A zero-one integer programming model for open pit mining sequences

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Synopsis

The aim of short-term production scheduling is to decide which blocks of ore and waste to mine in which time period (shift, days, weeks or months) so that several operational and geometrical constraints can be satisfied simultaneously. Since 1960s several mathematical programming approaches were developed for solving production scheduling problems that are based on a combination of various operational research approaches such as linear programming (LP), integer programming (IP), dynamic programming (DP), etc. A number of models have been developed in the past three decades but some models have limited application especially to geometrical mining constraints. One of the geometrical constraints is block accessibility. All blocks must be accessible to mining equipments on the same bench. In this paper a binary integer programming model is developed in order to incorporate block accessibility constraints in an efficient manner. This mathematical model insures that each block has been open and can be loaded and transported easily by shovels and trucks.

Introduction

The open pit mine production scheduling can be defined as:

Specifying the sequence of block extraction from the mine to give the highest NPV, subject to a variety of production, grade blending and pit slope constraints.

Production scheduling over a certain period is known as the scheduling horizon. Production scheduling typically encompasses three time ranges for decision making: long-term, medium-term and short-term. Long-term can range from 20 to 30 years depending on the situation. The 20 to 30 year periods are broken into several smaller periods with a duration between one and five years. A medium-term schedule ranges from one to five years, which gives more detailed information, allowing for more accurate design such as extracting from a special area of a mine, substituting the equipment and purchasing of the needed resource capacities. The one to five year period is broken into one to six month periods for more detailed scheduling in the intermediate-term model. Finally, short-term production planning duration is between one month and one year. Similarly, this period is divided into daily, weekly or monthly sub-periods. During short-term scheduling, detailed design of the mine takes place and the long-term plans, typically annual plans, are implemented on a level of detail suitable for guiding operations in a time frame that meets the needs of production planning at the mine. At this level of design, the goals are:

- ➤ Properly utilize production equipment by avoiding idle time and excessive moves to different working levels
- ➤ Ensure that stripping proceeds in advance of ore production
- ➤ Maintain working slope angles
- ➤ Provide haul road access to all working benches
- ➤ Maintain ore blend to avoid excessive quality and quantity fluctuations at the mill
- ➤ Minimum deviation from the long-term and medium-term plans
- ➤ To ensure a production schedule that is practical in terms of mining operations
- ➤ To ensure maximal flexibility of the system.

When a non-homogeneous deposit is to be mined, a detailed short-term production schedule becomes a necessity in order to provide a homogeneous concentrator feed. If the deposit is extremely non-homogeneous, blending facilities are usual required between the mine and concentrator.

Because of the very short time usually available for short-term production scheduling, it is nearly impossible to consider all factors in a manual scheduling method. Also, a quick revision of the production schedule under emergency conditions, such as a breakdown of a shovel or unexpected changes in the ore grade, is impossible in this method. Therefore, the mathematical programming method, such

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as linear programming, integer programming, etc., can be used to obtain an optimal production schedule subject to geometrical and operational constraints. In order to generate a schedule, a great number of factors should be taken into account, such as:

- The characteristics of waste and ore blocks exposed to mining
- Capacities of the available equipment
- The actual state of the system as a whole (mine, crusher, blending bed, concentrator, etc.)
- Long-term and medium-term production plans
- Enough working space for shovel and trucks.

A number of linear programming models have been developed in the past three decades[1–3] but some models have limited application especially in geometrical mining constraints. One of the geometrical constraints is block accessibility. All blocks must be accessible to mining equipment on the same bench. In this paper a binary integer programming model is developed in order to incorporate block accessibility constraints in an efficient manner.

Formulation of short-term production scheduling problem

Most of the current open pit design and scheduling processes begins with a geologic block model obtained by dividing the deposit into a three-dimensional grid of fixed size blocks, as shown in Figure 1. Block dimensions are selected according to the exploration drilling pattern, orebody geology and mine equipment size. After establishing the dimensions of the block model, the geological characteristics of each block (grade) are assigned using available estimation techniques such as the inverse distance weighted interpolation technique, weighted moving averages, Kriging, etc.

Before determination of the extraction sequences of these blocks in short-term periods, some symbols are defined.

Symbols

t: Scheduling period index, $t = 1, 2,\ldots, T$.
T: Total number of periods in the scheduling horizon.
i,j,k: Block identification number in x, y and z directions respectively.
I,J,K: Total number of blocks in block model in x, y and z directions respectively.
$x^t_{i,j,k}$: A binary decision variable, which is equal to 1 if block i, j, k is to be mined in period t and 0 otherwise.

