Interaction of slurry pipe flow with a stationary bed

by V. Matoušek*

Synopsis

The paper addresses pipe flows of aqueous medium-sand slurries at velocities lower than the deposition-limit velocity. Laboratory test results including measurements of particle concentration distributions are used in order to describe a flow with a (pseudo-) linear concentration gradient above a stationary bed. Particle dispersion within the flow is evaluated and a stratification-ratio equation is formulated, estimating a portion of contact load in the flow. The formula is incorporated in the proposed predictive model for the hydraulic gradient for a medium-sand flow above a stationary bed.

Introduction

The pipe experiments on a slurry flow over a stationary bed were carried out in two laboratories, the SRC laboratory (Pugh, 1995) and the Dredging laboratory of Delft University of Technology (Matoušek, 2004, 2006). The experimental data from both laboratories included concentration profiles across slurry pipes. Pughs experiments were analysed in a series of articles in the nineties (e.g. Pugh and Wilson, 1999a, 1999b). In this paper Pughs original data from his PhD thesis are compared with the Delft data.

Experimental data

Laboratory tests

During the 2004 tests in the Dredging laboratory at Delft University of Technology, a new beam collimator was used for the radiometric concentration profiler. It made it possible to produce concentration profiles of accuracy and quality comparable with the profiles from the earlier SRC tests. In Delft, medium sand of median-diameter $d_{50} = 0.37$ mm was tested in a 150-mm pipe (a circular pipe of internal diameter D = 150 mm). Ten tests runs were collected for different slurry-flow conditions. Pughs SRC tests were carried out in a 105-mm pipe for two fractions of sand and for one fraction of bakelite. All the results of Pugh's tests for the two sand fractions (d_{50} = 0.3 mm (6 test runs), and $d_{50}0 = 0.56$ mm (4 test runs) are used here for comparison with the sand-test results from the Delft laboratory. In both laboratories, the measured quantities in addition to local concentrations were the mean velocity in the entire pipe cross section, V_m, the hydraulic gradient, I_m, and the mean delivered volumetric concentration of transported particles, C_{vd}. The mean spatial concentration of particles, C_{va}, in a discharge area above a top of the bed was obtained from the integration of a concentration profile across a discharge area. The Delft tests covered a broad range of the Shields number values ($3 < \theta_b < 21$) and included flows with fairly high average concentrations of solids in flow above the bed (C_{vd} up to 0.26, C_{va} up to 0.35). The SRC tests produced data for $3 < \theta_b < 14$ and C_{vd} up to 0.15 (C_{va} up to 0.25).

Determination of bed shear stress

A standard method is used to determine the shear stress, τ_b, at the top of a stationary bed of the thickness y_b from the measured integral quantities (V_m, I_m) of the tested flows. The method requires the mean velocity of the flow above the bed, V_a, and it is determined from the measured y_b and V_m using the continuity equation. The discharge area A_a, i.e. the area with the bottom boundary given by y_b in the pipe cross section, $A_a = f_n(y_b, D)$, is confined by two boundaries of two different values of the hydraulic roughness. A pipe wall is a smooth boundary; its Blasius-type friction law is calibrated for each particular pipe using water-flow data (see coefficients α and β later

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in the text). The top of a stationary bed is the other boundary with its own friction law. The discharge area A_a is divided into two areas, each associated with one of the boundaries (see Figure 1). In the area associated with the pipe wall, A_{aw}, the flow is influenced by friction at the pipe wall, in the area associated with the top of the bed, A_{ab}, the flow is influenced by bed friction. Obviously, $A_a = A_{aw} + A_{ab}$.

It is assumed that the hydraulic gradient and mean velocity of flow are equal in both areas. The balance of driving and resisting forces in a pipe section of a unit length holds for both areas, i.e. $\rho_f.g.I_m.A_a = \tau_w.O_w + \tau_b.O_b$, $\rho_f.g.I_m.A_{ab} = \tau_b.O_b$, $\rho_f.g.I_m.A_{aw} = \tau_w.O_w$. In the equations, O is the perimeter of a particular boundary and τ is the shear stress at a particular boundary. The boundary shear stress is related to the mean velocity of the flow through λ, the Darcy-Weisbach friction coefficient for a particular boundary. The equations read

$$\tau_w = \frac{\lambda_w}{8} \cdot \rho_f \cdot V_a^2, \text{ and } \tau_b = \frac{\lambda_b}{8} \cdot \rho_f \cdot V_a^2$$

Further, each particular area is characterized by its hydraulic radius, $R_{hw} = A_{aw}/O_w$ and $R_{hb} = A_{ab}/O_b$. A determination of the shear stress at the pipe wall, τ_w, is based on the assumption that the same pipe-wall friction law applies to flow of water through the pipe free of solid particles and for flow of slurry above a stationary bed in the same pipe. In other words, it is assumed that particles in a pipe do not alter the friction law of a pipe wall. The friction law for a smooth pipe wall relates the friction coefficient to the Reynolds number of the flow through the area associated with the pipe wall,

$$\lambda_w = \frac{\alpha}{Re_w^\beta} = \frac{\alpha}{\left(\dfrac{V_a \cdot 4 \cdot R_{hw}}{\nu_f}\right)^\beta}$$

