Net present value maximization model for optimum cut-off grade policy of open pit mining operations

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Synopsis

The optimum cut-off grade policy maximizes the net present value (NPV) of an open pit mining operation subject to the mining, processing, and refining capacity constraints. The traditional approaches to cut-off grade determination ignore the escalation of the economic parameters such as metal price and operating costs during life of an operation, and consequently lead to unrealistically higher values of the objective function. Further, the NPV of a mining operation declines due to the depletion of the available reserves, causing a decline in the optimum cut-off grade, i.e. higher cut-off grades in the early years of an operation and lower cut-off grades during the later years. Hence, low grade material mined in the earlier years may be stockpiled for processing during later years to offset the effect of escalating economic parameters on NPV. This paper demonstrates the combined impact of introducing economic parameters, escalation and stockpiling options into the cut-off grade optimization model. The model promises an enhancement in NPV as illustrated in a case study incorporating practical aspects of an open pit mining operation.

Keywords
mining, modelling, cut-off grade, stockpiling, optimization.

Introduction

Cut-off grade is the criterion that discriminates between ore and waste within a given mineral deposit[1,2]. If material grade in the mineral deposit is above cut-off grade it is classified as ore, and if material grade is below cut-off grade, it is classified as waste. Ore, being the economically exploitable portion of the mineral deposit, is sent to the processing plant for crushing, grinding, and concentration of the metal content. The product of the processing plant is called concentrate, which is fed to the refinery for production of refined metal. Hence, an ideal open pit mining operation consists of three stages i.e. mine, processing plant, and refinery[3,4].

Long-range production planing of an open pit mining operation is dependent upon several factors; however, cut-off grade is the most significant aspect, as it provides a basis for the determination of the quantity of ore and waste in a given period[5]. Eventually, the profit over time may be enhanced only by flow of high grade material to the processing plant. This strategy supports the objective function and, depending upon the grade-tonnage distribution of the deposit, higher NPV may be realized during earlier years to recover the initial investment[6,7]. However, as the deposit becomes depleted, the NPV as well as the cut-off grade decline; hence, cut-off grade policy and the production plan defined as a result of this policy dictate phenomenal influence on the overall economics of the mining operation[8,9].

The optimum cut-off grades, which are dynamic due to the declining effect of NPV, not only depend on the metal price and cash costs of mining, processing, and refining stages, but also take into account the limiting capacities of these stages and grade-tonnage distribution of the deposit. Therefore, the technique that determines the optimum cut-off grade policy considers the opportunity cost of not receiving future cash flows earlier during mine life, due to the limiting capacities of any of mining, processing, or refining stages[10,11]. However, metal price and operating costs of mining, processing, and refining change during mine life, and this happens quite often, due to the longer life of most of the open pit mining operations. Ignoring the effect of these changes in the economic parameters on the optimum cut-off grade policy would lead to unrealistic production plans[12,13].

Further, the declining effect of NPV allows higher cut-off grades in the early years of mine life and lower cut-off grades in the later years,

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Net present value maximization model for optimum cut-off grade policy

due to depletion of high grade material. Therefore, depending upon the existing circumstances in an open pit mining operation, the production plans may also include the flexibility of providing stockpiles of low grade ore mined in the earlier years to be processed later as it becomes economical to do so. This enhances not only the life, but also the NPV of a mining operation. The management of stockpiles of low grade ore is possible using the following two options[14]:

1. The stockpile is utilized parallel to the mining operation. This means that material is sent to the processing plant either from mine or stockpile. This decision is based on the overall economy/profitability of the operation.
2. The stockpile is utilized after the mine is exhausted. This simplifies the decision-making, since the stockpile acts as an additional portion of the deposit, where all available material is economical. However, the high grade material in the stockpile is scheduled to be utilized earlier than the low grade material.

In this study, the second case is chosen owing to ease of operation. Therefore, keeping in view the prospect of a contribution to the mining industry, we propose an extension in the established Lane's theory of optimum cut-off grades[15,16]. The proposed cut-off grade optimization model considers not only dynamic metal price and cost escalation, but also results in the creation of a stockpile of low grade ore during the mine life and its utilization as ore after the exhaustion of the deposit. Lane's original theory has been modified by Dagdelen[3,4], Dagdelen and Kawahta[6], Bascetin[9], Osanloo and Ataei[11], Asad[12,13,14], Dagdelen and Asad[17], Ataei and Osanloo[18], Osanloo *et al.*[19], and King[20], but these studies did not attempt to analyse the combined impact of economic parameters escalation and stockpiling on NPV. We implement the iterative algorithmic steps of the modified model in Visual C++ programming language to develop alternative cut-off grade policies in a case study of a hypothetical copper deposit. The results demonstrate the effect of the change in economic parameters and the stockpiling option on mine planning with an increase/decrease in NPV.

The model

Prerequisites for the application of a cut-off grade optimization model include the development of ultimate pit limit or pit extent and pushback (a manageable portion of the deposit inside the ultimate pit limit that may be mined, processed, and refined in a number of years/periods) design, ore reserves in terms of mineral grade and tonnage distribution in each pushback, and mining, processing, and refining stage capacities, the operating costs of these stages, and the current metal price.

The objective function of cut-off grade optimization model is to maximize the NPV of the operation subject to mining, processing, refining, and stockpile capacity constraints, which may be represented mathematically as follows:

$$Max\ NPV = \sum_{n=1}^{N} \frac{P_n}{(1+d)^n} \qquad [1]$$

Subject to:

$$Qm_n \leq M\ \forall n \qquad [2]$$

$$Qc_n \leq C\ \forall n \qquad [3]$$

$$Qr_n \leq R\ \forall n \qquad [4]$$

$$\sum_{n=1}^{Nm} Qs_n \leq S \qquad [5]$$

Here,

$$P_n = \left[\left[(p_n - r_n)Qr_n \right] - \left[c_n Qc_n \right] - \left[m_n Qm_n \right] - f_n \right] \qquad [6]$$

where n = period (year) indicator, N = total life of operation (years), Nm = mine/deposit life (years), P = profit ($/year), d = discount rate (%), M = mining capacity (tons/year), C = concentrating or milling capacity (tons/year), R = refining capacity (tons /year), S = stockpile capacity (tons), p = metal selling price ($/ton of product), m = mining cost ($/ton of material mined), c = concentrating or milling cost ($/ton of ore), r = refining cost ($/ton of product), f = administrative/ fixed cost ($/year), Qm = quantity of material mined (tons/year), Qc = quantity of ore processed (tons/year), Qr = quantity of concentrate refined (tons /year), Qs = quantity of material stockpiled (tons/year).

