Incorporation of ore grade uncertainty into the push back design process

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Synopsis

In open pit mine planning and design, push back mine sequencing is widely used for long-term production planning. In push back design, a series of nested pits is generated and then considered as possible push backs with regard to the minimum push back width requirements. These nested pits are produced by parameterizing on one economic parameter like commodity price, mining or processing cost, stripping ratio, etc. The most important characterization of these push backs is that the innermost push back has the highest value per ton, the second innermost push back has the second highest value per ton, and so on. These nested pits are generated based on the best estimates of grade within a representation of the orebody in a three-dimensional matrix form. Therefore, the inherent uncertainty associated with reserve estimation (for our discussion, grade uncertainty) is not taken into account when push backs are designed. These methods do not attempt to establish which part of the deposit is likely to be worth most. This paper introduces a new algorithm that incorporates ore grade uncertainty during the push back design process. The suggested strategy tries to seek to schedule risky blocks later in the extraction sequence.

Keywords: open pit mine, push back, ore grade uncertainty.

Introduction

Long-term production planning design is a major step in mine planning, because it determines the economic outcome of a project. Also long-term planes act as a guide for medium and short-term production scheduling. Because there are a large number of blocks within the ultimate pit limit, the pit can be divided into a series of sub-pits commonly called *push backs, cut backs or phases*. These push backs are designed with haul road access and act as a guide during the yearly scheduling process. Therefore, pushback design plays a key role in defining annual cash flows to be generated from a mining operation. There may be many alternative push back sequences for an open pit, which can result in obtaining different NPV from a mining project. Since the 1960s many push back design algorithms have been developed. Two strategies were followed through the previous push back design algorithms:

1. Determining the push backs with the assumption that the ***best ore*** is the ore that has the *highest grade* and needs to have a *very low stripping ratio* to be extracted. Economic parameterization methods are classified in this group. In these methods a series of pits are generated by applying the ultimate pit limit algorithms (Learchs and Grossman's algorithm[1,2] and network flow one[3]) on different economic block models of the deposit. These economic models can be generated by changing some economic parameters such as commodity price, cut-off grade or mining and processing costs. The most widely used algorithm in this regard is Whittle's method[4,5]. Wang and Sevim[6] proposed a push back design algorithm using reserve parameterization instead of economic parameterization. Their algorithm was based on obtaining maximum metal pit containing *M* blocks through *M+N* blocks in the model. In 1986 a heuristic approach was formulated by Gershon[7] which considers the quality of the ore, the position of the ore block and the quality of the blocks under the desired ore block.

Experience showed that defining the best ore as above may not lead to maximizing the

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achievable NPV of a mining project, because this strategy does not necessarily take into account the stripping required to get the blocks with the highest ore grade[8].

2. Determining the push backs with the assumption that the ***best ore*** is the *ore that has a high grade* and its extraction *needs to have the least amount of waste stripping*. Ramazan and Dagdelen[8] proposed the *minimum stripping ratio push back design algorithm* based on this strategy. Their algorithm finds the push backs that each of them has the minimum stripping ratio among all possible push backs with the same size. They claimed that the schedule obtained by this algorithm will reach the ore blocks faster than previous algorithms. This leads to increasing the NPV of a mining project.

The optimal scenario for push backs is sensitive to the uncertainties concerned with the inputs to the optimization model. Dimitrakopoulos[9] classified the uncertainties of mining projects as follows:

- ➤ Uncertainty of the orebody model and related *in situ* grade variability and material type distribution.
- ➤ Uncertainty of technical mining specifications such as slope constraints, excavation capacities, etc.
- ➤ Uncertainty of economic issues including capital and operating costs, and commodity prices.

Among these uncertainties, the uncertainty related to the orebody model and *in situ* grade variability is the major contributor causing expectations not to be met in the early stages of a projects. Valee[10] reported that in the first years of operation after start-up, 60% of the surveyed mines had an average rate of production less than 70% of the designed capacity. All the above methods and algorithms work with the best estimate of grade ignoring the uncertainty that accompanies the estimated grades.

This paper introduces a new method that incorporates grade uncertainty into the push back generation process.