The values of the empirical coefficients α and β are found from water-flow tests in a pipe free of particles ($\alpha = 0.244$ and $\beta = 0.212$ for the 150-mm pipe in the Delft laboratory). This equation, together with the momentum equation,

$$I_m = \frac{\tau_w}{\rho_f \cdot g \cdot R_{hw}}$$

and the shear-stress equation,

$$\tau_w = \frac{\lambda_w}{8} \cdot \rho_f \cdot V_a^2$$

enables one to determine an R_{hw} value for observed slurry-flow conditions characterized by certain measured values of y_b, V_a, and I_m. From the R_{hw} value all other required parameters are determined, including R_{hb} and τ_b.

Slip ratio

During the tests in both laboratories the values of delivered volumetric concentration, C_{vd}, were measured in observed slurry flows. These can be compared with the values of spatial volumetric concentration, C_{va}, of particles in the discharge area above the bed. The comparison provides interesting information about slip in the observed flows. Generally, the C_{vd} differs from the C_{va} provided that there is a slip between the phases in a flow. The flows of all tested sand fractions exhibited differences between C_{vd} and C_{va}, that were much larger than those in flows without a stationary bed in the pipes. The observed trend in a development of the slip ratio C_{vd}/C_{va} was not surprising (Figure 2). The C_{vd}/C_{va} values grew towards one with the increasing values of the u^*_b/v_t ratio (u^*_b is the bed friction velocity, $u^*_b = (\tau_b/\rho_f)^{0.5}$, v_t is the terminal settling velocity of solid particle). This meant that flows with steeper concentration gradients exhibited smaller slip ratio than flows with less steep gradients.

Solids distribution in slurry flow

Shape of concentration profile

For nine concentrations profiles obtained in flows of $C_{vd} < 0.18$, the Delft tests confirmed Pugh's observations (Pugh, 1995, all ten sand-slurry Pugh profiles were for $C_{vd} < 0.16$) that the concentration profiles in the flow above the top of the

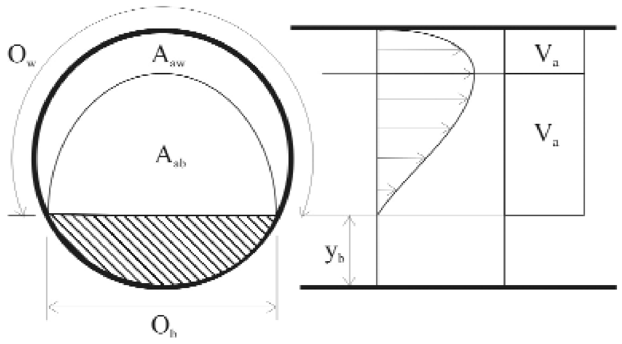

Figure 1—Schematic geometry of pipe cross-section and schematic distribution of velocity in flow of slurry through discharge area above stationary bed

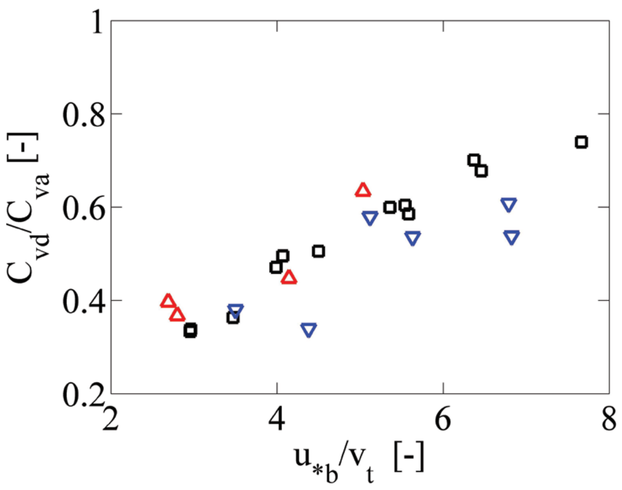

Figure 2—Slip ratio in flow above stationary bed (squares—0.37-mm sand in the 150-mm pipe, downward-pointing triangle—0.3-mm sand in the 105-mm pipe, upward-pointing triangle—0.56-mm sand in the 105-mm sand)

