Support pressure estimation for circular and non-circular openings based on a parametric numerical modelling study

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Synopsis

Closed form solutions assume opening shape to be circular to simplify the calculation of the ground response curve for the design of underground openings. However, in mining and civil engineering works, non-circular openings are commonly excavated. Therefore, there is a need to estimate the ground response curve for such openings. In this respect, a simple model for circular, arch and rectangular shape underground openings subject to a hydrostatic stress field is the aim of this study. A comprehensive numerical modelling, using the finite difference method, based on numerical modelling code FLAC version 4.0, was carried out. In addition to shape of an opening, the effect of rock mass strength, opening size, *in situ* stress on strain (tunnel deformation to tunnel radius) of the opening in response to applied support pressures was investigated parametrically by developing models for each specific condition. The strain results for each excavation shape obtained from this large number of models against the normalized strength of rock mass and internal support pressure with *in situ* stress magnitude were evaluated by multiple regression analysis. As a result, a model that was well correlated to data was developed for an overall estimate of the ground response curve depending on relevant effecting parameters. The regression coefficients of the model were given for estimating both roof and sidewall response curve of each excavation shape. The model was verified by statistical tests and by an analytical solution. Diagrams were also established to highlight the influence of relevant parameters and to estimate the support pressure for a stable opening staying within the acceptable limits of strain. Strains occurring in a rectangular opening are found to be larger than those in the arch and circular openings for the other fixed parameters. In the mean time, larger strains occur around an arch shape opening than a circular opening.

Introduction

Underground structures are subject to a primary state of stress with no deformation before any excavation is carried out. This natural stress state is disturbed by an excavation and so deformation of the rock starts to occur ahead of the opening face. With the advance of the opening, deformation of the rock around the opening gradually increases with decreasing face restraint behind the face. Support pressure provided by the face allows enough time for installing the support. In addition to face restraint, deformation of the opening depends mainly on the properties of rock mass, stress state, dimensions and shape of the opening. All these factors could be incorporated into a simple curve showing the relation between deformation of the opening and support pressure for a particular problem. This curve is commonly known as a ground response curve and could be used for preliminary estimation of necessary support pressure to stabilize an opening at an acceptable deformation level.

Researchers to calculating the ground response curve have presented a wide range of solutions. A detailed review of these solutions proposed in the literature can be found in Brown *et al.* (1983) and Alonso *et al.* (2003) references. Solutions are generally based upon a two-dimensional plane strain model with a circular shape opening excavated in homogenous, isotropic and elasto-plastic material subject to a hydrostatic stress field. Differences mainly arising in these solutions are how they treat elasto-plastic material behaviour in the post-failure region and the yielding behaviour, which is generally assumed according to Mohr-Coulomb failure criterion (Brady and Brown, 1993 ; Hoek *et al.*, 1995) or Hoek-Brown failure criterion (Hoek and Brown, 1980; Brown *et al.*, 1983; Wang, 1996; Carranza-Torres and Fairhurst, 1999). After yielding, the behaviour of rock material is assumed to be perfectly plastic (Hoek *et al.*, 1995; Carranza-Torres and Fairhurst, 1999), perfectly brittle (Hoek and Brown, 1980; Brown *et al.*, 1983; Brady and Brown, 1993; Wang, 1996) or strain softening (Brown *et al.*, 1983; Duncan Fama, 1993; Brady and Brown, 1993; Alonso *et al.*, 2003). Compared to other work, Hoek (2000) and Hoek and Marinos (2000) proposed more simple ground response curve estimation models for circular shape openings under a

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 SA ISSN 0038–223X/3.00 + 0.00. Paper received Jan. 2005; revised paper received Jun. 2005.

uniform stress field. These models are statistically derived from the results of parametric studies. Hoek (2000) investigated the effect of parameters on the strain state of a circular opening by means of a closed form analytical solution, while Hoek and Marinos (2000) investigated it by means of a finite element model. Asef *et al.* (2000) modelled a circular tunnel, 10 m in diameter, subject to hydrostatic stress conditions using the finite difference code 'FLAC' and proposed a ground response curve prediction chart for RMR values ranging from 20 to 80 for a specific problem.

Opening cross-sections are commonly non-circular in practice, for example, mining galleries in Turkey, which are mostly either arch or rectangular in shape with different cross-sectional areas depending on their purpose (Biron and Arioglu, 1983). As highlighted before, closed form analytical solutions, including the numerical modelling studies carried out for determining the ground response curve, assume the opening shape to be circular.

To estimate the ground response curve for a non-circular opening from the present closed form solutions, Curran *et al.* (2003) propose a method of predicting support pressure by approximating the non-circular cross-section with an equivalent circular tunnel of the same cross-sectional area. Although this method could be used for an overall preliminary prediction of support measure, its application to openings deviating greatly from a circular shape could result in an erroneous prediction (Curran *et al.*, 2003).

The aim of this study is to develop a simple ground response curve estimation model for circular, horseshoe and rectangular shape openings under a hydrostatic stress field. For this purpose, a large number of numerical models investigating the effect of relevant parameters on the strain state of the opening were carried out. Strain data obtained from these parametric models were then evaluated for each opening shape and two different rock mass classes by means of multiple regression analysis.

Method of investigation

The effect of the opening shape, together with the other parameters influencing the deformation response of the opening, was investigated through a parametric numerical modelling program. The reason behind the selection of numerical modelling was its advantages over empirical design methods or closed form solutions due to its capacity to explicitly simulate any aspect of the problem (Coulthard, 1999). All the computations were carried out by the finite difference based two dimensional continuum modelling program FLAC version 4.0 (Fast Lagrangian Analysis of Continua) (ITASCA, 2000).

This code is based on an explicit solution technique, in which the evolution of a system is computed by means of a time-stepping numerical integration of Newton's equations of motion for grid points within the model (ITASCA, 2000). A key advantage of this approach is that the underlying equations for the mechanics are not based on an initial assumption that the system is in equilibrium, as in implicit methods (ITASCA, 2000). The numerical solution is thus able to follow the development of a failure and even the ultimate collapse in a system. In cases of quasi-static loading, damping is added to allow the numerical solution to come smoothly to equilibrium or to a steady state of failure (ITASCA, 2000; Coulthard, 1999).

Models were used to perform a parametric study, providing insight into possible range of strain responses of an opening, given the likely ranges for the various parameters. The tunnel strain data produced from the models were then evaluated by multiple regression analysis to establish a ground response curve estimation model for circular and non-circular openings surrounded by a weak quality rock mass.