The model relies on the fact that the capacities of the mining, processing, and refining stages limit the operation either independently or jointly. While an individual stage causes constrained production, it leads to the determination of refinery limiting economic cut-off grades for mining, processing, and refining, represented as γ_m, γ_c, and γ_r, respectively. However, if a pair of stages is limiting the operation, then the output from each constraining stage must be balanced to utilize the maximum capacity of these stages. This requires the determination of three balancing cut-off grades pairing mine–processing plant, mine–refinery, and processing plant–refinery, represented as γ_{mc}, γ_{mr}, and γ_{cr}, respectively. Ultimately, the optimum cut-off grade γ is selected between the limiting economic and balancing cut-off grades.

As the grade and amount of low grade stockpile material in a period n is also dependent upon the determination of optimum cut-off grade, the solution to this problem may be presented in two sequential steps. The first step determines the optimum cut-off grade, and the second step defines the grade and amount of stockpile material.

Optimum cut-off grade

Dynamic metal prices and operating costs influence limiting economic cut-off grades, while the grade-tonnage distribution of the deposit is the only factor affecting balancing cut-off grades[13]. The optimum cut-off grade among six limiting economic and balancing cut-off grades is calculated as follows:

Assuming that the grade-tonnage distribution of a pushback consists of K grade increments i.e. (γ_1, γ_2), (γ_2, γ_3), (γ_3, γ_4), $---$, (γ_{K-1}, γ_K), and, for each grade increment, there exist t_k tons of material. In general, if $\hat{k}$ represents grade increment (γ_k, γ_{k+1}) and the lower grade in $\hat{k}$ i.e. γ_k is

Net present value maximization model for optimum cut-off grade policy

considered as the cut-off grade, then quantity of ore t_o, quantity of waste t_w, and the average grade of ore $\bar{\gamma}$ are the given in Equations [7], [8], and [9]:

$$t_o(\gamma_k) = \sum_{k=k^*}^{K} t_k \quad [7]$$

$$t_w(\gamma_k) = \sum_{k=1}^{k^*-1} t_k \quad [8]$$

$$\bar{\gamma}(\gamma_k) = \frac{\sum_{k=k^*}^{K}\left[t_k \left(\frac{\gamma_k + \gamma_{k+1}}{2} \right) \right]}{t_o(\gamma_k)} \quad [9]$$

If y is the metallurgical recovery, then Qm_n, Qc_n, and Qr_n are sequentially determined according to any one of the following three conditions:

1. Set:

$$Qm_n = M, \ Qc_n = Qm_n\left[\frac{t_o}{t_o + t_w} \right], \ \text{and} \ Qr_n = Qc_n\bar{\gamma}y$$

2. If $Qc_n > C$ or $Qr_n > R$ from condition 1, then set:

$$Qc_n = C, \ \Theta m_n = Qc_n\left[1 + \frac{t_w}{t_o} \right], \ \text{and} \ Qr_n = Qc_n\bar{\gamma}y$$

3. If $Qr_n >$ or $Qm_n > M$ from condition 2, then set:

$$Qr_n = R, \ Qc_n = \frac{Qr_n}{\bar{\gamma}y}, \ \text{and} \ Qm_n = Qc_n\left[1 + \frac{t_w}{t_o} \right]$$

Mining the next Qm_n amount of material may require time τ. For calculating the profit generated from Qm_n at the end of time τ, Equation [6] may be updated as:

$$P_n\left[\left[\left(\left((p_n - r_n)\bar{\gamma}y \right) - c_n \right)Qc_n \right] - \left[m_nQm_n \right] - \left[f_n\tau \right] \right] \quad [10]$$

Since the objective function is to maximize the NPV of future profits, assuming that ς is the maximum possible net present value of future profits at time zero (i.e. now) and Ω is the maximum possible net present value of future profits ($P_{\tau+1}$ to P_N) at time τ, then the scenario may be presented as shown on the time diagram in Figure 1[21].

Knowing the discount rate d:

$$\Omega = \left[\frac{P_{\tau+1}}{(1+d)^{\tau+1}} + - - - - + \frac{P_N}{(1+d)^N} \right] \quad [11]$$

$$\varsigma = \left[\frac{P_n + \Omega}{(1+d)^\tau} \right] \quad [12]$$

The increase in present value v is realized through mining the next Qm_n of material and the difference of ς and Ω represents this increase. Knowing that τ is the short interval of time, Equation [12] may be written as:

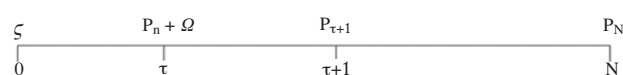| ς	$P_n + \Omega$	$P_{\tau+1}$		P_N
0	τ	$\tau+1$		N

Figure 1—Time diagram of present value of future profits at time zero and τ

$$v = [\varsigma - \Omega] = [P_n - d\varsigma\tau] \quad [13]$$

Substituting Equation [10] into Equation [13] yields the basic present value expression that dictates the calculation of the limiting economic cut-off grades:

$$v = \left[\left[\left(\left((p_n - r_n)\bar{\gamma}y \right) - c_n \right)Qc_n \right] - \left[m_nQm_n \right] - \left[(f_n + d\varsigma)\tau \right] \right] \quad [14]$$

Mining, processing, or refining capacities define time τ, leading to three values depending upon the actual constraining capacity i.e. $\frac{Qm_n}{M}$, $\frac{Qc_n}{C}$, or $\frac{Qc_n\bar{\gamma}y}{R}$, respectively. Substituting these values into Equation [14] generates the basic equations for limiting economic cut-off grades:

$$v_m = \left[\left[\left(\left((p_n - r_n)\bar{\gamma}y \right) - c_n \right)Qc_n \right] - \left[\left(m_n + \frac{f_n + d\varsigma}{M} \right)Qm_n \right] \right] \quad [15]$$

$$v_c = \left[\left[\left(\left((p_n - r_n)\bar{\gamma}y \right) - \left(c_n + \frac{f_n + d\varsigma}{C} \right) \right)Qc_n \right] - \left[m_nQm_n \right] \right] \quad [16]$$

$$v_r = \left[\left[\left(\left(p_n - r_n - \frac{f_n + d\varsigma}{R} \right)\bar{\gamma}y \right) - c_n \right)Qc_n \right] - \left[m_nQm_n \right] \right] \quad [17]$$

