Geostatistical estimation of mineral resources with soft geological boundaries: a comparative study

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Synopsis

Mineral resource evaluation requires defining geological domains that differentiate the types of mineralogy, alteration and lithology. Usual practice is to consider the domain boundaries as hard, i.e. data from across the boundaries are disregarded when estimating the grades within a given domain. This practice may hinder the quality of the estimates when a significant spatial correlation of the grades exists across the domain boundaries.

In this paper, several geostatistical methodologies to handle soft geological boundaries are compared through a case study on a copper mine in Chile. The estimation methods considered are: ordinary kriging using hard boundaries between geological domains; ordinary kriging omitting the geological boundaries; traditional and standardized ordinary cokriging of the grades assayed in different domains; ordinary kriging within dilated geological domains, that is, incorporating samples from adjacent domains up to a given radius from the boundary of the domain being considered. The estimations are performed using a set of exploration data (drill hole samples) and are validated using a set of production data (blast hole samples collected for grade control).

Results indicate that kriging with dilated domains performs better than ordinary kriging using hard boundaries or no boundaries and better than the ordinary cokriging approaches. It therefore appears as a simple alternative to global kriging (without considering the geological domains) and allows accounting for changes in the grade average, dispersion and spatial continuity with the geological characteristics of the deposit, via the use of grade variograms proper to each geological domain. The maximal distance for searching samples from adjacent domains should be chosen according to the spatial correlation of the grades across the domain boundaries, or via a cross-validation at an early stage of the mine or a jack-knife if production data are available.

Keywords: Kriging; cokriging; soft boundaries; hard boundaries; geological control; geological domains.

Introduction

Every mining project requires defining the mineral resources prior to mine design and planning. This definition is based on geological knowledge of the orebody and on sample information from an exploration drilling grid, usually with infill drilling in the areas to be extracted during the first years of the project. The first step in resources estimation is an exploratory analysis aimed at understanding the characteristics of the available data and at identifying homogeneous geological domains within the deposit, according to the spatial continuity of grades and to geological features such as lithology, mineralogy and alteration.[1–4]

Three problems arise during this process:

- The definition of geological domains relies on the subjective interpretation of the mining geologist and on his understanding of the genetic processes that caused the mineralization. Various interpretations are therefore possible.
- The delineation of the geological domains is always subject to errors, since only fragmentary information is available through a finite set of samples drilled in the deposit. Delineating the domains must be done carefully, accounting for geological knowledge about the deposit genesis. Alternatively, one could consider modelling the uncertainty in the boundaries layout by resorting to a geostatistical simulation technique.[5,6]
- The boundaries that define the contact between adjacent geological domains are seldom 'hard', particularly for porphyry copper mineralization, that is, the grades measured at either side of a boundary are not independent. Besides, the boundary may be defined by a change in the local mean grade, which is usually gradational rather than abrupt.

Normal practice is to estimate the grades and to assess the mineral resources within each geological domain independently.[7,8] This approach implies that:

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- grades within a geological domain are not influenced by information across the boundary, i.e. there is no spatial correlation of the grades across the boundary (hard boundary)
- grades are assumed stationary within each domain, that is, they do not show a significant change in the local mean as one gets closer to the boundary.

Alternatively, several authors suggested considering a single domain and incorporating auxiliary variables that codify the geological characteristics into the grade estimation process.[9,10] However, so far, such an approach has not seen a wide application in the mining industry.

This paper focuses on the problem of grade estimation in the presence of soft or hard boundaries between geological domains. To address this problem, one must know when a boundary can be called soft or hard and, if a soft boundary exists, the question is how to incorporate information from across this boundary to estimate the grades in a particular geological domain. In the following, several geostatistical approaches to geological control are compared through a case study from a copper mine in Chile.

Methodology

Hard or soft boundaries?

A contact between two geological domains, from an alteration, mineralogy or lithology point of view, is considered hard if there is an abrupt change in mineralogy or grade without a transition at the scale of observation. On the contrary, a soft boundary shows a transition zone between the two domains, making it difficult to identify the exact layout of the limit.[11]

Structural control of grades, such as faulting and displacements, will normally preclude a smooth transition between geological domains. Abrupt changes in the petrophysical properties of adjacent domains frequently translate into a hard boundary. In particular, if the porosity of the rock to the mineral bearing fluids is heterogeneous from one domain to another, the grade is likely to change abruptly.