Modelling parameters

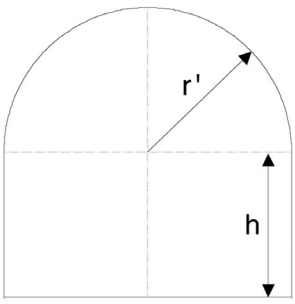
In the determination of ground response curve for the preliminary design of underground openings based on rock-support interaction analysis, models should take geometry and dimensions of the opening, initial stress field and properties of the rock mass into account.

Geometry and dimensions of modelled openings

Three different opening shapes, commonly practised in mining and civil engineering underground constructions, were modelled. These are circular, horseshoe and rectangular in shape. The diameter of the circular openings modelled ranged from 2 to 10 metres in 2-m intervals. In studying the dimensions of the horseshoe and rectangular shape openings, commonly practised dimensions of mining galleries serving the extraction of ore reserves in Turkey were considered. The idealized excavation geometry and dimensions of horseshoe shaped openings for the models, coded according to their cross-sections as B-5, B-8, B-10, B-14 and B-18 (Biron and Arioglu, 1983), are given in Figure 1, while the rectangular shape opening modelled has commonly practised dimensions, 4.8 m in width and 2.4 m in height.

Initial stress field

Two main *in situ* stress parameters, which are vertical stress and stress ratio (horizontal stress / vertical stress), should be considered in parametric modelling. In the current work, the number of stress parameters considered assumes excavating a tunnel in a hydrostatic stress field. The far-field stress magnitude applied to the boundary of each model was in the range 5 to 30 MPa in 5 MPa intervals. These are equivalent to depths ranging from 185 m to 1110 m for an average unit weight 0.027 MN/m^3 of the rock mass overlying the excavation. For shallower depths with weak rock mass conditions, rock support interaction analysis may not be the correct method due to the possible extent of the failure zone up to the surface. For these cases, rock load calculation could give better results.



Code	r' (m)	h (m
B-5	1.3	1.0
B-8	1.7	1.2
B-10	2.0	1.2
B-14	2.5	1.3
B-18	2.9	1.3

Figure 1—Dimensions of horseshoe cross-section openings modelled

Rock properties

In analysing the tunnel deformation under different conditions, estimating the properties of rock mass is very important. For systematic data production, the most widely accepted system proposed by Hoek and Brown was used (Hoek and Brown, 1997). The general form of the failure criterion for the determination of rock mass properties is given by the equation:

$$\sigma_1 = \sigma_3 + \sigma_c\left(m_b \frac{\sigma_3}{\sigma_c} + s\right)^a \qquad [1]$$

Where σ_1 and σ_3 are maximum and minimum principle stresses, σ_c is the uniaxial compressive strength of intact rock, m_b is the rock mass material constant determined from material constant m_i for intact rock and Geological Strength Index (GSI), s and a are constants which depend upon the characteristics of the rock mass.

The deformation behaviour of weak rock masses surrounding the opening are of interest here due to the stability problems encountered mostly in excavations surrounded by these type of rock masses. Lower bound quality ratings for poor and fair quality rock masses with GSI values of 23 and 44 (Hoek and Brown, 1988), respectively, were adopted in the calculation of rock mass parameters from the Hoek-Brown failure criterion. The rock mass was treated as a homogenous, isotropic, elastic-perfectly plastic material yielding according to the Mohr-Coulomb failure criterion with no volume change. This assumption could be made for relatively weak rock masses (Hoek and Brown, 1997) due to its relative simplicity and the rather small number of easily interpretable parameters appearing in it (Anagnostou and Kovari, 1993).

In the determination of rock mass properties, the GSI, uniaxial compressive strength of intact rock (σ_c) and material constant (m_i) of intact rock are essential parameters to be determined. According to intact rock classification, the uniaxial compressive strength of intact rocks stays within the range of 25 to 5 MPa for weak rock and 50 to 25 MPa for medium strong rocks (Hoek and Brown, 1997). The material constant m_i changes from 7 to 25 depending on the type and characteristics of intact rock (Hoek and Brown, 1988). These parameters were incorporated into the generation of rock mass data in a way that it was possible to evaluate the effect of rock mass strength from worst to best condition on the deformation of the tunnel (see Table I). The calculation of equivalent Mohr-Coulomb strength parameters from the Hoek-Brown failure criterion was based on the procedure suggested by Hoek and Brown (1997) and the cohesion (c) and internal friction angle (ϕ) values found in this way are given in Table I. A Poisson ratio of 0.25 was considered representative for most rocks, while the deformation modulus of rock mass (E_m) was calculated from the following equation (Hoek and Brown, 1997):

$$E_m = \sqrt{\frac{\sigma_c}{100}}\,10^{(GSI-10/40)} \qquad [2]$$

The global strength of rock mass (σ_m) for each data set, determined by applying the following well-known transformation yielded a data range for the modelling of 9.6 MPa to 0.5 MPa (Table I).

$$\sigma_m = \frac{2c\cos\phi}{1-\sin\phi} \qquad [3]$$

Model specification

A half section of openings was modelled due to symmetry. It was assumed in models that rock properties did not change in longitudinal direction and a two-dimensional plane strain condition with no strain occurring perpendicular to the model plane was applicable. This assumption is adequate for ground response curve determination in this study (Hoek and Marinos, 2000; Hoek, 2001; Asef, 2000). However, 3-dimensional modelling is necessary to determine the ground deformations that occur before the support installation in order to establish the reaction curve of the support in rock-support interaction analysis (Carranza-Torres and Fairhurst, 1999; Curran *et al.*, 2003). Boundary conditions and generated mesh were the same for all opening shapes and are shown in Figure 2 for a horseshoe shape opening. The number of zones was 10 032 in all models. The aspect ratio of the zones was kept close to unity, with increasing size away from the boundary of the opening. The initial stress field was assumed to be hydrostatic and uniform. At the far-field boundaries, the applied stresses are equal to the initial stress (p_o) and kept constant during calculations for each model. A symmetrical boundary was fixed so as not to deform the width of openings in the radial half. Excavation of the tunnel was simulated through a gradual unloading of the tunnel boundary from its initial stress value to null in order to generate ground response curve for the specified conditions. The support pressure (p_i) was assumed to be uniform along the boundary of the opening and so uniformly distributed radial support pressure along the opening boundary was applied.