From the mentioned algorithms in the previous paragraphs, Whittle's is one of the most widely used one in the push back design process. Because our work is based on Whittle's algorithm, this model will be reviewed here. To demonstrate Whittle's idea, let us define the block economic value of the i^{th} block (BEV_i))[9]:

$$BEV_i = (TO_i * G_i * R * P) - (TO_i * PC) - (TR_i * MC) \quad [1]$$

TO_i: The total amount of ore in the block *i*.
G_i: Estimated grade of i^{th} block.
R: The proportion of the product recovered by processing the ore.
P: The price obtainable per unit of product less any delivery costs.
PC: The extra cost per ton of mining and processing of a block as ore rather than treating it as waste.
TR_i: The total amount of rock (ore and waste) in the i^{th} block.
MC: The cost of mining and removing a ton of waste.

Whittle reduced the number of economic variables by dividing Equation [1] by *MC*:

$$\frac{BEV_i}{MC} = \left(TO_i * \overline{G}_i * R * P/MC\right) - \left(TO_i * PC/MC\right) - \left(TR_i\right) \quad [2]$$

P/MC is the amount of product that should be sold to pay for mining a ton of material. This is the only significant variable, because PC/MC is not expected to change significantly unless there is a considerable change in the cost components, or a new mining or processing method is introduced[5]. Equation [2] can be rewritten as:

$$V_i = (TO_i * \overline{G}_i * R * \lambda) - (TO_i * \theta) - (TR_i) \quad [3]$$

Where V_i is the value generated per unit cost of mining, λ and θ are equal to *P/MC* and *PC/MC* respectively.

In order to generate push backs, λ is changed in each step and then the Learchs and Grossman algorithm is implemented on each economic converted block model.

New definition of the best ore

We set up our definition about the best ore as:

Material that has high grade, a low amount of uncertainty and needs a low waste stripping for extraction.

According to the above definition the best place to start the mining is a push back that is likely to be worth most. Before complete explanation of the suggested algorithm, an example is given. Assume that there are two blocks in an orebody block model: block *A* and *B*. Also the probability distribution of the grade at each block is assumed to be the classical normal distribution with the mean and variance of being equal to the Kriging estimate ($\overline{G}_i$) and estimation variance (σ_i^2) respectively (these two values can also be obtained by using geostatistical simulation techniques). These two values for blocks *A* and *B* are listed below:

	$\overline{G}_i$(%)	σ_i^2(%)^2
Block *A*	6	6.25
Block *B*	5	1

According to the probability theories when the normal curve is applicable, with the probability of 95% the true mean grade of each block (G_i) will fall within the standard deviation of the mean (σ_i)[11]:

$$\Pr[\overline{G}_i - 1.96\sigma_i \le G_i \le \overline{G}_i + 1.96\sigma_i] = 0.95 \quad [4]$$

In the other words, with a 95% confidence level the true mean grade will place inside $\overline{G}_i \pm 1.96\sigma_i$ or:

$$\overline{G}_i - 1.96\sigma_i \le G_i \le \overline{G}_i + 1.96\sigma_i \quad [5]$$

Hence, the confidence intervals of the mean grade for these two blocks can be calculated as:

$$6 - 1.96 \times 2.5 \le G_A \le 6 + 1.96 \times 2.5 \quad \Rightarrow \quad 1.1 \le G_A \le 10.9$$

$$5 - 1.96 \times 1 \le G_B \le 5 + 1.96 \times 1 \quad \Rightarrow \quad 3.04 \le G_B \le 6.96$$

Where, G_A and G_B are the true but unknown grade of blocks *A* and *B* respectively. As is clear from the above calculation, with the probability of 95% the least amount of grade for block *A* (1.1) is less than that of block *B* (3.04). Now if the cut-off grade is assumed to be 1.5%, then removing block *A* before block *B* has more risk than extraction of block *B* before *A*, because there is a probability that block *A* is a waste block. Therefore, despite that block *A* has more mean grade value than block *B*, it may be better to extract block *B* before block *A*. Now assume that block *A* has a standard deviation of 1.3%. Then we have:

$$6-1.96\times 1.3 \le G_A \le 6+1.96\times 1.3 \quad \Rightarrow \quad 3.45 \le G_A \le 8.548$$

In this case it is more favourable to extract block *A* before block *B*. The suggested algorithm will determine these preferences.

In order to determine push backs with the above objectives, first of all a set of (up to 100) nested pits should be generated. The correct combination of these nested pits leads to generating reasonable push backs for manoeuvering space and fewer costs. In the following section we outlined a new method to generate nested pits in accordance with our definition of the best ore.