In many cases, however, changes in the grade behaviour are not so abrupt and one observes a transition zone where the local mean grades vary smoothly between one domain and the other. This trending behaviour translates into a correlation between data located on each side of a boundary (generally inferred by an expert geological interpretation), which can be used to improve the estimation of mineral resources near the domain boundaries. The possible trend in the grades is handled by using robust estimation techniques such as ordinary kriging, in which a local mean is implicitly calculated for every location where the grade is being estimated.[12]

The tools usually considered to quantify the spatial correlation are the direct variogram and, when dealing with multiple variables (in this case study, the grades within different geological domains), the cross variogram.[13] The latter is defined as:

$$\gamma_{XY}(\mathbf{h}) = \frac{1}{2} E\left\{\left[X(\mathbf{u}+\mathbf{h}) - X(\mathbf{u})\right]\cdot\left[Y(\mathbf{u}+\mathbf{h}) - Y(\mathbf{u})\right]\right\}$$

where $\mathbf{u}$ and $\mathbf{u} + \mathbf{h}$ are two locations separated by the lag vector $\mathbf{h}$, X is a random field representing the grade within a geological domain and Y is a random field for the grade within a different domain. Given that the geological domains are, by definition, disjoint, X and Y are never simultaneously known at the same location. Accordingly, the cross variogram cannot be inferred from the data. For this reason, we use another tool, known in geostatistical applications as the pseudo-cross variogram[14,15], which can be calculated even when there are no matching samples between X and Y:

$$\gamma^*_{XY}(\mathbf{h}) = \frac{1}{2} E\left\{\left[X(\mathbf{u}+\mathbf{h}) - Y(\mathbf{u})\right]^2\right\}$$

This formula is meaningful, since both random fields X and Y refer to the same physical variable (the grade of the element of interest), expressed in the same unit, within different geological domains.

Once all the pseudo-cross variograms are calculated, a visual examination of their plots indicates whether a significant correlation in the grades across the boundaries exists or not and, if so, up to which distance the data across the boundary should be considered for grade estimation in the domain of interest.

Reference cases

In order to assess the quality of the geostatistical methods that will be proposed for geological control, two reference cases are considered:

- *Case 1*—Ordinary kriging is performed within each geological domain by using hard boundaries, that is, the block grade in a domain is calculated using only the drill hole samples from the same domain and a variogram calculated exclusively from the samples within this domain. This case represents the common practice in the mining industry.
- *Case 2*—Ordinary kriging of block grades is performed disregarding all boundaries and geological information. The grade variogram is inferred from all the samples, irrespective of the geological domains they belong to.

Accounting for information across the boundaries

To assess the relevance of using data from different geological domains, the following approaches are considered:

- *Case 3*—Traditional ordinary cokriging of the block grades, considering the grade within each geological domain as a separate variable[16]. The cokriging system is based on the direct and pseudo-cross variograms of the different variables, which are fitted by using a linear model of coregionalization[8,13]. This cokriging type uses as many unbiasedness constraints as variables are considered (see Appendix).
- *Case 4*—Standardized ordinary cokriging of the block grades, which uses a single unbiasedness constraint (Appendix).[7,8,17] This cokriging type implicitly assumes that the mean grades are the same on both sides of the domain boundaries, at least at the scale of the cokriging neighbourhood, an assumption that seems reasonable when considering soft geological boundaries.

➤ *Case 5*—Ordinary kriging within each geological domain, akin to Case 1, but considering all the data within a dilated neighbourhood of the domain volume. This allows data from adjacent domains to be incorporated in the kriging system (provided that they are close enough to the boundary) as if they belonged to the domain of interest.

Prior to applying the five methodologies to the case study, one could expect Case 1 to outperform Case 2, since the latter does not account for the geological information. However, this situation might change if the grade transitions are smooth near the boundaries and the kriging search parameters (number of samples and search radii) are appropriately defined.

Further improvements are expected with Case 3 and Case 4, since the incorporation of secondary information (via the use of covariates) reduces the theoretical estimation variance and therefore provides more precise results. However, these improvements may be counterbalanced by the greater difficulty in fitting simultaneously the grade variograms and pseudo-cross variograms with a linear model of coregionalization[8,13]. A poor fit of these variograms may degrade the quality of grade estimates.

Finally, Case 5 represents an intermediate situation in between Cases 1 and 2, where information from across the boundary is considered only up to a finite distance. The use of a grade variogram for each geological domain provides an advantage over Case 2, for which a single (global) variogram model is used.