In Equation [15], the mine has a bottleneck that limits the operation and therefore delays the opportunity of achieving future positive cash flows. Hence, the opportunity cost $\frac{f_n + d\varsigma}{M}$ is distributed per ton of material mined. In this scenario, ore may be processed and refined as soon as material is mined. Therefore, cut-off grade should be such that the processing and refining costs are covered. This shows that every unit of material for which $[(p_n - r_n)\gamma_m y]$ is greater than the processing cost c_n, should be classified as ore. Thus, the mine limiting cut-off grade, which invokes constraint 1 (Equation [2]), becomes:

$$\gamma_m = \frac{c_n}{(p_n - r_n)y} \quad [18]$$

Similarly, in Equation [16] the processing plant has a bottle-neck that delays the operation, and the opportunity cost $\frac{f_n + d\varsigma}{C}$ is distributed per ton of ore processed. The cut-off grade is chosen such that in addition to processing and refining costs, it pays the opportunity cost of not receiving the future cash flows. Thus, the processing plant limiting cut-off grade, which invokes the second constraint (Equation [3]), becomes:

$$\gamma_c = \frac{c_n + \frac{f_n + d\varsigma}{C}}{(p_n - r_n)y} \quad [19]$$

Also, in Equation [17] the refinery is responsible for delaying the future cash flows, and the opportunity cost $\frac{f_n + d\varsigma}{R}$ is distributed per unit of concentrate refined.

Therefore, the refinery limiting cut-off grade, which invokes the third constraint (Equation [4]), becomes:

Net present value maximization model for optimum cut-off grade policy

$$\gamma_r = \frac{c_n}{\left(p_n - r_n - \dfrac{f_n + d\varsigma}{R}\right)y} \qquad [20]$$

The balancing cut-off grades depend upon the grade tonnage distribution of an individual pushback. Therefore, these cut-off grades are deduced from the grade-tonnage distribution curves representing quantity of ore per unit of material mined, recoverable metal content per unit of material mined, and the recoverable metal content per unit of ore, as given in Figures 2, 3, and 4, respectively.

The mine and processing plant balancing cut-off grade is the one which invokes the first and second constraints (Equations [2] and [3]). The mine and processing plant will be in balance when quantity of ore per unit of material mined equals the ratio C/M.

For the grade category k^*, the ratio of ore tons to total tons mined, represented as $mc(k^*)$ is:

$$mc\left(k^*\right) = \frac{t_o\left(\gamma_k\right)}{t_o\left(\gamma_k\right) + t_w\left(\gamma_k\right)} \qquad [21]$$

Knowing this ratio, the mine and processing plant balancing cut-off grade is determined from the curve presented in Figure 2. As ratio C/M lies between $mc(k^*)$ and $mc(k^*+1)$ on the y-axis, the corresponding value on the x-axis representing the mine and processing plant balancing cut-off grade γ_{mc} is determined by linear approximation as follows:

$$\gamma_{mc} = \frac{\dfrac{C}{M} - mc\left(k^*\right)}{\dfrac{mc\left(k^*+1\right) - mc\left(k^*\right)}{\gamma_{k+1} - \gamma_k}} + \gamma_k \qquad [22]$$

Similarly, the mine and refinery balancing cut-off grade is the one that invokes the first and third constraints (Equations [2] and [4]). The mine and refinery will be in balance when the recoverable metal content per unit of mined material equals the ratio R/M.

For the grade category k^*, the ratio of recoverable metal content to the total tons mined, represented as $mr(k^*)$ is:

$$mr\left(k^*\right) = \frac{t_o\left(\gamma_k\right)\overline{\gamma}\left(\gamma_k\right)\gamma}{t_o\left(\gamma_k\right) + t_w\left(\gamma_k\right)} \qquad [23]$$

Knowing this ratio, the mine and refinery balancing cut-off grade is determined from the curve presented in Figure 3. As ratio R/M lies between $mr(k^*)$ and $mr(k^*+1)$ on the y-axis, the corresponding value on the x-axis representing the mine and refinery balancing cut-off grade γ_{mr} is determined by linear approximation as follows:

$$\gamma_{mr} = \frac{\dfrac{R}{M} - mr\left(k^*\right)}{\dfrac{mr\left(k^*+1\right) - mr\left(k^*\right)}{\gamma_{k+1} - \gamma_k}} + \gamma_k \qquad [24]$$

Also, the processing plant and refinery balancing cut-off grade is the one that invokes the second and third constraints (Equations [3] and [4]). Therefore, the processing plant and refinery will be in balance when the recoverable mineral content per unit of ore equals the ratio R/C.

For the grade category k^*, the recoverable metal, represented as $cr(k^*)$ is:

$$cr\left(k^*\right) = \overline{\gamma}\left(\gamma_k\right)\gamma \qquad [25]$$

Knowing $cr(k^*)$, the processing plant and refinery balancing cut-off grade is determined from the curve presented in Figure 4. As ratio R/C lies between $cr(k^*)$ and
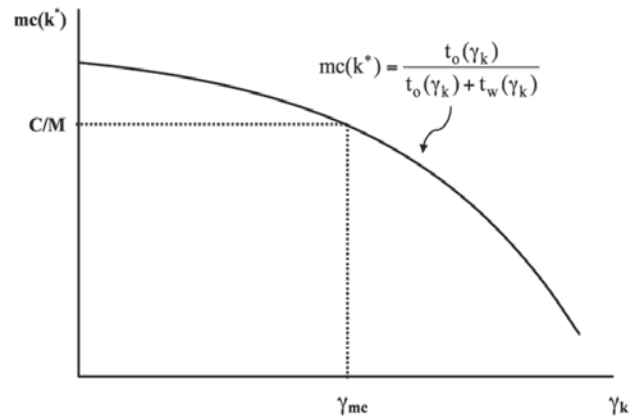

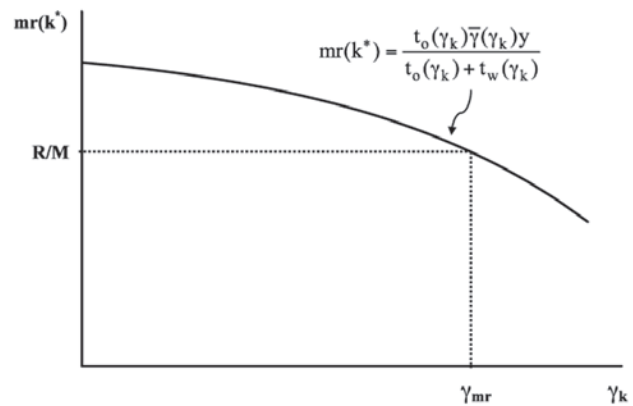Figure 2—Grade-tonnage curve for mine and processing plant balancing cut-off grade