Table I

Numerical modelling input data for rock mass properties

Rock mass class	GSI	σ_c (MPa)	m_i	c (MPa)	ϕ (Degrees)	E_m (MPa)	σ_m (MPa)
Fair quality	44	50	25	2.35	38	5000	9.6
		44	17	1.88	35	4696	7.2
		38	15	1.59	33	4364	5.9
		32	10	1.21	30	4000	4.2
		25	7	0.87	27	3540	2.8
Poor quality	23	25	25	0.73	31	1057	2.6
		21	17	0.53	29	969	1.8
		18	15	0.43	27	897	1.4
		14	10	0.29	24	791	0.9
		10	7	0.18	21	668	0.5

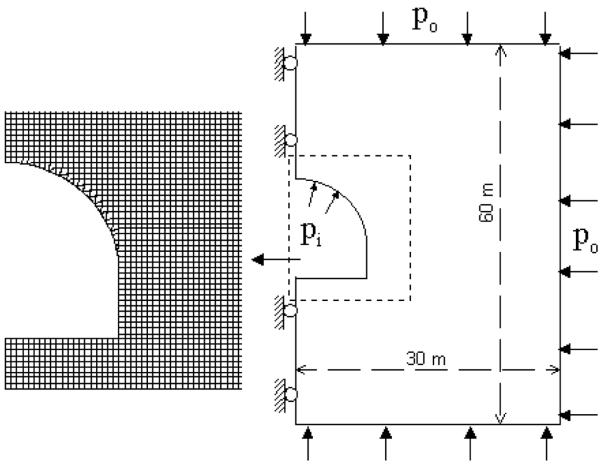


Figure 2—Details and conditions for models

Model results

For six different far-field stress parameters and ten different material property parameters (see Table I), 300 models for circular openings with diameters which are 2, 4, 6, 8 and 10 m, 300 models for horseshoe shape openings with dimensions shown in Figure 1, and 60 models for a rectangular opening were arranged. In all these models, initial support pressure applied to the boundary of the opening was equal to the far-field stress magnitude and was then gradually reduced, in steps of 0.2 MPa, to zero. In each support pressure step, the final deformation satisfying the equilibrium condition of the model was recorded automatically by implementing this procedure to FLAC through an ad-in programming language FISH. This procedure allowed the generation of a ground response curve for each model case that differed from other cases by one parameter.

The extent of the failure zone and displacements along the boundary of the circular opening were uniform and displacements were radial to the opening wall. However, the non-circular opening cross-section caused a non-uniform extent of the failure zone and distribution of displacements. This can be seen from Figures 3a and b showing the direction and magnitude of the displacement vectors around an unsupported horseshoe and rectangular opening excavated in rock mass having 5.9 MPa compressive strength (see Table I) subject to 20 MPa field stress, respectively. In all model cases of rectangular and horseshoe shape openings, the extent of the yielding zone and the deformations on the roof is larger than those on the sidewall with the decrease in strength of rock mass. Generation of a single ground response curve was possible for circular opening, whereas determination of a family of curves was required for the other opening shapes because the variation in deformation magnitude occurred along the opening boundary at various support pressures.

Given the shape of an opening and the quality of the surrounding rock mass, the deformation obtained from model openings was investigated according to the following parameters,

$$u = f(\sigma_m, p_o, p_i, r) \qquad [4]$$

where u is the deformation of the opening, σ_m is the compressive strength of the rock mass, p_o is the *in situ* stress magnitude, p_i is internal support pressure and r is the radius of the opening. Analysis of output deformation data showed that the deformation of the opening increased linearly with the opening size when the other parameters were taken as equal. So the strain of an opening (u/r) could be determined by applying the following transformations to the independent parameters (Hoek, 2000 and Hoek, 2001)

$$\frac{u}{r} = f\left(\frac{\sigma_m}{p_o}, \frac{p_i}{p_o}\right) \qquad [5]$$

Where σ_m/p_o is the competency factor of the rock mass, which is also commonly used as a squeezing indicator for rock mass (Barla, 2001), and this input data varied from 0.52 to 0.03 for poor quality rock and from 1 to 0.1 for fair quality rock mass. p_i/p_o is the ratio of support pressure to *in situ* stress and was gradually reduced in all models from 1 for initial case to 0 for unsupported case with no face restraint. u/r is the resulting strain of the opening depending on the σ_m/p_o and p_i/p_o ratios and varied in the results of the numerical models from zero for p_i/p_o =1 to 1 for unsupported opening excavated in very weak rock mass. In the calculation of strain (u/r) from the deformation data of numerical modelling for horseshoe and rectangular openings, an approximation of the non-circular cross-section with an equivalent circular opening of the same cross-sectional area was made. The radius of the equivalent circular opening (r) was calculated from the relationship, $r = \sqrt{A/\pi}$, where A is the area of horseshoe or rectangular opening.

By taking the natural logarithm of both the u/r and σ_m/p_o data, a smooth data distribution was observed despite the wide range of parameters evaluated in models, as shown in Figures 4 and 5 for all shape openings surrounded by either fair or poor quality rock masses, respectively.

Regression analysis of strain data

By regression analysis of data shown in Figures 4 and 5 with a large number of equations, the following exponential function model with the Taylor series polynomial exponent was found to be the best one describing the strain change depending on competency factor and the ratio of support pressure to *in situ* stress. The equation representing the model can be written in the following form:

$$\frac{u}{r} = e^{\beta_o + \beta_1 \ln\left(\frac{\sigma_m}{p_o}\right) + \beta_2\left(\frac{p_i}{p_o}\right) + \beta_3\left(\ln\left(\frac{\sigma_m}{p_o}\right)\right)^2 + \beta_4\left(\frac{p_i}{p_o}\right)^2 + \beta_5 \ln\left(\frac{\sigma_m}{p_o}\right)\left(\frac{p_i}{p_o}\right) + \beta_6\left(\ln\left(\frac{\sigma_m}{p_o}\right)\right)^3 + \beta_7\left(\frac{p_i}{p_o}\right)^3 + \beta_8 \ln\left(\frac{\sigma_m}{p_o}\right)\left(\frac{p_i}{p_o}\right)^2 + \beta_9\left(\ln\left(\frac{\sigma_m}{p_o}\right)\right)^2\left(\frac{p_i}{p_o}\right)} \qquad [6]$$

where β_0, β_1, β_2, β_3, β_4, β_5, β_6, β_7, β_8, and β_9 are regression coefficients. This equation could be converted into linear form with the following transformations in order to estimate the regression coefficients from the least square method (Snedecor, 1989):

$$Y = \beta_o + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \beta_7 X_7 + \beta_8 X_8 + \beta_9 X_9 \qquad [7]$$

where $Y = \ln(u/r)$, $X_1 = \ln(\sigma_m/p_o)$, $X_2 = p_i/p_o$, $X_3 = (\ln(\sigma_m/p_o))^2$, $X_4 = (p_i/p_o)^2$, $X_5 = \ln(\sigma_m/p_o)(p_i/p_o)$, $X_6 = (\ln(\sigma_m/p_o))^3$, $X_7 = (p_i/p_o)^3$, $X_8 = \ln(\sigma_m/p_o)(p_i/p_o)^2$, $X_9 = (\ln(\sigma_m/p_o))^2(p_i/p_o)$.