Case study: porphyry copper deposit

Presentation of the data

Data from a porphyry copper deposit in Chile are now used to compare the performance of the aforementioned approaches to resource estimation in the presence of soft geological boundaries. The data correspond to 2 376 composites from an exploration diamond-drill hole campaign. The composites are 12 m long, which corresponds to the bench height in the mine. Part of the drill holes are located on a relatively regular grid, spaced approximately 40 m in the horizontal plane, completed by infill drills in the central part of the deposit.

Each composite is assigned a geological domain, which in this case relates exclusively to lithology, since mineralogy and alteration are not relevant factors for the definition of geologically and statistically consistent domains. Although the rock codes were originally six, they were grouped into three main types (Figure 1a), namely:

➤ *Granodiorite* (code 1). This is the host rock where breccias intruded. It is located in the eastern and southern parts of the deposit.

➤ *Tourmaline breccia* (code 2). This breccia has granodiorite clasts with cement composed by tourmaline and sulphides such as chalcopyrite, pyrite, molybdenite, and some bornite. Its emplacement is related to the main alteration-mineralization event. This rock has the highest mean grade (about 1.20 per cent) and is centrally located in the deposit.

➤ *Other breccias* (code 3). They are composed by three different breccia types and outcrop in the western and southern areas of the deposit. Their emplacement is simultaneous or more recent than the intrusion of tourmaline breccia, relocating and diluting it.

In addition to the drill hole database, production data are available from several benches (Figure 1b). These correspond to 12 793 samples taken at blast holes over 13 benches of the mine. Although the original rock code was not logged on these samples, for the validation of the proposed methodologies, a rock code has been assigned, based on the closest neighbour sample from the drill hole database.

Blast hole samples show a higher mean grade than drill holes (1.20 per cent versus 1.05 per cent), but this is due to their central location where copper grades are higher. The sampling procedures for drill holes and blast holes have been subject to a careful quality assurance and quality control programme to ensure that there is no systematic bias and that the variance of the errors is within accepted industry ranges. The summary statistics from all the data sets are shown in Table I.

Variogram analysis and modelling

When considering the cokriging approaches (Cases 3 and 4) the direct and pseudo-cross variograms are required. All these variograms are modelled with a nugget effect and two exponential nested structures, as shown in Figure 2 and

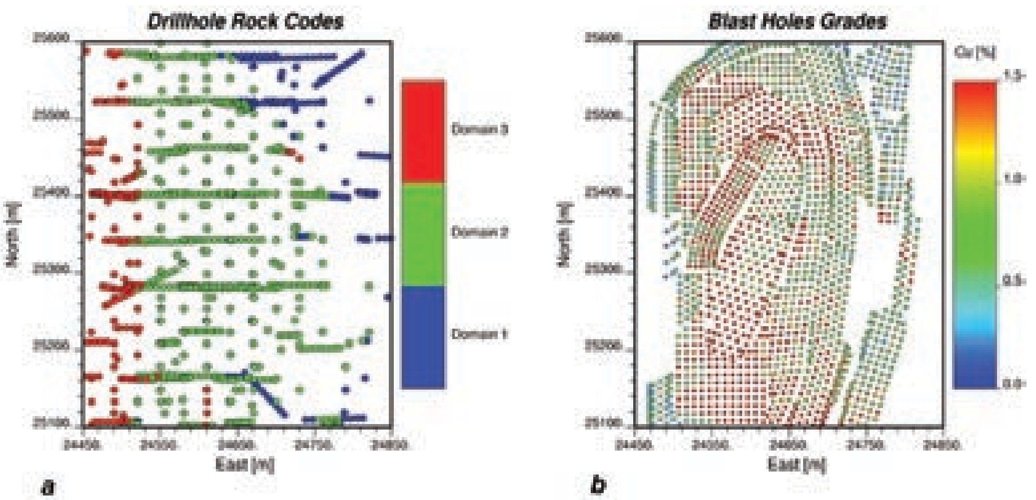

Figure 1—(a) Location map of drill hole samples coded by rock type; (b) Location map of blast hole samples coded by copper grade for a particular bench

Table I

Basic statistics from drill hole and blast hole data sets.