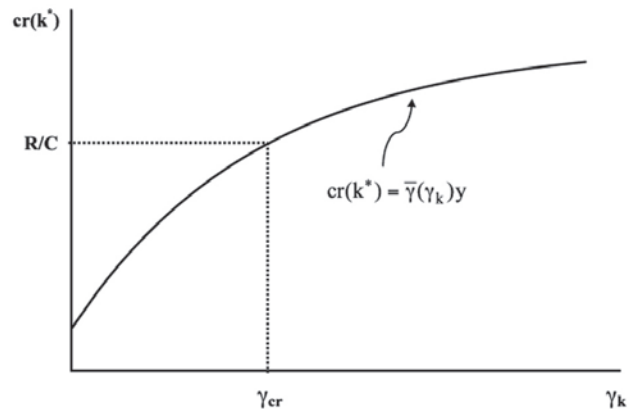Figure 3—Grade-tonnage curve for mine and refinery balancing cut-off grade

Figure 4—Grade-tonnage curve for processing plant and refinery balancing cut-off grade

Net present value maximization model for optimum cut-off grade policy

cr(k^*+1) on the y-axis, the corresponding value on x-axis representing the processing plant and refinery balancing cut-off grade γ_{cr} is determined by linear approximation as follows:

$$\gamma_{cr} = \frac{\dfrac{R}{C} - cr(k^*)}{\dfrac{cr(k^*+1) - cr(k^*)}{\gamma_{k+1} - \gamma_k}} + \gamma_k \qquad [26]$$

Once the three limiting economic cut-off grades i.e. γ_m, γ_c, and γ_r, and three balancing cut-off grades i.e. γ_{mc}, γ_{mr}, and γ_{cr} are determined, the optimum cut-off grade γ is selected from among them. The equations of limiting economic cut-off grades reveal that for a mining operation, the optimum cut-off grade may never be less than γ_m, since it represents the lowest (break even) cut-off grade. Also, the optimum cut-off grade may never exceed γ_c, since this may schedule some of the valuable ore to the waste dumps. Therefore, the optimum cut-off grade γ lies between γ_m and γ_c i.e. $\gamma_m \leq \gamma \leq \gamma_c$. If $\tilde{m}$ represents the median value, then the following criterion dictates the selection of the optimum cut-off grade:

$$\gamma = \tilde{m}\left[\tilde{m}(\gamma_m, \gamma_c, \gamma_{mc}), \tilde{m}(\gamma_m, \gamma_r, \gamma_{mr}), \tilde{m}(\gamma_c, \gamma_r, \gamma_{cr})\right] \quad [27]$$

Creation of stockpiles

The creation of stockpiles follows the determination of the optimum cut-off grade γ. The optimum cut-off grade classifies the following:

1. The material above optimum cut-off grade i.e. tons of ore $t_o(\gamma)$. This material is sent to the processing plant
2. The intermediate grade stockpile material i.e. tons of potential ore between the lowest cut-off grade γ_1 and the optimum cut-off grade γ, represented as $t_s(\gamma_1, \gamma)$
3. The material below the lowest cut-off grade γ_1 i.e. tons of waste $t_w(\gamma_1)$. This material is sent to the waste dumps.

As described in the previous section, the grade-tonnage distribution of the pushback consists of K grade increments i.e. $[\gamma_1, \gamma_2], [\gamma_2, \gamma_3], [\gamma_3, \gamma_4], \ldots, [\gamma_{K-1}, \gamma_K]$, where each grade increment of a pushback consists of t_k tons of material. If the optimum cut-off grade γ exists in the k' grade increment i.e. $[\gamma_{k'}, \gamma_{k'+1}]$, and the lowest cut-off grade γ_1 exists in k'' grade increment i.e. $[\gamma_{k''}, \gamma_{k''+1}]$ and assuming that optimum cut-off grade $\gamma = \gamma_{k'}$ and lowest cut-off grade $\gamma_1 = \gamma_{k''}$, then:

$$t_o(\gamma) = \sum_{k=k'}^{K} t_k \qquad [28]$$

$$\overline{\gamma}(\gamma) = \frac{\sum_{k=k'}^{K}\left[t_k\left(\dfrac{\gamma_k + \gamma_{k+1}}{2}\right)\right]}{t_o(\gamma)} \qquad [29]$$

$$t_s(\gamma_1, \gamma) = \sum_{k=k''}^{k'-1} t_k \qquad [30]$$

$$t_w(\gamma_1) = \sum_{k=1}^{k''-1} t_k \qquad [31]$$

If T represents the total available tons in the pushback, then:

$$T = t_o(\gamma) + t_s(\gamma_1, \gamma) + t_w(\gamma_1) \qquad [32]$$

Similarly, the quantities mined Qm_n, processed Qc_n, and refined Qr_n may be defined as a function of optimum cut-off grade γ using the three conditions given in the previous section.

The grade-tonnage distribution of stockpiles i.e. the grade increments and available tons in each grade increment is deduced from the grade-tonnage distribution of the pushback. If α represents the difference between the lowest cut-off grade increment i.e. k'' and that of optimum cut-off grade increment i.e. k', then:

$$\Rightarrow \quad \alpha = k' - k'', \text{ where } \alpha \geq 0 \qquad [33]$$

Now, if $\alpha > 0$, then the stockpile tons for respective grade increments are determined using Equations [34], [35], and [36]:

1. The tons of material in the first stockpile grade increment, which is same as that of lowest cut-off grade, i.e. k'', may be determined as:

$$t_s(k'') = \left[\left[t_{k'}\left(\frac{Qm_n - Qc_n}{T - t_o(\gamma)}\right)\right] - \left[\frac{\left(\dfrac{\gamma_1 - \gamma_{k''}}{\gamma_{(k''+1)} - \gamma_{k''}}\right)}{\left(t_{k''}\left(\dfrac{Qm_n - Qc_n}{T - t_o(\gamma)}\right)\right)}\right]\right] \qquad [34]$$

2. The tons of material in stockpile grade increments from $(k''+1)$ to $(k'-1)$, represented as k''', are:

$$t_s(k''') = \left[t_{k'''}\left[\frac{Qm_n - Qc_n}{T - t_o(\gamma)}\right]\right] \qquad [35]$$

3. The tons of material in the last stockpile grade increment, which is same as that of the optimum cut-off grade, i.e. k', may be determined as:

$$t_s(k') = \left[\left[t_{k'}\left(\frac{Qm_n - Qc_n}{T - t_o(\gamma)}\right)\right]\left[\frac{\gamma - \gamma_{k'}}{\gamma_{(k'+1)} - \gamma_{k'}}\right]\right] \qquad [36]$$