For the 9 variables in n data points, the model equation for the ith observation could be written as follows:

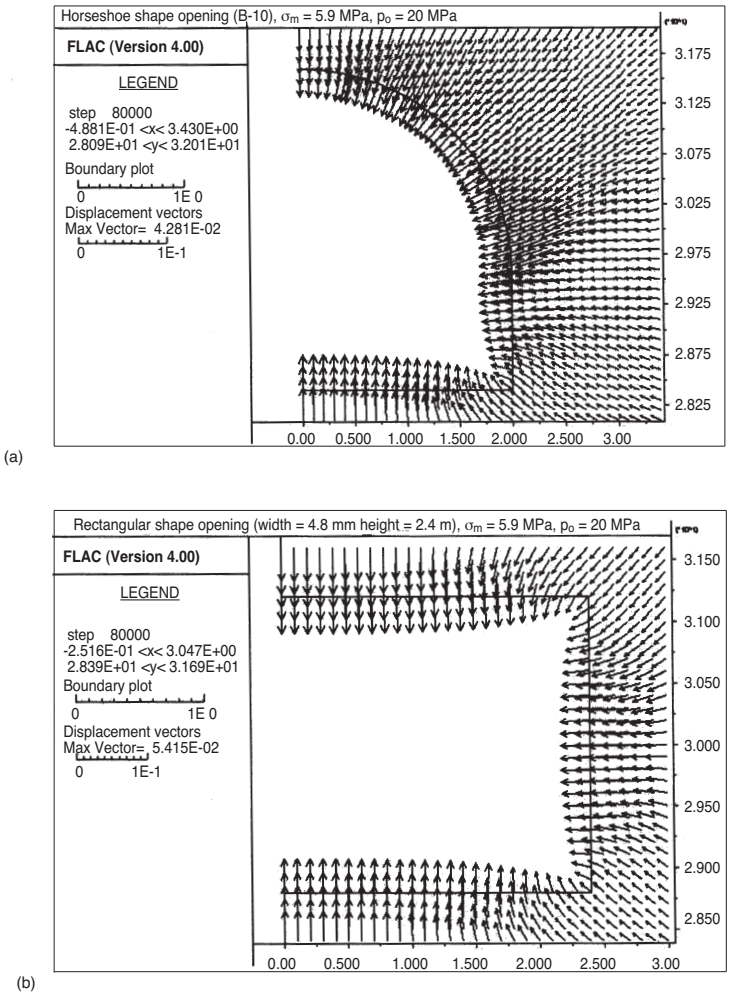


Figure 3—Examples of deformations occurring around non-circular openings (a) horseshoe shape (b) rectangular shape

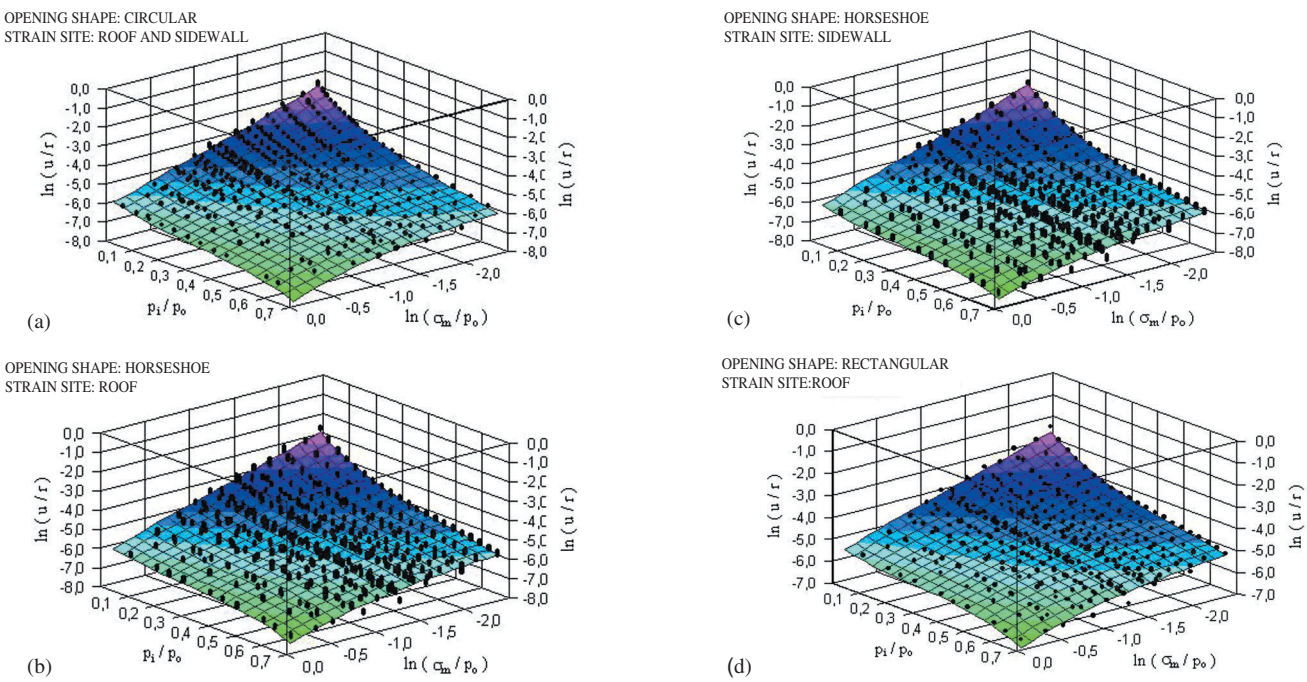
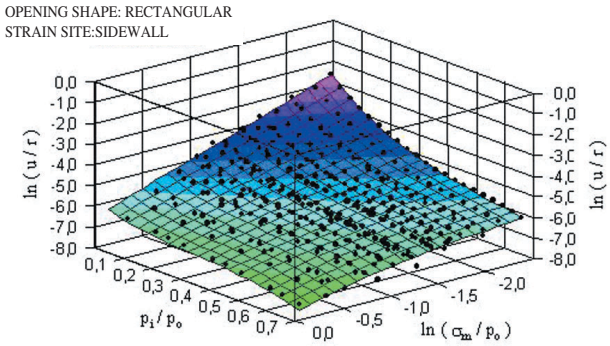
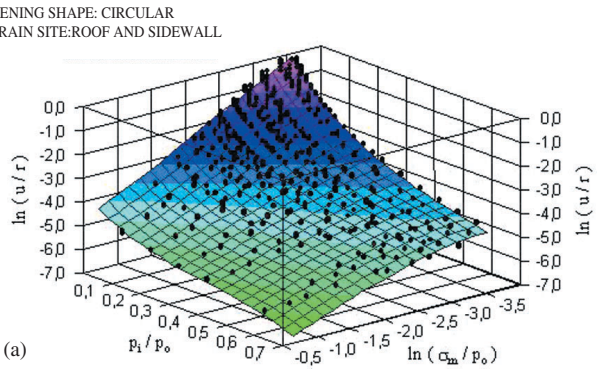