		Number of data	Mean (%Cu)	Std. dev. (%Cu)	Coef. of Var.	Min (%Cu)	Lower quartile (%Cu)	Median (%Cu)	Upper quartile (%Cu)	Max (%Cu)
Drill holes	All data	2 376	1.054	0.645	0.612	0.120	0.625	0.940	1.330	7.240
	Granodiorite	354	0.709	0.423	0.604	0.120	0.400	0.600	0.920	3.650
	Tourmaline breccia	1 635	1.196	0.661	0.552	0.160	0.783	1.080	1.450	7.240
	Other breccias	387	0.769	0.515	0.670	0.140	0.410	0.620	0.990	3.710
Blast holes	All data	12 793	1.200	0.621	0.517	0.110	0.790	1.090	1.480	7.710
	Granodiorite	1531	0.825	0.445	0.539	0.170	0.500	0.730	1.050	4.400
	Tourmaline breccia	10 028	1.287	0.620	0.482	0.110	0.870	1.160	1.550	7.710
	Other breccias	1 414	0.991	0.578	0.583	0.160	0.560	0.870	1.310	4.200

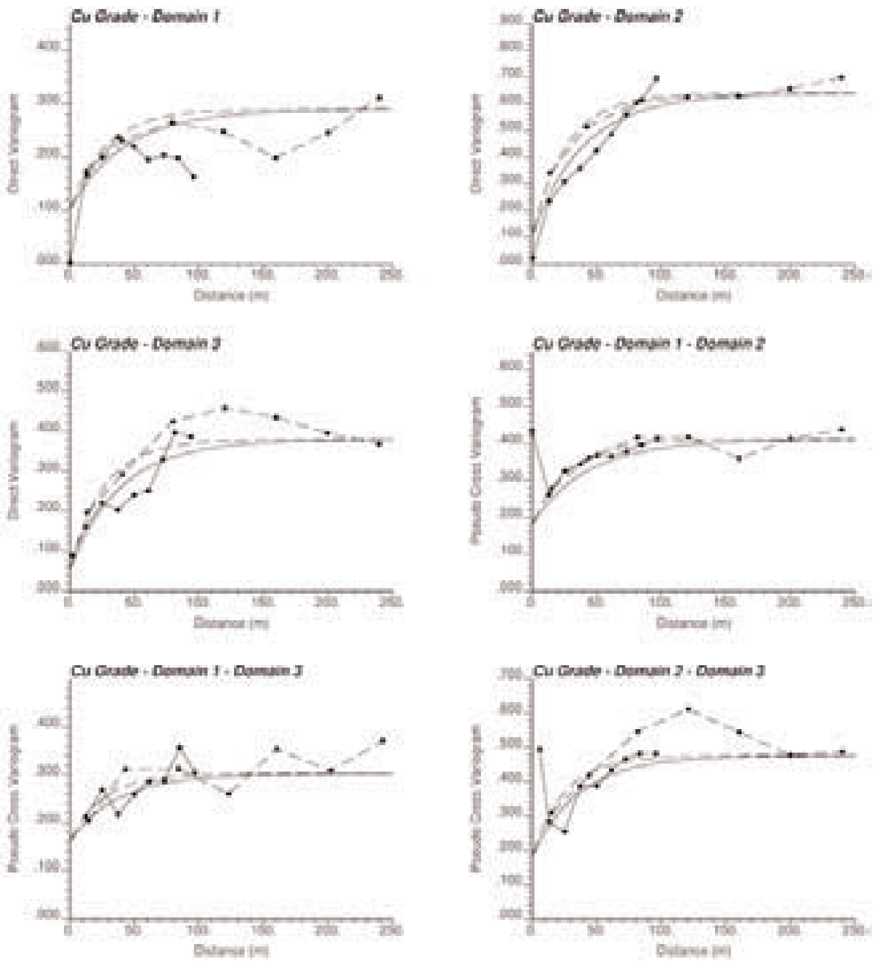

Figure 2—Direct and pseudo-cross variograms of the copper grade for the different geological domains fitted with a linear model of coregionalization. Sample variograms are shown as lines with dots at the calculated lag distances, while the models are presented as lines without dots. The horizontal variograms are represented by dashed lines and the vertical variograms by solid lines

summarized in Table II. The mathematical consistency of the model is checked by verifying that the matrices of contributions to the sill for the different nested structures are positive definite, that is, the eigenvalues of the matrices of sill contributions are positive.[13] One observes that, at a lag distance of 100 m, all the pseudo-cross variograms are close to their sills, which indicates that the correlation becomes very small. Therefore, this distance is chosen as the maximal search radius to use in Case 5, in which the grade estimation within a given geological domain incorporates data from adjacent domains.

The direct variograms obtained from the joint fitting are considered in the cases when the pseudo-cross variograms are not used (Cases 1 and 5). Also, it should be noted that the nugget components of the pseudo-cross variograms are absent from the cokriging systems, since there is never a collocated secondary sample at the location where the primary variable is estimated (proof in Appendix). Accordingly, the linear model of coregionalization is relevant only in the structured portion of the pseudo-cross variograms.