Similarly, if $\alpha = 0$, i.e. the lowest cut-off grade and the optimum cut-off grade exist in the same grade increment, which may be represented as k'''', then the tons of material in the stockpile only grade increment are:

$$t_s(k'''') = \left\{\begin{array}{l}\left[t_{k''}\left(\dfrac{\gamma - \gamma_{k''}}{\gamma_{(k''+1)} - \gamma_{k''}}\right)\left(\dfrac{Qm_n - Qc_n}{T - t_o(\gamma)}\right)\right] - \\[2ex] \left[\left(\dfrac{\gamma_1 - \gamma_{k''}}{\gamma_{(k''+1)} - \gamma_{k''}}\right)\left(t_{k''}\left(\dfrac{\gamma - \gamma_{k''}}{\gamma_{(k''+1)} - \gamma_{k''}}\right)\left(\dfrac{Qm_n - Qc_n}{T - t_o(\gamma)}\right)\right)\right]\end{array}\right\} \qquad [37]$$

A demonstration of the computations presented in Equations [34], [35], [36], and [37] is offered through an example in the next section.

Copper deposit case study

Consider a hypothetical copper deposit divided into three pushbacks[15]. Table I presents capacities, price of copper,

Net present value maximization model for optimum cut-off grade policy

Parameter	Unit	Quantity
Mine capacity	tons/year	20 000 000
Mill capacity	tons/year	10 000 000
Copper refining capacity	tons/year	90 000
Stockpile capacity	tons	60 000 000
Price of copper	$/ton	2100.00
Mining cost	$/ton	1.05
Milling cost	$/ton	2.66
Refining cost	$/ton	100.00
Fixed cost	$/ton	4 000 000
Copper price escalation	%/year	0.80
Mining cost escalation	%/year	2.50
Milling cost escalation	%/year	3.00
Refining cost escalation	%/year	2.50
Fixed cost escalation	%/year	2.50
Recovery of copper	%	90
Discount rate	%	15

operating costs, and escalation rates for this open pit mining operation. Table II gives the grade-tonnage distribution within ultimate pit limits for all three pushbacks.

The process of determination of the optimum cut-off grade and the creation of stockpiles presented in the previous sections is computation-intensive; therefore a dialogue-based application in Visual C++ implementing the iterative algorithmic steps is used for the development of optimum cut-off grade policies in this case study. The algorithmic iterations continue in anticipation of the NPV convergence, i.e. the calculation of the optimum cut-off grade for a period n is repeated until no further improvement in NPV is possible. A description of these steps is as follows:

1. Set n to 1 and iteration i to 1
2. Compute available reserves Q_n. If $Q_n = 0$, then go to step 10, otherwise go to next step
3. If $i = 1$, set V to 0
4. Set $\varsigma = V$
5. Compute:
   a. γ_m, γ_c, γ_r, γ_{mc}, γ_{mr}, and γ_{cr}
   b. γ using Equation [27]
   c. $t_o(\gamma)$, $t_w(\gamma)$, and $\bar{\gamma}(\gamma)$ using Equations [7], [8], and [9], respectively
   d. Qm_n, Qc_n, and Qr_n using the conditions described in the model
   e. N based on the limiting capacity identified in step 5(d)
   f. P_n using Equation [10]
   g. $V = \dfrac{P_n\left((1+d)^N-1\right)}{d(1+d)N}$.
6. If $i = 1$, check for ς convergence (i.e. compare V (step 5(g)) with previous V (step 4). If ς is converged (within some tolerance, say $500 000.00$), then go to step 7, otherwise go to step 4
7. Knowing γ, compute stockpile grade-tonnage distribution using Equations [34], [35], [36], and [37]
8. Knowing that Qm_n is mined and Qc_n is processed, adjust the grade-tonnage distribution of the deposit
9. Set $n = n + 1$, go to step 2
10. If $i = 1$, then knowing P_n from period 1 to N, find the Ω_n i.e. present value of future cash flows at period n, and go to step 11. If $i = 2$, then stop

11. Compute the optimum cut-off grades policy using $\varsigma = \Omega_n$ for corresponding year n, and go to step 4.

The steps in the algorithm generate alternative policies presented in Tables III, and IV. Table III shows the optimum policy without escalation and stockpile consideration. As indicated in Table III, the optimum cut-off grade in year 1 is 0.50%. At this cut-off grade, 17.85 million tons of material is mined, and 10 million tons of ore is processed which results in 90 000 tons of refined copper. Hence, the operation has excess mining capacity, while processing plant and refinery are limiting the operation. Therefore, 0.50% optimum cut-off grade refers to the processing plant and refinery balancing cut-off grade (Equation [26]). This pattern continues until year 6, and it is worth mentioning that in the same year the reserves in pushback 1 are exhausted and mining from pushback 2 is commenced. From year 7 through year 10, mine and processing plant are limiting the operation and the 0.53% optimum cut-off grade refers to mine and processing plant balancing cut-off grade (Equation [22]). It is worth clarifying that from one year to next, the grade-tonnage distribution dictating the balancing cut-off grades is adjusted uniformly (i.e. without any change in the structure of distribution) among the intervals. Consequently, the optimum cut-off grade corresponding to the limitation of similar pair of stages remains constant as observed from years 1 to 6 and years 7 to 10. Similarly, from year 11 to the life of operation, i.e. year 17, the flow of material from the mine to refinery is limited due to full utilization of the processing plant capacity. Hence, the optimum cut-off grade during these years refers to the processing plant limiting economic cut-off grade (Equation [19]), and it is declining with exhaustion of the reserves and a consequent decline in the present value of the remaining reserves. The objective function, i.e. maximum NPV of the open pit mining operation, is predicted to be $735 770 000 as shown in the optimum policy in Table III.