New suggested push back design algorithm

In order to generate nested pits with the mentioned objectives, the concept of parametric analysis is applied. As stated before, in this concept nested pits are generated by the gradual modification of one or more key parameters and then the LG algorithm is implemented to seek the corresponding pit outline for each modification. In this algorithm the economic value of each block in the model would be reduced in each step in a manner such that this reduction is proportional to the block uncertainty. This means that if there are two blocks in the model with the same average grade and different degrees of uncertainty, the reduction in the economic value of that block with the greater amount of uncertainty will be more than the other one.

Block grade standard deviation can be considered as an indicator for block grade uncertainty. In this case the block economic value should be reduced in each step with an amount proportional to the block grade standard deviation. In order to introduce this idea into block economic value we suggest the following formulation to calculate the block economic value:

$$BEV(i) = [TO_i * (\overline{G}_i - n\sigma_i) * R * P] - (TO_i * PC) - (TR_i * MC) \qquad [6]$$

where:

n: is a constant number whose variation results in generating different pits.

σ_i: standard deviation of i^{th} block grade.

The approach is to set up the block values using the lowest value of *n* (zero) and then to obtain the optimal pit. This pit is the ultimate pit and all blocks outside it can be excluded from further calculation. If we increase *n* in the next stage, the average grade of ore block and, consequently, the block economic value is reduced. Higher standard deviation results in a greater decrease in the value of a block. Therefore, smaller nested pits contain more valuable and less uncertain ore blocks than bigger pits. Applying this formulation results in generating a range of optimal pits (nested pits) covering the likely range of values of average ore grade.

Let us assume that we generate an optimal pit (pit I in Figure 1) using Equation [4]. Now we want to find the smaller pit with more valuable and certain blocks. If we increase *n* in Equation [4] and recalculate the block values, it is clear that the value of every block in the model will either increase or stay the same (waste blocks), but this reduction is more in the blocks with the greater amount of uncertainty (blocks with higher standard deviation). Another optimization using the new values results in another pit outline (shown as pit II in Figure 1), which is included by pit I. Pit I consist of pit II plus some additional ore blocks and their attendant waste.

It should be noted that it may be possible that a small increase in *n* value does not lead to creating a smaller pit. Now if we step the *n* through a series of ascending values

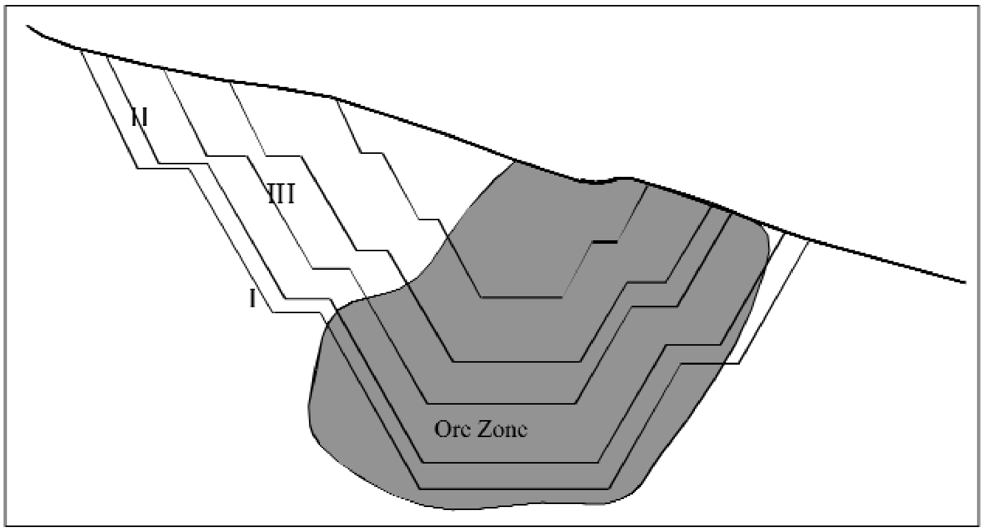

Figure 1—A cross-section of pit showing the relationships of nested pits to the deposit

and then, do an optimization for each generated economic block model, we obtain a set of nested pit outlines. It may be possible, for example, that after 100 steps, only 40 or 50 nested pits are generated. The smallest shell contains the blocks that are probably the most valuable ones. These shells are actual pit approximations with various degrees of risk but are not called 'push backs' because of the lack of smoothing, benching or access road. A set of shells that incrementally produces a certain tonnage of ore and meets some minimum mining width criteria serves as a push back.