Table II

Parameters of the linear model of coregionalization fitted to the sample direct and pseudo-cross variograms of the copper grades

Head domain	Tail domain	Nugget effect variance	Exponential model sill Horizontal range = 70 m Vertical range = 100 m	Exponential model sill Horizontal range = 100 m Vertical range = 160 m
Domain 1 (Granodiorite)		0.100	0.090	0.100
Domain 2 (Tourmaline breccia)		0.100	0.340	0.200
Domain 3 (Other breccias)		0.005	0.220	0.110
Domain 1	Domain 2	0.095	0.104	0.128
Domain 1	Domain 3	0.006	0.109	0.034
Domain 2	Domain 3	0.006	0.240	0.056

Case 2 requires the knowledge of the grade variogram as if no geological boundaries existed. All drill hole data are then pooled together for the calculation and fitting of the sample variogram. The resulting model has a nugget effect of 0.02 and two nested exponential structures, the first one with a contribution to the sill of 0.200, horizontal range of 50 m and vertical range of 90 m, and the second one with a sill contribution of 0.37 and ranges of 70 m in the horizontal plane and 180 m in the vertical direction (Figure 3). One observes that the overall sill is higher than the data variance (Table I). This may be due to the effect of combining all the domains into a single one, or to the relatively small size (with respect to the correlation range) of the sampled domain. The impact of this higher sill on kriging estimates is not relevant, since kriging weights are independent of the scaling of the variogram.

Estimation and validation using jack-knife

The prediction of the grade values for blocks over a regular grid with a mesh of 10 m x 10 m x 12 m is done for the five cases, using the information from the drill hole data set and an assumed rock type model. The resulting maps (Figure 4) show clear differences, particularly in the areas where copper grades are extrapolated and, most importantly, near the assumed boundaries of the geological domains. The application of hard boundaries to the estimation of mineral resources (Case 1) imposes artefacts that are highly dependent on the geological model and on the layout of the boundaries, which in reality are but an interpretation from the sample information and are inevitably subject to error.

The same effect can be observed when differentiating the geological domains and incorporating data from across the boundaries as covariates (cokriging approaches). However, when the standardized ordinary cokriging is considered (Case 4), the grade estimates are significantly smoother near the boundaries than in the case of traditional ordinary cokriging (Case 3). This stems from the fact that standardized cokriging uses the same local mean at both sides of the boundary and therefore assumes grade continuity. In contrast, traditional ordinary cokriging assumes that the (unknown) mean grades of the geological domains are not the same, which produces discontinuities in the grade estimates near the domain boundaries.

If no geological control is accounted for (Case 2), the map looks smooth and free of artefacts related to the definition of the geological boundaries. Incorporating the geology by modelling a grade variogram in each domain, but allowing data from adjacent domains to be considered (Case 5) also frees the resulting estimation map of the strong dependence upon the assumed layout of the boundaries, as seen in Cases 1 and 3.

To assess the performance of the different methodologies beyond a visual check of the block grade models, jack-knife is used against the blast hole data set. The scatter diagrams between true and estimated grades are shown in Figure 5 . These diagrams indicate that none of the proposed approaches suffers from a significant conditional bias, as the regressions of the true grades upon the estimated grades are fairly close to the first bisector line.

In addition, statistical comparisons are made, based on statistics such as the correlation between the estimated and true sample values, the mean error, mean absolute error and mean squared error. The results are presented in Table III . According to this table, the ordinary kriging approach with hard boundaries (Case 1) provides the worst results (highest mean absolute and mean squared errors). This is explained by the fact that the geological boundaries in the deposit under study are not 'hard' and are associated with gradational transitions in the mean copper grade, except maybe the boundary between domain 2 (tourmaline breccia) and domain 3 (other breccias) in the western part of the deposit. Regarding the other methods, one observes that