Table IV presents the optimum policy allowing escalation of economic parameters without the stockpiling option. The price escalation is 0.80% per year, which indicates that in year 1 the metal price is $2100.00 per ton of copper. However, it escalates to $2405.00 per ton of copper in year

Table II

Grade-tonnage distribution of copper deposit

Copper (%)	Tons		
	Pushback 1	Pushback 2	Pushback 3
0.00–0.15	14 400 000	15 900 000	17 900 000
0.15–0.20	4 600 000	5 100 000	5 500 000
0.20–0.25	4 400 000	4 900 000	5 400 000
0.25–0.30	4 300 000	4 700 000	5 300 000
0.30–0.35	4 200 000	4 500 000	4 900 000
0.35–0.40	4 100 000	4 400 000	4 700 000
0.40–0.45	3 900 000	4 300 000	4 600 000
0.45–0.50	3 800 000	4 100 000	4 500 000
0.50–0.55	3 700 000	3 900 000	4 200 000
0.55–0.60	3 600 000	3 800 000	3 900 000
0.60–0.65	3 400 000	3 600 000	3 800 000
0.65–0.70	3 300 000	3 500 000	3 700 000
> 0.70	42 300 000	37 300 000	31 600 000

Net present value maximization model for optimum cut-off grade policy

Table III

Life of operation optimum production schedule without escalation and stockpiling option

Year	Pushback	Cut-off Grade (%)	Average Grade (%)	Qm (tons)	Qc (tons)	Qr (tons)	Profit ($ million)
1	1	0.50	1.00	17 850 000	10 000 000	90 000	130.65
2	1	0.50	1.00	17 850 000	10 000 000	90 000	130.65
3	1	0.50	1.00	17 850 000	10 000 000	90 000	130.65
4	1	0.50	1.00	17 850 000	10 000 000	90 000	130.65
5	1	0.50	1.00	17 850 000	10 000 000	90 000	130.65
6	1	0.50	1.00	10 760 000	6 030 000	54 280	78.80
6	2	0.53	0.95	79 400 00	3 970 000	34 060	47.64
7	2	0.53	0.95	20 000 000	10 000 000	85 820	120.04
8	2	0.53	0.95	20 000 000	10 000 000	85 820	120.04
9	2	0.53	0.95	20 000 000	10 000 000	85 820	120.04
10	2	0.53	0.95	20 000 000	10 000 000	85 820	120.04
11	2	0.49	0.93	12 060 000	6 380 000	53 350	74.52
11	3	0.47	0.85	7 240 000	3 620 000	27 550	36.42
12	3	0.45	0.83	19 190 000	10 000 000	74 690	98.63
13	3	0.41	0.80	17 920 000	10 000 000	72 270	95.12
14	3	0.36	0.77	16 690 000	10 000 000	69 690	91.25
15	3	0.31	0.74	15 510 000	10 000 000	66 900	86.92
16	3	0.26	0.71	14 340 000	10 000 000	63 820	81.98
17	3	0.21	0.67	9 110 000	6 880 000	41 660	52.70

NPV = $735.77 million

Table IV

Life of operation optimum production schedule considering escalation without stockpiling option

Year	Pushback	Cut-off Grade (%)	Average Grade (%)	Qm (tons)	Qc (tons)	Qr (tons)	Profit ($ million)
1	1	0.50	1.00	17 850 000	10 000 000	90 000	130.65
2	1	0.50	1.00	17 850 000	10 000 000	90 000	130.46
3	1	0.50	1.00	17 850 000	10 000 000	90 000	130.32
4	1	0.50	1.00	17 850 000	10 000 000	90 000	130.14
5	1	0.50	1.00	17 850 000	10 000 000	90 000	129.93
6	1	0.50	1.00	10 760 000	6 030 000	54 280	78.21
6	2	0.53	0.95	79 400 00	3 970 000	34 060	46.97
7	2	0.53	0.95	20 000 000	10 000 000	85 820	117.93
8	2	0.53	0.95	20 000 000	10 000 000	85 820	117.47
9	2	0.53	0.95	20 000 000	10 000 000	85 820	116.97
10	2	0.53	0.95	20 000 000	10 000 000	85 820	116.43
11	2	0.49	0.92	12 060 000	6 390 000	53 390	71.89
11	3	0.47	0.85	7 220 000	3 610 000	27 490	34.26
12	3	0.45	0.83	19 380,000	10 000 000	75 030	92.60
13	3	0.42	0.81	18 310 000	10 000 000	73 040	88.98
14	3	0.38	0.79	17 290 000	10 000 000	70 970	85.07
15	3	0.35	0.76	16 300 000	10 000 000	68 800	80.84
16	3	0.31	0.74	15 340 000	10 000 000	66 480	76.18
17	3	0.27	0.71	6 150 000	4 280 000	27 350	30.37

NPV = $723.35 Million

17. Similarly, operating and fixed costs escalate from year 1 through year 17. As specified in Table IV, the trend of optimum cut-off grades from years 1 through 17 follows the same model as presented in the previous policy. However, the minimum optimum cut-off grade has increased from 0.21% to 0.27% in year 17. This shows that the operation ensures flow of comparatively high grade material even in the final years to pay off the escalated operating and fixed costs. The impact of price and cost escalation leads to 1.7% decrease in maximum NPV in year 1, i.e. from $735 770 000 to $723 350 000.

Table V shows a comprehensive breakdown of the optimum policy, allowing both escalation of economic parameters and stockpiling option. Owing to the similar grade-tonnage distribution and initial values for the economic parameters, an equivalent trend is followed for selection of the optimum cut-off grades and the resultant flow of material from mine to refinery. Table V indicates that NPV (last column) is declining with exhaustion of reserves (column 2 and 3) from year 1 to year 22. It describes the process of discounting the annual cash flows to calculate Ω_n (last column corresponds to step 11 of the algorithm) for a

Net present value maximization model for optimum cut-off grade policy

Table V

Life of operation optimum production schedule considering escalation and stockpiling option

