc., New Jersey, 1975.
15. Wismer, D and Chattergy, R. *Introduction to Nonlinear Optimization: A* Problem Solving Approach, Elsevier North-Holland, New York, 1978.
16. Wolfe, P. The simplex method for quadratic programming, *Econometrica*, 1959, 27, 382-398. ◆