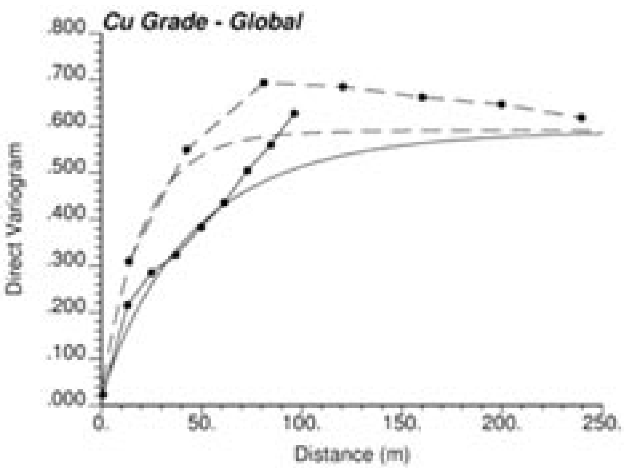

Figure 3—Copper grade variogram disregarding geological boundaries. The sample variogram is shown as lines with dots at the calculated lag distances and the model is presented as lines without dots. The horizontal variograms are represented by dashed lines and the vertical variograms by solid lines

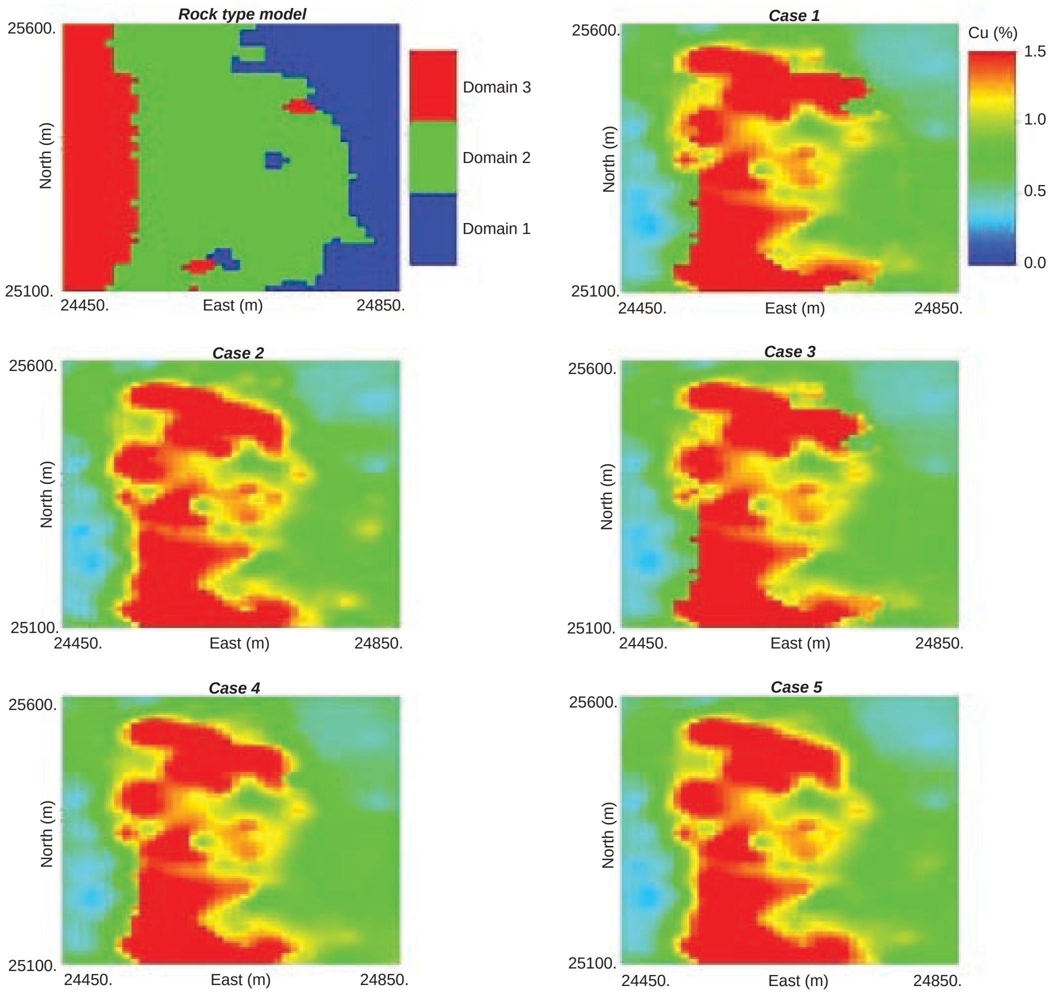

Figure 4—Plan views of the geological model showing the three geological domains and of the estimated grades obtained with all five methodologies considered