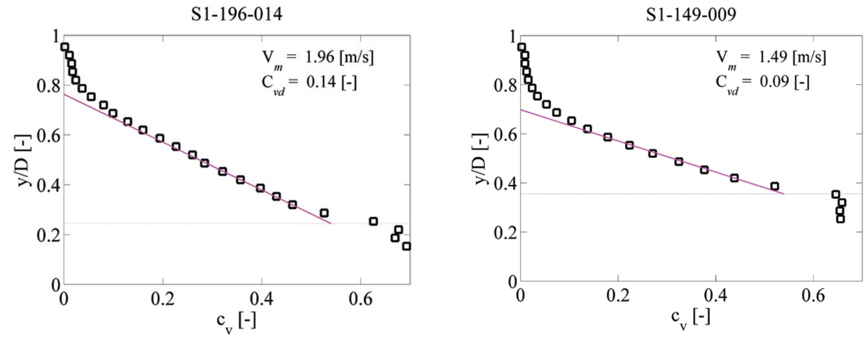

Figure 3a—Concentration profiles in the 150-mm pipe (squares—measured local concentrations, lines—best-fit lines)

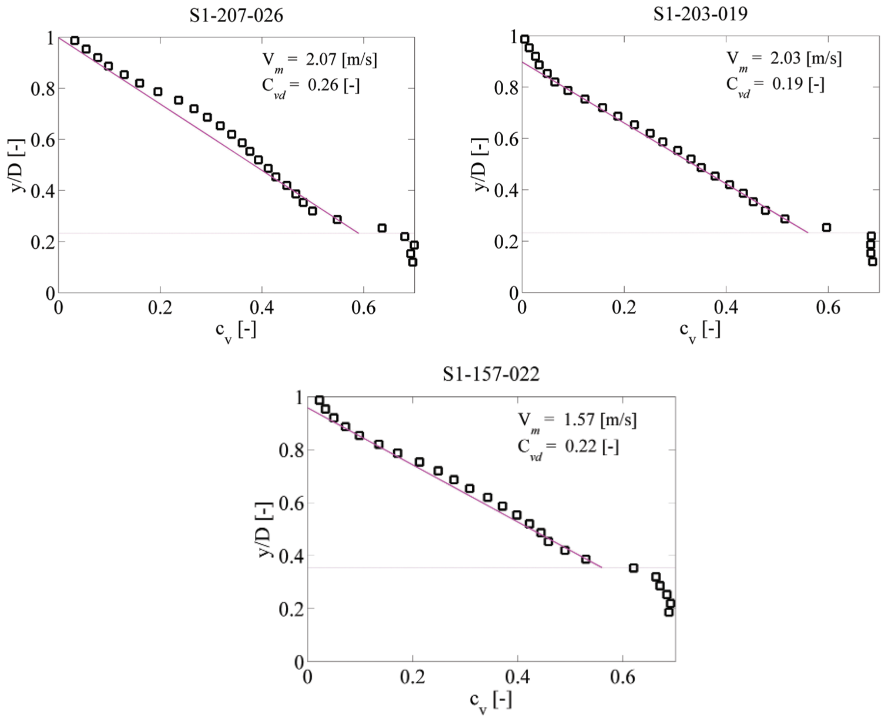

Figure 3b—Concentration profiles in the 150-mm pipe (squares—measured local concentrations, lines—best-fit lines)

bed could be approximated by a line from which the measured local concentrations diverted just around and above the top of the linear profile (Figure 3a). Above the top of the approximated linear profile, a real profile formed a low-concentration 'tail' that shifted the position at which the local concentration reached zero to some vertical position above that proposed by the linear profile. Typically, the measured profiles started to divert from the proposed line at positions where the local concentration fell below 0.05–0.10.

The Delft tests produced three additional concentration profiles in higher concentrated flows of $C_{vd} > 0.18$ (and $u^*b/v_t > 6$) and those exhibited shapes that could not be simply approximated by a line (Figure 3b). Instead, the local concentrations in the mid-region of the flow were higher than the estimated linear profile would suggest.

Furthermore, the Delft tests revealed that attention had to be paid to the way how a linear profile was linked to the top of a stationary bed (see both Figures 3a and 3b). Pugh suggested that a linear profile started at the local concentration, c_{vb}, equal to that typical for an entire stationary bed (typically $c_{vb} = 0.65$). However, shapes of the Delft profiles suggest that the local concentration, c_{Ob}, at some position just above the top of the bed is more suitable to link a linear profile with the top of the bed. Its value can be considerably smaller than 0.65, typically about 0.54.