Figure 4—Data distribution and regression curve for strain response of openings excavated in fair quality rock mass (a) on the roof and sidewall of circular shape (b) on the roof of horseshoe shape (c) on the sidewall of horseshoe shape (d) on the roof of rectangular shape

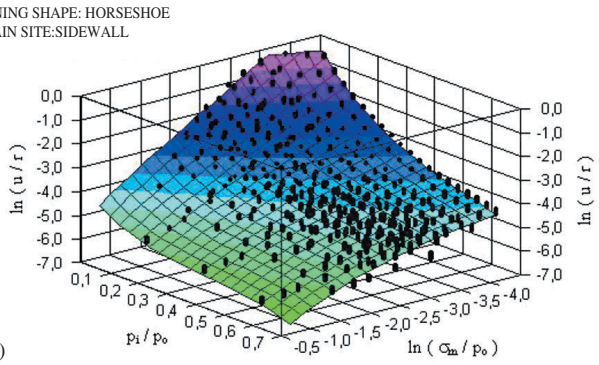


(e)

Figure 4 (continued)—Data distribution and regression curve for strain response of openings excavated in fair quality rock mass (e) on the sidewall of rectangular shape



(a)



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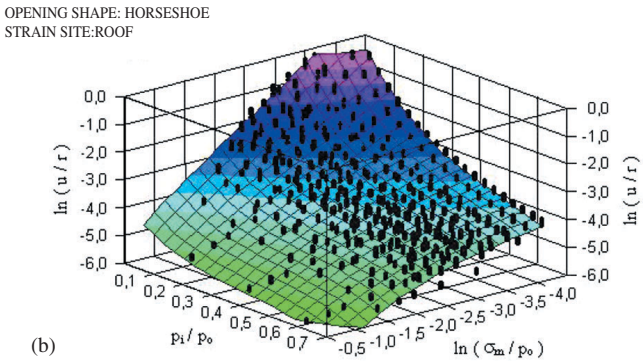

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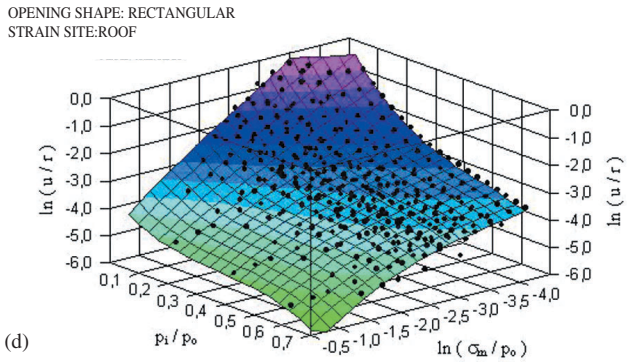

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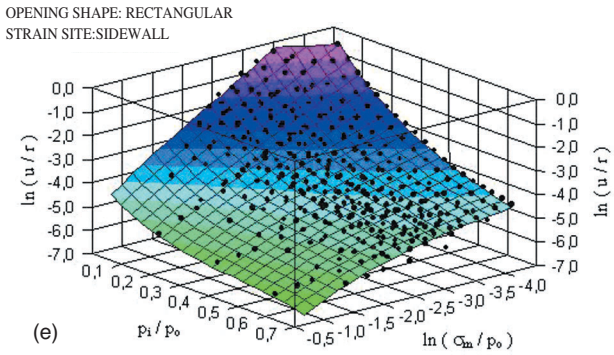

(e)

Figure 5. Data distribution and regression curve for strain response of openings excavated in poor quality rock mass (a) on the roof and sidewall of circular shape (b) on the roof of horseshoe shape (c) on the sidewall of horseshoe shape (d) on the roof of rectangular shape (e) on the sidewall of rectangular shape

$$Y_i = \beta_o + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \beta_5 X_{i5} + \beta_6 X_{i6} + \beta_7 X_{i7} + \beta_8 X_{i8} + \beta_9 X_{i9} \quad [8]$$

There are n such equations, one for each data point. The method of least squares estimates the coefficients, which minimize the sum of squared deviations between the fitted and actual strain data, from the following equation (Snedecor, 1989):

$$\begin{bmatrix} \beta_0 \\ \beta_1 \\ . \\ . \\ . \\ \beta_9 \end{bmatrix} = \left(X^T X\right)^{-1} X^T Y \quad [9]$$

where Y is a 9x1 matrix and X is an n x 10 matrix for this problem. That is,

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ . \\ . \\ . \\ Y_n \end{bmatrix} \text{ and } X = \begin{bmatrix} 1 & X_{11} & X_{12} & . & . & X_{19} \\ 1 & X_{21} & X_{22} & . & . & X_{29} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 1 & X_{n1} & X_{n2} & . & . & X_{n9} \end{bmatrix} \quad [10]$$

The best estimate of a coefficient is the value minimizing the residual sum of squares in the regression model to bring the curve close to the data points. From the scattered data illustrated in Figures 4 and 5, regression coefficients providing a best fit surface curve to data—as shown in the same figures for circular, horseshoe and rectangular openings , quality of rock mass and site of the deformation (roof or sidewall)—are given in Table II.

Statistical testing for the model validity

Equation [6]was found to be the best model with the lowest standard error of estimate (S) and higher coefficient of multiple determination (R^2) among the large number of equations fitted to the data. S, which is an important measure for showing how close the actual data points fall to the predicted values on the regression curve, and multiple R^2, which measures the proportion of variation in the data points explained by regression model, are given in Table II separately for each opening shape, rock mass quality and deformation site around the opening. The S values in comparison to the mean of the predicted values of the strains for each case are less than 10% of the mean. The R^2 of the model is higher than 0.959 for describing the strain data. These measures show that the model is useful to describe the strain of openings against the σ_m/p_o and p_i/p_o with regression coefficients given in Table II for different shape of opening, rock mass quality and deformation site.