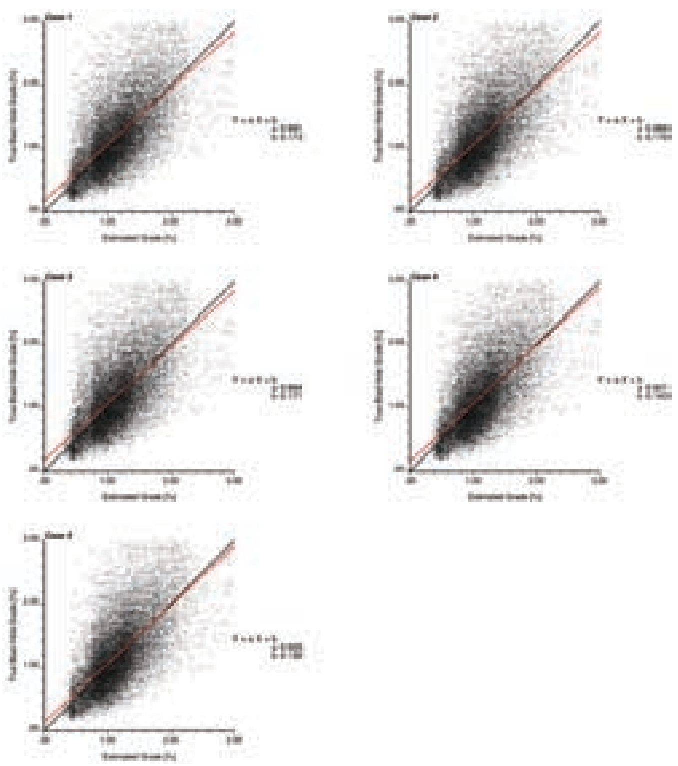

Figure 5—Scatter plots comparing the actual blast hole grades from the production data set with the grades estimated by all five methods studied

ordinary kriging disregarding the boundaries (Case 2) or 'dilating' the geological domains (Case 5) outperforms ordinary cokriging (Cases 3 and 4). A reason could be the difficulty in representing the spatial correlation of the grades within and across the geological domains through a consistent multivariate variogram model.

On the whole, the use of dilated geological domains (Case 5) gives the most precise grade estimates, as it reduces the mean absolute and mean squared errors with respect to the other approaches and improves the correlation coefficient between estimated and true grades, especially in the areas close to the geological boundaries (Table III). This method is quite simple to implement and, unlike global kriging (Case 2), it uses a variogram model proper to each geological domain and can therefore account for changes in the grade spatial continuity with the geological characteristics of the deposit. The maximal distance for searching samples from adjacent domains should be chosen on the basis of the spatial correlation of the grades across geological domains (analysis of the pseudo-cross variograms) or via a cross-validation or a jack-knife procedure. This distance can also be adapted to the boundary type and be smaller when the grade transitions are sharper.

Conclusions

The definition of geological domains is highly relevant due to the recoveries that ores with distinct geological properties have. Accordingly, the geological information and interpretation must be included in the estimation of mineral resources. The case study presented in this paper indicates that the effort needed to perform a cokriging approach brings little improvement over simpler kriging methods. The necessity of a linear model of coregionalization for cokriging often makes the fitting of the sample direct and cross variograms poorer (and harder) and tends to deteriorate the final results.

It is reasonable to expect that a clear geological reason should support the definition of hard boundaries for grade estimation. In many mineral deposits, the mineralization is disseminated and the metal is relocated many times by different physical and chemical processes, resulting in smooth grade transitions from one geological domain to another. Consequently, the use of the information across the boundaries of the geological model is often relevant. This can be done in a rather simple fashion, by allowing some samples of adjacent domains (up to a certain distance from the boundaries) to be used when estimating the grade in a given domain. The choice of the distance to which data from across a boundary are used can be determined by examining the pseudo-cross variogram (or, alternatively, the lagged scatter-plots) between grades sampled on each side of the boundary and by determining the maximal distance for which the correlation remains significant. Another reason to go for simpler techniques is that current software for performing geostatistical analysis and cokriging with more than two variables is scarce and the results are still hard to validate.