Interval estimate of n

The values of *n* which are selected for parametric analysis should be such that they reflect the block grade uncertainty correctly. In the other words, each assigned block grade in each step should be one of the possible grade realizations that is expected to be reached after block extraction.

From Equation [4] it is clear that the true ore block grade is greater than $\bar{G}_i - 1.96\sigma_i$. With reference to Equation [6], *n* should be less than 1.96. Also, it should be noted that each possible average block grade should be greater than or equal to zero, i.e.:

$$\bar{G}_i - n\sigma_i \geq 0 \quad \Rightarrow \quad n \leq \frac{\bar{G}_i}{\sigma_i} \qquad [7]$$

Hence:

$$n \in [(0 \leq n \leq 1.96) \cap (n \leq \frac{\bar{G}_i}{\sigma_i})] \qquad [8]$$

For example, assume that for block *i* we have $\bar{G}_i$ and σ_i = 12%, then we have:

$$n \in [(0 \leq n \leq 1.96) \cap (n \leq \frac{54}{12})] = [(0 \leq n \leq 1.96) \cap (n \leq 4.5)]$$

$$\Rightarrow \quad 0 \leq n \leq 1.96$$

If we have $\bar{G}_i - n\sigma_i < 0$ then, $\bar{G}_i - n\sigma_i$ is set to zero.

In some cases we do not have any information about the probability distribution of block grade, but we know its mean and variance. With this limited information we can also estimate the interval for *n* by using Chebyshev's Theorem[11]:

Let *X* be any random variable for which the expected value (μ) and variance (σ^2) may be found. Then:

$$\Pr[\mu - k.\sigma \leq X \leq \mu + k.\sigma] \geq 1 - \frac{1}{k^2} \qquad [9]$$

This means that the probability that a random variable will fall within $\pm k$ standard deviation of its mean is at least $1 - \frac{1}{k^2}$.

According to the above theorem, if we put *k*=4.5, then we have:

$$\Pr[\mu - 4.5\sigma \leq X \leq \mu + 4.5\sigma] \geq 0.95 \qquad [10]$$

From Equation [10], it is clear that the ore block grade is greater than that with a probability greater that of 0.95.

With reference to the Equation [6], *n* should be less than 4.5.

As stated before, if we use kriging in order to block grade estimation, $\bar{G}_i$ is kriging estimation and is the minimum estimation variance or Kriging variance[12].

Kriging is a geostatistical method that estimates the ore block grade so that the mean squared error is minimized; therefore, the variance of value estimated by kriging is smaller than the real but unknown variance. This smoothing of true variability of the grade leads to overestimation of low grades and underestimation of high grades. Additionally, smoothing is a function of data density and configuration: areas of greater density will show more local variability while areas having sparse data will be more uniform[13]. Hence, production schedules that are based on kriging cannot account for probable deviation in production targets and obtained plans will provide only erroneous conclusions. Multiple indicator kriging works on a probabilistic basis to define the distribution of grade of samples within each search window, providing a discrete approximation to the conditional cumulative distribution function for each block[14]. Rather than kriging, although the probabilistic estimates produced by the multiple indicator kriging method often reduces the smoothing effect, but the local grade variability may still be incorrectly characterized.

The best way to quantify grade uncertainty is conditional simulation. Conditional simulation is a generalization of the Monte Carlo type simulation approach, which considers three-dimensional spatial correlation[12,15]. It produces independent equally probable images of *in situ* orebody grades which have the following characteristics[16]:

- At the sampled location, the simulated values of each variable are the same as the measured values of those variables.
- All the simulated values of a given variable have the same spatial relationships as observed in the data value.
- All the simulated values of any pair of variables have the same spatial interrelationships as observed in the data values.
- The histograms of the simulated values of all variables are the same as those observed for the data images.

Each simulation run produces an image or a realization of the deposits that correctly reflect the statistical and spatial variability of the real data. Performing several independent simulations is required to assess the impact of local variability.

If we have the results of simulation, then we have several grade samples for each block whose mean and variance can be used in the risk analysis process.

Illustration of an example

A 2D block model configuration containing the average grade and standard deviation of each block grade is shown in Figures 2 and 3 respectively.

In calculating the values per ton of blocks, it is assumed that the copper price is \$1100/ton, mining cost is \$5/ton of material, processing cost is \$20/ton of ore, the recovery factor is 80% and the specific weight of ore and waste is 2.9 and 2.3 ton/m^3 respectively.