Delimitation of shear layer

Pugh and Wilson suggested that all particles within their linear layers (called shear layers) were supported exclusively by mutual contacts. Thus the intergranular normal stress that those particles exerted against the top of the bed produced a solids shear stress that was equal to the bed shear stress, i.e. the shear stress opposing the flow at the top of the stationary bed (e.g. Pugh, 1995, Pugh and Wilson, 1999a). From this relationship the height of the region with the linear profile, H_{lin}, could be determined for the known bed friction velocity, u^*_b, mean volumetric concentration of particles within the region ($c_{Ob}/2$) and the friction coefficient tanφ' in which φ' was the dynamic version of the static friction angle of the transported solids. The equation for H_{lin} reads

$$H_{lin} = \frac{2 \cdot u_{*b}^2}{(S_s - 1) \cdot g \cdot c_{0b} \cdot \tan\varphi'} \qquad [1]$$

and S_S is the relative density of solid particles, c_{Ob} the local concentration at the origin of the linear profile above the top of the bed and H_{lin} the height above the top of the bed of the position at which the linear profile reaches the zero concentration. Provided that the height H_{lin} is estimated from the measured shape of a concentration profile, Equation [1] can serve to estimate the tanφ' value required to establish the linear profile of the particular height H_{lin} produced by the particular bed friction velocity u^*_b. An analysis of Delft and SRC profiles showed that for a majority of the measured profiles the value of tanφ' had to be taken very low in order to produce linear profiles obtained as best-fit lines from the measured profiles. Instead of expected values of about 0.32–0.4 or more, the Delft tests suggested the tanφ' values round 0.21–0.25. Pugh's SRC tests with slurries of a similar sand (0.3-mm sand) suggested values far below 0.3 too. Only Pugh's coarse-sand tests (0.56-mm sand) gave the tanφ' values within the expected range.

The surprisingly low values of tanφ' obtained from the linear profiles processed using Equation [1] indicate that constructed linear layers would be rather 'heavy' if they contained only particles supported by interparticle contacts. The normal stress exerted against the top of a bed by the submerged weight of the particles occupying a linear layer was big. Therefore low values of the friction coefficient tanφ' had to be taken to produce values of bed shear stress equal to those determined from the measurements.

The balance between the solids shear stress and shear stress in flow must hold not only at the bottom of a linear layer but also at each vertical position within the linear layer. This means that no particles could be supported by mutual contacts at the vertical position of the zero flow shear stress (i.e. the vertical position of the maximum local velocity) and above. From this point of view, the estimated tops of the pseudo-linear profiles presented in Figure 3b can hardly represent upper boundaries of shear layers containing particles supported exclusively by interparticle contacts.

These observations suggest that the layers delimited by the boundaries of the measured (pseudo-) linear concentration profiles may contain a certain portion of particles supported by some dispersion mechanism different from interparticle contacts. It seems that a turbulent dispersal mechanism can support particles above a shear layer (a low-concentration 'tail' above the top of a linear profile due to the turbulent dispersal support reported by Pugh and Wilson (1999a)). It would be interesting to test theoretically whether the region of acting turbulent dispersal support can penetrate the upper boundary of a (pseudo-) linear layer and expand down to a certain portion of the layer. It is obvious that this expansion must have a limit; the turbulent dispersal mechanism cannot act in a region of high local concentration of particles.

Particle dispersion within (pseudo-) linear layer

Concentration profile gradients due to a turbulent dispersive mechanism support of transported particles seem to depend primarily on the ratio $v_t/(u^*_b.R_{hb})$. A concentration gradient across a flow in which solid particles are supported by turbulent eddies is often described using the Schmidt-Rouse equation with the implemented effect of hindered settling,

$$-\frac{dc}{dy} = \frac{v_t \cdot c \cdot (1-c)^m}{\varepsilon_S}$$

Earlier Delft tests of flows of medium-sand slurries (including the 0.37-mm sand discussed in this paper) through a 150-mm pipe without a stationary bed showed a virtually uniform distribution of the dispersion coefficient, ε_S, across the core of the flow. The mean value of the dispersion coefficient for different flow conditions could be estimated using

$$\frac{\varepsilon_{S,\text{mean}}}{u_* \cdot R} \approx 0.11$$

(Matoušek, 2000). A combination of the above equations suggests that the concentration gradient due to turbulent dispersion of medium-sand particles is primarily a function of the ratio $v_t/(u^*.R)$,

$$-\frac{dc}{dy} = \frac{c \cdot (1-c)^m}{const} \cdot \frac{v_t}{u_* \cdot R}$$

For concentration profiles in turbulent suspension flow above a stationary bed it seems appropriate to replace the $v_t/(u^*.R)$ ratio with the parameter $v_t/(u^*b.R_{hb})$ so that the concentration slope produced by the turbulent-diffusion model at any vertical position y_O above the top of the bed

$$-\frac{dc}{d(y_0/D)} = \frac{c \cdot (1-c)^m \cdot D}{K_{turb}} \cdot \frac{v_t}{u_{*b} \cdot R_{hb}} \qquad [2]$$

Recent comparisons of the Delft concentration profiles with predictions using the simple turbulent dispersal model (Equation [2]) showed that for certain conditions the predicted concentration gradients could reasonably follow the measured concentration gradients relatively deep into a (pseudo-) linear layer, even down to the vicinity of the centre of the linear layer (Matoušek, 2006). The model predictions diverted sharply from the measured gradients at lower vertical positions where local concentrations exceeded values of about 0.27–0.3. This simple test indicated that for (pseudo-) linear profiles with small gradients the region in which particles are supported exclusively by interparticle contacts may be confined to an area considerably smaller than that delimited by a best-fit linear profile.