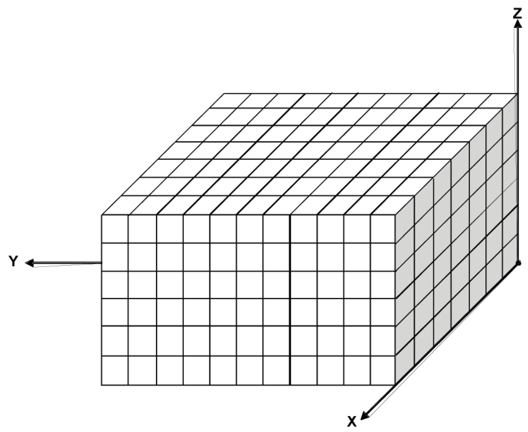

Figure 1—Isometric view of block model

$g^c_{i,j,k}$: Grade of element c in blocks i, j, k. Where $c = 1,2,\ldots,C$ is the index of contaminate element in block i, j, k and $c = C+1, C+2,\ldots,P$ is index for useful element in the block.
TO_{ijk}: The total amount of ore (tonnages) in block i,j,k.
TW_{ijk}: The total amount of waste (tonnages) in block i,j,k.
G^c_u: The upper bound average grade of material sent to the mill with regard to element c.
G^c_l: The lower bound average grade of material sent to the mill with regard to element c.
$OC_{max}{}^t$: The upper bound total tons of ore processed in period t.
$OC_{min}{}^t$: The lower bound total tons of ore processed in period t.
$TC_{max}{}^t$: The upper bound total amount of material (waste and ore) to be mined in period t.
$TC_{min}{}^t$: The lower bound total amount of material (waste and ore) to be mined in period t.
d: Discount rate.

From the definition described above, the multi-period short-term production scheduling model will be formulated as follows:

Objective function

Mathematical programming approach to production scheduling aims to find the combination of mining blocks in an optimal way. Optimization can be based on many different objective functions. In this paper two kinds of objective functions are reviewed. One approach is to use a single objective that will give us a solution toward a long-term goal while including constraints that will produce a blend that is within certain limits. An example of this can be taken from mining high sulphur coal. In this case, the goal may be to maximize the production of sulphur while remaining within quality limits. This allows higher sulphur coals to be mined early on and extends the life of the operation. Constraints are used to limit sulphur content to no more than the maximum allowed and set additional limits on other production characteristics. In this case long-term plans typically seek to maximize the value of the project. The second approach is goal programming in which the objective is to minimize the deviation of a number of goals from their target values. These goals are represented by deviation variables, which are defined by goal constraints[1].

In this paper we use the first approach. The objective function is to maximize the production of contaminate elements (such as sulphur, phosphor, etc.).

$$Max\ Z = \sum_{t=1}^{T}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\frac{x^t_{ijk}\cdot g^c_{ijk}\cdot TO_{ijk}}{(1+d)^t} \quad [1]$$

In order to determine the amount of contaminate element for a block in different periods, a discount rate can be applied, which is different from the usual economic discount rate. If the higher discount rate is used, the difference between the per cent of these elements in different periods is higher; therefore this discount rate is a parameter forcing the model to mine the blocks with higher contaminate materials in the earlier production periods.

Grade blending constraints

The average grade of the contaminate element in the ore sent to the mill has to be less than an upper bound (G^c_u) in each period:

$$\begin{bmatrix} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} \frac{x^t_{ijk}.g^1_{ijk}.TO_{ijk}}{x^t_{ijk}.TO_{ijk}} \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} \frac{x^t_{ijk}.g^2_{ijk}.TO_{ijk}}{x^t_{ijk}.TO_{ijk}} \\ . \\ . \\ . \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} \frac{x^t_{ijk}.g^C_{ijk}.TO_{ijk}}{x^t_{ijk}.TO_{ijk}} \end{bmatrix} \le \begin{bmatrix} G^1_u \\ G^2_u \\ . \\ . \\ . \\ G^C_u \end{bmatrix} \Rightarrow \qquad [2]$$

$$\begin{bmatrix} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} (G^1_u - g^1_{ijk}).x^t_{ijk}.TO_{ijk} \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} (G^2_u - g^2_{ijk}).x^t_{ijk}.TO_{ijk} \\ . \\ . \\ . \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} (G^C_u - g^C_{ijk}).x^t_{ijk}.TO_{ijk} \end{bmatrix} \ge \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix}$$