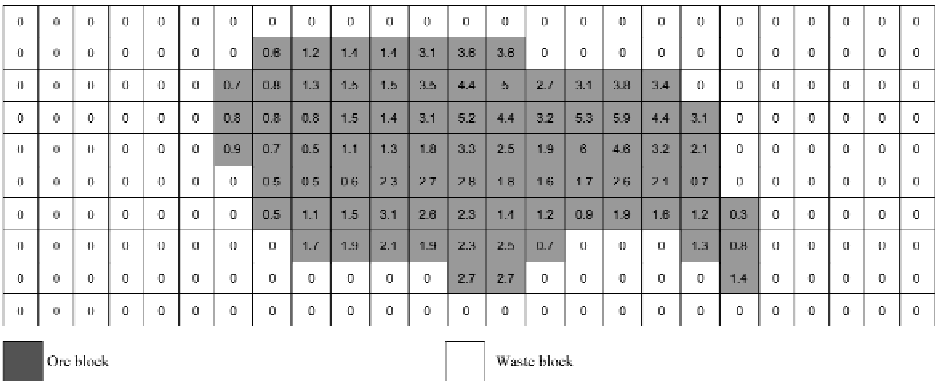

Figure 2—Cross-section view of a 2D example showing the average block grade (%)

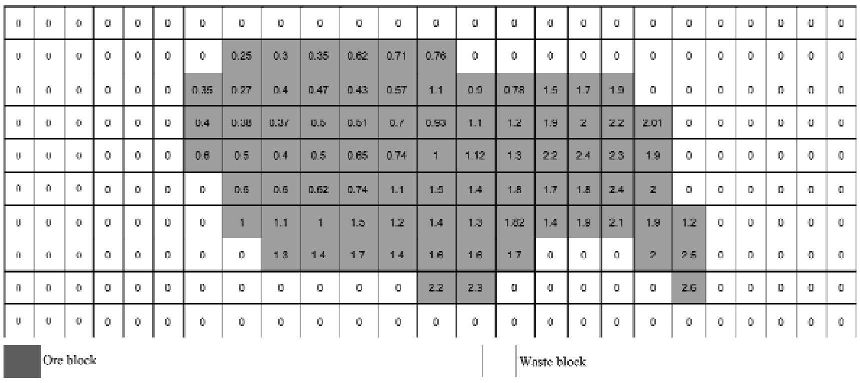

Figure 3—Cross-section view of a 2D example showing the standard deviations (%^{2})

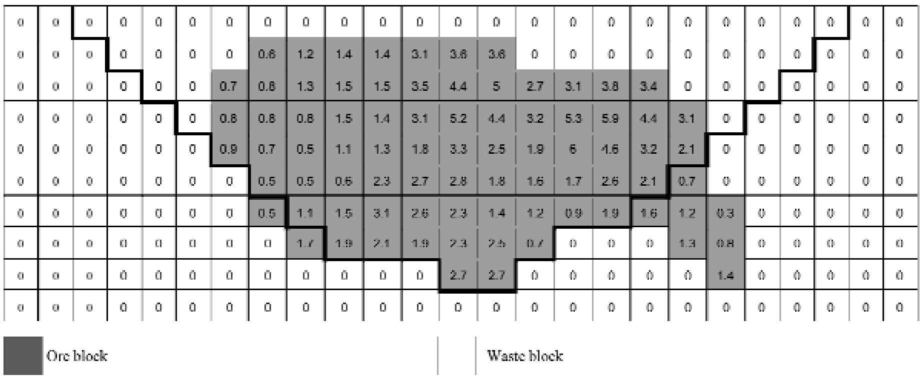

Figure 4—Cross-section view of the optimal pit

First of all, the ultimate pit limit should be determined. This can be performed by applying the 2D LG algorithm on the economic block model. The ultimate pit limit of this example is shown in Figure 4.

When Whittle's algorithm is implemented in the example, no push back is obtained until λ reaches 54. Note that θ is kept constant at 4. The generated push back is shown in Figure 5.

As can be seen from Figure 5, the first push back generated using Whittle's method is very large. In other words, there is a big jump from no pit to the first push back. This problem is called the 'gap problem'. This problem causes some difficulties in the effective application of the push backs to the yearly production scheduling.

By increasing the step-by-step of the λ, ultimately 9 push backs are generated, as shown in Figure 6.