A possible presence of turbulent support seems to be associated with best-fit profiles of small gradients. Equation [2] is used to estimate a threshold value of the slope of a best-fit linear layer, the upper half of which may be occupied by particles supported by turbulent dispersion. The best-fit-linear-profile gradients ($c_{Ob}.D/H_{lin}$) of the Delft profiles matched reasonably the theoretical gradients calculated for the centre of the linear profiles using Equation [2] ($K_{turb}/D \approx 0.24$) for gradient values smaller than say 2. This suggested that for the tested profiles with concentration gradients smaller than 2 the concentration profile in the discharge area above the centre of the linear layer could be approximated using the turbulent diffusion model. In the rest of the discharge area (i.e. at the vertical positions between the centre and the bottom of the linear layer), the concentration profile could be approximated by the line of the gradient given by Equation 2 [for] the centre of the linear layer.

A future work will investigate whether (pseudo-) linear layers developed in flows of high shear stress and having their tops near the top of a pipe cross-section can be seen as combinations of two sublayers with two different prevailing particle-support mechanisms. A preliminary hypothesis based on the simple considerations presented here states that a layer delimited by a best-fit linear profile may be split into a lower sublayer containing particles supported exclusively by mutual contacts and an upper turbulent-suspension sub-layer. An interface between the layers may be virtual rather than real and its vertical position above a bed variable with flow conditions. The problem of appropriate linking of a concentration profile above a bed with the top of the stationary bed (the discrepancy between c_{vb} and c_{Ob}) will be a part of the investigation.

Prediction of hydraulic gradient

Contact-load portion (stratification ratio)

The above discussion on solids distribution in flows over a stationary bed suggests that not all particles within a (pseudo-) linear profile may contribute to the contact load. Consequently, the linear-profile parameters H_{lin} and c_{Ob} may not always be suitable to quantify the number of particles that are transported as a contact load in a flow above a bed. For the purpose of a formulation of a predictive model for the hydraulic gradient for slurry flow above a stationary bed, we need an alternative method for the determination of the contact-load portion from the total concentration of transported particles in the flow. As stated earlier, the method for the determination of u^*_b from the measured flow quantities (y_b, V_m, I_m) requires that all transported particles contribute to total friction through u^*_b. This means that all particles should occupy the area associated with the top of the bed, A_{ab}. Therefore $C_{va}.A_a = C_{vab}.A_{ab}$. If we assume that not all particles contribute to friction through interparticle contacts, then the $C_{vab} = C_{vabc} + C_{vabt}$, where the concentration C_{vabc} represents the portion of particles contributing to contact load within the area A_{ab} and C_{vabt} the portion of particles supported by turbulent dispersal mechanism within the area A_{ab}. For a determination of the contact-load portion of the total amount of transported particles, the slurry-flow literature introduced the parameter called the stratification ratio, C_c/C_{vd}. In the stratification ratio, C_c represents the contact-load part of the C_{va} and is related with C_{vabc} through $C_c.A_a = C_{vabc}.A_{ab}$. The C_{vd} is usually used in the stratification ratio instead of C_{va} for practical reasons; the C_{vd} is an input parameter to predictive models for the hydraulic gradient in slurry pipes. The following reasoning shows how the values of the stratification ratio can be processed from the Delft and SRC database.

The bed shear stress is composed of two components, $\tau_b = \tau_{bs} + \tau_{bf}$, in which τ_{bs} is the bed shear stress acting as a result of the particulate normal stress exerted on the top of the bed by the contact-load particles travelling in the flow above the bed and τ_{bf} the fluid-like shear stress opposing the flow of carrying liquid and solid particles that are suspended in the flow and do not contribute to the contact load.

In a unit length of a pipe, the submerged body of contact-load particles exerts the submerged weight $\rho_f.g.(S_s-1).C_{vabc}.A_{ab}$ against the top of the bed. The particulate normal stress that the submerged weight produces at the width of the top of the bed, O_b, is $\sigma_{bs} = \rho_f.g.(S_s-1).C_{vabc}.A_{ab}/O_b = \rho_f.g.(S_s-1).C_{vabc}.R_{hb}$. The Coulombic equilibrium requires that the particulate normal stress at the top of the stationary bed is related with the bed shear stress through the friction coefficient μ and therefore $\tau_{bs} = \mu.\rho_f.g.(S_s-1).C_{vabc}.R_{hb}$. The τ_{bs} is usually much higher than the τ_{bf} in flows with bed Shields number $\theta_b > 2$ and thus in the following considerations the simplification is adopted that $\tau_b = \tau_{bs}$. A combination of the equation $\tau_b = \mu.\rho_f.g.(S_s-1).C_{vabc}.R_{hb}$ with the momentum equation $\tau_b = \rho_f.g.R_{hb}.I_m$ gives $\mu.C_{vabc} = I_m/(S_s-1)$. An implementation of $C_c.A_a = C_{vabc}.A_{ab}$ to the equation and further rearrangements produce the equation

$$\frac{C_c}{C_{vd}} = \frac{I_m \cdot A_{ab}}{A_a \cdot \mu \cdot C_{vd} \cdot (S_s - 1)}$$