Also the average grade of the useful element in the ore sent to the mill has to be less than an upper bound (G^c_u) and more than a lower bound (G^c_l) in each period:

$$\begin{bmatrix} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} (G^{C+1}_u - g^{C+1}_{ijk}).x^t_{ijk}.TO_{ijk} \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} (G^{C+2}_u - g^{C+2}_{ijk}).x^t_{ijk}.TO_{ijk} \\ . \\ . \\ . \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} (G^P_u - g^P_{ijk}).x^t_{ijk}.TO_{ijk} \end{bmatrix} \ge \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix} \qquad [3]$$

and:

$$\begin{bmatrix} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} (G^{C+1}_l - g^{C+1}_{ijk}).x^t_{ijk}.TO_{ijk} \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} (G^{C+2}_l - g^{C+2}_{ijk}).x^t_{ijk}.TO_{ijk} \\ . \\ . \\ . \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} (G^P_l - g^P_{ijk}).x^t_{ijk}.TO_{ijk} \end{bmatrix} \le \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix} \qquad [4]$$

Ore production constraints

The total tons of ore produced in each period should be more than a lower bound (OC_{min}^t) and less than an upper bound (OC_{max}^t):

$$\begin{bmatrix} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x^1_{ijk}.TO_{ijk} \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x^2_{ijk}.TO_{ijk} \\ . \\ . \\ . \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x^T_{ijk}.TO_{ijk} \end{bmatrix} \le \begin{bmatrix} OC^1_{\max} \\ OC^2_{\max} \\ . \\ . \\ . \\ OC^2_{\max} \end{bmatrix} \qquad [5]$$

and:

$$\begin{bmatrix} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x^1_{ijk}.TO_{ijk} \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x^2_{ijk}.TO_{ijk} \\ . \\ . \\ . \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x^T_{ijk}.TO_{ijk} \end{bmatrix} \ge \begin{bmatrix} OC^1_{\min} \\ OC^2_{\min} \\ . \\ . \\ . \\ OC^2_{\min} \end{bmatrix} \qquad [6]$$

Mining capacity constraints

The total tons of rock (waste and ore) to be mined should be more than a lower bound (TC_{min}^t) and less than an upper bound (TC_{max}^t):

$$\begin{bmatrix} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x^1_{ijk}.(TO_{ijk} + TW_{ijk}) \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x^2_{ijk}.(TO_{ijk} + TW_{ijk}) \\ . \\ . \\ . \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x^T_{ijk}.(TO_{ijk} + TW_{ijk}) \end{bmatrix} \ge \begin{bmatrix} TC^1_{\min} \\ TC^2_{\min} \\ . \\ . \\ . \\ TC^2_{\min} \end{bmatrix} \qquad [7]$$

and:

$$\begin{bmatrix} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x^1_{ijk}.(TO_{ijk} + TW_{ijk}) \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x^2_{ijk}.(TO_{ijk} + TW_{ijk}) \\ . \\ . \\ . \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x^T_{ijk}.(TO_{ijk} + TW_{ijk}) \end{bmatrix} \ge \begin{bmatrix} TC^1_{\min} \\ TC^2_{\min} \\ . \\ . \\ . \\ TC^2_{\min} \end{bmatrix} \qquad [8]$$

Slope constraints

These constraints ensure that all blocks that directly restrict the mining of a given block *ijk* must be completly mined out before mining block *ijk*. To represent the restricting blocks a cone template is made that contains 9 blocks above block *ijk*. These constraints can be written as:

$$9x^t_{ijk} - \sum_{l=1}^{t}(x^l_{i-1,j-1,k+1} + x^l_{i-1,j,k+1} + x^l_{i-1,j+1,k+1} + x^l_{i,j-1,k+1} + x^l_{i,j+1,k+1} + x^l_{i+1,j-1,k+1} + x^l_{i+1,j,k+1} + x^l_{i+1,j+1,k+1}) =$$

$$9x^t_{ijk} - \sum_{l=1}^{t}\sum_{m=i-1}^{i+1}\sum_{n=j-1}^{j+1} x^l_{mnk+1} \le 0 \qquad [9]$$

for $t = 1, 2,\ldots, T$; $i = 1,2,\ldots,I$; $j = 1,2,\ldots,J$; $k = 1,2,\ldots,K$.
or:

$$\begin{bmatrix} 9x^1_{ijk} - \sum_{m=i-1}^{i+1}\sum_{n=j-1}^{j+1} x^1_{mnk+1} \\ 9x^2_{ijk} - \sum_{l=1}^{2}\sum_{m=i-1}^{i+1}\sum_{n=j-1}^{j+1} x^l_{mnk+1} \\ . \\ . \\ . \\ 9x^T_{ijk} - \sum_{l=1}^{T}\sum_{m=i-1}^{i+1}\sum_{n=j-1}^{j+1} x^l_{mnk+1} \end{bmatrix} \le 0 \qquad [10]$$