F statistics was performed for testing the global usefulness of the model. This test is widely used in regression and analysis of variance (Snedecor, 1989). The null hypothesis for this test is H_0: $\beta_1 = \beta_2 = ... = \beta_9 = 0$ against the alternative hypothesis H_1: at least one of β_1, β_2, ..., β_9 not equal to zero. The calculated F ratios of the model for different parameters (opening shape, rock mass quality, etc.) are given in Table II. For a significance level of 1%, the tabulated F ratio with the numerator degrees of freedom of 9 and with the denominator degrees of freedom, larger than 350 (e.g. rectangular shape opening) is 2.41. In comparison to the tabulated F ratio, computed F ratios of the model in all cases are quite large enough to reject the null hypothesis (see

Table II

Results from regression analyses of data

Opening shape	Rock mass quality	Site for ground response curve determination		Regression coefficients										Model		
				β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	R^2	S (x10^{-3})	F-ratio
Circular	Fair	Roof and sidewall	Value (x10^{-3})	-5936.18	-1566.88	-4491.41	-150.37	9567.47	1663.23	-52.22	-9903.18	-2822.51	-547.82	0.987	1.18	14185
			S_{pi} (x10^{-3})	0.32	0.60	2.10	0.46	5.31	1.47	0.12	4.79	1.55	0.39			
			t–ratio	-18800	-2619	-2134	-328	1802	1128	-452	-2065	-1825	-1400			
	Poor	Roof and sidewall	Value (x10^{-3})	-4930.95	-969.74	-7063.51	298.61	14435.48	1613.49	43.97	-13466.50	-2536.02	-298.15	0.968	22.74	5088
			S_{pi} (x10^{-3})	19.06	22.46	72.17	9.62	128.55	36.28	1.38	100.43	29.91	5.92			
			t–ratio	-258	-43	-97	31	112	44	31	-134	-84	-50			
Arch	Fair	Roof	Value (x10^{-3})	-5824.60	-1700.22	-4029.12	-256.05	9773.28	2180.75	-72.32	-10584.00	-2993.48	-394.04	0.962	1.64	4966
			S_{pi} (x10^{-3})	0.60	0.95	3.52	0.65	8.03	2.21	0.15	6.46	2.03	0.52			
			t–ratio	-9665	-1780	-1144	-389	1216	986	-452	-1638	-1471	-756			
		Sidewall	Value (x10^{-3})	-5971.93	-1765.78	-4636.39	-181.12	11989.71	2813.15	-54.43	-12440.80	-3504.89	-377.75	0.989	0.86	18352
			S_{pi} (x10^{-3})	0.31	0.50	1.85	0.34	4.23	1.16	0.08	3.40	1.07	0.27			
			t–ratio	-18808	-3510	-2499	-522	2832	2415	-646	-3654	-3270	-1376			
	Poor	Roof	Value (x10^{-3})	-5642.47	-2186.81	-6856.58	-227.83	17165.70	2228.43	-31.13	-17002.00	-3033.07	-267.58	0.982	15.90	10885
			S_{pi} (x10^{-3})	14.31	13.07	50.87	4.55	86.29	21.49	0.56	64.07	17.04	3.03			
			t–ratio	-394	-167	-134	-50	198	103	-55	-265	-177	-88			
		Sidewall	Value (x10^{-3})	-5608.73	-2172.71	-7649.32	-207.27	17107.88	2171.21	-26.4222	-14875.80	-2477.47	-214.11	0.991	11.50	22309
			S_{pi} (x10^{-3})	10.35	9.46	36.78	3.29	62.45	15.55	0.40	46.35	12.21	2.18			
			t–ratio	-541	-229	-207	-62	273	139	-64	-320	-202	-98			
Rectangular	Fair	Roof	Value (x10^{-3})	-5338.21	-1609.36	-3571.34	-275.01	9169.01	1920.74	-73.51	-10554.23	-2854.80	-377.15	0.959	2.26	897
			S_{pi} (x10^{-3})	1.85	2.94	10.84	2.02	24.73	6.80	0.49	19.89	6.26	1.60			
			t–ratio	-2876	-547	-329	-135	370	282	-149	-530	-455	-235			
		Sidewall	Value (x10^{-3})	-5985.80	-1849.77	-5091.71	-159.78	12525.58	3053.36	-50.05	-12323.14	-3534.52	-365.91	0.995	0.67	8073
			S_{pi} (x10^{-3})	0.55	0.88	3.25	0.60	7.42	2.04	0.14	5.97	1.88	0.48			
			t–ratio	-10743	-2095	-1564	-262	1686	1494	-338	-2062	-1879	-760			
	Poor	Roof	Value (x10^{-3})	-5273.23	-2197.55	-6425.48	-246.00	18187.72	2361.33	-32.23	-19249.77	-3392.57	-266.67	0.994	10.20	6087
			S_{pi} (x10^{-3})	20.53	18.76	73.03	6.54	123.97	30.87	0.81	92.68	25.15	4.47			
			t–ratio	-256	-117	-87	-37	146	76	-39	-207	-134	-59			
		Sidewall	Value (x10^{-3})	-5483.82	-2161.67	-8187.08	-194.09	16238.49	2009.62	-24.05	-12559.57	-1978.04	-186.50	0.993	10.56	5845
			S_{pi} (x10^{-3})	21.25	19.42	75.57	6.76	128.25	31.95	0.83	95.74	25.60	4.53			
			t–ratio	-258	-111	-108	-28	126	62	-28	-131	-77	-41			

Table II). From this it can be concluded that model is valid and significant to calculate the strain of an opening against the σ_m/p_o and p_i/p_o.

Hypothesis testing on the significance of individual coefficients was performed by *t*-test. The null hypothesis for this test is H_0: $\beta_i = 0$ for each coefficient against the alternative hypothesis H_0: $\beta_i \neq 0$. If one of the parameters is not important in the model then the *b* value of that parameter is found to be zero from this test. The t-ratio for each estimated value of β_i found by dividing the β_i to its standard deviation ($S\beta i$) are given in Table II. The tabulated *t*-ratio for 1% significance level was found to be 2.326 from the table and was same for all cases. As could be seen from Table II, the absolute values of computed *t*-ratios are significantly larger than the tabulated ratio, suggesting that all the coefficients in the model are not equal to zero. It can be concluded from this that the parameters incorporated into the model are all relevant and significant for the calculation of opening strain and the model is valid.

Comparison of the model with an analytical solution

The validity of the current model described by Equation [6] was checked with an analytical solution suggested by Hoek *et al.* (1995). Comparison was made just for circular opening due to there being no available solution for non-circular openings to calculate the ground response curve. In the derivation of the equations predicting the ground response curve, this solution assumed similar conditions to this work, e.g. a circular tunnel surrounded by weak rock mass under hydrostatic stress, elastic-perfectly plastic material with no volume change and Mohr-Coulomb yielding behaviour. Comparison was made for two different material properties and stress conditions. Firstly, a circular tunnel of 9 m in diameter surrounded by a fair quality rock mass with 5.9 MPa compressive strength subjected to a 28 MPa stress field, and secondly, a circular tunnel of 2 m in diameter surrounded by a poor quality rock mass with 1.8 MPa compressive strength subjected to a 15 MPa stress field, were considered. Results both from this solution and model are illustrated in Figure 6, showing that the support pressure-deformation relation suggested by the model is in close agreement with the analytical solution.