During the experiments in the Delft laboratory and the SRC laboratory the values of I_m, A_a, and C_{vd} were measured and the values of the dependent parameter A_{ab} were determined from the measured parameters. Therefore the values of

$$\mu \cdot \frac{C_c}{C_{vd}} = \frac{I_m \cdot A_{ab}}{A_a \cdot C_{vd} \cdot (S_s - 1)} \qquad [3]$$

were known for each test run (12 runs in the Delft laboratory and 10 runs in the SRC laboratory). The measured data from all test runs suggest that this parameter can be successfully correlated with reasonably correlated with a simple power-law function (K and n are empirical coefficients) at least for medium-sand fractions. Figure 4 shows the correlation for K = 730 and n = 2,

$$\mu \cdot \frac{C_c}{C_{vd}} = K \cdot \left(\frac{V_a}{V_t}\right)^{-n} \qquad [4]$$

Equations [3] and [4] cannot be used directly for a prediction of the pressure drop in a flow of certain V_m, C_{vd} and y_b/D. An additional equation is required to produce a value for the area associated with the top of the bed, A_{ab}, or other interrelated parameter, like λ_b, for the purpose of the I_m prediction.

Friction coefficient at top of bed

The friction coefficient at the top of the bed, λ_b, relates the bed shear stress, τ_b, with the mean velocity, V_a, via

$$\lambda_b = \frac{8 \cdot \tau_b}{\rho_f \cdot V_a^2}.$$

Its combination with the momentum equation $\tau_b = \rho_f.g.R_{hb}.I_m = \rho_f.g.A_{ab}/O_b.I_m$ gives

$$A_{ab} = \frac{\lambda_b \cdot V_a^2 \cdot O_b}{8 \cdot g \cdot I_m}.$$

An implementation of this equation to Equation [3] gives after rearrangements the formula that predicts the λ_b directly from the model-input quantities (D, y_b, C_{vd}, S_s, μ, v_t, and V_a, V_m respectively) and from C_c (the output of Equation [4]),

$$\lambda_b = \frac{A_a}{O_b} \cdot C_{vd} \cdot (S_s - 1) \cdot \mu \cdot \frac{C_c}{C_{vd}} \cdot \frac{8 \cdot g}{V_a^2} \quad [5]$$

Additional equations are required to link the friction coefficient λ_b with the hydraulic gradient I_m. The earlier work (Matoušek, 2004, Matoušek, 2005) on the slurry flow of the 0.37-mm sand above a stationary bed in a 150-mm pipe slurry showed that the friction law for the top of the stationary bed could be expressed by the Nikuradze equation for the hydraulically rough boundary,

$$\frac{V_a}{u_{*b}} = \sqrt{\frac{8}{\lambda_b}} = 2.46 \cdot \ln \frac{14.8 \cdot R_{hb}}{k_s} \quad [6a]$$

provided that the relative hydraulic roughness, k_s/d_{50}, was considered a function of the bed Shields number, θ_b. Our data base showed that for qb> 3 an appropriate correlation is

$$\frac{k_s}{d_{50}} = 1.3 \cdot \theta_b^{1.65} \quad [7]$$

Pugh's data exhibit the same trend in the logarithmic plot of k_s/d_{50} vs. θ_b and collapse to the same line with the Delft data (Figure 5), if the coefficient 14.8 is replaced with 4.4 in the Nikuradze equation, i.e.

$$\sqrt{\frac{8}{\lambda_b}} = 2.46 \cdot \ln \frac{4.4 \cdot R_{hb}}{k_s} \quad [6b]$$

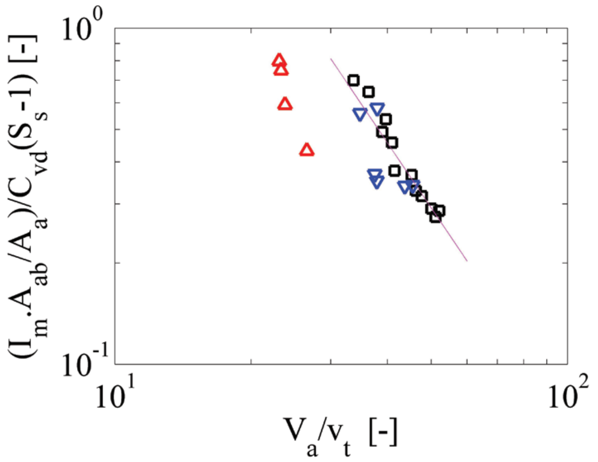

Figure 4—Stratification ratio (µ - 0.37 mm sand in the 150 mm pipe, Ñ - 0.3 mm sand in the 105 mm pipe, Δ - 0.56 mm sand in the 105 mm sand, line-correlation, Equation [4], for K = 730 and n = 2)