For all *i,j,k.*

Reserve constraints

Reserve constraints insure that any block in the model mined at most once:

$$\sum_{t=1}^{T} x^t_{ijk} \le 1 \qquad [11]$$

for $t = 1, 2,\ldots, T$; $i = 1,2,\ldots,I$; $j = 1,2,\ldots,J$; $k = 1,2,\ldots,K$.

Accessibility constraints

Each block on an open pit bench has eight neighbouring blocks (Figure 2). It is assumed if not less than three contiguous blocks have been mined or selected for mining, block *ijk* is considered to be accessible to the mining equipment.

In order to set up the accessibility constraints, a horizontal cone is built on block *ijk* (Figure 3).

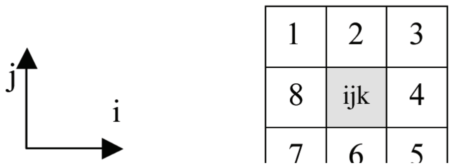

Figure 2—Blocks surrounding block *ijk* to establish the accessibility constraints

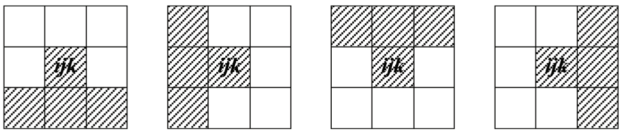

Figure 3—Accessibility cone templates and the directions of free faces for block *ijk*

Figure 3 shows that block *ijk* can be accessed from four directions: right, left, up and down. If the blocks located in at least one cone template are mined before or simultaneously with block *ijk*, block *ijk* is accessible. It should be emphasized that only one direction is enough for accessibility. Therefore the accessibility constraints can be written as follows:

$$3x^t_{ijk} - \sum_{l=1}^{t}(x^l_{i+1,j-1,k} + x^l_{i+1,j,k} + x^l_{i+1,j+1,k}) \le My^t_1 \qquad [12]$$

$$3x^t_{ijk} - \sum_{l=1}^{t}(x^l_{i+1,j+1,k} + x^l_{i,j+1,k} + x^l_{i-1,j+1,k}) \le My^t_2 \qquad [13]$$

$$3x^t_{ijk} - \sum_{l=1}^{t}(x^l_{i-1,j-1,k} + x^l_{i-1,j,k} + x^l_{i-1,j+1,k}) \le My^t_3 \qquad [14]$$

$$3x^t_{ijk} - \sum_{l=1}^{t}(x^l_{i-1,j-1,k} + x^l_{i,j-1,k} + x^l_{i+1,j-1,k}) \le My^t_4 \qquad [15]$$

$$y^t_1 + y^t_2 + y^t_3 + y^t_4 \le 3 \qquad [16]$$

M is a big number and y^t_i are binary variables. Constraints 16 ensure that at lease one of constraints 12 to 15 is active. For example, suppose that $y^t_1 = 1$, $y^t_2 = 1$, $y^t_3 = 0$ and $y^t_4 = 1$. Therefore, constraints 14 are active and block *ijk* will be accessible from its left direction. Now if $y^t_1 = 1$, $y^t_2 = 0$, $y^t_3 = 0$ and $y^t_4 = 1$, constraints 13 and 14 are active and block *ijk* will be accessible from its left and up directions. As a matter of fact, y^t_i will determine the direction of the block access.

These constraints should be written for all the selected blocks for the scheduling process, regardless of the types of benches. We have two benches in production scheduling, an open bench and a closed bench. An open bench has been opened before and there are some accessible blocks on it, but a closed level has not been opened and therefore there is not any accessible block on it. This problem is called the sinking cut problem. In this case, the above constraints will force the model to mine block *ijk* and one of the three contiguous blocks simultaneously in the same period with no free face, typically the bottom of the pit or pushback.

Conclusion

In this paper a binary integer programming model was developed for a short-term production scheduling model in open pit mines. In order to achieve this goal, new binary variables are introduced in the original model. In spite of this, this kind of modelling leads to increasing the size of the problem, but this model ensures that each block in the block model has a free face for loading and transportation equipment. Also this model ensures that in each period enough ore with the predetermined quality is prepared for the mill.

References

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