Effect of parameters on the deformation response of the opening

Model results show that the lower rock mass strength, the higher the *in situ* stress, the larger the opening diameter, and the lower the support pressure conditions, then the greater deformations around the opening. On the other hand, the opening shape analysed and its effect contained in the model with coefficients had a significant effect on the deformation level. Figure 7 illustrates this effect with the plotted ground response curves generated for the roof of circular, horseshoe and rectangular openings excavated in fair or poor quality rock mass. As expected, a rectangular opening causes the occurrence of more deformation than a horseshoe shape opening relative to a circular shape opening. However, its effect is not more significant than the quality of rock mass and σ_m/p_o ratio, as can be seen from Figure 7.

Support pressure estimation

One useful way of predicting the necessary support pressure within an acceptable level of tunnel strain and for seeing the effect of different parameters, is to construct dimensionless plots. These dimensionless plots were established from the model and are illustrated in Figures 8 and 9 for the roof of an opening excavated in fair or poor quality rock mass conditions, respectively. For a given shape of an opening, rock mass quality, and rock mass strength to *in situ* stress ratio, the support pressure to *in situ* stress ratio required to maintain per cent strain can be estimated from these diagrams. The support pressure can then be readily calculated from the known *in situ* stress. Type and amount of support needed for the stability of opening can be estimated from the work of Hoek (1998) using the diameter of a circular opening or equivalent diameter of horseshoe and rectangular openings in conjunction with the support pressure readily available from these diagrams.

It is important to point out that allowing the tunnel to undergo deformations extensively to reach the minimum levels of support pressures is not possible due to the gravity effect, leading to unstable failure propagation on the roof. Therefore, there should be an allowable limit of strain before setting up the support. Field observations and measurements performed by Sakurai (1983) and followed by Chern *et al.* (1998) show that strain of the opening in excess of 1% is associated with the onset of tunnel instability and with difficulties in providing adequate support. Hoek (2001) reports that a 1% limit of strain is only an indication of

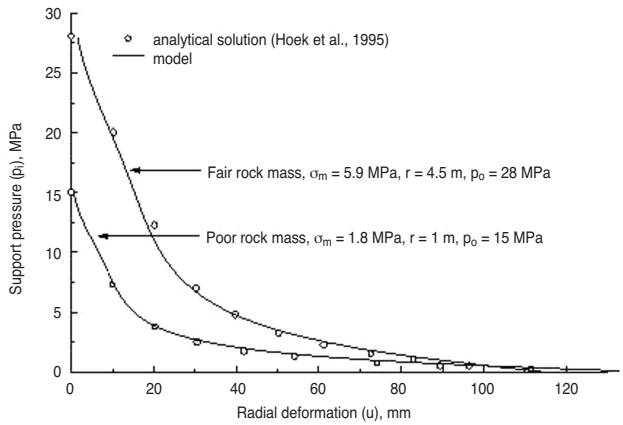

Figure 6—Comparison between model and an analytical solution

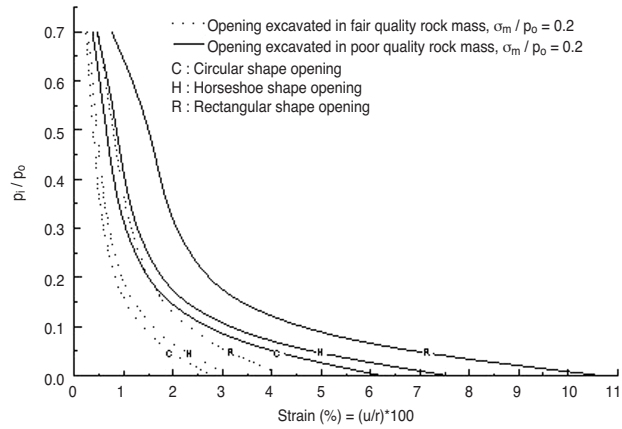

Figure 7—Effect of opening shape, rock mass quality and *pi/po* ratio on the strain response of the opening

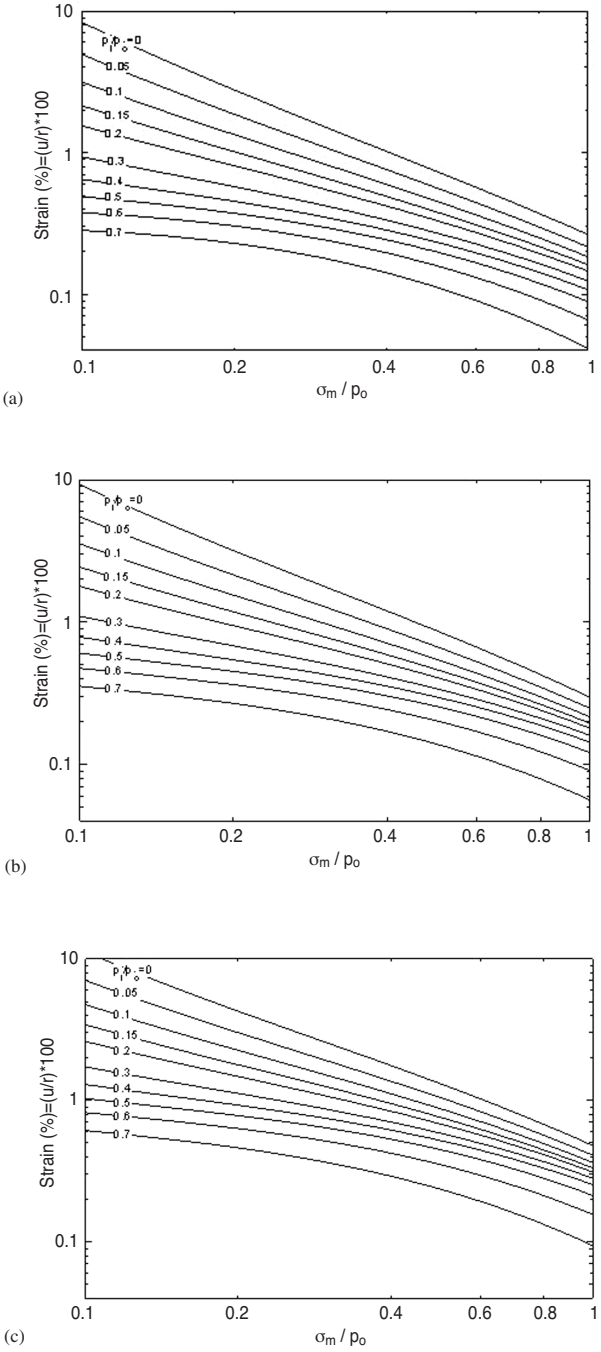

Figure 8—Support pressure estimation diagrams for the roof of openings excavated in fair quality rock mass (a) circular shape (b) horseshoe shape (c) rectangular shape

increasing difficulty to install the support and it should not be assumed that sufficient support should be installed to limit the opening strain to 1%. Based on measurements carried out in some tunnels, Hoek (2001) suggests that it is desirable to allow the tunnel to undergo strains of as much as 5%, in some cases, before activating the support. These tunnels were successfully completed without having stability problems, but construction problems increased with increasing strain levels.