Hydraulic gradient

The friction-law model (Equations [6] and [7]) provides a missing link between the bed friction coefficient and the hydraulic gradient. Its calculation gives a value of the R_{hb}. Finally, the hydraulic gradient is calculated using the momentum equation,

$$I_m = \frac{\tau_b}{\rho_f \cdot g \cdot R_{hb}}$$

with the implemented equation for the bed shear stress,

$$\tau_b = \frac{\lambda_b}{8} \cdot \rho_f \cdot V_a^2, \text{ i.e.} \quad [8]$$

$$I_m = \frac{\lambda_b}{4 \cdot R_{hb}} \cdot \frac{V_a^2}{2 \cdot g}$$

The proposed predictive model (Equations [4] to [8]) for the hydraulic gradient in pipe flows above a stationary bed approximates the measured I_m values for the tested sands in both pipes better than the relationship proposed by Pugh and Wilson (Pugh and Wilson, 1999b), Figure 6.

Conclusions

The observations in a 0.37-mm-sand flow over a stationary bed in a 150-mm pipe showed that for conditions given by high values of bed shear stress, the measured concentration profiles above the bed may not be linear. Further, the analysis of measured concentration profiles in flows of three different medium-sand fractions in two laboratory pipes (internal diameter 105 mm and 150 mm) suggested that for high-shear-stress flows interparticle contacts may not act as an exclusive particle-dispersion mechanism in an upper part of a discharge area above the bed. Therefore a stratification-ratio equation considering this effect was developed to determine the contact-load portion of the total concentration of particles transported in the flow above the bed. The slip ratio C_c/C_{vd} is dependent primarily on the velocity ratio V_a/v_t.

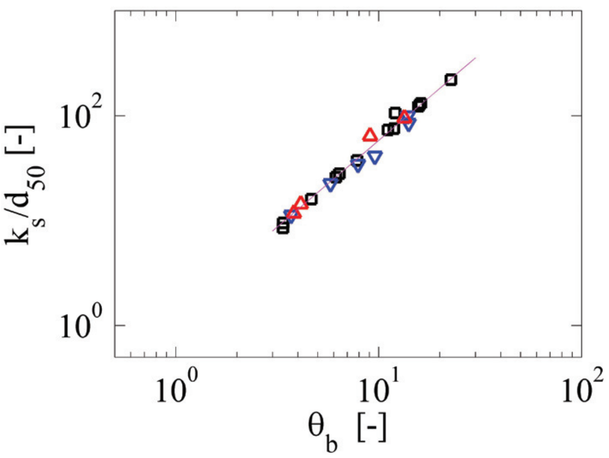

Figure 5—Relative hydraulic roughness of the top of the stationary bed (squares—0.37-mm sand in the 150-mm pipe, downward-pointing triangle—0.3-mm sand in the 105-mm pipe, upward-pointing triangle—0.56-mm sand in the 105-mm sand)

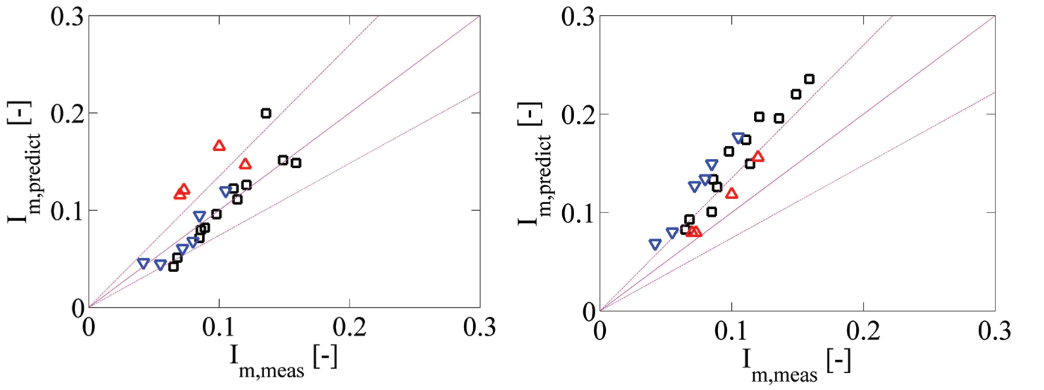

Figure 6—Comparison of measured and predicted I_m (left—here presented model, right—Pugh-Wilson model, dashed line—35% deviation from the $I_{m,predict} = I_{m,meas}$)

A predictive model was formulated for hydraulic gradient in a medium-sand-slurry flow above a stationary bed in a pipe. The model is composed of continuity, momentum, boundary-friction and stratification-ratio equations.