Year	Pushback	Available material (tons)		γ (%)	Available material (tons) @ γ			Material handled (tons)				Cash Flow	NPV ($)
		Pushback	*Pit*		*Ore*	*Waste/ stockpile*	*γ̄ (%)*	*Qm*	*Qs*	*Qc*	*Qr*	($)	@ 15%
1	1	100 000, 000	300 000 000	0.50	56 031 133	43 968 867	0.99996	17 847 221	3 363 999	10 000 000	89 996	130 653 218	730 419 555
2	1	82 152 779	282 152 779	0.50	46 031 133	36 121 646	0.99996	17 847 221	3 363 999	10 000 000	89 996	130 462 455	709 329 270
3	1	64 305 559	264 305 559	0.50	36 031 133	28 274 425	0.99996	17 847 221	3 363 999	10 000 000	89 996	130 318 433	685 266 205
4	1	46 458 338	246 458 338	0.50	26 031 133	20 427 205	0.99996	17 847 221	3 363 999	10 000 000	89 996	130 140 463	657 737 703
5	1	28 611 117	228 611 117	0.50	16 031 133	12 579 984	0.99996	17 847 221	3 363 999	10 000 000	89 996	129 927 358	626 257 896
6	1	10 763 897	210 763 897	0.50	6 031 133	4 732 763	0.99996	10 763 897	2 028 873	6 031 133	54 278	78 224 913	590 269 222
6	2	100 000 000	200 000 000	0.53	50 000 000	50 000 000	0.95355	7 937 733	3 792 637	3 968 867	34 061	46 953 073	590 269 222
7	2	92 062 267	192 062 267	0.53	46 031 133	46 031 133	0.95355	20 000 000	6 207 764	10 000 000	85, 820	117 925 765	553 631 620
8	2	72 062 267	172 062 267	0.53	36 031 133	36 031 133	0.95355	20 000 000	8 888 000	10 000 000	85 820	117 470 837	518 750 598
9	2	52 062 267	152 062 267	0.53	26 031 133	26 031 133	0.95355	20 000 000	8 888 000	10 000 000	85 820	116 973 712	479 092 351
10	2	32 062 267	132 062 267	0.53	16 031 133	16 031 133	0.95355	20 000 000	8 888 000	10 000 000	85 820	116 432 981	433 982 492
11	2	12 062 267	112 062 267	0.53	6 218 075	5 844 192	0.94043	12 062 267	6 937 294	6 218 075	52 629	70 972 442	382 646 884
11	3	100 000 000	100 000 000	0.47	50 000 000	50 000 000	0.84553	7 563 850	3 936 477	3 781 925	28 780	35 857 079	382 646 884
12	3	92 436 150	92 436 150	0.47	46 218 075	46 218 075	0.84553	20 000 000	5 259 183	10 000 000	76 098	94 058 057	333 214 397
13	3	72 436 150	72 436 150	0.44	38 036 117	34 400 032	0.82686	19 044 044	6 971 626	10 000 000	74 417	90 932 173	289 138 499
14	3	53 392 106	53 392 106	0.41	29 589 377	23 802 728	0.80579	18 044 349	5 620 662	10 000 000	72 521	87 362 426	241 577 101
15	3	35 347 756	35 347 756	0.38	20 689 726	14 658 030	0.78382	17 084 690	4 267 141	10 000 000	70 544	83 517 002	190 451 240
16	3	18 263 066	18 263 066	0.34	11 298 163	6 964 903	0.76092	16 164 633	2 968 633	10 000 000	68 483	79 383 037	135 501 924
17	3	2 098 433	2 098 433	0.30	1 373 839	724 594	0.73679	2 098 433	1 242 287	1 373 839	9 110	10 275 206	76 444 176
17	Stockpile	54 806 161	54 806 161	0.30	47 296 823	7 509 339	0.39422	8 626 161		8 626 161	30 606	19 576 283	76 444 176
18	Stockpile	46 180 000	46 180 000	0.29	41 071 325	5 108 675	0.39137	10 000 000		10 000 000	35 223	20 990 103	58 059 313
19	Stockpile	36 180 000	36 180 000	0.29	32 946 890	3 233 110	0.38919	10 000 000		10 000 000	35 027	19 390 025	45 778 108
20	Stockpile	26 180 000	26 180 000	0.28	24 339 885	1 840 115	0.38734	10 000 000		10 000 000	34 861	17 803 735	33 254 798
21	Stockpile	16 180 000	16 180 000	0.28	15 293 718	886 282	0.38592	10 000 000		10 000 000	34 733	16 253 390	20 439 284
22	Stockpile	6 180 000	6 180 000	0.27	6 180 000		0.37900	6 180 000		6 180 000	21 080	8 339 554	7 251 786

particular period, which is then used to compute the optimum cut-off grade. For example, processing capacity is limiting the operation during year 15, hence $\gamma = \gamma_c$, and keeping the escalated values of economic parameters, γ may be calculated as follows:

$$\gamma = \gamma_c = \frac{4.14 + \dfrac{5793192.67 + (0.15 \times 190451240)}{10000000}}{(2366.61 - 144.83) \times 0.90}$$

$$= 0.00379 = 0.38\%$$

Table V also demonstrates the accumulation of stockpile material from years 1 to 17. As the lowest cut-off grade remains 0.27% (from Table IV) and the optimum cut-off grade during year 1 is 0.5036%, they exist in 4th and 9th grade increments of the Pushback 1 (see Table II), respectively. Therefore, the stockpile material consists of six grade increments, presented as:

[0.27, 0.30], [0.30, 0.35], [0.35, 0.40], [0.40, 0.45], [0.45, 0.50], [0.50, 0.5036]

The amount of material in the first stockpile grade increment is determined using Equation [34]:

$$t_s(1) = \left\{ \begin{bmatrix} 4300000x \\ \left(\dfrac{17847221 - 10000000}{100000000 - 56031133} \right) \end{bmatrix} - \begin{bmatrix} \left(\dfrac{0.27 - 0.25}{0.30 - 0.25} \right)x \\ \left(\dfrac{4300000x}{\left(\dfrac{17847221 - 10000000}{100000000 - 56031133} \right)} \right) \end{bmatrix} \right\} = 460458.29$$

The tons of material from the 2nd to 5th stockpile grade increments are determined using Equation [35]:

$$t_s(2) = \left\{ \begin{bmatrix} 4200000 \times \\ \left[\dfrac{17847221 - 10000000}{100000000 - 56031133} \right] \end{bmatrix} \right\} = 749583.27$$

$$t_s(3) = \left\{ \begin{bmatrix} 4100000 \times \\ \left[\dfrac{17847221 - 10000000}{100000000 - 56031133} \right] \end{bmatrix} \right\} = 731736.05$$

Net present value maximization model for optimum cut-off grade policy

$$t_s(4) = \left\{ \begin{bmatrix} 3900000 \times \\ \left[\dfrac{17847221 - 10000000}{100000000 - 56031133} \right] \end{bmatrix} \right\} = 696041.60$$

$$t_s(5) = \left\{ \begin{bmatrix} 3800000 \times \\ \left[\dfrac{17847221 - 10000000}{100000000 - 56031133} \right] \end{bmatrix} \right\} = 678194.38$$

Similarly, the tons of material in the 6th stockpile grade increment are determined using Equation [36]:

$$t_s(6) = \left\{ \begin{bmatrix} 3700000 \times \\ \left(\dfrac{17847221 - 10000000}{100000000 - 56031133} \right) \times \\ \left[\dfrac{0.5036 - 0.50}{0.55 - 0.50} \right] \end{bmatrix} \right\} = 47985.22$$

Therefore, approximately 3.4 million tons of material is scheduled to stockpiles in year 1. A total of 54.81 million tons of raw material is accumulated in stockpiles from years 1 through 17, which otherwise is scheduled to waste dumps in the previous policies. Stockpiling option promises an increase of five years in the operation's life along with a 1% increase in NPV from $723 350 000 to $730 419 555.