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Table III

Performance comparison among the five cases studied. The statistics are given for the entire validation set (12 793 blast holes) and for a subset of it (2 248 blast holes located at less than 25 metres from a geological boundary)

		Case 1 (OK with hard boundaries)	Case 2 (OK without) boundaries)	Case 3 (Traditional OCK)	Case 4 (Standardized OCK)	Case 5 (OK with dilated domains)
Validation set (12 793 blast holes)	Correlation True-estimated	0.655	0.665	0.657	0.662	0.670
	Mean error	-0.049	-0.042	-0.050	-0.054	-0.044
	Mean absolute error	0.324	0.320	0.323	0.320	0.317
	Mean squared error	0.225	0.219	0.224	0.220	0.215
Validation subset (2 248 blast holes close to the geological boundaries)	Correlation True-estimated	0.485	0.560	0.500	0.543	0.563
	Mean error	-0.103	-0.081	-0.102	-0.110	-0.079
	Mean absolute error	0.379	0.351	0.377	0.355	0.348
	Mean squared error	0.287	0.246	0.281	0.257	0.242

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Appendix

The grade estimation by ordinary cokriging (Cases 3 and 4) uses data from P different geological domains. Henceforth, let Z_p be the random field representing the grade in the p-th domain and $\{\mathbf{u}_i^p, i = 1 \ldots n_p\}$ be the sample locations in this domain. The grade estimate at location $\mathbf{u}$ belonging to the p_0 -th domain is defined by a weighted average of all the available data:[8]

$$Z_{p_o}^{OCK}(\mathbf{u}) = \sum_{p=1}^{P}\sum_{i=1}^{n_p}\lambda_{i,p}^{OCK}(\mathbf{u})\cdot Z_p\left(\mathbf{u}_i^p\right) \quad \text{[A1]}$$

In the case of *traditional ordinary cokriging* (Case 3), there are as many unbiasedness constraints as the number of variables considered. The weights of the primary variable must addup to one, while for each secondary variable the weights addup to 0, which implies that some secondary data have negative weights:

$$\sum_{i=1}^{n_p}\lambda_{i,p}^{OCK\,trad}(\mathbf{u}) = \delta_{p_0,p} \quad \forall p = 1,\ldots,P \quad \text{[A2]}$$

Such constraints amount to hypothesizing that the random fields $\{Z_p, p = 1,\ldots, P\}$ have different unknown means, hence this type of ordinary cokriging should be applied only in case of abrupt grade transitions near the geological boundaries. The minimization of the error variance subject to constraints (A2) leads to the following system of linear equations, in which $C_{pq}(.)$ stands for the cross-covariance function between Z_p and Z_q, while $\mu_1,\ldots \mu_P$ are Lagrange multipliers:[8,13,17]

$$\sum_{p=1}^{P}\sum_{i=1}^{n_p}\lambda_{i,p}^{OCK\,trad}(\mathbf{u})\cdot C_{pq}\left(\mathbf{u}_i^p,\mathbf{u}_j^q\right) + \mu_q = C_{p_0q}\left(\mathbf{u},\mathbf{u}_j^q\right) \quad \forall q = 1,\ldots,P; \forall j = 1,\ldots,n_q \quad \text{[A3]}$$

An alternative to the traditional ordinary cokriging estimator is the standardized version of it (Case 4) that imposes a single unbiasedness constraint over all the available data:[7,8]

$$\sum_{p=1}^{P}\sum_{i=1}^{n_p}\lambda_{i,p}^{OCK\,std}(\mathbf{u}) = 1 \quad \text{[A4]}$$

In this case, secondary samples are more relevant (their average contribution is not zero) when estimating the primary variable at unsampled locations. The single unbiasedness constraint implicitly assumes that all the random fields $\{Z_p, p = 1,\ldots, P\}$ have the same unknown expected value, i.e. that the mean grades are the same (at least, at the scale of the cokriging neighbourhood) in the different geological domains. Such an assumption is consistent with the idea of soft geological boundaries. Apart from Equation [A4], the standardized ordinary cokriging weights are determined by the following system of linear equations:[8,17]

$$\sum_{p=1}^{P}\sum_{i=1}^{n_p}\lambda_{i,p}^{OCK\,std}(\mathbf{u})\cdot C_{pq}\left(\mathbf{u}_i^p,\mathbf{u}_j^q\right) + \mu = C_{p_0q}\left(\mathbf{u},\mathbf{u}_j^q\right) \quad \forall q = 1,\ldots,P; \forall j = 1,\ldots,n_q \quad \text{[A5]}$$

In systems [A3] and [A5], the left-hand side members contain the covariances between pairs of data, while the right-hand side members account for the covariances between the data and the unknown grade. If one assumes that location $\mathbf{u}$ does not coincide with a sample, it is seen that the cross-covariance terms are never used at the zero distance for the right-hand side member. Neither are they for the left-hand side member, since there is no matching samples between two different variables (by construction, the geological domains are disjoint). Accordingly, the cokriging systems and the resulting cokriging weights are the same if one adds nugget effects to the cross-covariances. This statement justifies why ordinary cokriging can be performed without specifying the nugget effects of the cross-structures. ◆