The new method is applied also on the 2D block model which was shown in Figure 2. If in Equation [4], n is set to zero, the ultimate pit limit (Figure 4) is generated. The first push back is generated at $n = 0.2$. This pit is shown in Figure 7.

With successive increasing of n, 12 push backs are generated, which are shown in Figure 8.

The smallest push back is in accordance with $n = 2.9$. As is clear from Figures 6 and 8; the smallest push back that is generated using the suggested algorithm is smaller than that of Whittle's algorithm. This means that the gap problem is reduced by using the new suggested method.

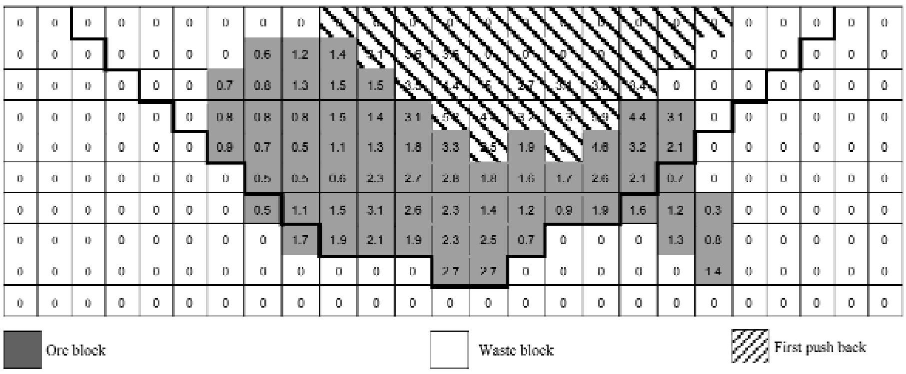

Figure 5—Cross-section view of the first push back generated using Whittle's method

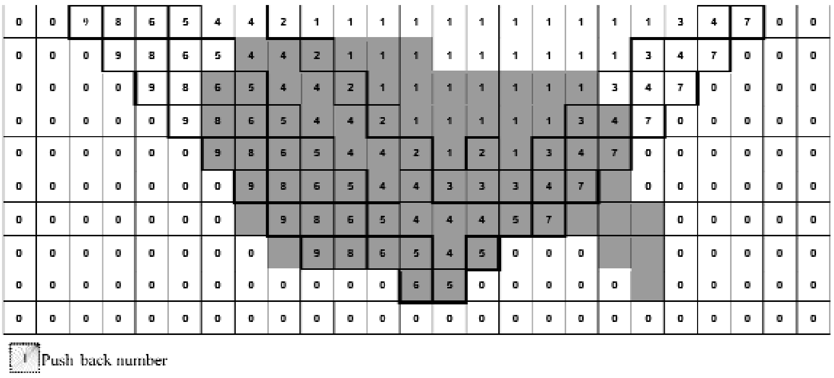

Figure 6—Cross-section view of the push backs generated using Whittle's method

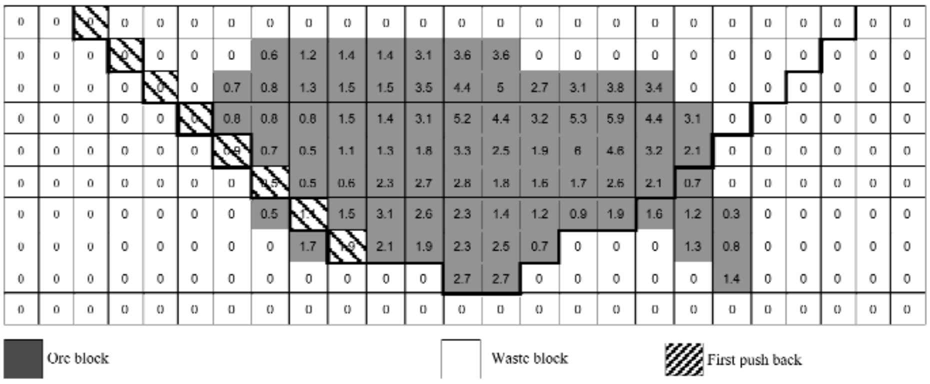

Figure 7—Cross-section view of the first push back generated using the suggested method