Conclusions

The optimum cut-off grade policies indicate that the impact of escalation on the objective function could be enormous in cases where operating and fixed costs are escalating at higher rates. This may change some of the economic open pit operations to an uneconomic scenario. It is observed that the operation presented in the case study becomes unprofitable during later years at an escalation rate of 6 per cent per year in the operating and fixed costs. Therefore, long term mining plans should always include escalation of economic parameters to establish the feasibility of the mining venture.

The results also reflect that the creation of stockpiles scheduled approximately 55 million tons of additional ore for processing, which facilitated in neutralizing the effect of escalating economic parameters through enhancement of the life of operations along with NPV. However, it is clear that allowing the creation of long-term stockpiles is a strategic decision, and exercising this option depends exclusively upon the operating conditions of an open pit mining operation. The proposed methodology is limited in its application to the metallic ores; therefore, while making this important decision, one must give a serious consideration to issues such as material deterioration and compaction during long exposure to the environment. Similarly, loss of values may take place due to leaching, and oxidation may introduce processing complexities and result in reduced recoveries.

The proposed cut-off grade optimization model considers the escalation of economic parameters, particularly, the operating and fixed costs, with a vision that mine planning activity is focused on survival strategies under harsh economic situations. As such, it is a contribution to the mine planning community in terms of facilitating the evaluation of different economic alternatives, ultimately ensuring the optimum utilization of resources coupled with appropriate policy formulation for making major mining investments.

The model does not consider uncertainty in economic parameters, especially, the uncertainty associated with the metal price. Also, it is limited to the creation of long-term stockpiles. Therefore, the development of cut-off grade optimization models taking into account metal price uncertainty and allowing the processing of stockpile material during mine life are some of the areas for future research.

References

1. TAYLOR, H.K. General background theory of cut-off grades. *Transactions of the Institution of Mining and Metallurgy Section A*, 1972. pp. 160–179.

2. TAYLOR, H.K. Cut-off grades—some further reflections. *Transactions of the Institution of Mining and Metallurgy Transactions Section A*, 1985. pp. 204–216.

3. DAGDELEN, K. Cut-off grade optimization. *Proceedings of the 23rd International Symposium on the Application of Computers and Operations Research in the Mineral Industry*, Tucson Arisona, 1992. pp. 157–165.

4. DAGDELEN, K. An NPV optimization algorithm for open pit mine design. *Proceedings of the 24th International Symposium on the Application of Computers and Operations Research in the Mineral Industry*, Montreal, Canada, 1993. pp. 257–263.

5. CETIN, E. and DOWD, P.A. The use of genetic algorithms for multiple cut-off grade optimization. *Proceedings of the 32nd International Symposium on the Application of Computers and Operations Research in the Mineral Industry*, Little, Colorado, 2002. pp. 769–779.

6. DAGDELEN, K. and KAWAHATA, K. Value creation through strategic mine planning and cut-off grade optimization. *Mining Engineering,* vol. 60, no. 1, 2008. pp. 39–45.

7. HE, Y., ZHU, K., GAO, S., LIU, T., and LI, Y. Theory and method of genetic-neural optimizing cut-off grade and grade of crude ore. *Expert Systems with Applications,* vol. 36, no. 4, 2009. pp. 7617–7623.

8. AKAIKE, A. AND DAGDELEN, K. A strategic production scheduling method for an open pit mine. *Proceedings of the 28th International Symposium on the Application of Computers and Operations Research in the Mineral Industry*, Golden, Colorado, 1999. pp. 729–738.

9. BASCETIN, A. Determination of optimal cut-off grade policy to optimize NPV using a new approach with optimization factor. *Journal of the Southern African Institute of Mining and Metallurgy,* vol. 107, no. 2, 2007. pp. 87–94.

10. ASAD, M.W.A. Development of generalized cut-off grade optimization algorithm for open pit mining operations. *Journal of Engineering and Applied Sciences,* vol. 21, no. 2, 2002. pp. 119–127.

11. OSANLOO, M. AND ATAEI, M. Using equivalent grade factors to find the optimum cut-off grades of multiple metal deposits. *Minerals Engineering,* vol. 16, no. 8, 2003. pp. 771–776.

Net present value maximization model for optimum cut-off grade policy

12. ASAD, M.W.A., Cut-off grade optimization algorithm for open pit mining operations with consideration of dynamic metal price and cost escalation during mine life. *Proceedings of the 32nd International Symposium on Application of Computers and Operations Research in the Mineral Industry* (APCOM 2005a), Tucson, Arizona, 2005. pp. 273–277.

13. ASAD, M.W.A. Optimum cut-off grade policy for open pit mining operations through net present value algorithm considering metal price and cost escalation. *Engineering Computations,* vol. 24, no. 7, 2007. pp. 723–736.

14. ASAD, M.W.A. Cut-off grade optimization algorithm with stockpiling option for open pit mining operations of two economic minerals. International *Journal of Surface Mining, Reclamation and Environment,* vol. 19, no. 3, 2005b. pp. 176–187.

15. LANE, K.F. Choosing the optimum cut-off grade. *Colorado School of Mines Quarterly,* vol. 59, 1964. pp. 811–829.

16. LANE, K.F. The Economic Definition of Ore, Cut-off Grade in Theory and Practice. Mining Journal Books, London, 1988.

17. DAGDELEN, K. and ASAD, M.W.A. Multi-mineral cut-off grade optimization with option to stockpile. *Society of Mining, Metallurgy, and Exploration Engineers (SME) Annual Meeting*, Preprint no. 97186. 1997.

18. ATAEI, M. and OSANLOO, M. Using a combination of genetic algorithm and grid search method to determine optimum cut-off grades of multiple metal deposits. *International Journal of Surface Mining, Reclamation and Environment,* vol. 18, no. 1, 2004. pp. 60–78.

19. OSANLOO, M., RASHIDINEJAD, F., and REZAI, B. Incorporating environmental issues into optimum cut-off grades modeling at porphyry copper deposits. *Resources Policy*, vol. 33, no. 4, 2008. pp. 222–229.

20. KING, B. Optimal mining principles. *Proceedings of the International Symposium on Orebody Modelling and Strategic Mine Planning. Australian Institute of Mining and Metallurgy*, Perth, Western Australia. 2009.

21. STERMOLE, F.J. and STERMOLE J.M. Economic Evaluation and Investment Decision Methods, 9th Edition, Investment Evaluation Corporation, Golden, Colorado, USA, 1996.   ◆