A similar observation is published by Singh *et al.* (1997) for tunnels in India. They define the critical strain, beyond which the necessary support pressure increases sharply, to be 5–6% due to unstable failure propagation initiated at this value, and the rock suddenly loses its cohesion. This corresponds to a jump of the ground response curve from the rock with the original cohesion to zero cohesion. These values yield first estimations of the optimal value of the tunnel convergence and can only be used only for a preliminary design purpose.

For an unsupported opening subject to hydrostatic stress, deformations increase very significantly when the ratio of rock mass strength to *in situ* stress level is less than 0.2 for circular shape, 0.25 for horseshoe shape and 0.3 for rectangular shape openings, respectively. According to the above field experiences, preliminary supporting of the opening should be done as close to the face as possible, as within 1% of tunnel strain, further deformation of the opening up to 5–6% strain level may be allowed under the control of support to reduce support pressure if the final size of the opening is not a major issue. Very weak rock mass surrounding the opening may be improved by grout injection, placement of grouted pipe forepoles, or reinforcement with fibreglass dowels if very high pressure is necessary for a stable condition on the roof of the tunnel near to the face and the final size of the opening is important.

Conclusions

A parametric numerical modelling study was carried out to establish a ground response curve prediction model for circular, horseshoe and rectangular shape openings. Parameters for the purpose of the developed models are shape and dimensions of opening, rock mass quality and strength, *in situ* stress magnitude for hydrostatic stress condition, and the roof and sidewall deformation response to the opening with respect to support pressure. Support pressure applied on a model opening boundary was uniform and reduced from *in situ* stress magnitude to zero in steps of equilibrium condition reached in the system. In spite of the wide range of parameters incorporated into the models, opening strain data (u/r) depended on dimensionless parameters (σ_m/p_o and p_i/p_o) exhibiting a smooth data distribution for each opening shape evaluated separately for each rock mass quality. For horseshoe and rectangular shapes, calculation of strain was done by the radius of the opening equivalent in area to the circular section. As a result of regression analysis and the statistical testing process, an exponential function model with the Taylor series polynomial exponent was found to be the best one for calculating the ground response curve. Regression coefficients of this model for each opening shape, rock mass quality and site of the deformation are given in Table II. Compared to an analytical solution, the ground response curve calculated from the model for a specific example proved that the model is valid. With the same conditions and an equal area, a rectangular section causes more deformation of the opening wall than a horseshoe section, whereas a horseshoe section causes more deformation than a circular section. On the other hand, the lower the rock mass strength, the higher the *in situ* stress, the larger the opening diameter, and the lower the support pressure conditions, then the greater deformations around the opening. Diagrams were also created to highlight the importance of each parameter and to quickly estimate the necessary support pressure for the tunnel roof within the acceptable limits of tunnel strain, which are up to 1% before

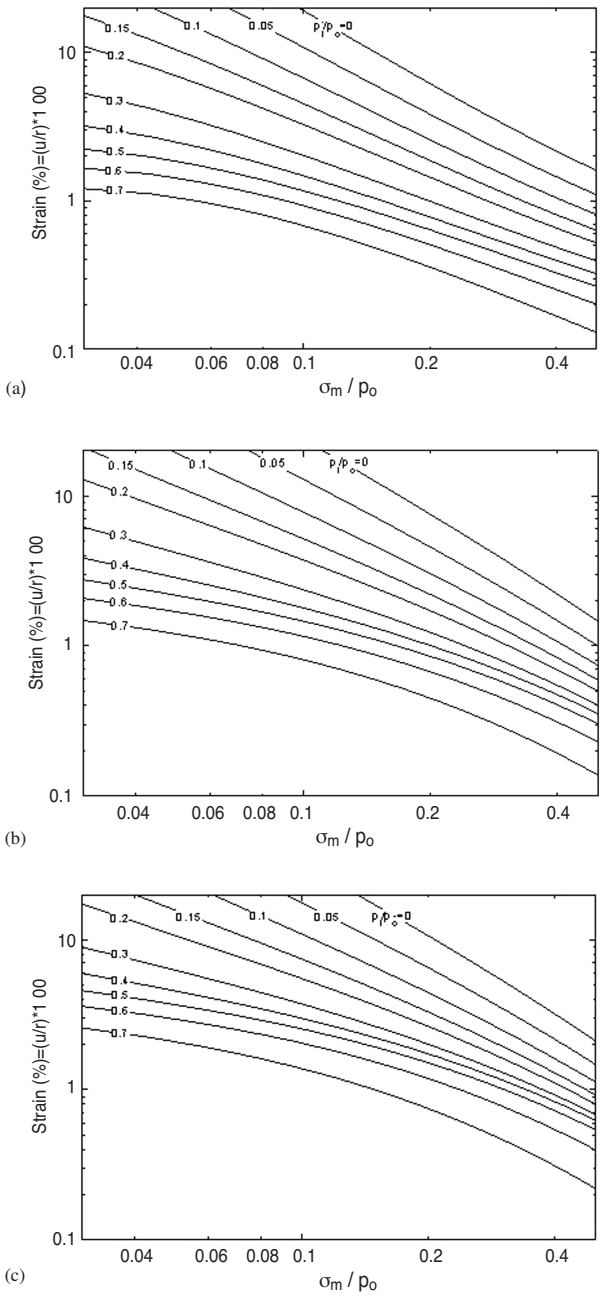

Figure 9—Support pressure estimation diagrams for the roof of openings excavated in poor quality rock mass a) circular shape b) horseshoe shape c) rectangular shape

support installation and up to 5–6% after installation of the support. In using these diagrams, previous field measurements should not be forgotten in that the opening roof requires a sharp increase in support pressure after 5–6% strain due to excessive material loosening and continuing increase in the upward extent of failure zone. For the worst ground conditions, ground improvement may be necessary to reduce the pressure imposed on supports and to provide a safe and stable tunnel condition.

Acknowledgement

The author would like to express his sincere gratitude to The Scientific and Technical Research Council (TUBITAK) of Turkey for providing financial support for this research study.

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