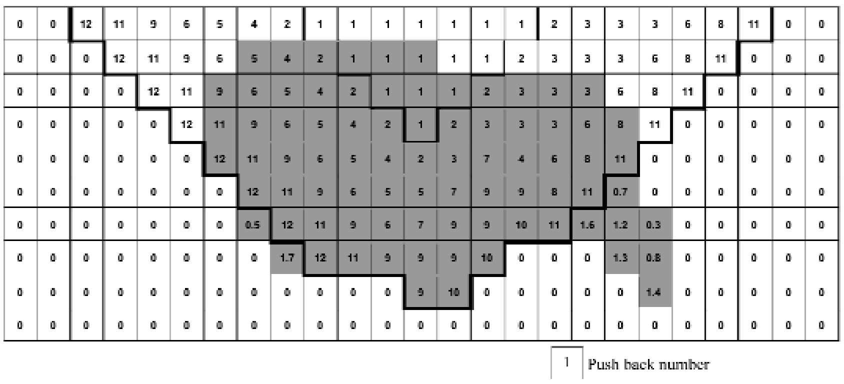

Figure 8—Cross-section view of the push backs generated using the suggested method

In order to compare the risk associated with these two methods, it is assumed that 5 blocks are mined in each year. Also for each block a risk indicator should be defined. This indicator is set to the estimation variance of each block (σ^2). Blocks with a higher amount of variance contain more risk. The sum of the undiscounted uncertainty associated with each push back is considered a 'push back risk indicator' (PRI) and can be calculated as follows:

$$PRI_k = \sum_{i=1}^{n_k} \sigma_i^2 + \sum_{i=1}^{n_k} \sum_{j=1}^{n_k} cov(i,j) \qquad i \neq j \qquad [11]$$

where:

PRI_k: undiscounted risk indicator for k^{th} push back; k=1,2,…,K. K is total number of push backs within optimal pit outline.

$Cov(i,j)$: the covariance between block i and block j within the k^{th} push back.

n_k: total number of blocks within the k^{th} push back.

Because the risk in the early years of the production period is more critical, a discount rate should be applied to each push back risk indicator. The 'discounted push back risk indicator' (DPRI) for a series of push backs can then be calculated as:

$$DPRI = \sum_{k=1}^{K} \frac{PRI_k}{(1+i_k)^{t_k}} \qquad [12]$$

where:

t_k: the year at which the extraction of the k^{th} push back is finished.

i_k: the discount rate for k^{th} push back. If we have the yearly discount rate and also each push back's life, then we can obtain i_k easily.

The discount rate of 12% is considered in order to calculate the DPRI associated with the schedules coming from both Whittle's and the suggested algorithms.

The number of ore and waste blocks within each push back, the life of each push back, the push backs risk indicator and discounted push back risk indicator for both Whittle's and the suggested algorithm are shown in Tables I and II respectively. It should be noted that the blocks' covariance is not considered in these calculations.

The cumulative amount of the discounted push back risk indicator in each period for these two methods is plotted in Figure 9.

As is clear from Figure 9, the cumulative amount of the discounted push back risk indicator in our method is less than that of Whittle's one. This indicates that the new algorithm extracts the more certain areas of deposit in earlier production periods and more uncertain areas are left for later periods, when additional information usually becomes available. Hence, implementing the yearly production schedule within push backs coming from the new method,

Table I

The undiscounted and discounted push back risk indicator using Whittle's algorithm

Push back number	Number of ore blocks	Number of waste blocks	Push back life time (years)	Undiscounted push back risk indicator	Discounted push back risk indicator
1	17	17	6.8	30.388	14.061
2	5	1	1.2	3.478	1.405
3	5	3	1.6	18.706	6.302
4	17	5	4.4	27.773	5.683
5	9	2	2.2	15.117	2.411
6	7	2	1.8	9.878	1.285
7	3	4	1.4	12.96	1.438
8	5	3	1.6	4.644	0.43
9	4	4	1.6	3.889	0.3
Total	72	41	22.6	126.833	33.315

Table II

The undiscounted and discounted push back risk indicator using the new algorithm

Push back number	Number of ore blocks	Number of waste blocks	Push back life time (years)	Undiscounted push back risk indicator	Discounted push back risk indicator
1	7	9	3.2	4.676	3.254
2	6	3	1.8	3.616	2.052
3	7	6	2.6	19.54	8.053
4	5	1	1.2	5.959	2.198
5	6	1	1.4	4.355	1.371
6	7	5	2.4	13.31	3.135
7	3	0	0.6	5.61	1.257
8	3	3	1.2	12.57	2.458
9	13	2	3	26.114	3.635
10	3	0	0.6	10.403	1.319
11	8	7	3	17.64	1.633
12	4	4	1.6	3.89	0.3
Total	72	41	22.6	126.833	30.665