Remark on prediction of thickness of stationary bed

A predictive model for flow of slurry above a stationary bed should not use the thickness of the stationary bed as an input parameter. The thickness of the bed that develops in a slurry pipe is a result of flow conditions in the pipe and therefore a complete predictive model should predict the thickness of the bed.

In principle, the thickness of the bed could be obtained from a predicted concentration profile across the discharge area above the bed. Further investigation is required to formulate this part of a complete predictive model. A preliminary procedure on how to incorporate a prediction of the thickness of the bed is based on the analysis described in the chapter 'Solids distribution in slurry flow' of this paper. According to the analysis, a simplified concentration profile across the medium-sand flow above the stationary bed can be considered as composed of two parts. In the upper part, the profile is represented by a curve described using the turbulent-diffusion model. In the lower part, the profile is represented by a line of the gradient equal to the gradient of the turbulent curve at the location at which both lines meet. This location is in the centre of a pseudo-linear layer. The typical value of the local concentration in this location is 0.27 ($c_{ob}/2$) approximately. The typical value of the local concentration at the top of the pipe cross section is zero. This information enables to determine the positions of both the centre and the bottom of the linear layer and thus the thickness of the stationary bed using the turbulent-diffusion model. The procedure is applicable to flows with concentration profiles the linear part of which has a value of the gradient $c_{Ob}.D/H_{lin}$ smaller than 2 and C_{vd} smaller than 0.25.

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Notation

A_a	area of pipe cross-section above bed	m2
A_{ab}	part of A_a associated with top of stationary bed	m2
A_{aw}	part of A_a associated with pipe wall	m2
c (c_v)	local volumetric solids concentration in pipe - cross-section	

c_{vb}	concentration of solids in the bed (at the top of stationary bed)	-
c_{0b}	local concentration at the bottom of linear-profile layer	-
C_c	mean concentration of contact-load particles, part of C_{va}	-
C_{va}	mean spatial volumetric solids concentration above stationary bed	-
C_{vab}	mean spatial volumetric solids concentration in part of discharge area associated with top of bed	-
C_{vd}	mean delivered volumetric concentration of solids	-
d	particle diameter	m
d_{50}	median particle diameter	m
D	internal pipe diameter	
g	gravitational acceleration	m/s^2
H_{lin}	thickness of layer with linear concentration profile	m
I_m	hydraulic gradient	-
k_s	characteristic grain roughness height (by Nikuradze)	m
K	proportional coefficient in stratification-ratio equation	-
K_{turb}	proportional coefficient in turbulent-difusion equation	-
m	exponent in equation for hindered settling velocity, (*m* calculated using Wallis equation)	-
n	proportional coefficient in stratification-ratio equation	-
O_b	perimeter of the top of bed	m
O_w	perimeter of the pipe wall above bed	m
R	internal pipe radius ($R = D/2$)	m
R_{hb}	hydraulic radius of area associated with the top of bed	m
R_{hw}	hydraulic radius of area associated with the pipe wall	m
Re_w	Reynolds number of flow through area A_aw	-

S_s	relative density of solids	-
u^*	shear velocity at pipe wall	m/s
u^*_b	shear velocity at the top of bed, (τ_b/ρ_f) 0.5	m/s
v_t	terminal settling velocity of solid particle	m/s
V_m	mean velocity of mixture in entire pipe cross-section	m/s
V_a	mean velocity of mixture in discharge area above stationary bed	m/s
y	vertical distance from pipe wall	m
y_b	thickness of stationary bed	m
y_0	vertical distance from the top of stationary bed	m
α	empirical coefficient in Blasius-type of friction law for smooth pipe wall	-
β	empirical coefficient in Blasius-type of friction law for smooth pipe wall	-
ε_s	solids dispersion coefficient in Schmidt-Rouse equation	m^2/s
$\varepsilon_{s,mean}$	average value of in core of flow	m^2/s
θ_b	Shields number, $u^*{}_b{}^2/((S_s-1).g.d_{50})$	-
λ_b	Darcy-Weisbach friction coefficient for the top of bed	-
λ_w	Darcy-Weisbach friction coefficient for pipe wall	-
μ	coefficient of mechanical friction between contact-load particles and top of bed	-
ν_f	kinematic viscosity of liquid	m^2/s
ρ_f	density of liquid	kg/m^3
σ_{bs}	solids normal stress at top of bed	$kg/(m.s^2)$
τ_b	shear stress at top of bed	$kg/(m.s^2)$
τ_{bf}	component of τ_b representing fluid-like (viscous) friction	$kg/(m.s^2)$
τ_{bs}	component of τ_b representing solids friction	$kg/(m.s^2)$
τ_w	shear stress at pipe wall	$kg/(m.s^2)$
ϕ'	dynamic friction angle of solids	deg