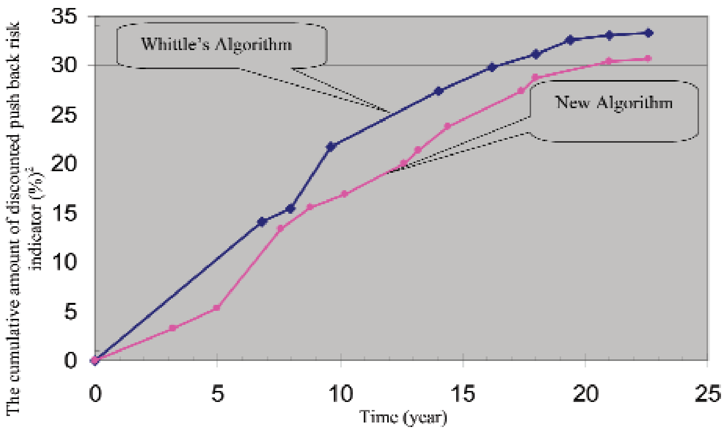

Figure 9—The cumulative amount of the discounted push back risk indicator in each period for Whittle's and new methods

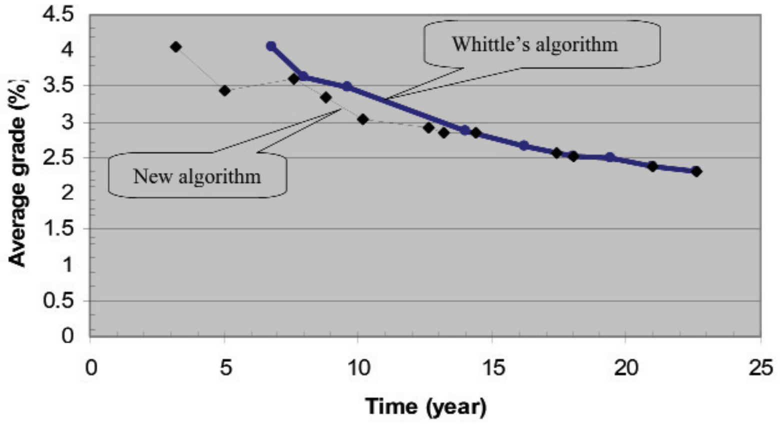

Figure 10—The average ore grade of each series of nested pits generated using both methods

results in producing schedules whose probability of meeting production targets is higher than that of Whittle's.

The average grade of the total amount of ore in each series of nested pits generated by using both methods is calculated and graphically shown in Figure 10.

The average grade of nested pits generated using the new method is less than that generated using Whittle's method before year 14. After that the nested pits generated using both methods have the same average grade.

The justification of this reality is that there are some blocks in the model that have a high amount of risk and contain medium to high grade ore. This model prefers to extract low risk and low–medium ore blocks instead of medium–high grade blocks that have a high amount of uncertainty in the earlier production periods. As a consequence, the theoretical (not achievable) NPV in Whittle's method may be higher than that of the new algorithm, but the achievable NPV using the new method will be more than that of Whittle's, because increasing the certainty of the extraction process can indirectly lead to increasing the achievable NPV.

Conclusion

In order to design push backs, a parametric method is developed that incorporates grade uncertainty during the push back design process. It is believed that the most important characterizations of the suggested method are:

- High-grade zones that have a low amount of uncertainty are selected for extraction in the earlier production periods.
- High-grade zones that have medium amount of uncertainty may also be selected for extraction in earlier production periods. In those parts of the deposit, the average grade of each block is high and then, subtracting it from a coefficient of standard deviation will not cause a high reduction in the ore values.
- High-grade zones with a high amount of uncertainty are left to be extracted in the middle life of mine when n in Equation [4] is still at the middle of its range.
- Low to medium-grade zones with a low amount of uncertainty may also be valuable when extracted at the middle life of mine.

- Low to medium-grade zones with a high amount of uncertainty are left to be extracted at the end of mine life, because in this case the reduction due to increasing of *n* is large and it may even be possible that a block grade is reduced to an amount less than the cut-off grade.

It is shown that the number of push backs generated using this method is more than that of Whittle's method. This indicates that the gap problem is eliminated or at least reduced using the suggested method.

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